

# OPTIMIZATION OF ENERGY CONSUMPTIONS IN SENSOR NETWORKS

Hongbing Cheng, Yu-Dong Yao  
Department of ECE  
Stevens Institute of Technology  
Hoboken, NJ 07030  
Email: {hcheng, yyao}@stevens.edu

Shafik Quoraishee  
Acoustics and Networked Sensors Division  
US Army RDECOM-ARDEC  
Picatinny, NJ 07806-5000  
Email: shafik.quoraishee@us.army.mil

## Abstract

The purpose of this paper is to minimize the energy consumption for a single link communication in wireless sensor network. The quadrature amplitude modulation (QAM) and auto repeat request (ARQ) with finite retry limit and various packet size are considered in the system model. We formulate the optimization problem to be a nonlinear integer programming with modulation order and packet size as variables. To solve this problem, the continuous relaxation based branch and bound method and the successive quadratic programming (SQP) algorithm is used. Further, a variable transformation method is adopted to convexify the problem and guarantee the global optimization. A numerical example is given to illustrate the solution, which suggests that the joint optimization of packet size and modulation order for various communication distance can make the sensor nodes work in a more energy efficient way.

## I. INTRODUCTION

In recent years, wireless networks have emerged as an efficient real time communication standard for both commercial and military networks. There has been an urgent need, due to the demand for applications that utilize data from multiple remote sources to classify targets and communicate with high priority responders, to make communications on wireless networks as seamless and uncorrupted as possible. Unfortunately, the task of optimizing communications becomes difficult when the sensor nodes are small, and have a low-battery life [Akyildiz et al., 2002]. Mission critical communications in the battlefield could be disrupted if a sensor network stops working

suddenly. In situations where the soldier might have to identify friend from foes, intruders, or even simply obtain readings from sensor nodes as to the conditions of a particular unmanned environment, sensors on a network such as distributed Unattended Ground Sensors that have failed or are failing tend to contribute erroneous data thereby providing compromised situational awareness. Thus, a false alarm for a target-rich zone could be returned to the warfighter, who would enter the zone, unprepared for conflict. With these concerns in mind, research into the realm of energy-efficient communication strategies from the perspective of cross communication layer optimization has been conducted. Many energy-efficient strategies have been proposed in different layers or by the use of cross-layer design, which is well surveyed in [Goldsmith et al., 2002].

In physical layer, the optimization of modulation order is proposed in [Cui et al., 2005] to achieve a balance between circuit energy consumption and transmission energy consumption. The work is extended to the multiple access scenario by [Cui et al., 2004]. Further, [Prabhu et al., 2007] minimized energy consumption by joint modulation and power adaptation in consideration of auto retransmission request (ARQ) in data link layer. It assumed a simple ARQ scheme with infinite retransmission limit to simplify the problem. Unfortunately, considering only optimization of the modulation order omits features of the data itself such as cyclic redundancy codes and perhaps more complex strategies such as data compression, which might allow for high fidelity transmission at a lower energy cost. In data link layer, energy efficiency based packet-size optimization is implemented in [Sankarasubramanian et al., 2003]. Once again packet size optimization techniques based on parameters such as channel conditions does not provide a very complete energy efficient solution since without considering the setup of the underlying hardware, there is a fundamental limit to how energy-

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efficient transmissions can get, constrained by the physical configuration of the system. By introducing the cross-layer design scheme, [Kwon et al., 2006] considered power control in physical layer, retransmission limit in datalink layer and routing protocol in network layer to maximize the lifetime of WSN. However, the modulation order and packet size are not involved in the optimization. A general optimization model for bottom three layers of WSN is outlined in [Cui et al., 2007].

In this paper, we focus on the cross layer energy minimization for a single link of WSN in consideration of the modulation order in physical layer and packet size in data link layer. An ARQ scheme with finite retransmission limit is assumed. And the transmission power is adjusted according to the modulation scheme and packet size to approach the target packet loss rate with the given retransmission limit. This work is a basis of the whole network energy minimization and has so far been overlooked. The contribution of this paper is to establish the energy consumption model for a single WSN link with MQAM modulation and finite retransmission ARQ scheme and jointly optimize the modulation order and packet size. As a typical nonlinear integer programming, the optimization problem can be solved by the continuous-relaxation-based branch-and-bound method [Li et al., 2006]. However, it can be found that the problem is non-convex even after continuous relaxation. Variable transformation method is used to convexificate the continuous problem [Li et al., 2005]. Then the global optimization can be obtained by general nonlinear convex optimization algorithms, such as successive quadratic programming (SQP) method [Bazaraa et al., 2006].

The rest of the paper is organized as follows. Section II describes the system model and section III formulates the optimization problem as a nonlinear integer programming and discusses the optimization algorithm and convexification method. Some numerical results are given in Section IV and the conclusion is drawn in section V.

## II. SYSTEM MODEL

A single communication link connecting two wireless sensor nodes is considered. Supposed that the information bits are divided into packets with  $L = L_h + L_p$  bits/packet, where  $L_h$ , and  $L_p$  are the fixed overhead bit number and the payload, respectively. With a simple ARQ protocol, each packet will be transmitted repetitively until it is received successfully or the maximum transmission number  $N_r$  occurs. If the target packet loss rate is  $p_{loss}$ , the packet error rate (PER) of each transmission can be calculated by

$$p_{PER} = p_{loss}^{\frac{1}{N_r}}. \quad (1)$$

And the average transmission times for each packet is

$$\begin{aligned} \bar{N} &= 1 + p_{PER} + \cdots + p_{PER}^{N_r-1} \\ &= \frac{1 - p_{PER}^{N_r}}{1 - p_{PER}} = \frac{1 - p_{loss}^{\frac{1}{N_r}}}{1 - p_{loss}^{\frac{1}{N_r}}}. \end{aligned} \quad (2)$$

For each packet transmission in the link, the nodes will spend time  $T_{on}$  in the active mode for transmission or reception, then return sleep mode in a transient duration of  $T_{tr}$ . Assumed that the energy consumption in sleep mode is 0, the total energy consumption  $E$  required to send a packet successfully can be calculated by

$$\begin{aligned} E &= \bar{N}(P_{on}T_{on} + P_{tr}T_{tr}) \\ &= \bar{N}[(P_t + P_{c0})T_{on} + P_{tr}T_{tr}], \end{aligned} \quad (3)$$

where  $P_{on}$ ,  $P_{tr}$  are power consumption values for the active mode and the transient mode, respectively.  $P_t$  and  $P_{c0}$  are the transmission power and circuit power in active mode, respectively. Specially,  $P_{c0}$ , which consists of two parts of power consumptions, can be given by

$$\begin{aligned} P_{c0} &= P_{amp} + P_c \\ &= \alpha P_t + P_c, \end{aligned} \quad (4)$$

where  $P_{amp}$  and  $P_c$  are the power consumption of radio frequency (RF) power amplifier (PA) and other circuits, respectively.  $\alpha = \frac{\xi}{\eta} - 1$  with  $\eta$  the drain efficiency of the RF PA and  $\xi$  the signal peak-to-average ratio (PAR).

Based on the model outlined above, the energy consumption for the successful communication of each information bit in the link can be given by

$$E_b = \frac{E}{L_p(1 - p_{loss})} = \frac{\bar{N}[(\frac{\xi}{\eta}P_t + P_c)T_{on} + P_{tr}T_{tr}]}{(L - L_h)(1 - p_{loss})}. \quad (5)$$

In physical layer, we assume that the bandwidth is  $B$  and the data rate is  $r_b$  bits/symbol. It is obvious that the packet size  $L$  should be an integer times of  $r_b$  to get more energy efficient. Thus the symbol number in each packet is  $L_s = L/r_b$  and the time duration of a packet is

$$T_{on} = L_s T_s \approx \frac{L_s}{B}, \quad (6)$$

where  $T_s$  is the time duration of a symbol. And the target symbol error rate (SER) can be calculated by

$$\begin{aligned} p_{SER} &= 1 - (1 - p_{PER})^{\frac{1}{L_s}} \\ &= 1 - (1 - p_{loss}^{\frac{1}{N_r}})^{\frac{1}{L_s}}. \end{aligned} \quad (7)$$

To simplify the model, forward error control (FEC) is not considered in our work. The information bits are

MQAM modulated with rectangle constellation of size  $M = 2^{r_b}$ . The PAR of the modulated symbol is

$$\xi = 3 \frac{2^{r_b/2} - 1}{2^{r_b/2} + 1}. \quad (8)$$

Supposed that the system works on an additive white Gaussian noise (AWGN) channel. The bound of the SER of MQAM is [Proakis, 2001]

$$\begin{aligned} p_{SER} &\leq 4Q\left(\sqrt{\frac{3\gamma}{2^{r_b} - 1}}\right) \\ &\leq 4e^{-\frac{3}{2^{r_b} - 1} \frac{\gamma}{2}}, \end{aligned} \quad (9)$$

where  $Q(x) = \int_x^\infty (1/\sqrt{2\pi})e^{-u^2/2}du$ ;  $\gamma$  is the signal to noise ratio (SNR) with  $\gamma = P_r/(2B\sigma^2 N_f)$ , where  $P_r$  is the received signal power,  $\sigma^2$  is the power spectral density of the AWGN, and  $N_f$  is the receiver noise figure defined as  $N_f = N_{total}/(2B\sigma^2)$  in which  $N_{total}$  is the power of the noise introduced by the receiver front end.

Considering a propagation path-loss model, the transmission power can be calculated by

$$P_t = G_d P_r, \quad (10)$$

where  $G_d = G_1 d^n M_l$  is the power gain factor in which  $M_l$  is the link margin compensating the hardware process variations and other additive background noise or interference and  $G_1$  is the gain factor at  $d = 1m$ , which is dependent on the antenna gain, carrier frequency and other system parameters;  $n$  is the path-loss factor.

According to (9) and (10), by approximating the bound as an equality, the required transmission power is

$$P_t \approx \frac{4}{3} G_d N_f B \sigma^2 (2^{r_b} - 1) \ln \frac{4}{p_{SER}}. \quad (11)$$

By taking (6), (8) and (11) into (5), we can get

$$\begin{aligned} E_b &= \frac{L_s}{L_s r_b - L_h} \left\{ A_1 (2^{\frac{r_b}{2}} - 1)^2 \ln \frac{4}{1 - (1 - p_{loss}^{\frac{1}{N_r}})^{\frac{1}{L_s}}} \right. \\ &\quad \left. + A_2 + \frac{A_3}{L_s} \right\}, \end{aligned} \quad (12)$$

with

$$A_1 = \frac{4G_d N_f \sigma^2}{\eta(1 - p_{loss}^{\frac{1}{N_r}})}, \quad (13)$$

$$A_2 = \frac{P_c}{B(1 - p_{loss}^{\frac{1}{N_r}})}, \quad (14)$$

$$A_3 = \frac{P_{Tr} T_{tr}}{1 - p_{loss}^{\frac{1}{N_r}}}. \quad (15)$$

### III. PROBLEM FORMULATION AND RESOLUTION

#### A. Problem Formulation

The objective of this paper is to minimize the energy consumption for the communication of each bit in a single WSN link considering the constraint of maximum transmission power and average delay. The problem can be written as

$$\begin{aligned} \min_{r_b, L_s} \quad & E_b, \\ \text{s.t.} \quad & P_t \leq P_{max}, \\ & \tau \leq \tau_{max}, \\ & L_s r_b > L_h, \\ & r_b, L_s \in \mathbf{Z}_+, \end{aligned} \quad (16)$$

where  $P_{max}$  and  $\tau_{max}$  are the maximum transmission power and maximum average delay, respectively;  $\tau$  is the average delay for a packet to be received successfully, which can be approximately calculated by

$$\tau = \bar{N} \left( \frac{L_s}{B} + T_{tr} \right). \quad (17)$$

Therefore, the delay constraint corresponds to the upper bound on  $L_s$  with  $L_s \leq L_{smax} = \lfloor \frac{B(\tau_{max} - T_{tr})}{\bar{N}} \rfloor$ , where  $\lfloor x \rfloor$  denotes the maximum integer less than or equal to  $x$ .

Define  $\mathbf{x} = [x_1, x_2]$  with  $x_1 = r_b$  and  $x_2 = L_s$ , the optimization problem can be easily formulated to

$$\begin{aligned} \min_{\mathbf{x}} \quad & E_b = f_0(\mathbf{x}), \\ \text{s.t.} \quad & f_1(\mathbf{x}) \leq 0, \\ & f_2(\mathbf{x}) \leq 0, \\ & \mathbf{lb} \leq \mathbf{x} \leq \mathbf{ub}, \\ & \mathbf{x} \in \mathbf{Z}_+^2, \end{aligned} \quad (18)$$

with

$$f_0(\mathbf{x}) = \frac{A_1 (2^{\frac{x_1}{2}} - 1)^2 x_2 h(x_2) + A_2 x_2 + A_3}{x_1 x_2 - L_h}, \quad (19)$$

$$f_1(\mathbf{x}) = \frac{4}{3} G_d N_f B \sigma^2 (2^{x_1} - 1) h(x_2) - P_{max}, \quad (20)$$

$$f_2(\mathbf{x}) = \frac{1}{x_1 x_2} - \frac{1}{L_h}, \quad (21)$$

$$h(x_2) = \ln \frac{4}{1 - (1 - p_{loss}^{\frac{1}{N_r}})^{\frac{1}{x_2}}}, \quad (22)$$

where  $\mathbf{lb} = [r_{bmin}, L_{smin}]$  and  $\mathbf{ub} = [r_{bmax}, L_{smax}]$  are the lower and upper bounds of  $\mathbf{x}$ , respectively. Assuming that 4-QAM is the smallest constellation size available [Cui et al., 2005], we set  $r_{bmin} = 2$ ,  $r_{bmax} = +\infty$ ,  $L_{smin} = 1$  and  $L_{smax} = \lfloor \frac{B(\tau_{max} - T_{tr})}{\bar{N}} \rfloor$ .

## B. Continuous-based Branch-and-Bound method

Obviously, problem (18) is a nonlinear integer programming (NIP), which generally can be solved by the continuous-relaxation-based branch-and-bound method, whose detailed process is outlined below [Li et al., 2006].

The algorithm starts by finding an optimum solution  $\mathbf{x}^*$  and the minimized object value  $f_{lb}$  of the continuous relaxation of the NIP problem. If  $\mathbf{x}^*$  is integral, then it is also optimum for the NIP problem. Otherwise, we approximate the integer optimum solution as  $\tilde{\mathbf{x}}^* = \lfloor \mathbf{x}^* \rfloor$  with corresponding object value  $f_{ub} = f_0(\tilde{\mathbf{x}}^*)$ . If the relative error  $\varsigma = \frac{f_{ub} - f_{lb}}{f_{lb}}$  caused by the approximation is less than a given tolerance  $\epsilon$ ,  $\tilde{\mathbf{x}}^*$  is regarded as solution of the NIP problem. Otherwise, select a fractional variable of  $\tilde{\mathbf{x}}^*$ , e.g.  $\tilde{x}_i^*$ . Two new subproblems are generated by adding variable constraints  $x_i \leq \tilde{x}_i^*$  and  $x_i \geq \tilde{x}_i^* + 1$ , respectively. For each subproblem, we apply the same method to find  $f_{lb}$  and  $f_{ub}$ . Choosing the subproblem that gives the minimum  $f_{lb}$ , we can continue to divide it until  $\varsigma \leq \epsilon$  or the problem is infeasible.

## C. Variable transformation convexification

It can be found that the above algorithm is based on the solution of the continuous relaxation of the NIP problem. However, even after continuous relaxation, problem (18) is non-convex. Therefore, the global optimization can not be guaranteed by convex optimization algorithms. Fortunately, we find that this problem can be convexified in some region by a variable transformation defined by

$$\mathbf{y} = \mathbf{t}(\mathbf{x}), \quad (23)$$

with

$$y_1 = t_1(x_1) = x_1^{\frac{1}{3}}, \quad (24)$$

$$y_2 = t_2(x_2) = x_2^{-\frac{1}{4}}. \quad (25)$$

Obviously, the transformation is an one-to-one mapping, so the continuous-relaxation of problem (18) is equivalent to [Li et al., 2005]

$$\begin{aligned} \min_{\mathbf{y}} \quad & E_b = \tilde{f}_0(\mathbf{y}), \\ \text{s.t.} \quad & \tilde{f}_1(\mathbf{y}) \leq 0, \\ & \tilde{f}_2(\mathbf{y}) \leq 0, \\ & \mathbf{lb}_{\mathbf{y}} \leq \mathbf{y} \leq \mathbf{ub}_{\mathbf{y}}, \end{aligned} \quad (26)$$

with

$$\begin{aligned} \tilde{f}_0(\mathbf{y}) &= f_0(\mathbf{t}^{-1}(\mathbf{y})) + \frac{A_3}{L_h}, \\ &= \frac{1}{1 - z(\mathbf{y})} [A_1 g_1(y_1) \tilde{h}(y_2) + \frac{A_2}{y_1^3} + \frac{A_3}{L_h}], \end{aligned} \quad (27)$$

$$\begin{aligned} \tilde{f}_1(\mathbf{y}) &= f_1(\mathbf{t}^{-1}(\mathbf{y})), \\ &= \frac{4}{3} G_d N_f B \sigma^2 g_2(y_1) \tilde{h}(y_2) - P_{max}, \end{aligned} \quad (28)$$

$$\tilde{f}_2(\mathbf{y}) = f_2(\mathbf{t}^{-1}(\mathbf{y})) = z(\mathbf{y}) - 1, \quad (29)$$

$$g_1(y_1) = \frac{(2^{\frac{y_1^3}{2}} - 1)^2}{y_1^3}, \quad (30)$$

$$g_2(y_1) = 2^{y_1^3} - 1, \quad (31)$$

$$\tilde{h}(y_2) = h(t_2^{-1}(y_2)) = \ln \frac{4}{1 - (1 - p_{loss}^{\frac{1}{N_r}}) y_2^4}, \quad (32)$$

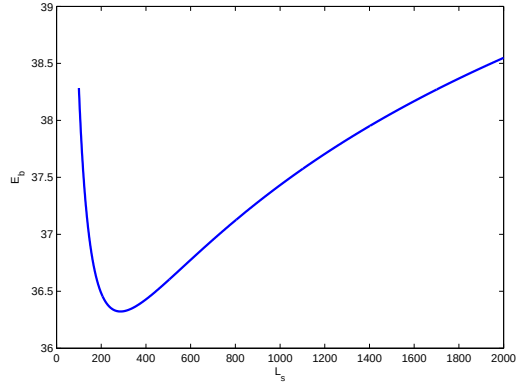
$$z(\mathbf{y}) = L_h \frac{y_2^4}{y_1^3}, \quad (33)$$

where  $\mathbf{lb}_{\mathbf{y}}$  and  $\mathbf{ub}_{\mathbf{y}}$  are the lower and upper bounds of  $\mathbf{y}$ , respectively.

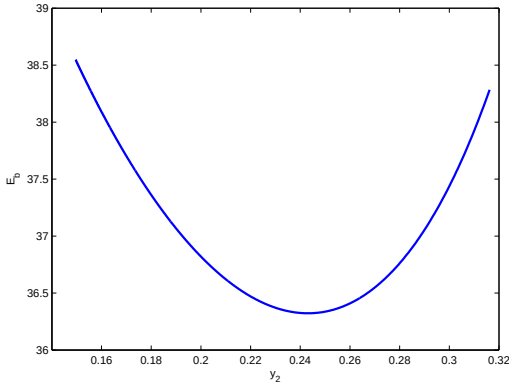
To analyze the convexity of problem (26), two important properties of convex functions will be used: (1) The sum of convex functions are convex; (2) The product of log-convex functions are convex [Bazaraa et al., 2006].

By calculating the second-order derivations or Hessian matrices, it is easy to prove that the functions  $\ln g_1(x)$ ,  $\ln g_2(x)$ ,  $\ln \frac{1}{1-x}$ ,  $\ln \frac{1}{x^3}$  and  $z(\mathbf{y})$  are convex. In addition,  $\ln \frac{1}{1-x}$  is nondecreasing, therefore,  $\ln \frac{1}{1-z(\mathbf{y})}$  is convex. Further, the calculation of  $\frac{d^2 \ln \tilde{h}(x)}{dx^2}$  indicates that  $\tilde{h}(x)$  is convex when  $1 - (1 - p_{loss}^{\frac{1}{N_r}}) x^4 \leq e^{-4}$ , which is equivalent to  $L_s \geq L'_{smin} = \lceil \frac{\ln(1 - p_{loss}^{\frac{1}{N_r}})}{\ln(1 - e^{-4})} \rceil$  in our model, where  $\lceil x \rceil$  denotes the minimum integer larger than or equal to  $x$ . Thus, according to the property (1) and (2),  $\tilde{f}_0(\mathbf{y})$ ,  $\tilde{f}_1(\mathbf{y})$  and  $\tilde{f}_2(\mathbf{y})$  are convex, i.e. problem (26) is convex, in the region of  $[\mathbf{lb}_{\mathbf{y}}, \mathbf{ub}_{\mathbf{y}}]$ , with  $\mathbf{lb}_{\mathbf{y}} = [2^{1/3}, L'_{smax}]$  and  $\mathbf{ub}_{\mathbf{y}} = [+ \infty, L'_{smin}]$ .

In other words, the continuous relaxation of energy minimization problem (16) can be transformed to an equivalent convex problem (26) when  $L_s \in [L'_{smin}, L'_{smax}]$ . So conventional nonlinear convex optimization algorithms, such as successive quadratic programming (SQP) method [Bazaraa et al., 2006], can be adopted to get the continuous optimum point and the branch-and-bound method can be used to search the integer optimum value of  $r_b$  and  $L_s$  in this region. Moreover,  $L'_{smin}$  is very small in general, e.g. when  $p_{loss} = 10^{-4}$ ,  $L'_{smin} = 1$  if  $N_r = 1, 2$  and  $L'_{smin} = 3$  if  $N_r = 3$ . Therefore, the exhaustive search method can be used to minimize the energy consumption for  $L_s \in [L'_{smin}, L'_{smax}]$ . Then a global optimization can be obtained by comparing the minimization results in the two regions.



(a) Original:  $E_b$  v.s.  $L_s$



(b) Transformed:  $E_b$  v.s.  $y_2$

Fig. 1. Objective function with  $r_b = 2$  at  $d = 50m$

#### IV. NUMERICAL RESULTS

For a numerical example, the following values are configured according to [Cui et al., 2005]. The system bandwidth  $B = 10kHz$ , circuit power  $P_c = 210mw$ , transient power  $P_{tr} = 100mw$ , transient time  $T_{tr} = 5\mu s$ ,  $G_1 = 30dB$ , path loss factor  $n = 3.5$ ,  $\sigma^2 = -174dBm/Hz$ ,  $N_f = 10dB$ ,  $\eta = 0.35$ . The overhead bit number is fixed to  $L_h = 32bits$ . The maximum transmit power  $P_{max} = 250mw$  and maximum average delay  $\tau_{max} = 500ms$ . The target packet loss rate is  $p_{loss} = 10^{-4}$  and the maximum retransmission number is  $N_r = 2$ .

Fig. 1(a) and Fig. 1(b) show the objective functions at  $d = 50m$  before and after the variable transformation with  $r_b = 2$  and variable packet size. It gives us an example to show that the non-convex objective function can be convexified efficiently by the variable transformation. Fig. 2 and Fig. 3 provides the optimum values for the modulation order  $r_b$  and packet size  $L = L_s r_b$ , respectively, with different communication distance  $d$ , where the SQP algorithm and branch-and-bound method with  $\epsilon = 10^{-4}$  is used to solve the optimization problem.

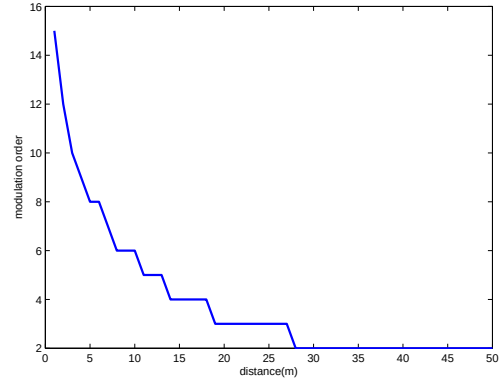


Fig. 2. Optimum modulation order v.s. Communication distance

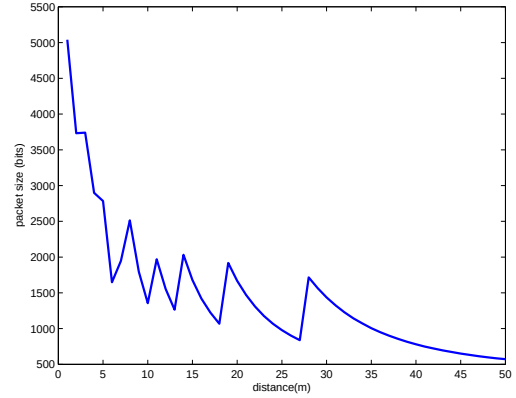


Fig. 3. Optimum packet size v.s. Communication distance

Fig. 4 compares the energy consumption of our joint optimization system with that of the system with fixed packet size and optimized modulation order as proposed in [Cui et al., 2005]. The comparison is measured by the use of energy saving percent defined as

$$\beta = \frac{E_f - E_{opt}}{E_f} \times 100\%, \quad (34)$$

where  $E_{opt}$  and  $E_f$  are the energy consumptions with joint optimization and modulation order optimization only, respectively.

Observing Fig. 4, one finds that the joint optimization saves about 6% energy at  $d = 50m$  in comparison with the fixed packet size  $L = 4000$  and save about 4% energy at  $d = 1m$  in comparison with  $L = 500$ . The experiment results indicate that the joint optimization can make the sensor nodes work in a more energy efficient way.

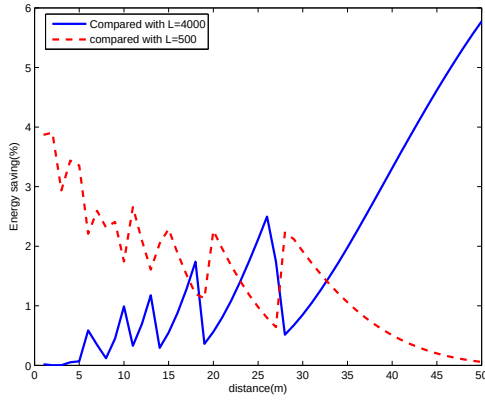


Fig. 4. Energy saving by joint optimization (in comparison with fixed packet size and optimized modulation order)

## V. CONCLUSION

In this paper, we derived an energy consumption model for WSN considering the parameters in the physical and data link layers. Based on the model, a NIP problem is formulated with the modulation order and packet size as variables and solved by the use of continuous-relaxation based branch-and-bound method, SQP algorithm and variable transformation convexification. It was found that the proposed joint optimization can improve the energy efficiency of WSN.

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