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# FATIGUE VARIABILITY OF A SINGLE CRYSTAL SUPERALLOY AT ELEVATED TEMPERATURE (PREPRINT)

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**Metals Branch**
**Metals, Ceramics and NDE Division**

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**14. ABSTRACT**

In order to develop more accurate life prediction tools, an improved understanding of the variability within the fatigue behavior of a material is required. Recent work has shown multiple failure mechanisms that drive the variability in fatigue life of polycrystalline titanium and nickel materials. In addition, the bimodal behavior in the fatigue response is not readily apparent when only a very small number of specimens are tested at each loading condition, as is normal practice. The objective of this work was to investigate the fatigue variability of a single crystal nickel-base superalloy at elevated temperature. PWA1484, a second generation single crystal alloy developed for advanced turbine airfoil applications, was the material of choice for this investigation. A large number of fatigue tests were performed at one condition (stress level, stress ratio, frequency and temperature) to determine the variability and identify the sources of uncertainty in life. Scanning electron microscopy was used to investigate the relationship between failure mechanisms and variability.

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# Fatigue Variability of a Single Crystal Superalloy at Elevated Temperature

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## Abstract

In order to develop more accurate life prediction tools, an improved understanding of the variability within the fatigue behavior of a material is required. Recent work has shown multiple failure mechanisms that drive the variability in fatigue life of polycrystalline titanium and nickel materials. In addition, the bimodal behavior in the fatigue response is not readily apparent when only a very small number of specimens are tested at each loading condition, as is normal practice.

The objective of this work was to investigate the fatigue variability of a single crystal nickel-base superalloy at elevated temperature. PWA1484, a second generation single crystal alloy developed for advanced turbine airfoil applications, was the material of choice for this investigation. A large number of fatigue tests were performed at one condition (stress level, stress ratio, frequency and temperature) to determine the variability and identify the sources of uncertainty in life. Scanning electron microscopy was used to investigate the relationship between failure mechanisms and variability. Crack growth analyses were used to predict lowest life estimates and were compared to experimental data. The results show large variability in fatigue life at fairly high stresses. Evaluation of the fracture surfaces indicated that microstructural features such as carbides and eutectics were responsible for the failures. In addition, the size of the feature responsible for fatigue failure could not be directly related to the

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fatigue life. The lowest expected life based on fatigue crack growth analyses did agree with the shortest life found experimentally. However, more testing and analysis is required.

**Keywords**

Fatigue, Variability, Single Crystal, Temperature

## 1. Introduction

The current life management approach for maintaining fracture critical aerospace turbine engine components in the U.S. Air Force is based on the specification known as the Engine Structural Integrity Program (ENSIP) [1]. ENSIP is based on traditional views of statistical behavior of materials, and hence, often very conservative, resulting in significant sustainment costs. Therefore, the Air Force has been pursuing the prognosis approach [2], which is based on the development and integration of new capabilities for real-time state awareness, physics-based damage and failure modeling, and autonomic reasoning. Reliable prognosis of the component depends on basic understanding and modeling of the sources of uncertainty in material behavior under service loading conditions. Many authors [3-16] have investigated the fatigue and crack growth behavior of single crystal superalloy materials. These deterministic approaches were aimed at understanding the general trend and mean behavior under expected service conditions. However, physics-based probabilistic models are essential for accurately predicting the fatigue life and the associated uncertainty in predicted life of single crystal materials. Since these models are based on observed damage mechanisms, prediction of the design fatigue life limit, e.g. 1 in 1000 probability of failure, can be expected to be more accurate compared to statistical estimation based on data. Recently, Jha et al. [17-18] showed that the fatigue life distribution on the cumulative distribution function (CDF) plot exhibited a dual failure mode (bi-modal distribution) at some stress levels, i.e. a single CDF could not describe all the data. Limited fatigue data [3-6] available for single crystal PWA 1484 does not exhibit the dual mode CDF observed by Jha et al. [17-18] in polycrystalline materials. However, these studies [3-6] did not investigate the material variability, and only tested 1-2 specimens under each loading condition. Jha et al. [17-18] also demonstrated that a small-crack-growth-based approach can be used to predict the minimum fatigue life in alpha+beta processed titanium and Ni-base superalloys. Hence, a study was initiated to conduct a detailed investigation of variability in fatigue life of single crystal superalloy PWA 1484 at the elevated temperature of 593°C. This paper will discuss the observed fatigue lifetime variability, crack initiation mechanisms, and the application of a small-crack-growth-based model to predict the limiting fatigue life.

## 2. Experimental Procedure

### *Material*

The material chosen for this study was a 2nd generation single crystal superalloy developed by Pratt & Whitney for use in turbine engine airfoil applications. It is a precipitation strengthened cast mono grain nickel alloy based on the Ni–Cr–Al system, with 60% by vol. γ' precipitates in a γ matrix. The material is characterized by continuous primary dendrites which span the casting in the direction of solidification. The primary dendrites are approximately 100μm in diameter with an average spacing between each of 450μm (Figure 1a). Because the material is HIPed, porosity is not apparent in the castings. However, eutectic γ/γ' regions and carbides, which can serve as initiation points during fatigue testing, can be found within the interdendritic regions. A close up view of a eutectic and carbides found in the interdendritic region is shown in Figure 1b and 1c. Finally, a secondary electron image of the γ' precipitates is shown in Figure 1d. Additional details concerning the microstructure of this material can be found in the literature [3, 4].

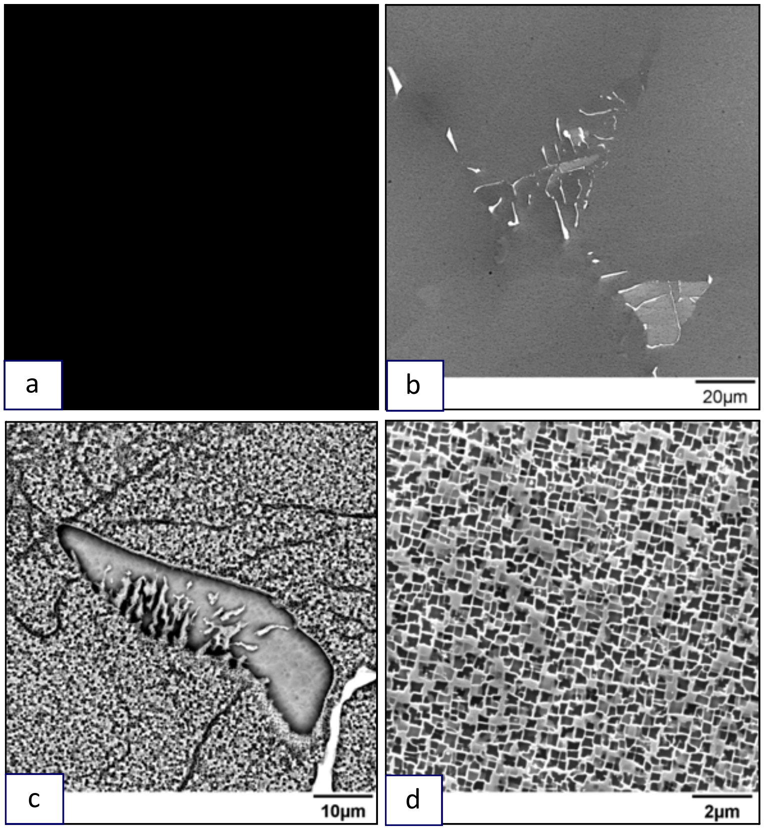

Figure 1: Microstructure of PWA 1484: a) primary dendrite core and interdendritic regions, b) script-type carbides, c) eutectic region and d) cuboidal $\gamma'$.

*Test Setup*

Testing parameters were chosen to correlate with previous results from other studies [4-5]. The objective was to determine the inherent variability and sources of initiation in the fatigue behavior of the material. The previous studies provided a baseline of S-N results from which appropriate testing conditions could be chosen, although there were no more than 2 data points at any specific loading condition. The stress ratio and maximum stress values were chosen to be at a stress level high enough to expect average failure lives of approximately $10^5$ to $5x10^5$ cycles but low enough to see significant variability, if it exists. The testing parameters chosen were: $\sigma_{MAX}$ = 860 MPa, R = 0.1, T = 593 C (1100 F), *f* = 20 Hz.

Test specimens for this study were machined from 152 cm x 7.6 cm x 1.6 cm (6” x 3” x .625”) cast slabs of PWA 1484 with the primary longitudinal axis in the <001> direction (±5 °). The dogbone specimens had a 6 mm gage length and 4 mm diameter, as shown in Figure 2. The specimens were low stress ground, and then electropolished, removing up to 25 μm of material from the surface.

All testing was performed using an MTS servohydraulic test system. An induction system was used to heat the specimen and a solid cylindrical susceptor was centered around the specimen to provide uniform heating. The coil design consisted of two turns above the gage section and three turns below. The susceptor and induction coils were on independent translational stages, which aided in adjusting the specimen temperature. The test system, including copper coils wrapped around the top and bottom of the system to cool the grips, can be seen in Figure 3.

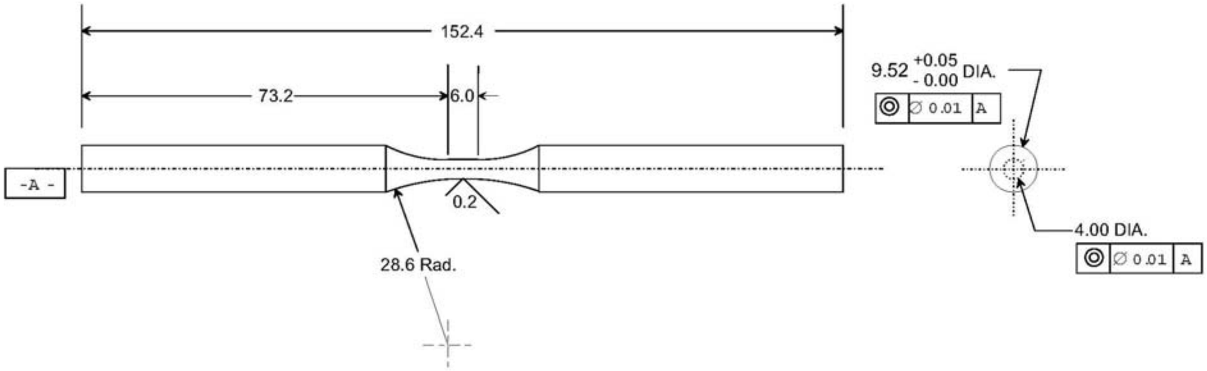

Figure 2: PWA 1484 test specimen with longitudinal axis oriented in <100> direction (all dimensions in mm).

In order to map out the temperature profile in the gage section, a specimen was fit with twelve thermocouples. Nine of the thermocouples were spaced along one side of the specimen, centered on the gage section. The remaining three thermocouples were located 180 degrees from the top, center, and bottom thermocouples. The top thermocouple, which was located at the outside edge of the radius where it meets the flat of the grip section, was used as a control. Figure 4 shows the measured temperature profile along the center of the specimen. The temperature within the gage section was consistently measured between 590 and 596 C. In addition, the thermocouples placed on the back side of the specimen always measured temperatures within 1 degree of the front side.

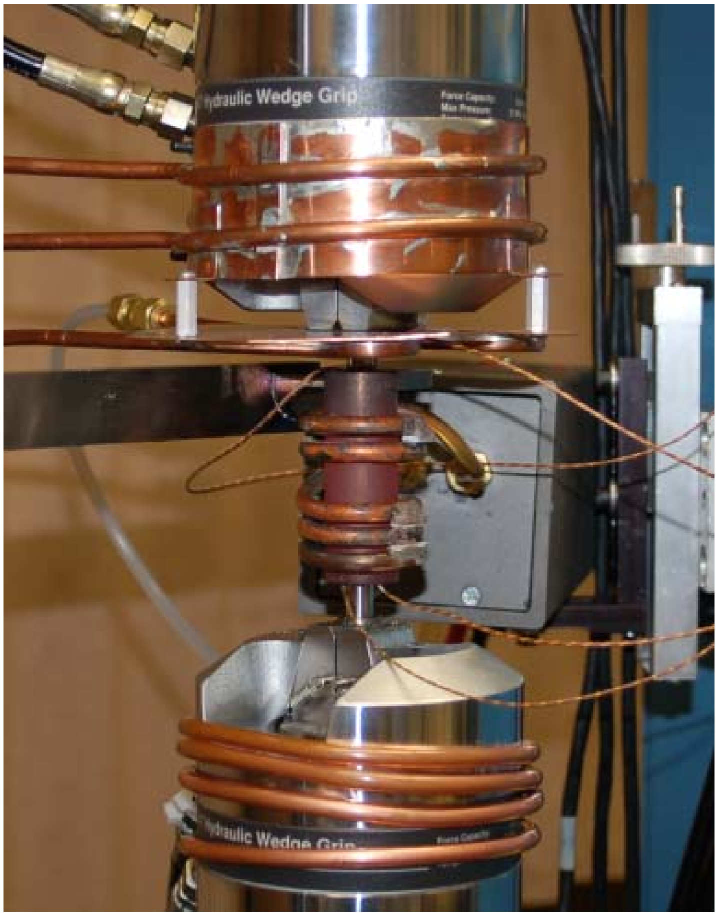

Figure 3: Test system setup.

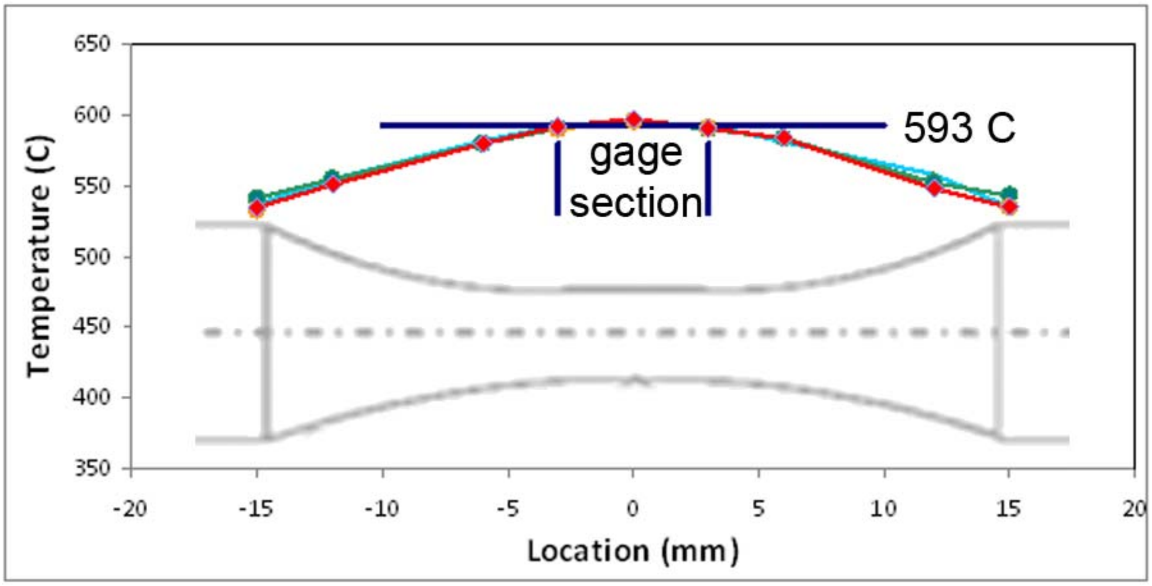

Figure 4: Specimen temperature profile.

## 4. Results and Discussion

There have been a number of studies in the literature concerning the fatigue properties of PWA 1484. It has been reported that fatigue failures often start from eutectics and carbides [4, 6-7]. In addition, fatigue failures tend to often occur along (111) planes, but also can occur along (001) planes [8,15**]**. Telesman and Ghosn [8] reported that the transition between failure along (111) planes and (001) planes is due to an environmental damage mechanism caused by oxygen embrittlement. It was shown that at lower temperatures and higher frequencies, fatigue crack growth occurred along (111) planes. However, at higher temperatures and/or lower frequencies, both of which exacerbate environmental damage, the fatigue crack growth transitions to (001) planes.

This study concentrated on one loading condition in order to determine the variability inherent in the material. A stress ratio, R, of 0.1 with a maximum stress of 860 MPa at a temperature, T, of 593 C (1100 F) was chosen as the loading condition. The results of eleven tests are shown on a stress-life plot in Figure 5, along with data from two other studies. In addition to these tests, three tests were conducted with a maximum stress of 760 MPa (R = 0.1, T = 593 C). Although there aren't enough data at this condition to understand the extent of the variability, it does show that the data in this study are consistent with the data from the previous studies. It can be seen from Figure 5 that the maximum stress level chosen for this study was well above the fatigue limit from earlier studies, and might be expected to produce lives of around $10^5$ cycles. What is immediately apparent, however, is that there is a large amount of variability in total life. Fatigue lives spanning over 2 orders of magnitude were seen, from 63,000 cycles to a runout of over 16 million cycles. This type of variability has not been reported in the past for single crystal materials, possibly because only 1 or 2 tests are typically performed at any loading condition.

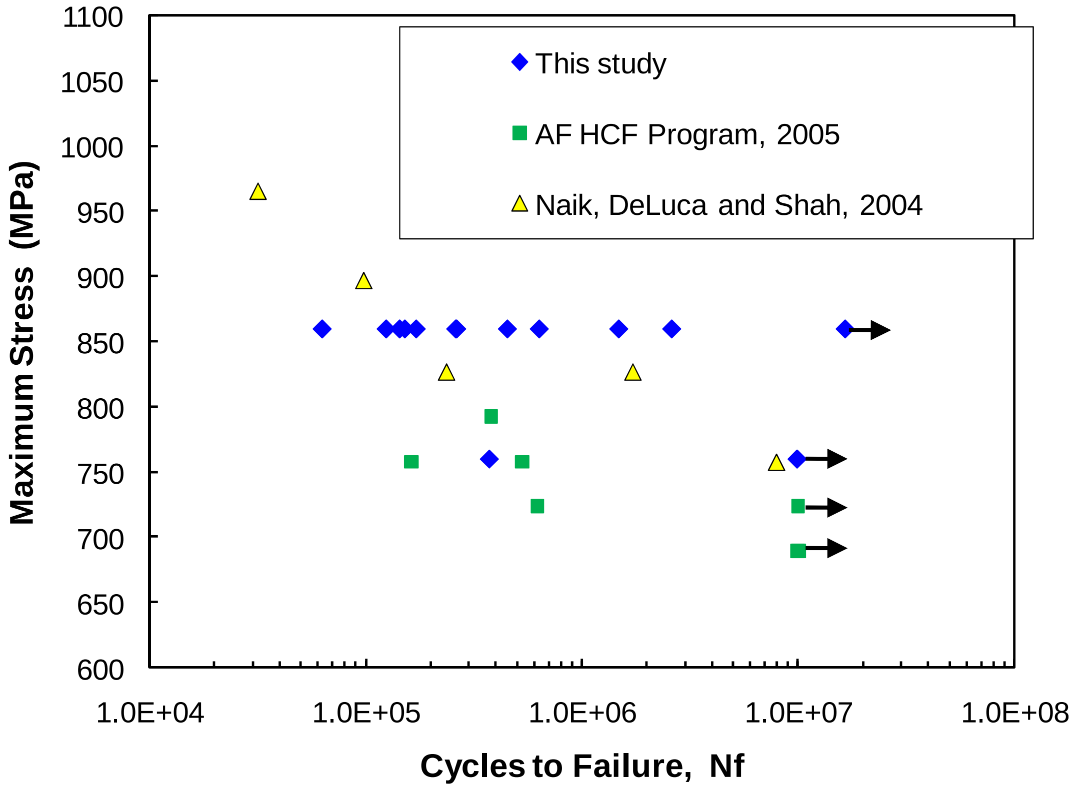

Figure 5: Stress-life plot of all data in this study along with data from the literature [4-5].

Similar results have been seen, however, on other nickel superalloys and titanium alloys [19-20], where it was reported that the large variability may be due to different failure mechanisms. In addition, when the fatigue lives were plotted on a probability plot it was apparent that they did not correspond to a single normal distribution. Jha *et al.* [19-20] reported that a bimodal distribution provided a more accurate description of the data.

In contrast, a bimodal description does not seem to apply to the data in this study. The results, plotted on a probability plot, are shown in Figure 6. While the results are not perfectly linear, they also do not appear to be a bimodal distribution such as those seen in other alloys. This may indicate that the failures are based on a single failure mechanism.

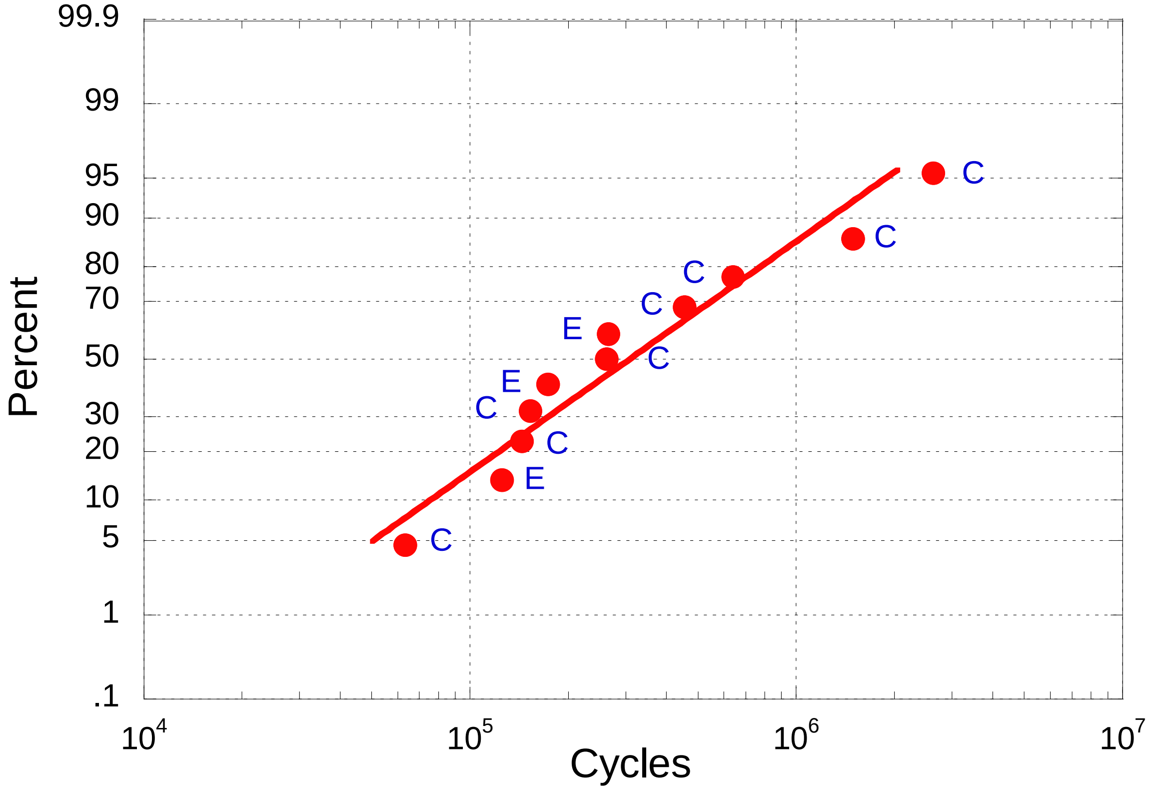

Figure 6: Probability plot of failures at constant amplitude $\sigma_{max}$ = 860 MPa, R = 0.1. ( “E” and “C” represent failure by eutectic and carbide, respectively)

*Fractography*

In order to understand the amount of variability seen in these results, a number of the failure surfaces were investigated. At a macro scale, all of the failure surfaces appear similar. Namely, initiation takes place internally, with fatigue crack growth occurring along 1 or more (111) planes. This is consistent with previous work [4]. When viewed at higher magnification, the initiation sites are located at either a carbide or a eutectic. Figure 7 shows the fracture surface from the lowest life specimen, $N_f = 63{,}000$ cycles. The specimen failed from an internal carbide, with fatigue crack growth along a (111) plane. The carbide had a length of approximately 10 μm, with a width of approximately 3-5 μm, and was typical of the carbides seen throughout this material. Figure 8 shows the fracture surface of a specimen that failed at 125,000 cycles. The initiation site was from a eutectic, with fatigue crack growth progressing along three separate (111) planes. The eutectic was significantly larger than the previous carbide, with dimensions approximately 15 μm by 20 μm. The eutectics were typically much larger than the carbides, and exhibited a coarse $\gamma/\gamma'$ microstructure. The eutectics also had dimensions that were more uniform, while the carbides had a larger length-to-width ratio. There did not seem to be any correlation, however, between cycles to failure and feature type (eutectic vs. carbide) or between cycles to failure and feature size. This can be seen in Figure 6, which identifies the failures as either carbide or eutectic. It is also consistent with the notion that the distribution governing the fatigue results is not bimodal. Instead, it may indicate that in some instances, fatigue crack growth from a microstructural feature begins almost immediately, while in other cases there is some finite incubation period before the crack starts to grow.

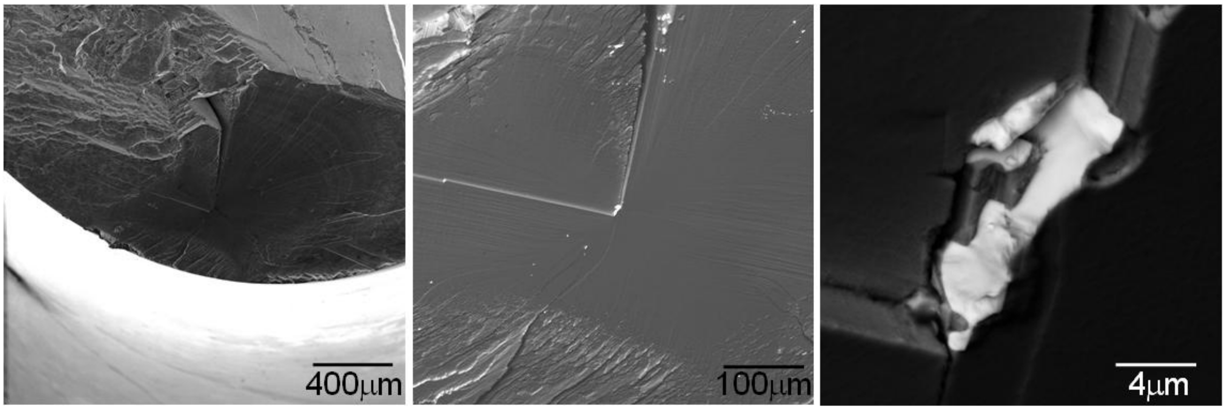

Figure 7: Fracture surface showing internal initiation from a carbide and fatigue crack growth along a (111) plane ($N_f$ = 63,000 cycles).

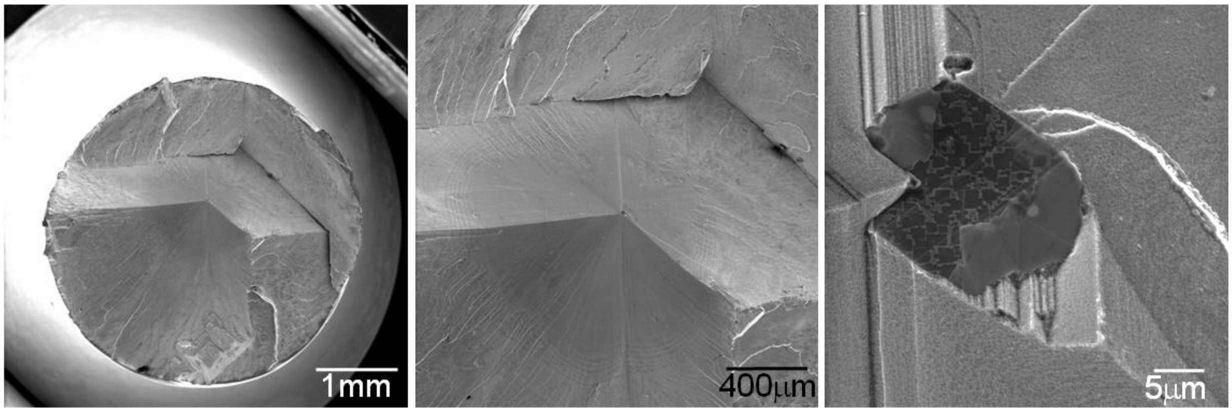

Figure 8: Fracture surface showing internal initiation from a eutectic and fatigue crack growth along multiple (111) planes ($N_f$ = 125,000 cycles).

### *Limiting Life Estimation*

Previous work on fatigue variability of nickel alloys and titanium alloys [17-20] has shown that the limiting (i.e. lowest) life of a material under a specific loading condition can be estimated using fracture mechanics based analysis of small cracks growing from observed microstructural features at the initiation sites. In the case of PWA 1484, the initiation size was estimated based on typical carbide and eutectic sizes. It was found that the typical crack radius was approximately 5.7 μm, assuming that the initiation area is equivalent to a circular crack.

This corresponds to an average equivalent initiation area of approximately 100 $\mu m^2$. As shown by Jha et al. [20], predictions of crack growth life from these observed initiation sites require the knowledge of small crack growth behavior in this material. Since fatigue crack growth testing was not performed in this study, long crack growth data from the National High Cycle Fatigue Program [4] was used to estimate the small crack growth behavior. Figure 8 shows the data, along with upper and lower curve fit estimates for da/dN vs. ΔK, assuming that the Paris regime of the long crack behavior represents the small crack growth as well.

The analysis assumes that a crack initiates from a carbide or eutectic on the first cycle. The initiation size (5.7 μm) is equal to the average initiation feature size measured from the failure surfaces. The stress intensity factor was determined using the solution for an embedded circular crack centrally located in a rod [21]. Small crack growth behavior is modeled as $da/dN = C(\Delta K)^n$, in which da/dN=crack growth rate, ΔK=applied stress intensity factor range, and C and n are constants. In Figure 8, n = 2.0 and 1.5 for the upper and lower bounds, respectively, while C = 1.8E-10 m/cycle for both. Environmental effects are neglected in the analysis, and crack growth in air is assumed to be the same as in a vacuum. This is necessary since the only crack growth data available was measured in air and the cracks grow internally. Crack growth was assumed to be mode I, with failure occurring at $K = 25\ MPa{*}m^{1/2}$.

The results of the crack growth analysis were used to predict the cycles to failure for a range of stresses using both the upper and lower bound crack growth rate estimates. Figure 9 shows the maximum stress-life plot including the fracture mechanics calculations for the range of limiting (lowest) life estimates. The analysis predicts that at a maximum stress of 860 MPa the lowest life should occur somewhere between 50,000 and 100,000 cycles. This predicted range agrees with the initial results obtained, which included a failure at 63,000 cycles. In order to increase confidence in the analysis, additional steps would need to be taken. First, crack growth data in a vacuum is needed to more accurately model the crack growth within the specimen. In addition, the analysis should include the mixed mode nature of the crack growth along (111) planes and account for the distribution of initiation sizes and variability in crack growth behavior. And finally, many more tests would need to be performed under the same conditions to provide a measure of the lowest life with statistical confidence.

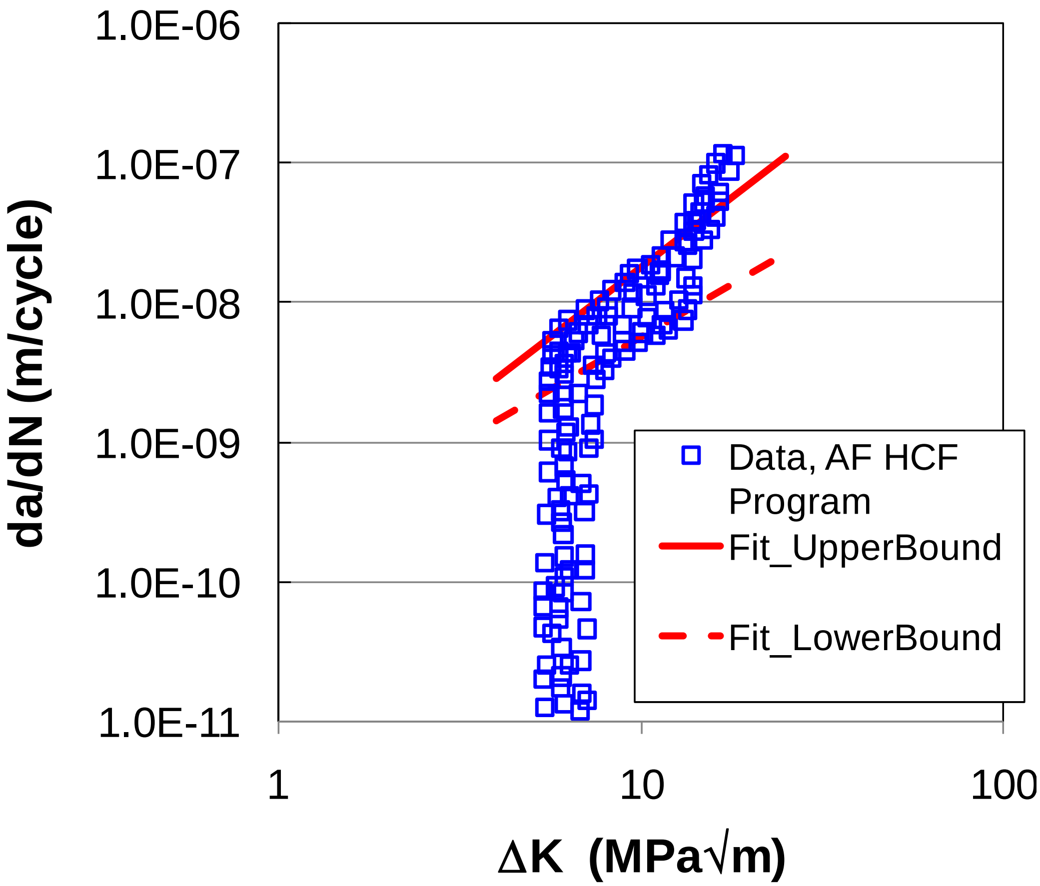

Figure 8: Fatigue crack growth data for PWA 1484 at R=0.1 from the HCF Program [4].

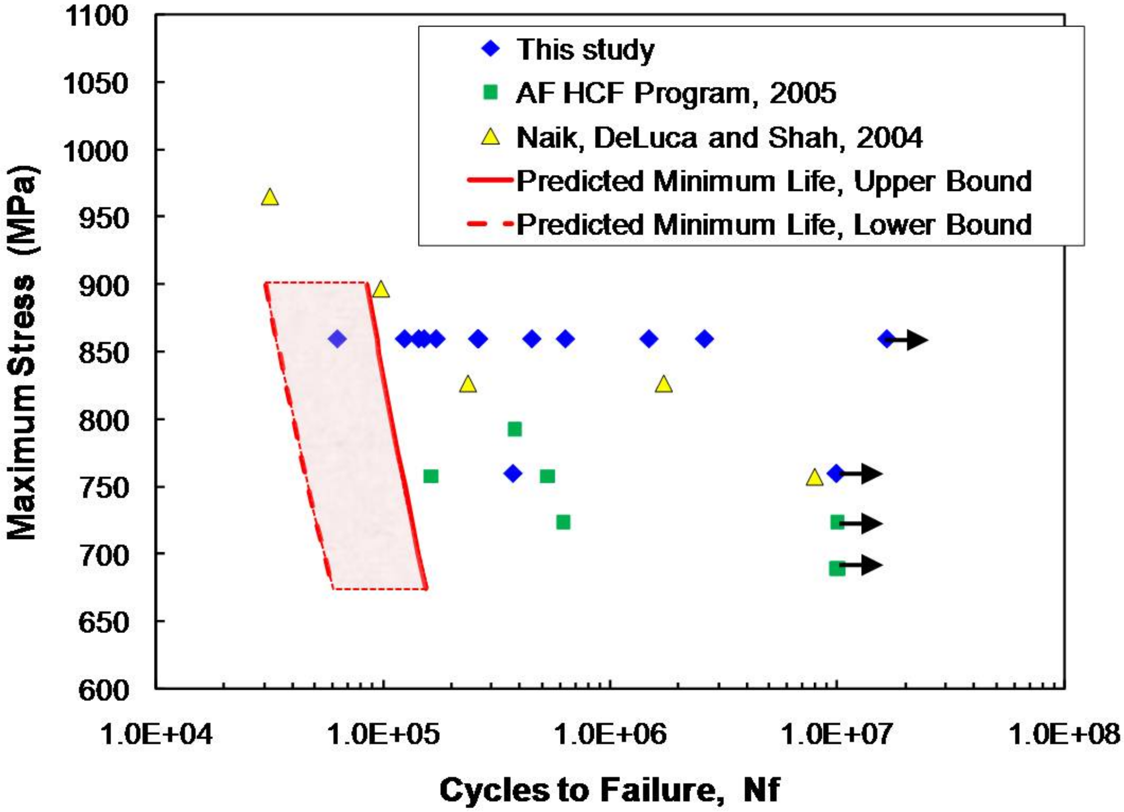

Figure 9: Stress-life plot including fracture mechanics predictions for lowest life failures.

## 5. Conclusions

In this study a 2$^{nd}$ generation single crystal superalloy, PWA 1484, was evaluated under one loading condition to determine the material variability. It was found that at a stress fairly close to yield, the variability in fatigue life was over two orders of magnitude, with one specimen not failing. In addition, there is no apparent difference in failure mechanism between short and long life specimens. Fatigue initiation is governed by microstructural features such as eutectics and carbides.

By employing fracture mechanics, a good estimate of the expected shortest life can be determined. The estimate is based on fatigue crack growth from a typical microstructural feature beginning at the first cycle. Employing such methods may lead to a higher design life than that which would be predicted using a normalized distribution covering all fatigue data. However, additional work is needed to determine a more accurate lowest life based on a more detailed crack growth analysis including fatigue crack growth data in a vacuum for short and long cracks as well as a mixed mode analysis based on the failure in (111) crystallographic planes.

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