BIPEDAL WALKING

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ABSTRACT

Mobility is the essence of warfare. Animal-like locomotion could provide leap-ahead capability to modern robotic systems. Legged vehicles are not limited to improved roads, but can operate in complex and urban terrain with ease. In this paper, we develop a mathematical argument for walking, and build a simple prototype to demonstrate walking behavior.

Fig. 1 Overall design

1. INTRODUCTION

It may seem preposterous, but under certain conditions, walking is faster than rolling. What may seem like a discrete difference between these two modes of transport is actually a smooth transition from walking to rolling as energy increases. In this paper, we examine the model of walking called the *rimless wheel*[1]; analyze its progression from a walking to a rolling state; and develop a bipedal mechanism to apply these results.

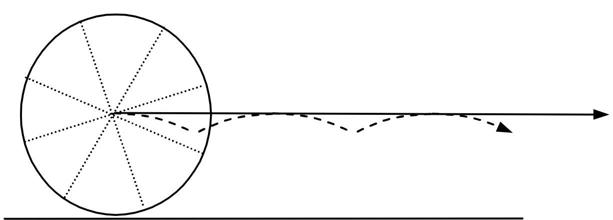

Fig. 2: Walking vs. rolling motion

2. TRANSPORTATION

When we think of transportation on flat terrain, we generally think of wheels, but let's back up for a moment, and examine our assumptions. What is transportation? A useful model might be: The problem of moving a mass *m* from point A to point B in time *t* expending energy *E*:

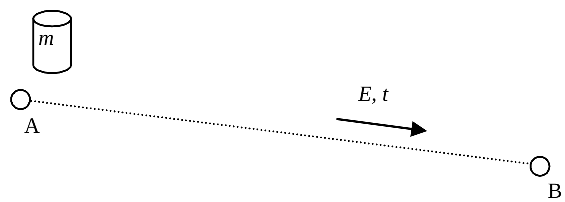

Fig. 3: The transportation problem

The point of transportation is to move *m* from A to B. How fast I do it and how much energy I expend while doing is the essence of the problem. If energy *E* is cheap, and time *t* is of the essence, my strategy will look different from the case where *E* is precious and we've got plenty of time *t*. Let's start with the case where *E* is not only precious, it must be conserved[2].

For a given E, the strategy becomes finding the least time path. What do I mean by least time path? Consider this conservative system: Let's say we wish to construct a ramp such that a mass rolls (or slides without friction) from rest starting at point A to point B in the least amount of time. The temptation is to draw a straight line from point A to point B, since obviously the shortest geometric distance between two points is a line. The problem with this temptation; of course, is that it's wrong.[3]

[1] Imagine a wheel where the spokes are rigid and massless. The intermittent motion is that of an inverted pendulum.

[2] A conservative system is one in which no dissipation occurs. In other words, no energy leaves the system, but no energy comes in either-it is conserved.

[3] The solution to the brachistochrone problem is a famous problem in mechanics, for the analysis of it led to the formulation of the calculus of variations, see Goldstein

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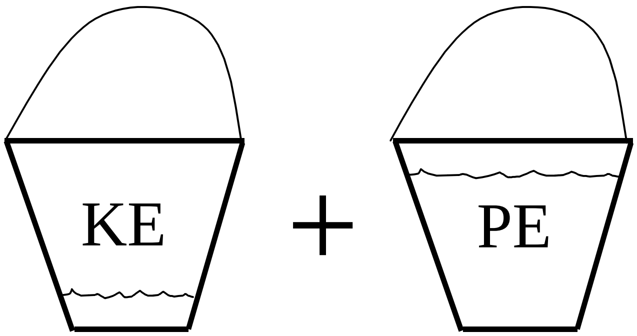

Figure 4: The energy buckets

Let’s look at this problem another way. Suppose you had a given amount of energy, *E*, distributed between two buckets. The Kintetic Energy (KE) bucket of the total energy, *E*, stored in the Potential Energy (PE) bucket. If my initial energy (velocity) is large, the KE bucket becomes larger than the PE bucket, and the tradeoff becomes less attractive because we are deviating from the straight line path[4]. This might correspond to the case of a light wheel rolling slowly at a constant axel height. By the principle of conservation of energy: energy can neither be created nor destroyed, but we can exchange energy from one type to another, just so long as the total amount remains constant. When constructing the least time path, we quickly trade some of our potential energy stored in the height of the axel in exchange for kinetic energy to pick up speed.

3. RIMLESS WHEEL

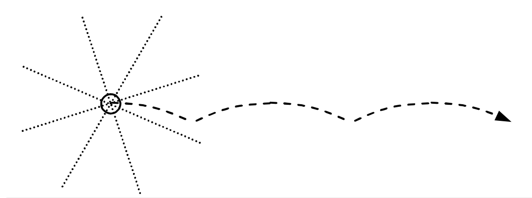

Fig. 4: Rimless wheel motion

To accomplish this energy trade, remove the rim from the wheel so that you are left with only the spokes. Now the axel is free to fall forward until it hits the next spoke. At this point in the traverse, we have to trade back kinetic energy to return to the initial height, but we do that only after we have reaped the time savings of the kinetic energy trade. As alluded to above, at larger energies, this tradeoff becomes less desirable, and the spokes gradually become closer and closer together until they become a wheel. In this analysis, we assume a fully regenerative spoke collision[5].

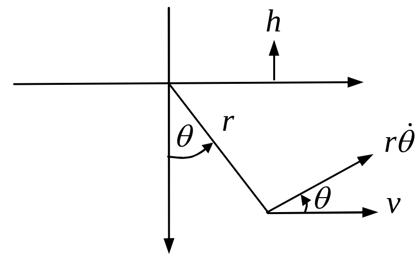

Fig. 5: Forward velocity

The total energy:

$$E = KE + PE \tag{1}$$
$$= \frac{1}{2}mv^2 + mgh$$
$$= \frac{1}{2}ml^2\dot{\theta}^2 - mgr\cos\theta$$

Divide through by *m* and set $g = r = 1$

$$E = \frac{1}{2}\dot{\theta}^2 - \cos\theta \tag{2}$$

and

$$\dot{\theta} = -\sqrt{2}\sqrt{E + \cos\theta} \tag{3}$$

where we took negative square root for forward walking

The velocity we are interested in is in the horizontal direction:

$$v = \dot{\theta}\cos\theta \tag{4}$$

and substituting (3) the forward velocity is

$$v = -\sqrt{2}\sqrt{E + \cos\theta}\,\cos\theta \tag{5}$$

We’ve given heuristic arguments that there exists a maximum energy where it makes sense to walk. Can we pinpoint the exact energy transition? If we take the derivative of *v* with respect to θ, we see that

$$\frac{dv}{d\theta} = \frac{\sqrt{2}}{2}\cos\theta\frac{\sin\theta}{\sqrt{E + \cos\theta}} + \sqrt{2}\sin\theta\sqrt{E + \cos\theta}$$

Simplifying:

$$\frac{dv}{d\theta} = \frac{\sqrt{2}}{2}\sin\theta\left(\frac{2E + 3\cos\theta}{\sqrt{E + \cos\theta}}\right) \tag{6}$$

You can see that at $\theta = \pi$ there is always extremum, because $\sin\pi = 0$. We can now examine the sign of $\frac{dv}{d\theta}$ to test if *v* has a minimum or a maximum at $\theta = \pi$.

[4] A large control force will have the same effect, heavily punishing deviations from the straight line path.

[5] This analysis is explained more fully in a previous paper - Pendulum Walker

Recall that if $\frac{dv}{d\theta}$ changes from + to – at $\theta = \pi$, then v has a local maximum there. Recall that $\sin\theta$ changes from + to – through $\theta = \pi$. So what happens there depends on the value of E in the numerator and denominator of the term in brackets. For the rimless wheel to be above the floor: $E > 1$ so that nothing strange happens under the square root. Now we are left with the numerator of (6): If the numerator, $2E + \cos\theta$, is positive then the sign of $\frac{dv}{d\theta}$ changes from + to – and we have a local maximum there. The condition for this local maximum is:

$$2E + 3\cos\pi > 0$$

simplifying:

$$E > \frac{3}{2} \quad (7)$$

above E=1.5, it makes no sense to take a step, so that for energies:

$$1 < E < \frac{3}{2} \quad (8)$$

walking is faster than rolling.

4. SIMPLE HUMAN MODEL

To determine the extent to which the above theory might be applied to actual physical motion, we built a simple bipedal simulator. The desire was to make a high center of gravity vehicle with pendulum-like motion. We split the development of the physical model into two stages. Stage one, which we have completed, was the initial design and construction of the device. Stage two, which we have now begun, is the comprehensive testing and algorithm refinement as well as result documentation..

We began by synthesizing the human gait from three distinct motions. First, the "powered phase" of the step, where energy is added to the system.

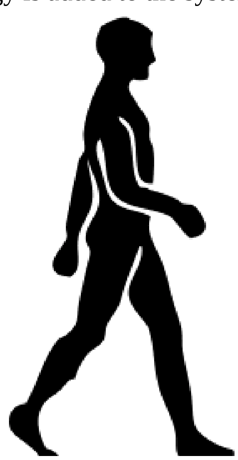

Fig. 6: Phase I - powered phase

Second, the muscles relax, allowing the leg to fall forward freely, carried in the direction of the step by momentum. This important phase is the phase of energy conservation as explained in section 3.

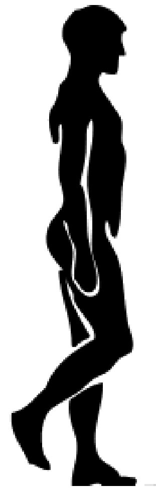

Fig. 7: Phase II - free fall phase

Finally, the swing foot impacts the ground, catches the body, thus halting forward motion. This now planted foot acts as a foundation off which the step cycle may be performed again with the opposite foot.

Fig. 8: Phase III - brake phase

5. MECHANICAL MODEL

Key to the mechanical design was the development of a three state clutch[6] located at the base of each leg. This clutch allows us to effectively simulate the three stages of human bipedal motion: Powered, Freefall, and Braked. Power is fed to the clutch through a drive shaft connected to a high torque electric motor located

[6] The motion of the clutch mechanism and its relation to the overall movement of the device was closely studied. The resulting animation of movement may be viewed as a video file upon request to the authors.

near the top of the leg. The slip clutch allows torque to be transmitted in one direction when the drive shaft turns clockwise, slip forward with minimal friction when the drive shaft is stopped, and engage the brake when the drive shaft turns counter-clockwise. This is accomplished through the use of two oppositely positioned needle roller bearings that only transmit torque in one direction. When torque is applied in the clockwise direction by the drive shaft, needle bearing A (See Fig. 9) rolls freely and needle bearing B locks and transmits torque to the wheels. When the drive shaft is stopped, both needle bearings roll freely, allowing the wheels to spin. Finally, when torque is applied in the counter-clockwise direction by the drive shaft, needle bearing A locks and engages the cam break.

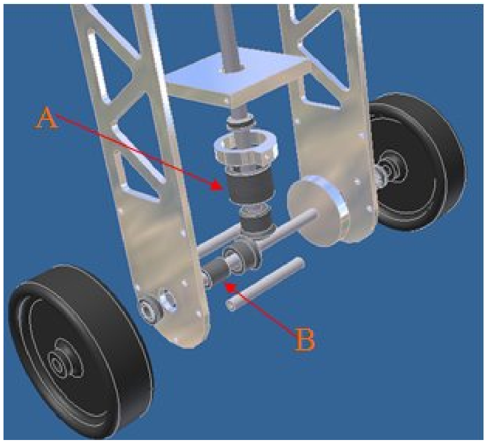

Fig. 9: Clutch explanation

CONCLUSIONS

We have explained the basic mechanics of walking,. We have shown that walking is faster than rolling when it acts as a conservative system. We have designed a device to demonstrate these concepts. Mobility is absolutely fundamental to the kind of asymmetric warfare we are faced with. We feel that legged robotics provides the kind of capability we as an Army will be looking for in the near future.

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