

Aspects of Multistatic Adaptive Pulse Compression

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ABSTRACT

It is well known that two or more radars operating in close proximity, at the same time, and in the same spectrum can severely interfere with one another despite the use of low cross-correlation waveforms. Recently, an approach was proposed called Multistatic Adaptive Pulse Compression (MAPC) based on a Minimum Mean-Square Error (MMSE) formulation which has been shown to suppress both range sidelobes and cross-correlation ambiguities to the level of the noise. This paper examines the performance of the MAPC approach under the effects of Doppler mismatch. It is demonstrated that for relatively high Doppler the MAPC approach experiences some Doppler-induced sidelobes around large targets yet is still substantially superior to the standard matched filter.

1. INTRODUCTION

The increasing demand for spectrum usage rights by the communications industry coupled with the requirement for wider instantaneous bandwidths for radar applications is creating an ever-growing need for more efficient use of the Radio Frequency (RF) spectrum. This is compounded by the fact that future sensing technology (of which radar will play a significant role) has been envisioned as taking the form of sensor networks comprised of several interacting sensors [1]. The inevitable result will be proximate radars which overlap (at least partially) their respective operating frequency bands and thereby become sources of mutual interference. It therefore becomes necessary to explore means of enabling concurrent, shared-spectrum multistatic radar operation in order to mitigate mutual interference, as well as to exploit the potential benefits that such an arrangement would provide such as aspect angle diversity and greater area coverage with shorter revisit times.

Typical multistatic radar configurations are comprised of only a single transmitter and multiple passive receivers [2,3] or employ sufficient frequency separation between the transmitters [4]. This is because two or more radars operating in close proximity, at the same time, and in the same spectrum are known to interfere with one another – often to the point of achieving complete RF fratricide. Essentially, the underlying problem is that no set of waveforms can guarantee orthogonal return signals from a complex scattering environment. Hence, standard pulse compression matched filtering [5] produces significant waveform cross-correlations which create target ambiguities at each multistatic receiver thereby inherently

limiting radar detection sensitivity. However, similar to what is done in multi-user communications [6], accurate joint estimation of multiple radar range profiles at a given radar receiver can be accomplished by iteratively canceling the interference the received return signals cause to one another. Specifically, the Multistatic Adaptive Pulse Compression (MAPC) algorithm [7,8] based on a Minimum Mean-Square Error (MMSE) formulation [9] was proposed recently as an alternative to the set of matched filters and has been shown to be capable of significantly suppressing both pulse compression range sidelobes and cross-correlation ambiguities.

This paper analyzes the MAPC algorithm for shared-spectrum radar by examining its performance in the presence of significant motion-induced target Doppler shift. The MAPC algorithm accounts for the presence of multiple, known transmitted waveforms that share the same spectrum and thereby jointly pulse-compresses the received signals by exploiting the known normalized cross-correlations and auto-correlations among the various multistatic waveforms. It is shown that while the standard matched filters are incapable of discerning targets simultaneously identified by different multistatic waveforms, the MAPC algorithm is able to estimate each individual range profile to the level of the noise floor. This paper generalizes previous results by including the effects of Doppler shift. Similar to the original monostatic Adaptive Pulse Compression (APC) algorithm [10-13], the MAPC algorithm is found to degrade gracefully with increasing Doppler and is rather robust even to relatively high target Dopplers.

The MAPC algorithm is based upon Reiterative Minimum Mean-Square Error (RMMSE) estimation originally developed in [10] where it was shown to be superior to both the matched filter [7] and the optimum Least-Squares mismatched filter [14]. The fundamental concept of RMMSE as it applies to monostatic pulse compression is to employ a different pulse compression receive filter for each individual range cell according to the relative locations and powers of targets in the surrounding range cells. As such, the pulse compression filter for a given range cell can place nulls at range cell offsets pertaining to large targets which would otherwise produce range sidelobes. Of course, this requires an initial estimate of the various range cell values which can be obtained via the standard matched filter. The application of the RMMSE concept to the multistatic scenario is a generalization whereby a different pulse compression filter is generated for each range cell for each received multistatic waveform. The RMMSE

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formulation thereby enables the placing of nulls to mitigate the interference from target returns received via the same waveform (*i.e.* self-interference) as well as the interference from target returns received via other waveforms as mutual interference.

It has been previously shown [11,12] that the monostatic APC algorithm is quite robust to target Doppler in that the Doppler-induced range sidelobes for APC remain well below that of the matched filter. The remainder of this paper presents the development of the MAPC algorithm followed by performance analysis of MAPC when subjected to target Doppler mismatch.

2. MULTISTATIC ADAPTIVE PULSE COMPRESSION

Consider K radars (designated individually as radar k , for $k = 1, 2, \dots, K$) that operate concurrently in the same spectrum each with a different transmitted waveform. We denote the discrete-time version of the k^{th} radar's transmit waveform as the N -length vector $\mathbf{s}_k$, and $\mathbf{r}_k = [1 e^{j\theta_k} \dots e^{j(M-1)\theta_k}]^T$ as the spatial steering vector corresponding to the angle-of-arrival (AOA) of the k^{th} radar return signal received at radar 1 (it is assumed for simplicity that the receive antenna is an M -length uniform linear array with digital beamforming capability). Note that without loss of generality the same processing is to be performed at each of the radars; thus for this development we consider only the 1st radar. It is assumed that the range profiles illuminated lie in the collective far field of the group of radars (for instance a cluster of ground-looking space-based radars) so that direct path interference between the radars can be mitigated (*e.g.* using spatial nulling). The ℓ^{th} time (range) sample of the K received radar return signals on the m^{th} antenna element (in general MAPC can be applied to any array geometry) is defined as

$$y_m(\ell) = \sum_{k=1}^K \mathbf{x}_k^T(\ell) \mathbf{s}_k e^{j(m-1)\theta_k} + v_m(\ell) \quad (1)$$

for $\ell = 0, \dots, L+N-2$ the indices of the received signal samples of interest (used to estimate the L -length processing windows of the respective range profiles) where $\mathbf{x}_k(\ell) = [x_k(\ell) x_k(\ell-1) \dots x_k(\ell-N+1)]^T$ is the N -length vector of discrete range profile samples at delay ℓ with which the discrete transmitted waveform $\mathbf{s}_k$ convolves, $v(\ell)$ is additive noise, and $(\bullet)^T$ is the transpose operation. The received radar return signals across the M antenna elements for the ℓ^{th} time sample comprise the vector $\mathbf{y}(\ell) = [y_0(\ell) y_1(\ell) \dots y_{M-1}(\ell)]^T$.

Let each antenna array element possess its own receive channel (frequency down-conversion, A/D converter, etc...) thus enabling digital beamforming. A separate beamformer is applied for each of the K received signals across the M outputs of the antenna array. Note that beamforming is not necessarily required for the MAPC algorithm but is employed to garner more separation between the received signals. To utilize beamforming it is assumed that each radar possesses knowledge of the respective AOAs of the received radar return

signals. The ℓ^{th} time (range) sample of the k^{th} received radar return signal after beamforming (and normalization) is denoted as

$$z_k(\ell) = M^{-1} \mathbf{r}_k^H \mathbf{y}(\ell) = \sum_{i=1}^K \eta_{ki} \mathbf{x}_i^T(\ell) \mathbf{s}_i + u_k(\ell), \quad (2)$$

where $u_k(\ell) = M^{-1} \mathbf{r}_k^H [v_0(\ell) v_1(\ell) \dots v_{M-1}(\ell)]^T$ is additive noise after normalized beamforming, $\eta_{ki} = M^{-1} \mathbf{r}_k^H \mathbf{r}_i$ is the normalized correlation between the k^{th} and i^{th} spatial steering vectors, and $(\bullet)^H$ is the complex conjugate transpose, or Hermitian, operation. By collecting N range samples of $z_k(\ell)$, the resulting signal model can be expressed as

$$\mathbf{z}_k(\ell) = \sum_{i=1}^K \eta_{ki} \mathbf{X}_i^T(\ell) \mathbf{s}_i + \mathbf{u}_k(\ell) \quad (3)$$

where $\mathbf{z}_k(\ell) = [z_k(\ell) z_k(\ell+1) \dots z_k(\ell+N-1)]^T$ is the received signal vector corresponding to the k^{th} waveform after beamforming, $\mathbf{u}_k(\ell) = [u_k(\ell) u_k(\ell+1) \dots u_k(\ell+N-1)]^T$ is a vector of additive noise after beamforming, and $\mathbf{X}_k(\ell) = [\mathbf{x}_k(\ell) \mathbf{x}_k(\ell+1) \dots \mathbf{x}_k(\ell+N-1)]$ is an $N \times N$ matrix comprised of N -length sample-shifted snapshots (in the columns) of the k^{th} range profile.

After beamforming, the standard matched filtering operation [5] is the convolution of the K beamformed received radar return signals with the time-reversed complex conjugates of the respective transmitted waveforms in order to obtain the K range profile estimates. The outputs from matched filtering can be expressed in the digital domain as

$$\hat{x}_{MF,k}(\ell) = \mathbf{s}_k^H \mathbf{z}_k(\ell) \quad (4)$$

for $k = 1, 2, \dots, K$ and $\ell = 0, 1, \dots, L-1$. However, since matched filtering assumes a point target from a single received signal in noise, it is expected that the matched filter will perform poorly in the multistatic scenario, especially for the denser target environments.

To accommodate for multiple, simultaneously received signals in the same spectrum, the Multistatic Adaptive Pulse Compression (MAPC) algorithm replaces the matched filter $\mathbf{s}_k$ in (4) with the multistatic RMMSE-based filter $\mathbf{w}_k(\ell)$ [10]-[13] which, for the k^{th} radar's waveform and ℓ^{th} range gate, minimizes the MMSE cost function [10]

$$J_k(\ell) = E \left[\left| x_k(\ell) - \mathbf{w}_k^H(\ell) \mathbf{z}_k(\ell) \right|^2 \right] \quad (5)$$

where $E[\bullet]$ denotes expectation. Assuming no correlation in range or among the K range profiles, the solution to (5) takes the form

$$\mathbf{w}_k(\ell) = \hat{\rho}_k(\ell) \left(\sum_{i=1}^K |\eta_{ki}|^2 \mathbf{C}_i(\ell) + \mathbf{R}_k \right)^{-1} \mathbf{s}_k \quad (6)$$

for each k , where $\hat{\rho}_k(\ell) = |\hat{x}_k(\ell)|^2$ is the estimated power of $x_k(\ell)$ and $\mathbf{R}_k = E[\mathbf{u}_k(\ell) \mathbf{u}_k^H(\ell)]$ is the temporal (range) noise

covariance matrix after beamforming in the direction of the k^{th} AOA. The matrix $\mathbf{C}_i(\ell)$ is defined as

$$\mathbf{C}_i(\ell) = \sum_{n=-N+1}^{N-1} \hat{\rho}_i(\ell+n) \mathbf{s}_{i,n} \mathbf{s}_{i,n}^H, \quad (7)$$

where $\mathbf{s}_{i,n}$ contains the elements of the waveform $\mathbf{s}_i$ shifted by n samples and the remainder zero-filled. For example, for $n=2$ we have $\mathbf{s}_{i,2} = [0 \ 0 \ s_i(0) \ \dots \ s_i(N-3)]^T$ and $n=-2$ yields $\mathbf{s}_{i,-2} = [s_i(2) \ \dots \ s_i(N-1) \ 0 \ 0]^T$.

To employ (6) and (7) requires initial estimates of the K range profiles as well as knowledge of the noise covariance matrices $\mathbf{R}_k$, $k=1,2,\dots,K$. Assuming the noise covariance is white Gaussian, $\mathbf{R}_k$ simplifies to $\sigma_v^2 \mathbf{I}$, where $\mathbf{I}$ is the $N \times N$ identity matrix and σ_v^2 is the noise power which can be assumed known since internal thermal noise dominates the external noise at microwave frequencies (where most radars operate) [6]. The initial estimates of the K range profiles can be obtained either by using standard matched filtering or by initializing the power estimates of all of the KL range cells to be equal and assuming the noise is negligible initially. In the latter case, (6) reduces to

$$\tilde{\mathbf{w}}_k = \left(\sum_{i=1}^K |\eta_{ki}|^2 \tilde{\mathbf{C}}_i \right)^{-1} \mathbf{s}_k \quad (8)$$

for $k=1,2,\dots,K$, where the matrix $\tilde{\mathbf{C}}_i$ is defined as

$$\tilde{\mathbf{C}}_i = \sum_{n=-N+1}^{N-1} \mathbf{s}_{i,n} \mathbf{s}_{i,n}^H. \quad (9)$$

The initialization MMSE filters from (8) are range invariant and can therefore be pre-computed. After (8) is applied, as in (4) with $\mathbf{s}_k$ replaced by $\tilde{\mathbf{w}}_k$, and the initial KL range cell power estimates have been obtained, (6) is subsequently used to estimate the refined receive filters which are then re-applied to the beamformed received signals and the range cell complex amplitudes are estimated again. The refined receive filters are better able to mitigate the masking effects caused by waveform cross-correlation and range sidelobes due to the fact that they are estimated based upon some *a priori* knowledge regarding the relative locations of larger targets, which were obtained by the previous stage. The re-estimation of the individual receive filters and range cells is repeated for a pre-determined number of stages. Hence, as long as sufficient adaptive degrees of freedom are available, the MAPC filters at each successive stage will further refine the estimate of the range profiles until reaching the noise floor.

The MAPC algorithm performance degrades gracefully when the spatial steering vectors for individual received signals become more closely aligned which effectively reduces the adaptive degrees of freedom. The MAPC algorithm still surpasses the performance of the standard matched filters as they represent the non-adaptive solution. Finally, the structure of MAPC enables fast implementation via the matrix inversion lemma through a straightforward extension of the method described in [12].

3. NUMERICAL STABILITY

The matrix $\left(\sum_{i=1}^K |\eta_{ki}|^2 \mathbf{C}_i(\ell) + \mathbf{R} \right)$ could potentially become ill-conditioned in the vicinity of very large targets or when small range cell estimates approach zero. However, the same heuristic approach described in [12] will also work for the multistatic case which is to replace $\hat{\rho}_k(\ell) = |\hat{x}_k(\ell)|^2$ with $\hat{\rho}_k(\ell) = |\hat{x}_k(\ell)|^\alpha$, (under the white noise assumption) replacing the noise power σ_v^2 in (6) with σ_v^α , and replacing $|\eta_{ki}|^2$ with $|\eta_{ki}|^\alpha$ for $0 \leq \alpha \leq 2$. For the case of large SNR targets, using $\alpha < 2$ reduces the effective SNR dynamic range and thereby alleviates the possibility of ill-conditioning. It has been found based upon extensive simulation that use of values of $1.1 \leq \alpha \leq 1.7$ with 2 to 4 stages of the MAPC algorithm (excluding the initialization stage) tends to yield the best results. Furthermore, α should be set at the high end (near 1.7) for the first stage to quickly drive down the sidelobes from large SNR targets and then decrease (to near 1.1 at the final stage). For the initialization stage using (8) α can be set to 2. It is a topic of future research to determine if optimal values of α can be found as a function of the surrounding range cell estimates.

An additional heuristic approach that can be used to alleviate ill-conditioning is to set a lower bound upon the magnitudes of the range cell estimates. This is done so that slightly larger values of α can be used to drive down the sidelobes from large targets more quickly without driving smaller range cell estimates to zero.

4. SIMULATION RESULTS

As in [7] we consider the simultaneous reception of two random-phase waveforms of length $N = 30$ received at angles of -20° and $+10^\circ$ off boresight of an 11-element uniform linear array. We shall examine three cases involving dense target scenarios. The first two cases apply MAPC with beamforming to compare performance with and without target Doppler while the third case ascertains the performance of MAPC for moving targets when beamforming is not employed. In all three cases four total stages of the MAPC algorithm are employed (including the initialization stage) with the parameter α set as 2, 1.7, 1.4, and 1.3 for the initialization stage and subsequent three adaptive stages. The autocorrelations of the random polyphase waveforms and their cross-correlation (neglecting spatial beamforming suppression) are depicted in Figs. 1-3. The waveforms both have normalized peak sidelobe levels of -11 dB and their cross-correlation peaks at -9 dB.

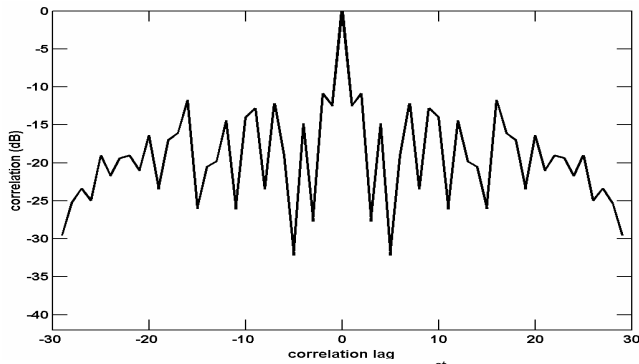


Fig. 1. Autocorrelation of the 1st waveform

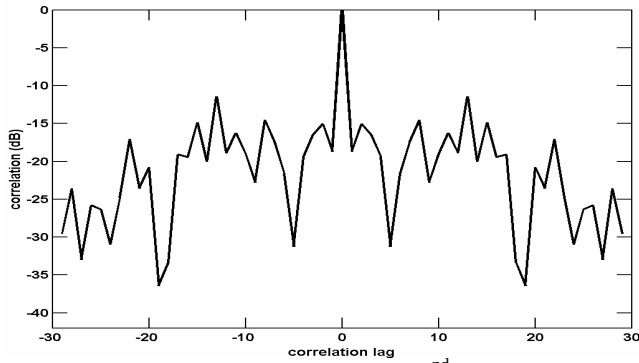


Fig. 2. Autocorrelation of the 2nd waveform

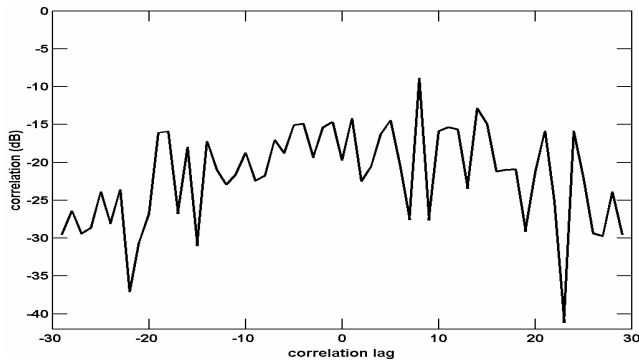


Fig. 3. Cross-correlation between waveforms

For the first case we consider range profiles in which the targets possess no Doppler. As is presented in Fig. 4, the ground truth of the respective range profiles (represented in black) is comprised of many closely spaced targets with highly disparate power levels and -60 dB noise (with respect to the largest target power). As expected, even after beamforming the matched filters (in blue) perform poorly due to the combined effects of range sidelobes and waveform cross-correlation. For the given scenario the MAPC algorithm suppresses both the range sidelobes and the cross-correlation interference to the level of the noise floor with the MAPC range profile estimates closely overlapping the ground truth. In terms of overall mean-square error (MSE), the (normalized) matched filters yield an MSE value of -12 dB while the MAPC algorithm achieves an MSE of -57 dB, an improvement of 45 dB.

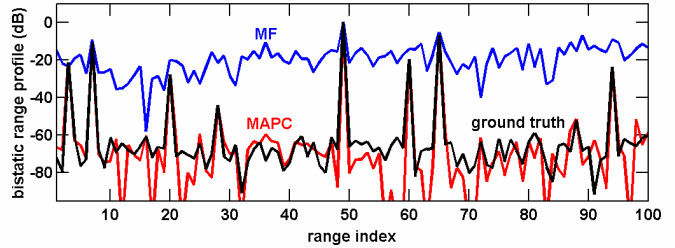
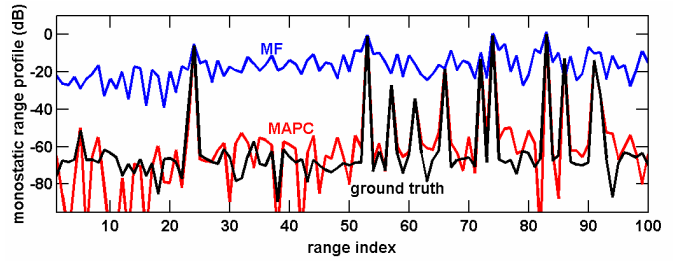


Fig. 4. Results for multistatic radar reception with no Doppler using spatial beamforming

To determine the Doppler tolerance of the MAPC algorithm we examine the same target scenario except now each target in the two range profiles possesses a randomly assigned Doppler shift (over the length of the waveform) chosen uniformly over the interval of $\pm 6^\circ$. For example, a 6° phase-shift would result for a Mach-2 target illuminated by a 1 μ s S-band pulse. As illustrated in Fig. 5, the MAPC algorithm experiences some degradation as a result of Doppler-induced sidelobes. Compared with Fig. 4, the Doppler-induced sidelobes levels are most noticeable around range cell 49 of the bistatic range profile for which the Doppler shift is $+5^\circ$ over the length of the waveform. In this case the MSE for matched filter remains the same at -12 dB while the MSE for MAPC increases to -44 dB. However, MAPC is still superior with a 32 dB lower MSE.

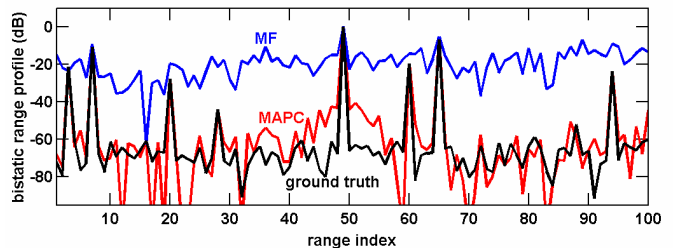
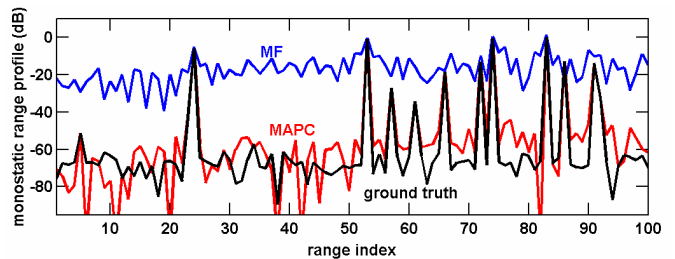


Fig. 5. Results for multistatic radar reception with Doppler using spatial beamforming

Finally, we examine the same target/Doppler scenario without the use of beamforming such as might occur if both

radars illuminated the same range profile from different aspect angles. Without beamforming, the spatial steering vector correlation becomes $\eta_{ki} \Rightarrow 1$ such that more adaptive degrees-of-freedom are needed to null nearby large targets and the mutual interference. As is shown in Fig. 6, the matched filter performs essentially the same as before while MAPC further degrades yet is still significantly better than the matched filter. The MSE for matched filter is -10 dB while the MSE for MAPC increases to -36 dB.

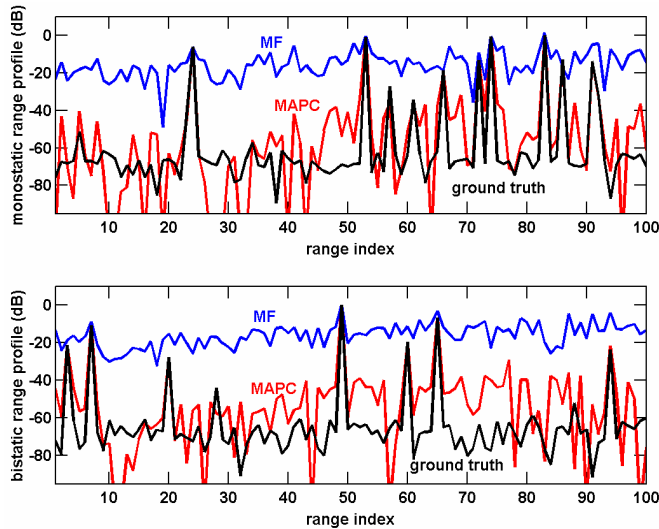


Fig. 6. Results for multistatic radar reception with Doppler and no spatial beamforming

CONCLUSIONS

In this work, Multistatic Adaptive Pulse Compression (MAPC) has been proposed as an adaptive alternative to the standard matched filter in order to enable shared-spectrum multistatic radar thereby potentially facilitating aspect angle diversity and greater area coverage with shorter revisit times. Based upon a Minimum Mean-Square Error (MMSE) formulation, the MAPC algorithm operating at each of the multistatic radar receivers jointly pulse compresses the multiple received radar return signals that result from the simultaneous transmission of different waveforms from each of the radars. It has been shown that, even in the presence of significant Doppler mismatch due to target motion, the MAPC algorithm is able to reduce the Mean-Square Error (MSE) of the estimates of the respective range profiles by several orders of magnitude compared to the matched filter which is itself fundamentally limited by the range and cross-correlation sidelobes of the transmitted waveforms. The reduction in MSE over the matched filter translates directly into substantially greater detection and identification performance for the MAPC algorithm in the shared-spectrum multistatic radar scenario.

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