PERFORMANCE EVALUATION OF THE GPS BLOCK IIR TIME KEEPING SYSTEM

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Abstract

The Time Keeping System (TKS) is essential to the GPS Block IIR total navigation payload and must provide an accurate time base for each GPS space vehicle. Performance analysis of the TKS has been reported previously for the case in which a rubidium (Rb) atomic frequency standard is used as a reference. In this paper the system error model of the TKS is developed and the performance of the TKS output using either a cesium (Cs) or a rubidium (Rb) atomic frequency standards as a reference is evaluated. The contributions to the TKS output Allan deviation was examined from each of the three noise sources: atomic frequency standard (either Cs or Rb), voltage control crystal oscillator (VCXO), and phase meter (PM). In addition, since the TKS does not have a baseplate temperature controller and the VCXO of the TKS is temperature-sensitive, a TKS error is induced when the VCXO is subject to the orbital temperature variations or the space vehicle umbra during solar eclipse. These two temperature-induced TKS output errors are also examined. Sensitivities of the Allan variance at the TKS output for each of its independent input noises are provided and they are valuable for design trade-off and troubleshooting. The results of this paper compare favorably with those obtained by the TKS system testing. The paper will show that when the TKS components meet or exceed their specifications, the RAFS TKS and the CAFS TKS should meet their specifications.

1 INTRODUCTION

The GPS Block IIR Time Keeping System (TKS) is used to generate a system output frequency of 10.23 MHz from an Atomic Frequency Standard (AFS) as an input reference. The output frequency of either the Cs AFS (CAFS) or the Rb AFS (RAFS) is approximately 13.4 MHz. Dither frequency for the Selective Availability (SA), if enabled, is added to the TKS output. The block diagram of the TKS system model is shown in Figure 1. Both CAFS and RAFS as frequency references to the TKS are considered in this paper and are referred as the CAFS TKS and the RAFS TKS respectively. A reference epoch is generated every 1.5 s based on the AFS frequency and another 1.5 s interval system epoch is generated by the 10.23 MHz system clock of the VCXO. Both epochs are input to the Phase Meter (PM), and the PM computes the timing error between the two epochs in terms of number of cycles of an asynchronous 600 MHz clock. Considering the timing error value, the loop adjusts the phase of the epoch from the VCXO so that the VCXO is phase-locked to the reference AFS.

In this paper the system error model of the TKS is developed and the performance of the TKS frequency output is evaluated from each of the three noise sources: atomic frequency

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standard (either Cs or Rb), voltage control crystal oscillator (VCXO), and phase meter (PM). In addition, since the VCXO is temperature-sensitive and the TKS does not have a baseplate temperature controller, the TKS error induced by the VCXO is also computed when the VCXO is subject to the orbital temperature variations or the space vehicle umbra during solar eclipse. The other intent of this paper is to provide as much relevant technical information as possible to the users of the GPS Block IIR Time Keeping System.

2 GPS TKS STABILITY REQUIREMENTS

The TKS stability requirements for the GPS Block II/IIA, Block IIR, and Block IIF using either a Rb AFS or a Cs AFS as an input reference is shown in Figure 2. It is interesting to note that the Allan deviation requirement of the Block II/IIA CFS TKS has a slope of $\tau^{-0.4247}$ instead of the theoretical value of $\tau^{-0.5}$. Also, the Block IIF TKS requirement is better than that of the Block IIR CAFS TKS and is not as good as that of the Block IIR RAFS TKS.

3 TKS BASIC NOISE MODELS

3.1 Phase Meter Phase Noise

The TKS phase meter noise was derived in [1] and it is shown to be a white phase noise. The power spectral density and the Allan variance are

$$S_p(f) = T_c^2/6$$

(1)

$$\sigma_p^2(\tau) = T_c^2/(6\tau^2)$$

where T_c is a cycle period of the 600 MHz clock.

3.2 VCXO Frequency Noise

The VCXO frequency noise is characterized by the following Allan variance[2]:

$$\sigma_V^2(\tau) = 10^{-24} + 10^{-27}\tau. \tag{2}$$

Assuming that the two terms of Eq. (2) are independent of each other, the power spectral density of the VCXO intrinsic frequency noise can expressed as[3]:

$$S_v(f) = 7.2134 \times 10^{-25}/f + 1.519 \times 10^{-28}/f^2 \tag{3}$$

3.3 AFS Frequency Noise

The Allan variances for both the CAFS and the RAFS are specified in [2,4].

$$\sigma^2_{AC}(\tau) = 9.0 \times 10^{-22}/\tau + 1.0 \times 10^{-26}, \text{for the CAFS noise}$$

(4)

$$\sigma^2_{AR}(\tau) = 1.0 \times 10^{-23}/\tau + 2.5 \times 10^{-27}, \text{for the RAFS noise}$$

The associated noise power spectral densities can be computed using well-known techniques[3]:

$$\sigma^2_{AC}(\tau) = 1.8 \times 10^{-21} + 7.213 \times 10^{-27}/f, \text{for the CAFS noise}$$

(5)

$$\sigma^2_{AR}(\tau) = 2.0 \times 10^{-23} + 1.803 \times 10^{-27}/f, \text{for the RAFS noise}$$

The Allan deviations of the PM phase noise, the VCXO frequency noise, the CAFS frequency noise, and the RAFS frequency noise are shown in Figure 3. Note that the PM phase noise is governing for low τ and the VCXO frequency noise is dominant for the $\tau > 60$ s.

4 TKS TRANSFER FUNCTIONS

The system model of the TKS as shown in Figure 1 results in the equivalent system error model indicated in Figure 4, obtained using the Z-transform formation. T_s is the sample period of the TKS, and is 1.5 s. Also, two delays of one epoch each are introduced in Figure 4 to account for the fact that the effects of the computed VCXO frequency modification in the current epoch will not show on the Phase Meter until two epochs later. SA (dither frequency) is excluded, and the VCXO gain is assumed to be local linear. This model is applicable to both the CAFS TKS and the RAFS TKS. The transfer functions relating the TKS output frequency error (T) to the input noises of the AFS (A) frequency, the VCXO (V) frequency, and the Phase Meter (P) phase are:

$$TKS/PhaseMeter = H_{TP}(Z) = [a(K_1 + K_2)Z^{-1} - a(K_1 + 2K_2)Z^{-2} + aK_2Z^{-3}]/\Delta \quad (6)$$

$$TKS/AFS = H_{TA}(Z) = [aT(K_1 + K_2)Z^{-1} - aTK_2Z^{-2}]/\Delta \quad (7)$$

$$TKS/VCXO = H_{TV}(Z) = [1 - (3 - a)Z^{-1} - (3 - 2a)Z^{-2} - (1 - a)Z^{-3}]/\Delta \quad (8)$$

where

$$\Delta = 1 - (3 - a)Z^{-1} + [(3 - 2a) + aT(K_1 + K_2)]Z^{-2} - (1 - a + aTK_2)Z^{-3} \quad (9)$$

The Bode plots (frequency responses) for these transfer functions with a TKS loop time constant of 150 s (a = 0.1199, K_1 = 8.2573 × 10^{-5}, and K_2 = 0.17341) are shown in Figure 5. A time constant of 150 seconds is chosen because it provides the best overall performance in terms of the TKS Allan deviation and transient response.[2] It is seen that $H_{TA}(Z)$ is a lowpass filter, $H_{TV}(Z)$ is a highpass filter, and $H_{TP}(Z)$ is a low-gain bandpass filter to suppress the PM phase noise.

5 ALLAN DEVIATIONS OF THE TKS OUTPUT FREQUENCY

Since the TKS error model is a linear system, the power spectral density at the TKS output can be computed with the TKS transfer functions as provided in Section 4 and the power spectral densities from the three noise sources: CAFS or RAFS, VCXO, and PM as provided in Section 3. The resulting power spectral density can then be used to compute the Allan variance at the TKS output. The details of how to compute the TKS output Allan deviation using the frequency domain techniques and to achieve the desired computation accuracy can be found in [5]. The frequency domain technique was used because it provides another independent evaluation of the TKS performance and because the flicker noise contained in the VCXO and AFS noises can be evaluated precisely without approximation.[6]

These Allan deviations at the TKS output are shown in Figure 6 and they can be considered as the sensitivities of the TKS output for each of the independent input noises. This technique can be used very effectively during the design, development, and testing phase of the program to determine the loop time constant, to define noise specifications, and to provide data for troubleshooting. It will be used later to identify the causes of some out of specification conditions. It is apparent from Figure 6 that for short average times the TKS performance is dominated by the VCXO and PM, while for long average times the TKS performance is governed by either the CAFS or the RAFS. The crossover average time is around 20 s for the CAFS TKS and around 1,700 s for the RAFS TKS.

Since the TKS noises are independent, the Allan deviation of the TKS can be obtained by root-sum-square of these independent TKS noises. The Allan deviations of the RAFS TKS output ($\sigma_{TR}(\tau)$), the RAFS TKS specification, and the RAFS input specification are shown in Figure 7. The figure shows that the RAFS TKS output is not as good as that of the RAFS input because of erosion from both the VCXO and the phase meter noises, as can be seen from the TKS sensitivity responses provided in Figure 6. The Allan deviations of the CAFS TKS output ($\sigma_{TC}(\tau)$) and the CAFS TKS specification are plotted in Figures 8. It shows that: the Allan deviation of the CAFS TKS output barely exceeds its specification for the average time from 170 s to 3,000 s. By examining the TKS sensitivity responses as provided in Figure 6, it is found that the out-of-specification condition is caused by the CAFS input noise. Since the Allan deviation test results of the GPS Block IIR CAFSs currently indicate that they are better than their specification, this should not present a problem. A sample of the measured Allan deviations of a CAFS and a RAFS are plotted in Figures 9 and 10, which are better than their specifications.

6 COMPARISON WITH THE TKS TESTING RESULTS

The Allan deviations of the GPS Block IIR TKS test results with a RAFS and a CAFS as an input are shown in Figures 7 and 8. The contribution from the house reference frequency standard (HP 5071A) has been removed from the test results. With the exception of the last test point of the CAFS TKS Allan deviation, the differences between the simulation results and the testing results are in good agreement and are less than 1 dB.

7 VCXO TEMPERATURE-INDUCED FREQUENCY NOISES

The VCXO is sensitive to temperature variations. Since the TKS does not have a baseplate temperature controller, a TKS error is induced when the VCXO of the TKS is subject to the orbital temperature variations or the space vehicle umbra during solar eclipse. These two temperature-induced TKS output errors are examined below.

7.1 TKS Error Due to the Orbital Temperature Variations

When the VCXO is subject to an orbital temperature variation estimated to be ± 2.5 degrees maximum with a period of 12 hours, the VCXO temperature-induced frequency noise with a temperature coefficient specification of 4 × 10^{-11} (df/f)/°C can be described as:

$$V_O(t) = 4 \times 10^{-11} \times 2.5 \times sin(2\pi f_o t + \Phi) \tag{10}$$

$$= 1x10^{-10} \times sin(2\pi f_o t + \Phi),$$

where f_o = 1/(12 × 3,600) is the orbital frequency with a period of 12 hours and Φ is an arbitrary starting phase angle. This TKS input frequency error with Φ = 90 degrees is plotted in Figure 11. The TKS output frequency error due to this VCXO input frequency noise can be computed using the transfer function of $H_T V(Z)$ and the result is provided in Figure 12. It is 180 degrees out of phase from the input frequency noise, as confirmed by the VCXO phase angle plot of the Bode diagram of Figure 5. The resulting TKS output timing error can be obtained by integrating the output frequency error and is shown in Figure 13. The Allan deviation of this timing error is computed using the overlapping samples technique and is shown in Figure 14. As can be seen, it is below the RAFS TKS specification line.

The TKS Allan deviation induced by orbital VCXO effect can also be determined analytically. The TKS loop gain from the VCXO to the TKS output at the orbital period of 12 hours can be obtained from its Bode plot, as shown in Figure 5 or can be determined by

$$Gain = |H_{TV}(e^{j2\pi f T_s})|_{f=1/12\times 3,600} \tag{11}$$

$$= 3.84 \times 10^{-4}.$$

Since the TKS error model is a linear system, the frequency error at the TKS output can be obtained as

$$V_{TO}(t) = 3.84 \times 10^{-4} \times 1 \times 10^{-10} \times sin(2\pi f_o t + \Phi) \tag{12}$$

$$= 3.84 \times 10^{-14} \times sin(2\pi f_o t + \Phi).$$

This is in perfect agreement with that shown in Figure 12 with $\Phi = 90$ degrees. The TKS output Allan variance from this sine frequency can be determined to be

$$\sigma_{TVT}^2(\tau) = 3.84^2 \times 10^{-28} \times sin^4(\pi f_o \tau)/(\pi f_o \tau)^2. \tag{14}$$

The square root of this Allan variance is shown in Figure 14 and clearly agrees with that obtained early by simulation.

7.2 TKS Error Due to the Space Vehicle Umbra During Solar Eclipse

The temperature profile due to the space vehicle umbra can be modeled as a 2 degrees/hour shift for a 1/2 hour and then a settling at a constant temperature for 1/2 hour before sloping back to its original temperature.[7] The resulting VCXO frequency error at the TKS input with a temperature coefficient of 4×10^{-11} (df/f)/°C is shown in Figure 11. The TKS output frequency error due to this VCXO input frequency noise can be computed using the transfer function of $H_{TV}(Z)$ and the result is provided in Figure 12. The resulting TKS output timing error is provided in Figure 13. The Allan deviation of this umbra error can be computed from this timing error and the result is shown in Figure 14. Again, this umbra-induced TKS error is below the RAFS specification line.

7.3 TKS Error Due to VCXO Out-of-Specification Temperature Coefficient

In the early delivery of the VCXOs, some of them have a temperature coefficient of 14×10^{-11} (df/f)/°C, that is, about 3.5 times of the specification. The TKS Allan deviations due to this out of specification for both the orbital and umbra effects are recomputed and are shown in Figure (15); as expected, they are much closer to the RAFS specification line. The Allan deviation of the RAFS TKS with the out-of-specification VCXO is also shown in Figure 15, and it meets the RAFS TKS specification.

8 CONCLUSIONS

The contributions to the TKS output Allan deviation from the three basic TKS noises as well as the VCXO orbital and umbra temperature-induced noises were computed. These formed the sensitivities of the Allan deviation at the TKS output for each of its input noises and they are valuable for design trade-off and troubleshooting. The results of this paper compare favorably with those obtained from the TKS system testing. When the TKS components meet their specifications, the RAFS TKS should meet its specification. The CAFS TKS should have no problem of meeting its specification, since the CAFS normally would exceed its specification.

9 REFERENCES

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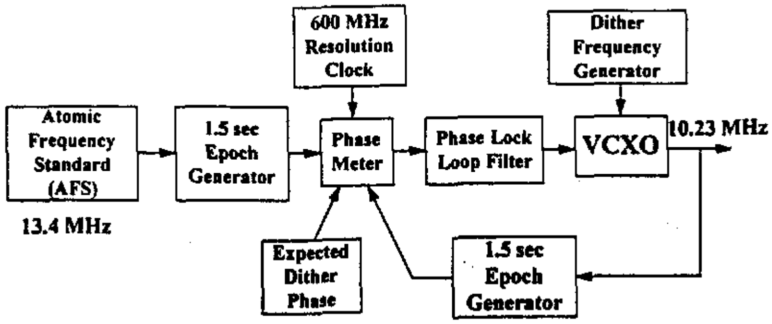


Figure 1. GPS Block IIR TKS System Model

Figure 2. GPS TKS Stability Requirements

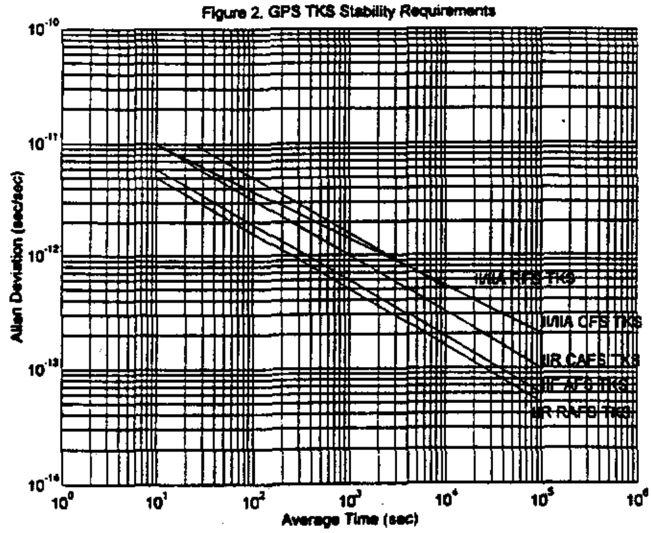
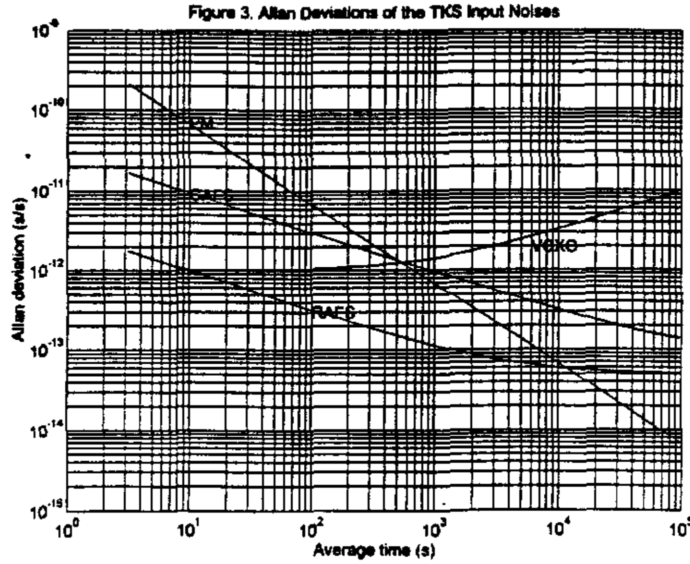


Figure 3. Allan Deviations of the TKS Input Noises



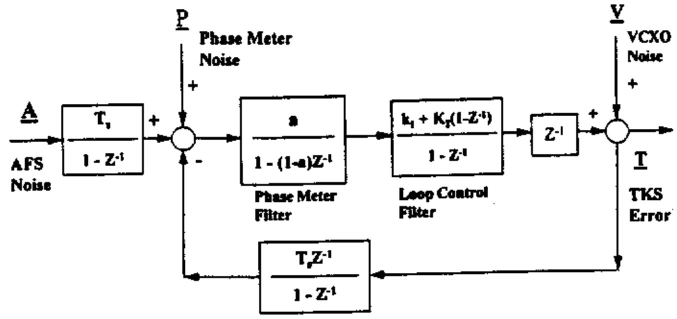


Figure 4. TKS Error Model

Fig. 5. Bode Plots of the TKS Transfer Functions

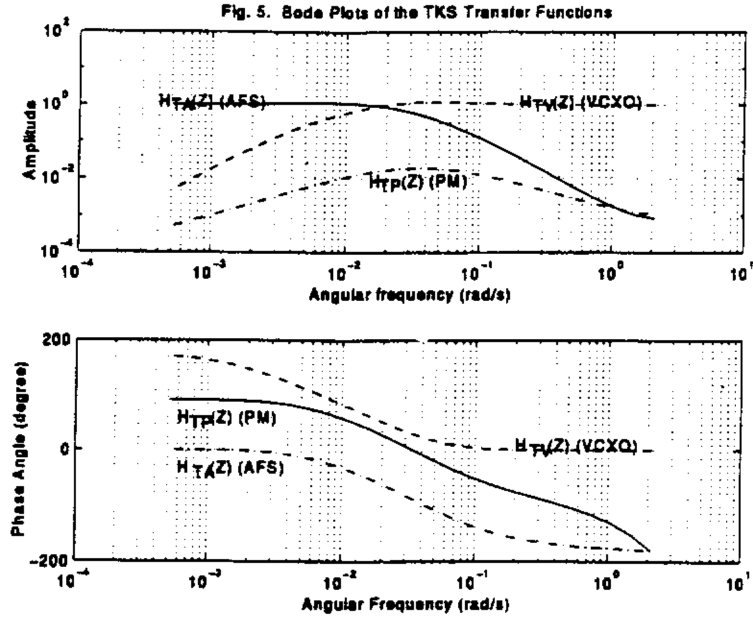


Figure 6. TKS Sensitivity Responses

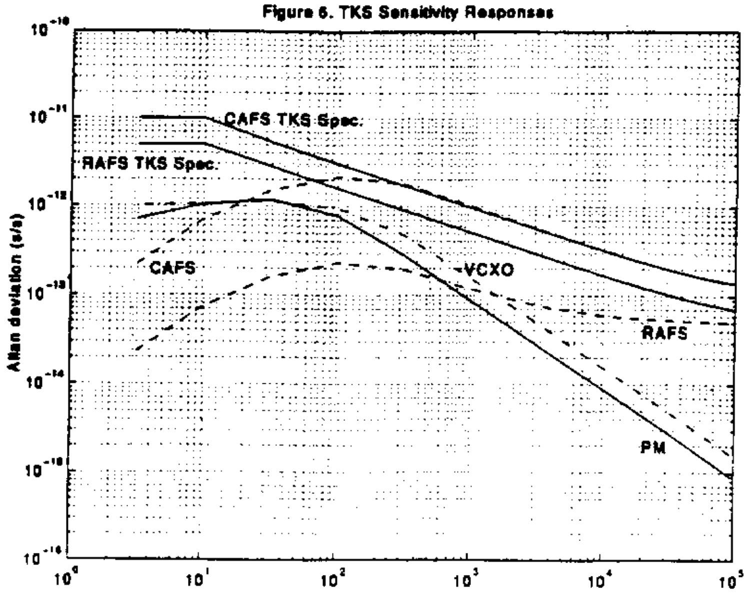


Figure 7. RAFS TKS Allan Deviations

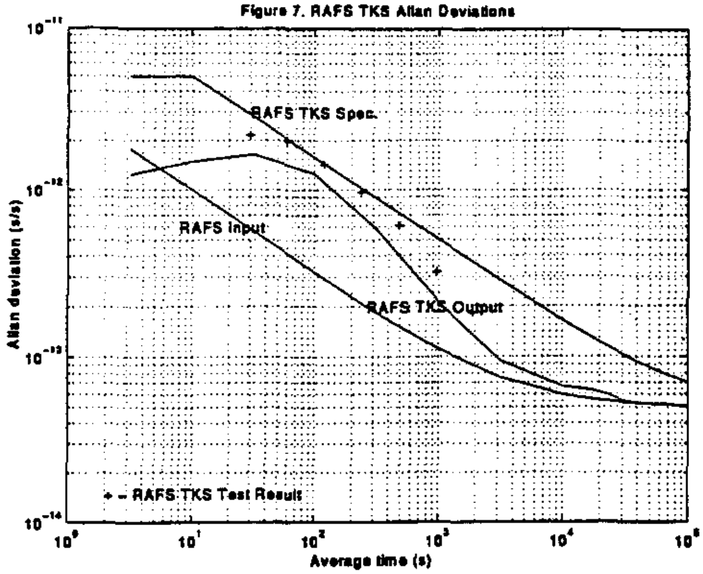
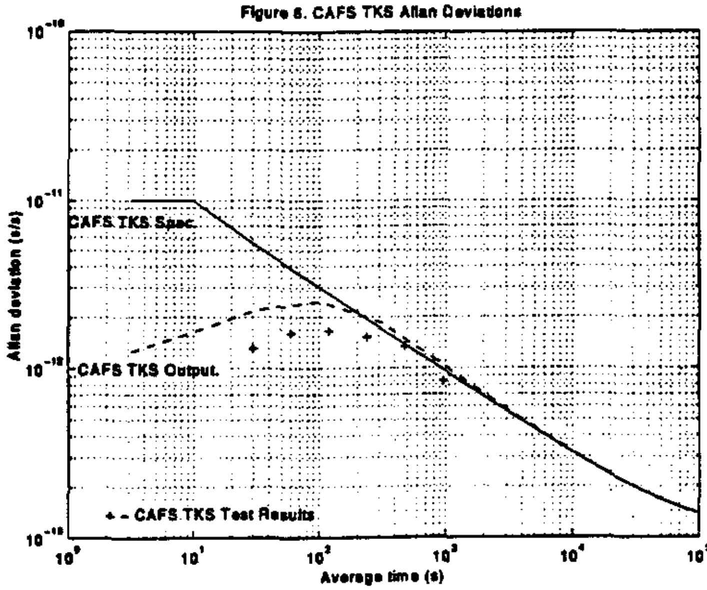


Figure 8. CAFS TKS Allan Deviations



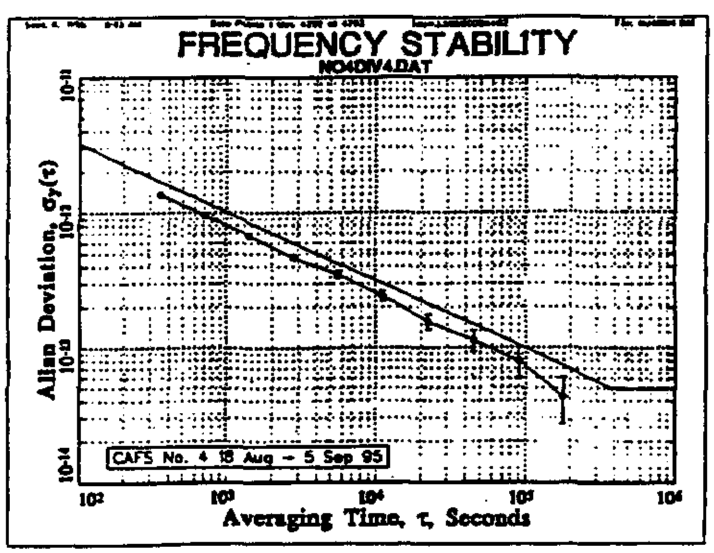


Figure 9. CAFS Test Result

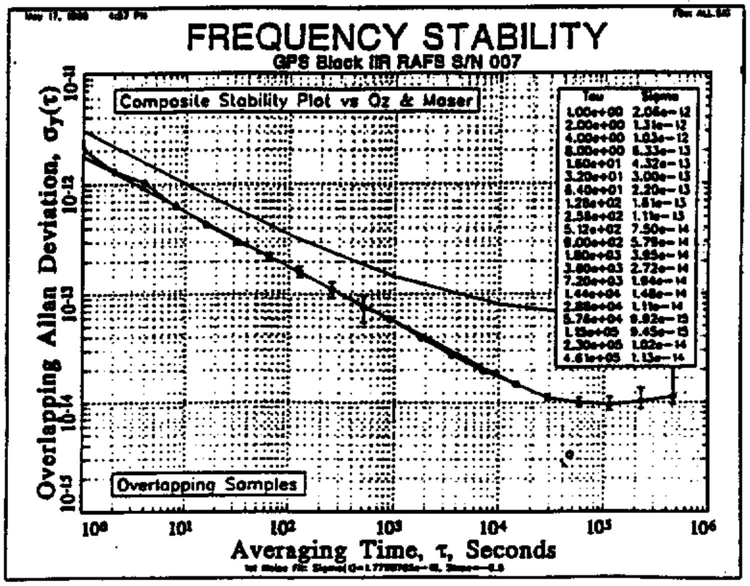
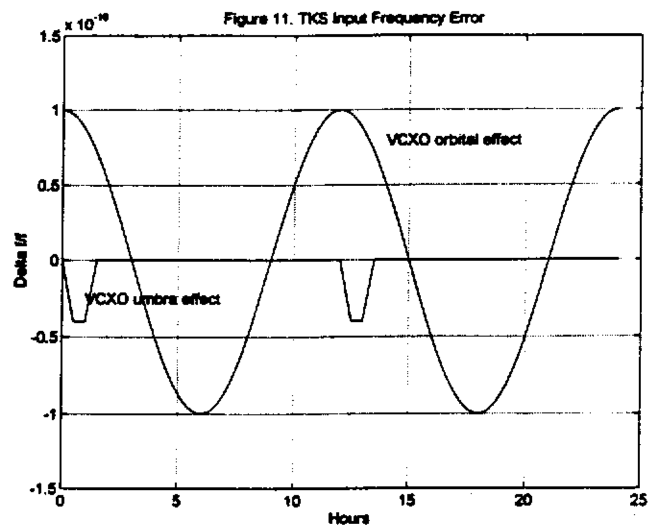


Figure 10. RAFS Test Result



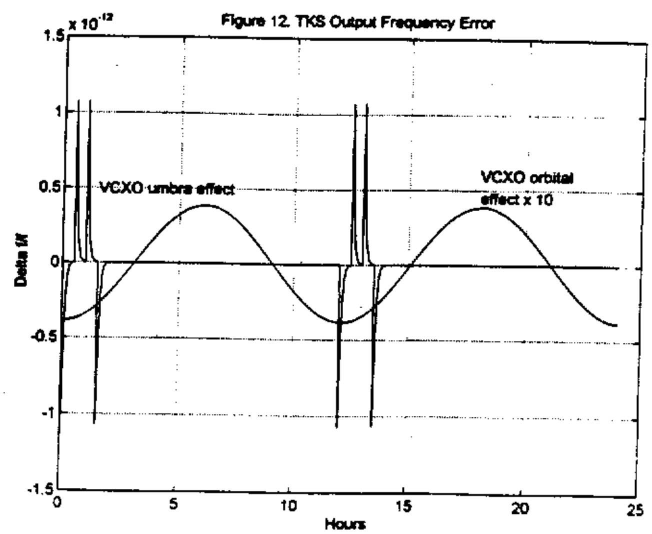


Figure 13. TKS Output Timing Error

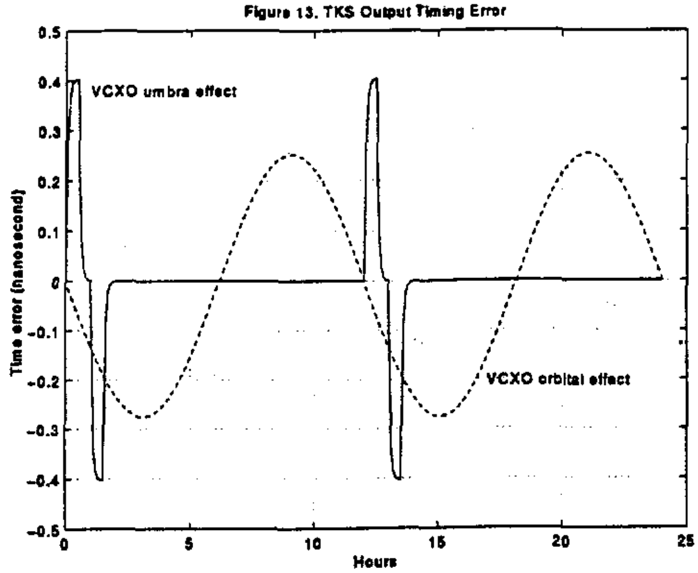


Figure 14. TKS Output Allan Deviations

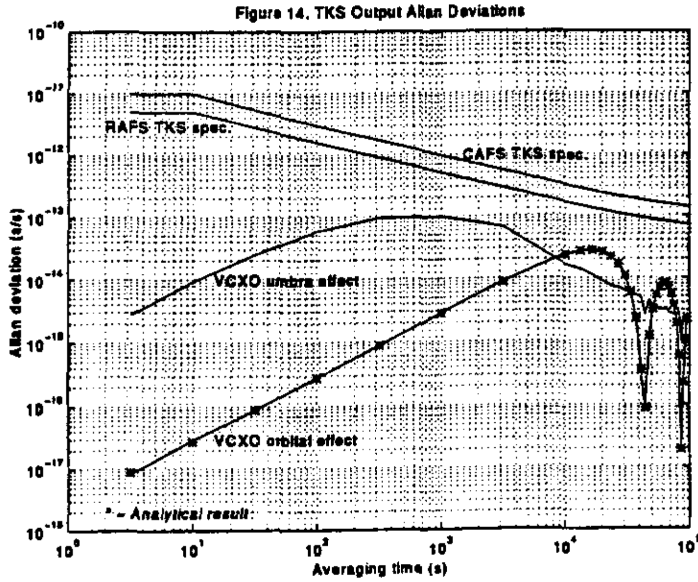
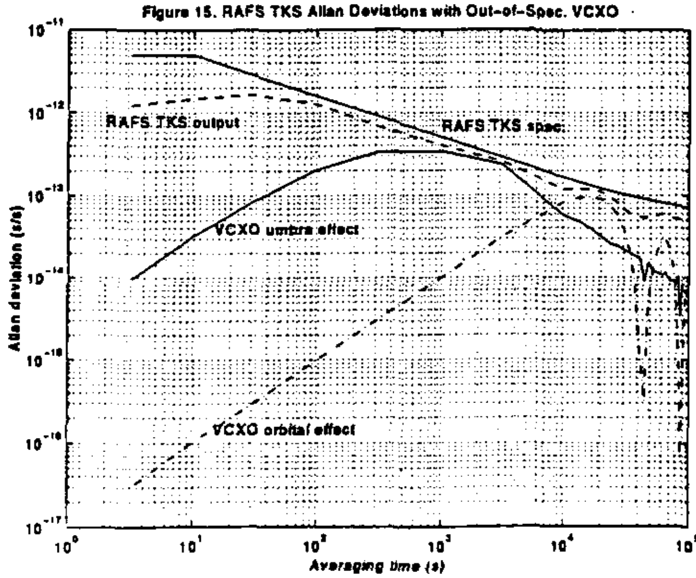


Figure 15. RAFS TKS Allan Deviations with Out-of-Spec. VCXO



Questions and Answers

STEVEN HUTSELL (USNO): There's an earlier plot that at least seems to indicate – and I'd like your opinion – that possibly with some manipulation of the gains, the servo could take better advantage of the physics package stability. I'd like your comment on that.

ANDY WU: The reason that 150 seconds was chosen was because the phase meter is so poor, having a resolution of 1.67 nanoseconds. So you need a long time constant to smooth it out. I did try one – let's say 12 or 15 seconds. And you're going to see the phase meter is going to pop over out here and you're going to have a problem.