Quantitative Biofractual Feedback Parts I-III

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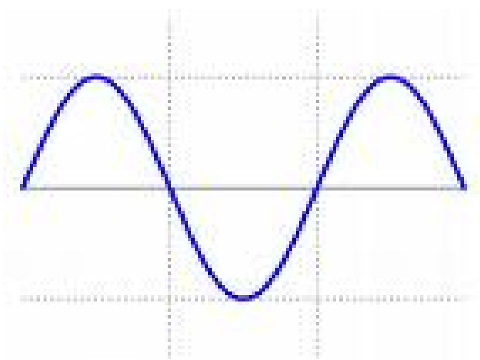
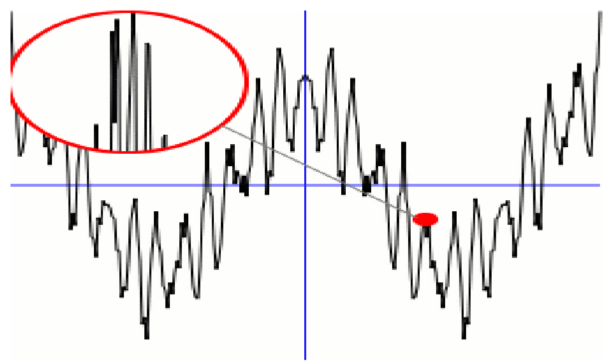
Overall Summary of Parts I, II, and III

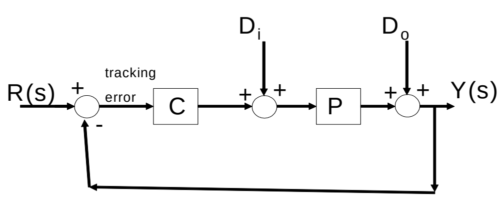
Part I: Fractional Dimension (Fractals, Bioinspired, Intelligent C.)

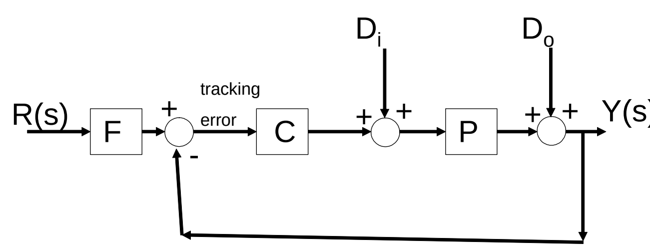
sine wave C^{∞} versus Weierstrass Function C^{0}





Part II: Quantitative Feedback Theory





Part III: A Common Problem – ***Diffusion Equation***

(a) Solve the classical way.

(b) Solve using Laplace Transforms.

(c) Solve using Fractional Calculus.

(d) Examine Robustness via Quantitative Feedback Theory.

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Quantitative Biofractual Feedback Part I

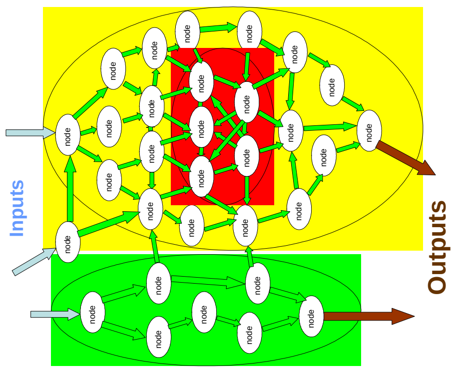


Figure 3 – The Original Network-Centric Distributed System

. We are now living in a world that is complex, distributed, but may be highly vulnerable.

. A better understanding of performance, and vulnerability of complex, distributed systems is required. How should we allocate resources for protection?

<u>The Part I talk will have four main components:</u>

(A) Pose the problem of performance and vulnerability in complex and distributed networks.

(B) Provide background material on some pertinent areas.

(C) Using Computational Intelligent methods, solve a related problem. This will be a “brute force” approach.

(D) Finally, hypothesize some theoretical approaches.

Part 1-A- Pose the Problem:

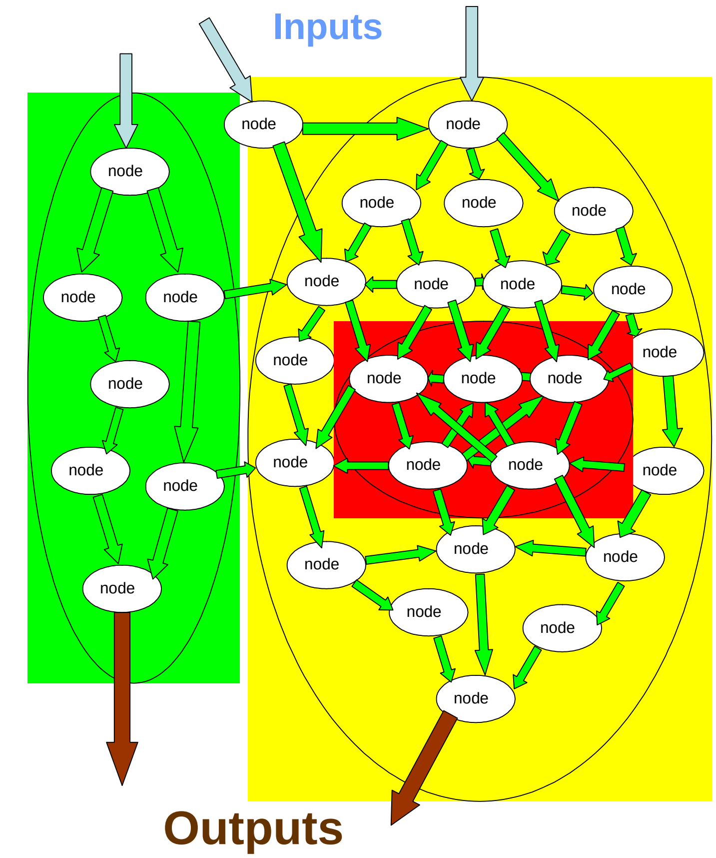
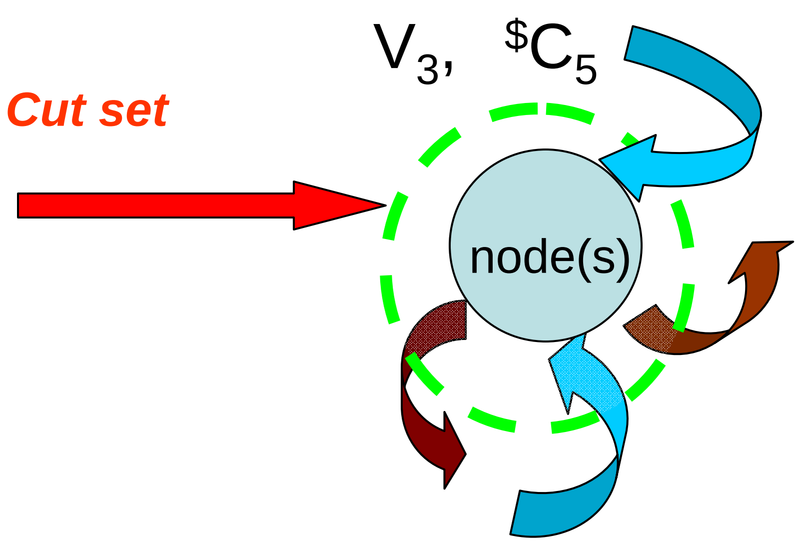


Figure 3 – The Original Network-Centric Distributed System

Performance: Rate of flow through the network.

Vulnerability: Sensitivity of performance to attack of node.



(absolute or relative objective measures of V_i, $^{\$}C_j$)

Part 1-A- Pose the Problem:

Some examples of important networks:

(1) Power grids, railroad tracks,

financial systems, etc..

(2) Flow of people, water, food, medicine.

(3) Communication systems.

(4) Information networks (Internet),

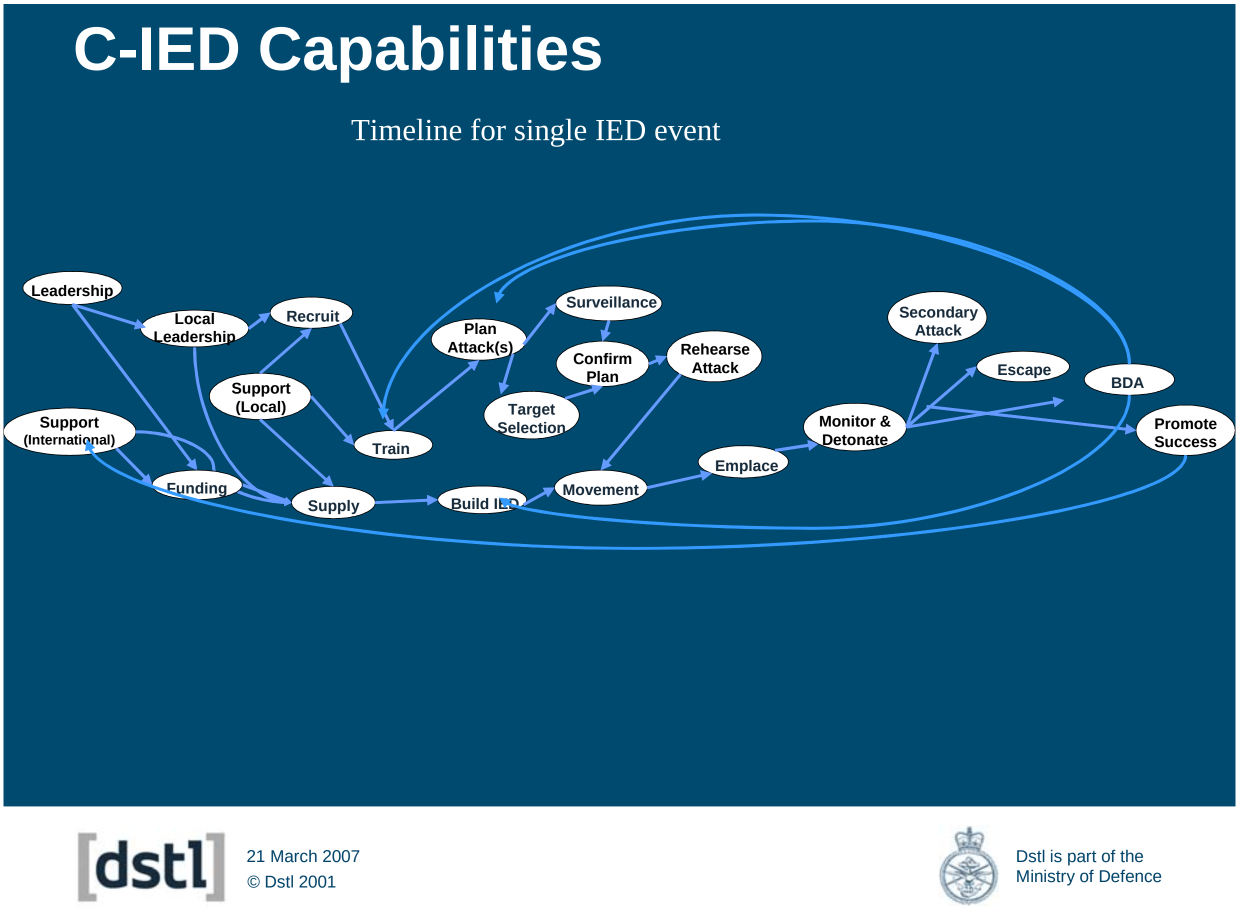
email systems.

(5) Physiological systems (blood, oxygen, heart attack, cell networks in biology).

(Some networks we may wish to destroy.)

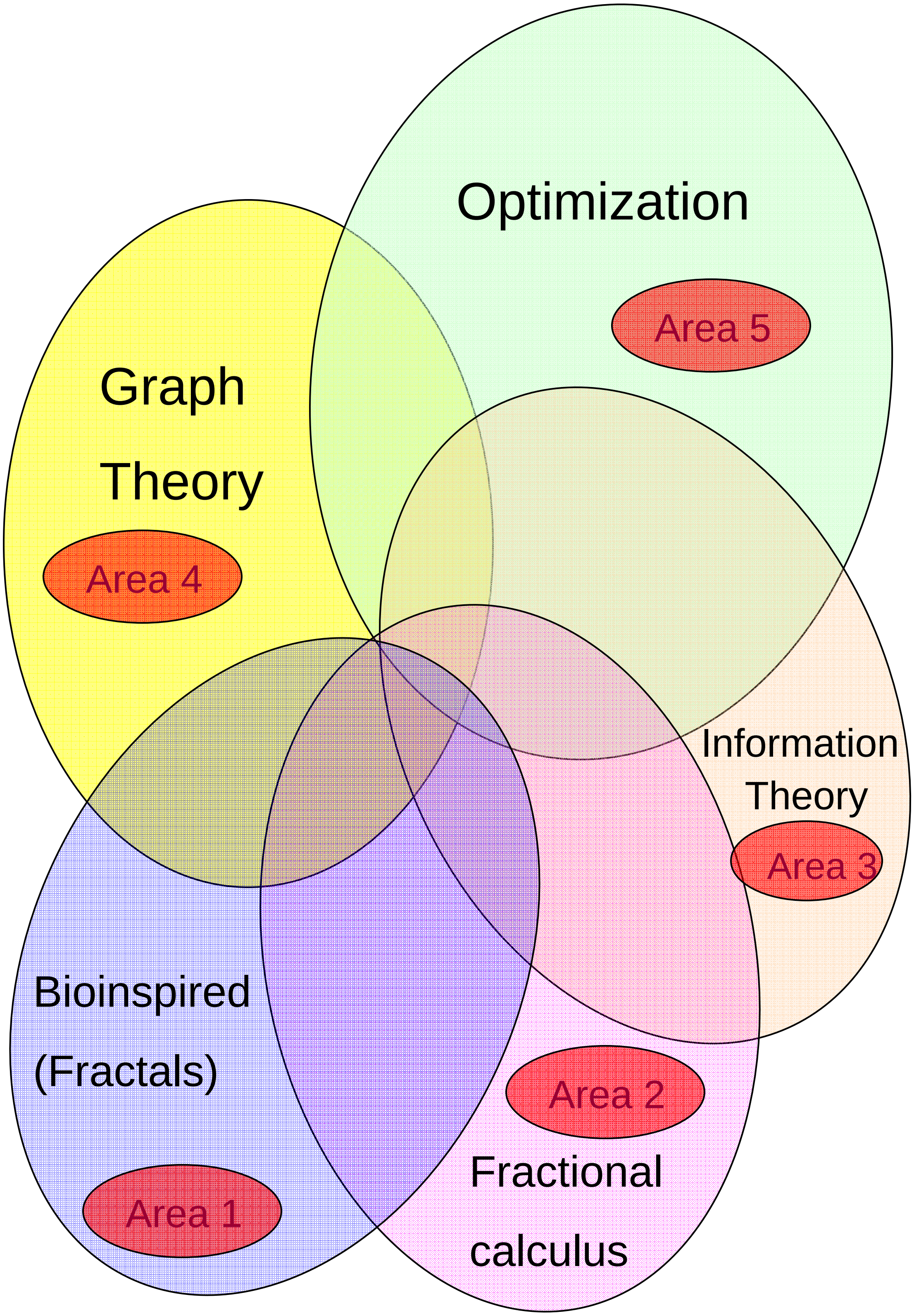
Part 1-A- Pose the Problem:

One Network we wish to destroy:



A second important network to introduce congestion or denial of service:

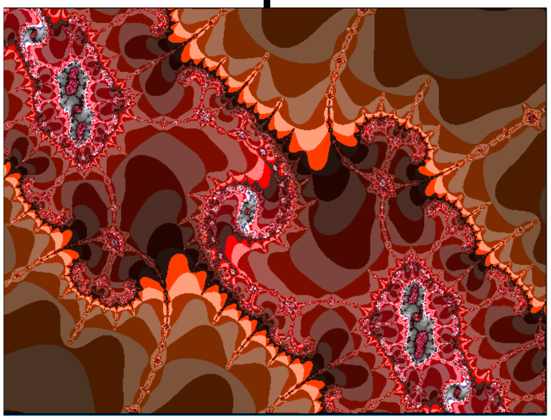
Part 1-B- Background Material

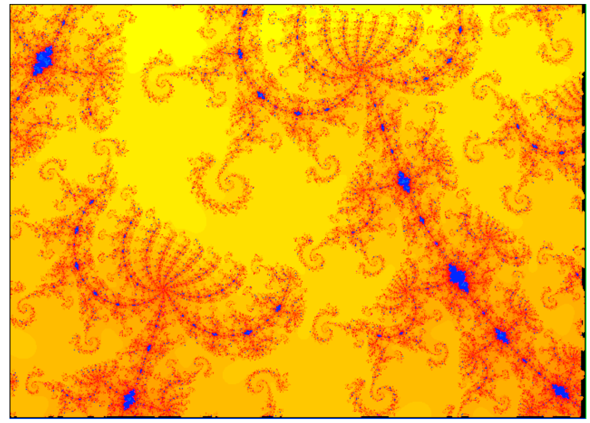


Part 1-B- Background Material

Area 1

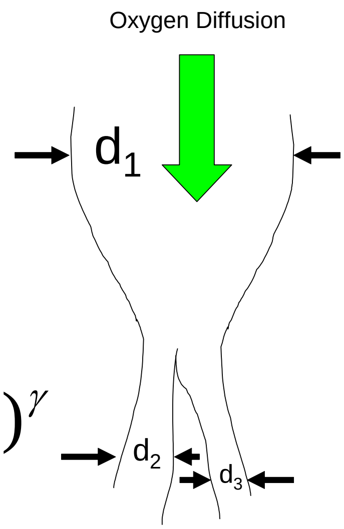
Bioinspired - Fractals



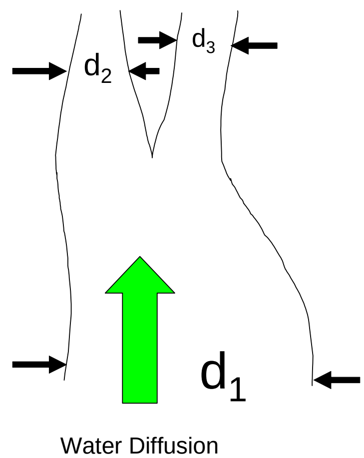


$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$

$$(d_1)^\gamma = (d_2)^\gamma + (d_3)^\gamma$$



Lung



Tree

γ = 2.5??

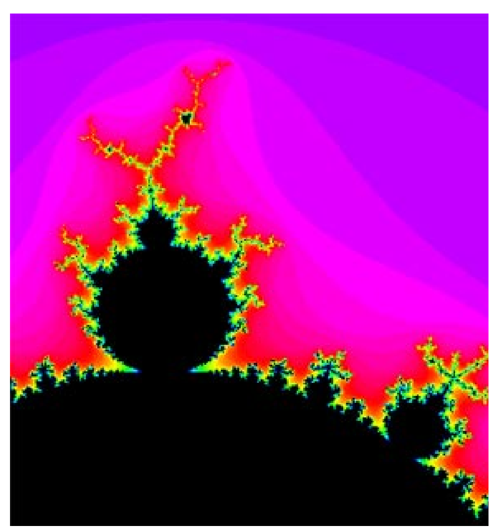
Fractional

Dimensions are ***NOT***

Minimum energy –

They are

Optimal for Diffusion



Part 1-B- Background Material

Area 1

Bioinspired - Fractals

. The Latin ***fractus*** = “broken” or “fractured”

. Fractals – scale free (self-similar), irregular overall length scales. (self similar means the structure is invariant to change in scale). ***Forever continuous but nowhere differentiable.***

. Fractals may have ***infinite circumference but finite area***.

. Fractals can have ***finite volume and infinite area.***

. A fractal can be defined in the sense of a recursive equation:

$$z_{n+1} = f(z_n)$$

. This is, apparently, the ***optimal way*** to distribute flow.

. Non Euclidean Geometry.

. Fractal examples (trees (branches), rivers, lighting bolts, cells, lung passageways, blood vessels, leaf patterns, cloud surfaces, molecular trajectories, neuron firing patterns, etc.).

Fractals – Lets Review the Area

Area 1

B. Mandelbrot (1960,s) asked the question: “How long is the coastline of Britain?”

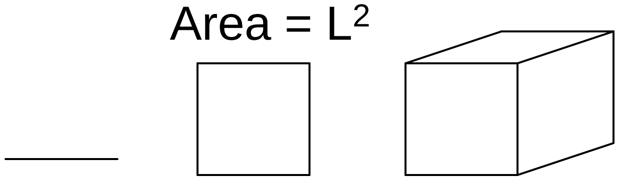
(Suppose we measured the coastline with a ruler that got smaller and smaller?)

A fractal has statistical self-similarity(power law, self affine).

A fractal has N identical parts with scale factor L.

The Hausdorff dimension is

Area = L^2



Length = L

Volume = L^3

(Measurement) $= L^D$

implies $\log(\mathit{Measurement}) = D(\log(L))$

$$D = \frac{\log(\mathit{Measurement})}{\log L} \neq \text{Integer}$$

Fractals – Lets Review the Area

Area 1

$$D = \frac{\log(Measurement)}{\log L}$$

(Measurement) = L ^{D}

L α A ^{½} α V$^{1/3}$

For irregular surfaces, we can define:

Let N = the number of divisions of fixed length.

Let r = length of a ruler.

$$D = \frac{\log(Total\ Length)}{\log(1/r)} \text{ as } r \to 0$$

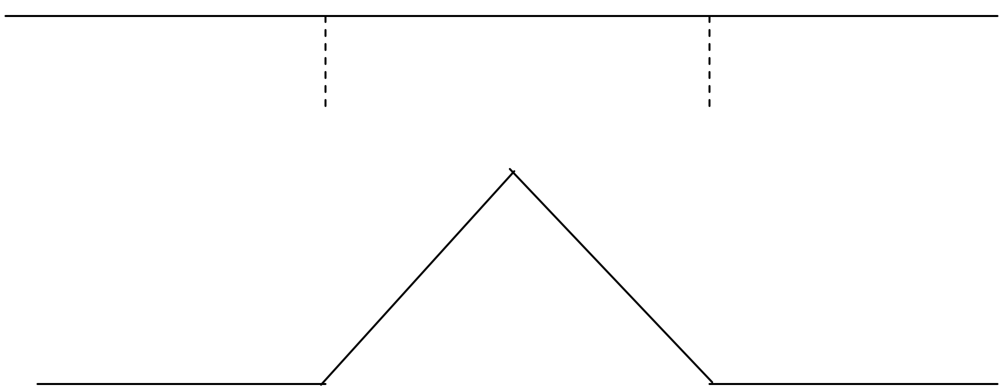
Fractals – Lets Review the Area

Area 1

$$D = \frac{\log(Measurement)}{\log L}$$

Total Length = L^D where $1 < D < 2$

Koch Snowflake



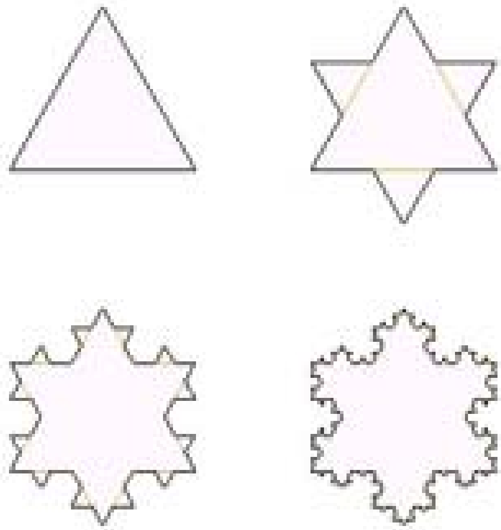
Length = 4 = measurement

Projection = topological dimension = 3

$$D = \frac{\log(4)}{\log(3)} = 1.26185...$$

Fractals – Lets Review the Area
Different versions of the Koch snowflake.

Area 1



Finite

Area

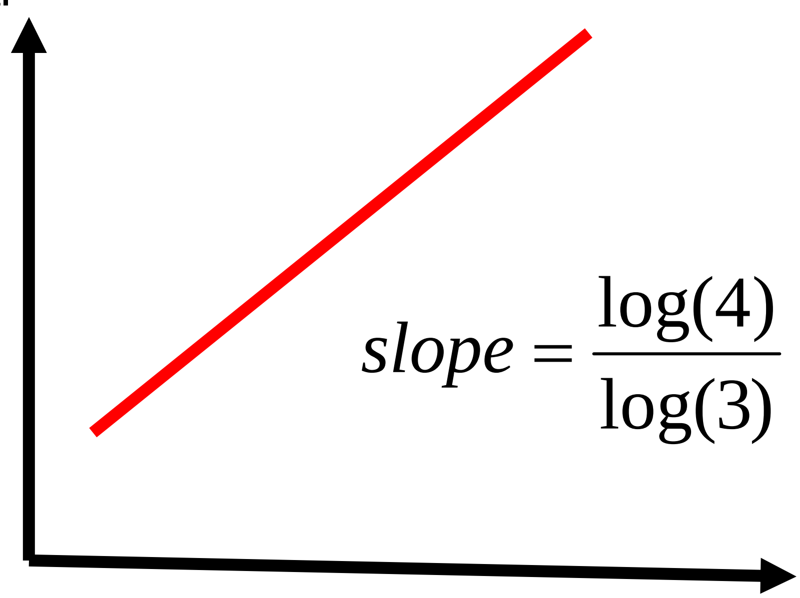
Circumference

= total length

= $(4/3)^n$

$$\lim_{n\to\infty} (\text{total length}) \to \infty$$

Power law



$$slope = \frac{\log(4)}{\log(3)}$$

Biofractals

21 orders of magnitude

Microbe = 10^{-13} g

Whale = 10^{8} g

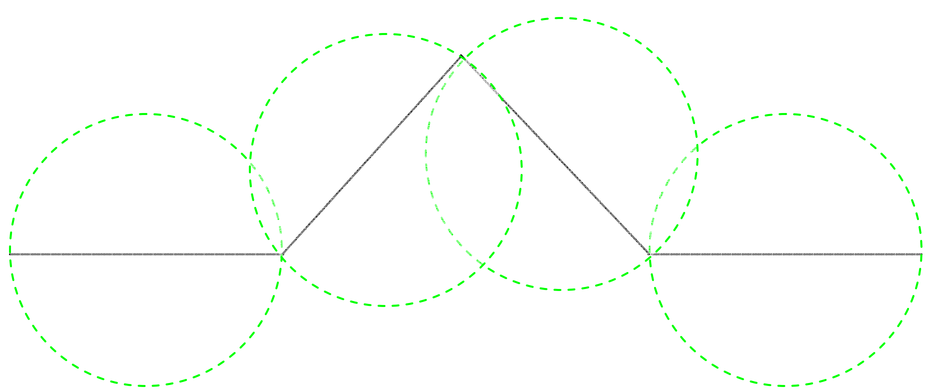
Fractals – Lets Review the Area.

Area 1

$$D = \frac{\log(Measurement)}{\log L}$$

How to determine Measurement?

We "cover" with boxes or disks.



Fractals – Cantor Set (Cantor Dust)

Area 1

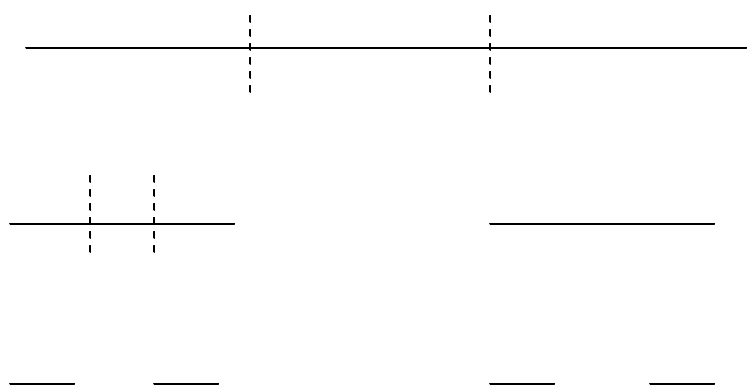
$$D = \frac{\log(Measurement)}{\log L}$$

(remove the middle third)

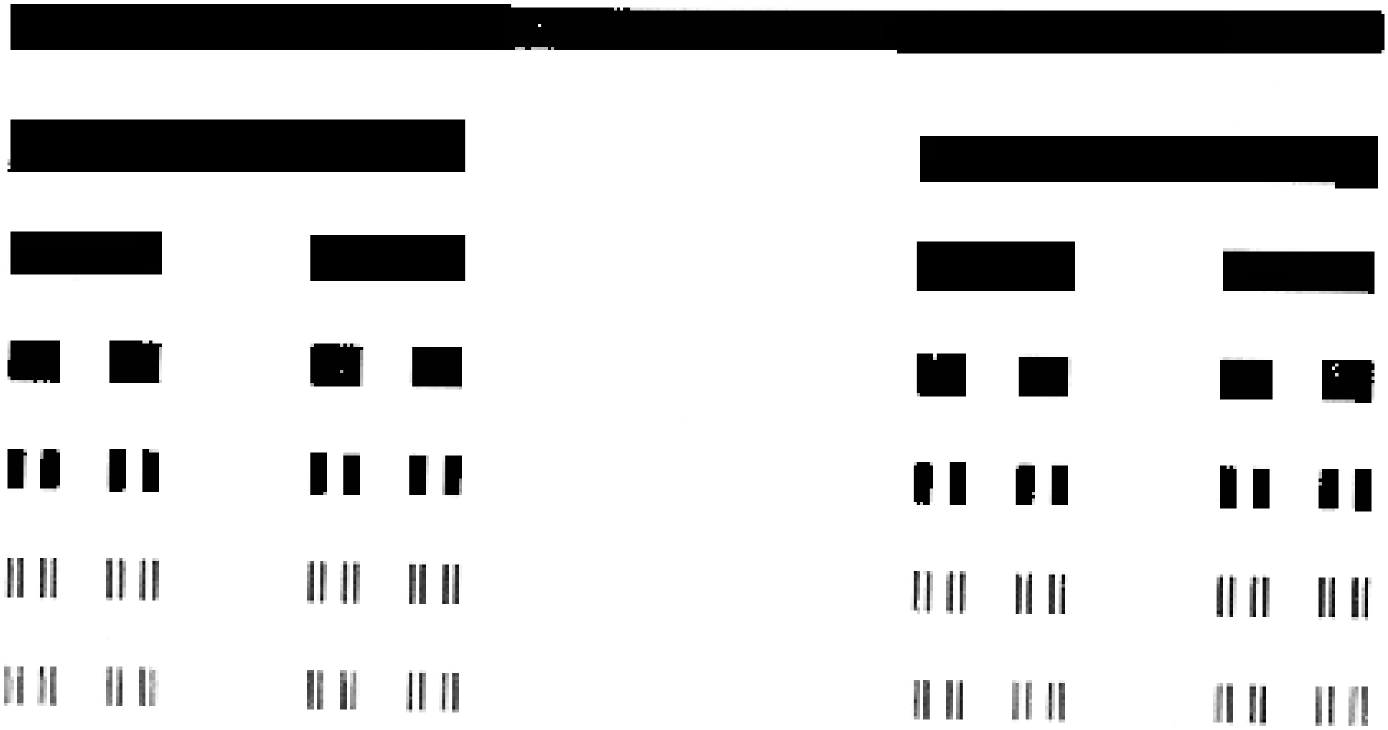
Basic

2/3

2/3 (2/3)



$$D = \frac{\log(2)}{\log(3)} = 0.63092...$$



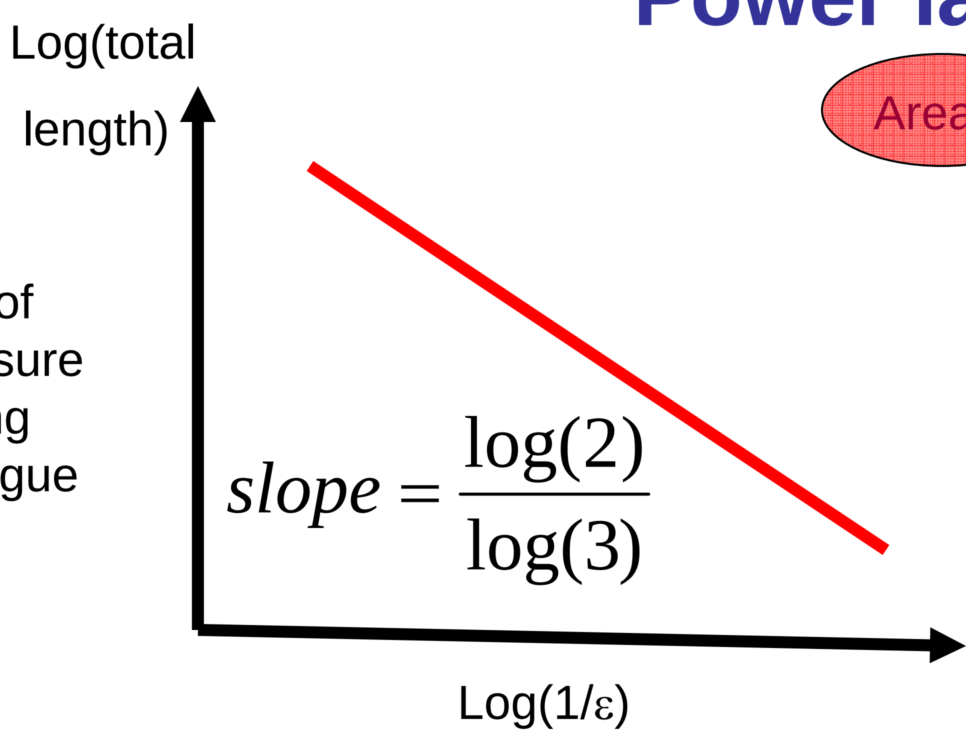
$\lim_{n\to\infty}$ (total length) $\to$ 0

Total length

$= (2/3)^n$

Power law

Area 1

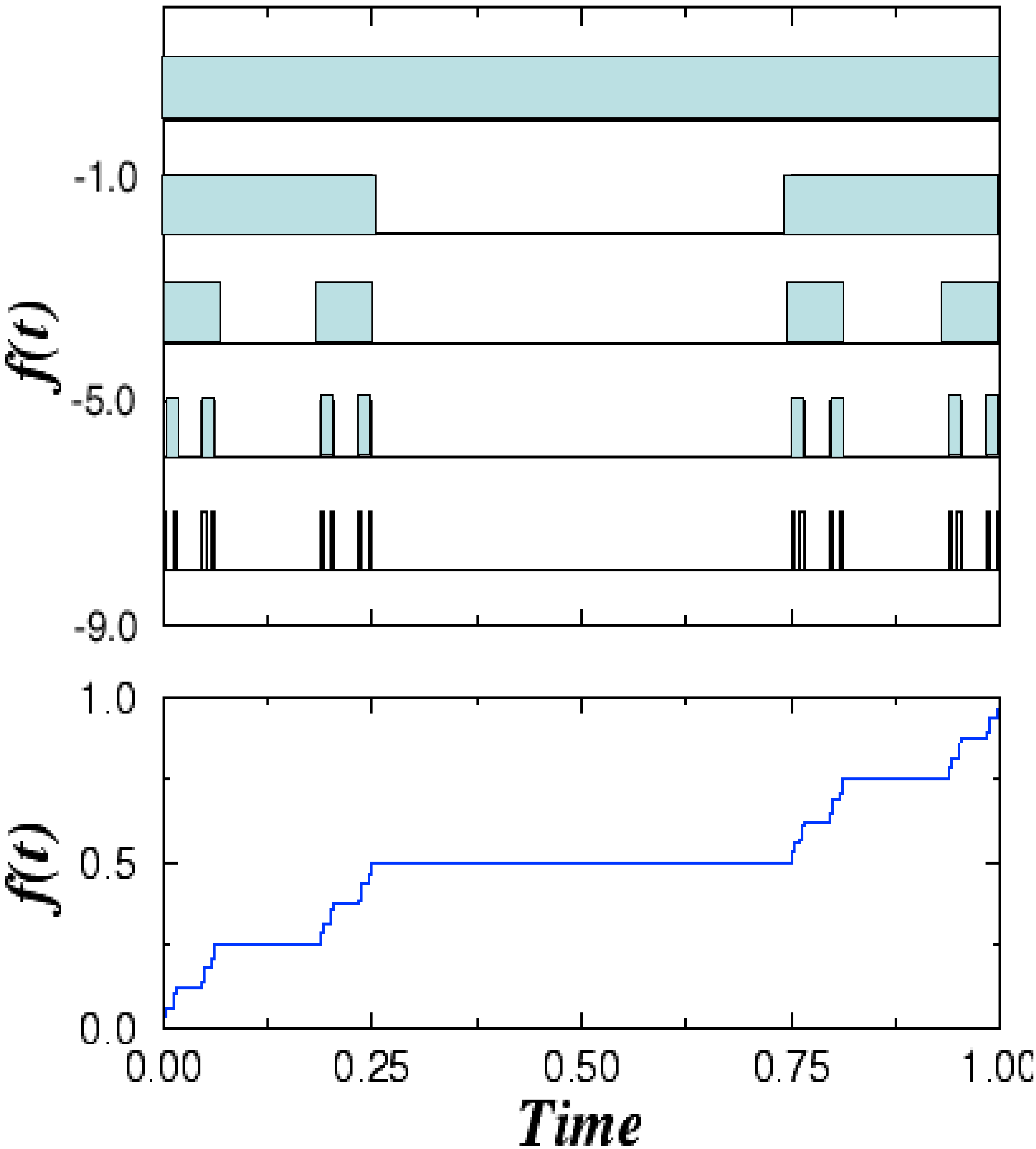


Deleted points of Lebesgue measure 1, the remaining points of Lebesgue measure 0.

What is the Complement of the Cantor Dust Set?

The set of deleted points of Lebesque measure 1

The remaining points of Lebesque measure 0.



Fractional Calculus – Main Points

(non Euclidean geometry)

Area 2

$$\frac{d^n y}{dt^n} = u(t)$$

(Notation invented by Leibniz)

What can *n* be?

Answer:

(In 1695, L'Hopital asked Leibniz, suppose n= ½?)

n = integer = 1, 2, 3 4,

n = negative integer = -1, -2, -3

n can be a non integer, n = ½, 5/6.

n can be a negative non integer, n = -.6, -3.4,

n can be irrational:

$$n = \sqrt{2}$$

n can be a complex number:

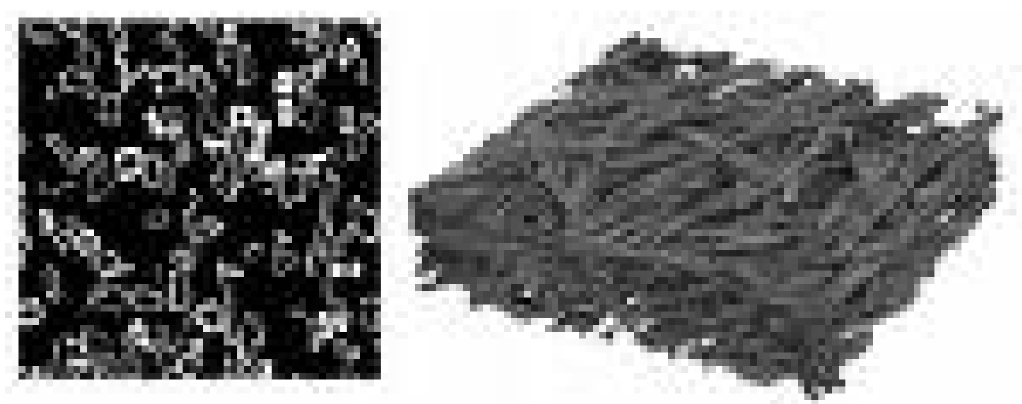
$$n = \sqrt{-1}$$

Fractional Calculus – Main Points

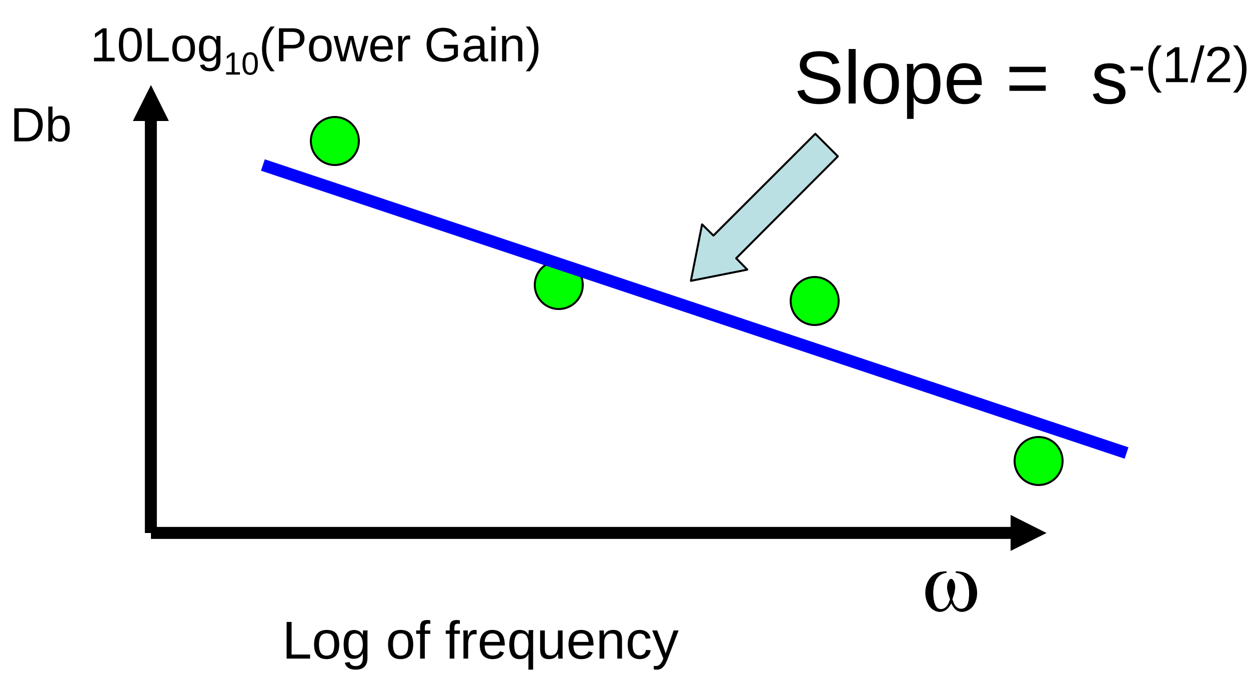
(non Euclidean geometry)

Area 2

Why Study Fractional Calculus?



Composite Materials



Fractional Calculus –Main Points

Area 2

Why use Fractional Calculus?

(1) It can deal with functions that are forever continuous and nowhere differentiable (fractals).

(2) It has the property of self similarity (scale invariance)

$$\frac{d^{5/2}(\alpha y)}{d(\alpha t)^{5/2}}+\frac{d^{3/2}(\alpha y)}{d(\alpha t)^{3/2}}+\frac{d^{1/2}(\alpha y)}{d(\alpha t)^{1/2}}=\frac{d^{3/2}(\alpha u)}{d(\alpha t)^{3/2}}+\frac{d^{1/2}(\alpha u)}{d(\alpha t)^{1/2}}$$

$$\frac{d^{q} f(bx)}{[dx]^{q}}=b^{q}\frac{d^{q} f(bx)}{[d(bx)]^{q}}$$

(3) It is also of the form:

$$z_{n+1} = f(z_n)$$

(Iterated function theory).

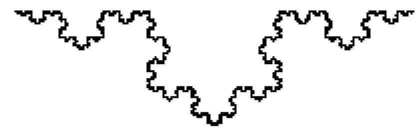
(4) It can also solve partial differential equations:

$$\frac{\partial^2 u(x,t)}{\partial x^2}=a^2\frac{\partial u(x,t)}{\partial t}$$

Fractional Calculus

Area 2

An Easier Way to View the Self Similarity Property



A power law $f(x) = x^a$ has the property that that the relative change in

$$\frac{f(kx)}{f(x)} = k^a$$

Is independent of x

In this sense, the functions lacks characteristic scale (scale free or scale invariant). Let us evaluate $\dfrac{f(kx)}{f(x)}$

Let $x = y^a$

Then

$$\frac{f(kx)}{f(x)} = \frac{(ky)^a}{y^a} = k^a \frac{\cancel{y^a}}{\cancel{y^a}} = k^a$$

Note: no dependence on x

Fractional Calculus –Main Points

(310 year old area). Non Euclidean

Area 2

Common Properties

(1) Scale Invariance – Self Similarity.

$$\frac{d^q f(bx)}{[dx]^q} = b^q \frac{d^q f(bx)}{[d(bx)]^q}$$

(2) Weierstrass Function:

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

(3) Solves Systems in Nature (Diffusion equation).

Fractional Calculus –Other Points

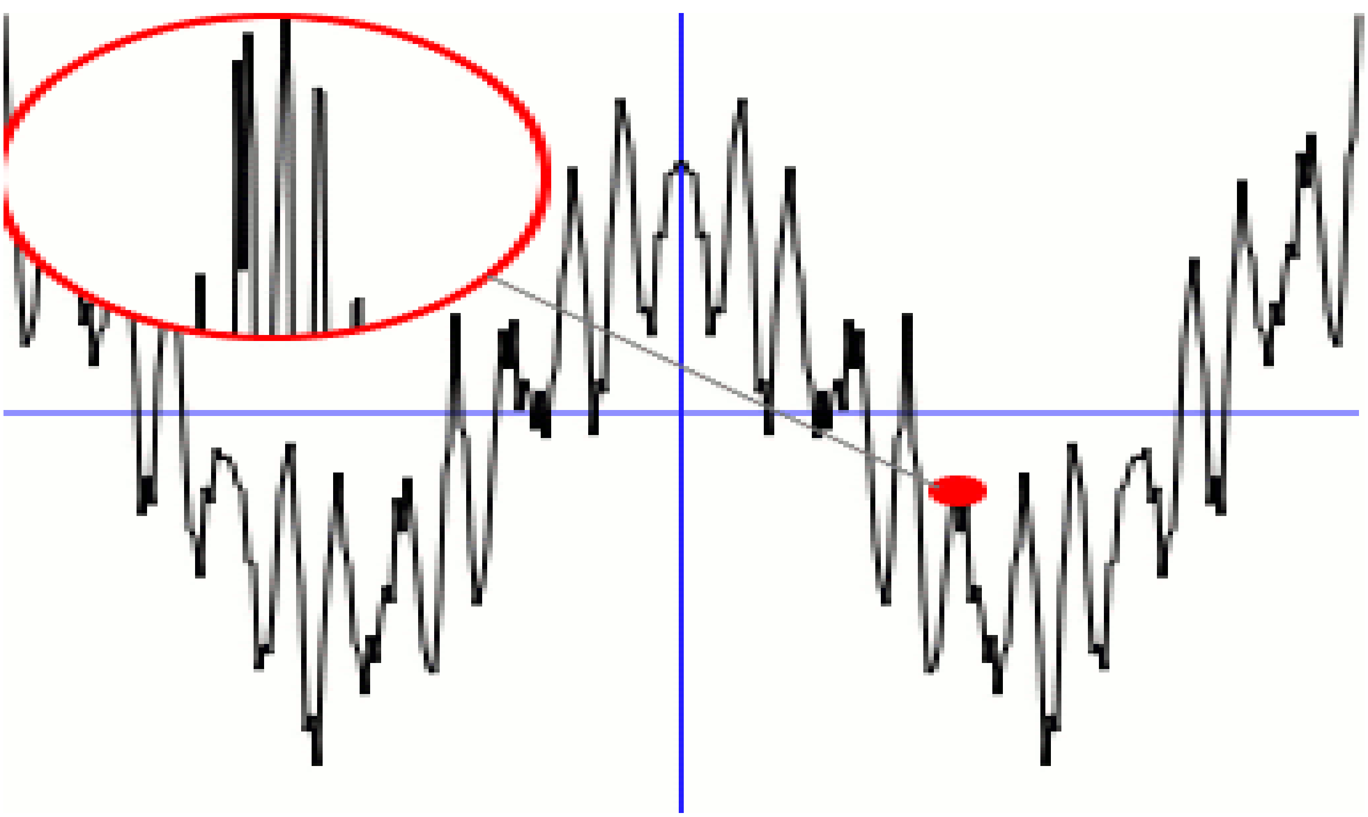
(310 year old area). Non Euclidean

Area 2

Forever continuous nowhere differentiable.

Weierstrass Function: $$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

0< *a* <1, *b* is a positive integer and *ab* > 1 + (3/2)π



Solves Systems in Nature (Diffusion equation).

Fractional Calculus –Other Points

Weierstrass Function (Why?):

Area 2

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

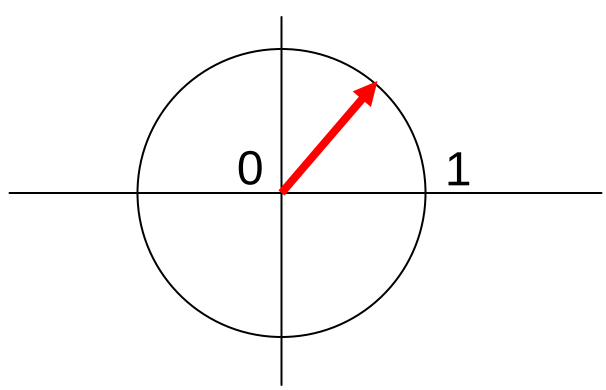
0< a <1, b is a positive integer and $ab > 1 + (3/2)\pi$

Step 1: We understand the radius of convergence:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \ldots$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

IFF $|x| < 1$



Fractional Calculus –Main Points

Area 2

(Solution of the Diffusion Equation)

$$\Gamma(z) = \int_0^\infty e^{-u} u^{z-1} du, \qquad \Gamma(1) = 1, \Gamma(z+1) = z\Gamma(z),$$

Thus: $\Gamma(z+1) = z!,$ $\quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$

Step 1 – Derivatives in x^m

$$\frac{d}{dx} x^m = m x^{m-1}, \qquad \frac{d^\beta}{dx^\beta} x^m = \frac{m!}{(m-\beta)!} x^{m-\beta}$$ but β may not be an integer

$$\frac{d^\beta}{dx^\beta} x^m = \frac{\Gamma(m+1)}{\Gamma(m-\beta+1)} x^{m-\beta}, \qquad \frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} x^1 = \frac{\Gamma(1+1)}{\Gamma(1-\frac{1}{2}+1)} x^{1-\frac{1}{2}} = \frac{2}{\sqrt{\pi}} x^{\frac{1}{2}}$$

This now ***generalizes*** for derivatives in e^{ax}

$$D^v e^{ax} = a^v e^{ax}$$

(v not an integer)

Generalizations to functions that can be written in a power series:

$$f_1(t) = \sum_{n=0}^{q} a_n + b_n x^n$$

Generalizations to functions that can be written in an exponential series:

$$f_2(t) = \sum_{n=0}^{q} a_n + b_n e^n$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Euler's Law:

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Fractional Calculus –Main Points

Area 2

(Solution of the Diffusion Equation)

Step 2 – Laplace Transform

$$F(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

Then: which holds if

$$e^{-\alpha t}\,|\,f(t)\,| \le M < \infty$$

$$L^{-1}[F(s)] = f(t)$$

$$L^{-1}\left(\frac{1}{s^{1+\beta}}\right) = \frac{t^{\beta}}{\Gamma(\beta+1)}, \beta > -1$$

$$L^{-1}\left[\frac{1}{s^{\frac{1}{2}}}\right] = \frac{t^{\frac{-1}{2}}}{\Gamma(\frac{1}{2})} = \frac{1}{(\sqrt{\pi})t^{\frac{1}{2}}}$$

Step 3 - Diffusion Equation:

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2\,\frac{\partial u(x,t)}{\partial t}$$

$$U(x,s) = L[u(x,t)] = \int_0^\infty e^{-st}u(x,t)dt$$

$$L\left[\frac{\partial u}{\partial t} - \frac{1}{a^2}\frac{\partial^2 u}{\partial x^2}\right] = sU(x,s) - f(x) - \frac{1}{a^2}\frac{\partial^2 U}{\partial x^2} = 0$$

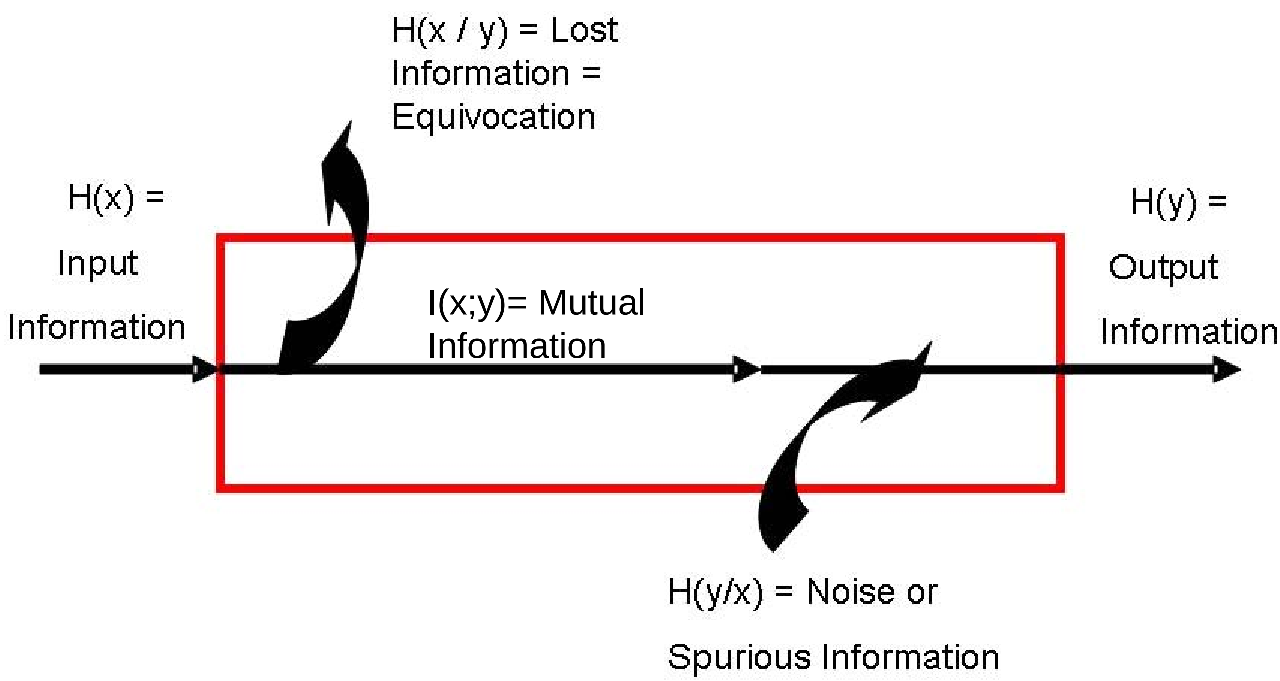
$$U(x,s) = Ae^{xas^{\frac{1}{2}}} + Be^{-axs^{\frac{1}{2}}} = \frac{1}{a^2 2\sqrt{s}}\int_{-\infty}^{\infty} e^{-\sqrt{s}|x-\tau|} f(\tau)d\tau$$

$$u(x,t) = \frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{\infty} e^{-\frac{-(x-\tau)^2}{4t}} f(\tau)d\tau$$

Part 1-B- Background Material

Information Theory

Area 3



$$D_R = H(x/y) + H(y/x) \quad \text{(metric not a measure)}$$

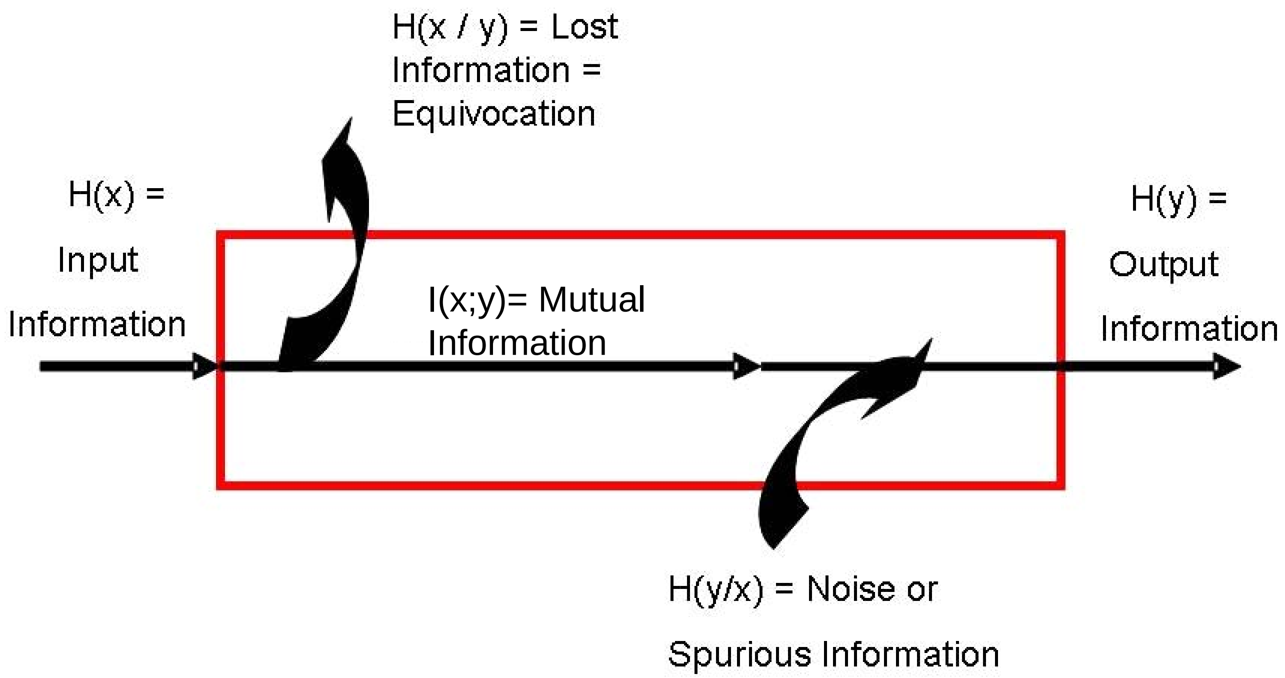
$\rho(x,y) \geq 0$ for all x and y. (non negativity)
$\rho(x,y) = \rho(y,x)$ (symmetry)
$\rho(x,z) \leq \rho(x,y) + \rho(y,z)$ (triangular inequality)
$\rho(x,y) = 0$ IFF x=y (identity of indiscernibles)

Mutual Information (I(x,y)) is well embraced by numerous disciplines. (MI is the ***reduction in uncertainty in an input object by observing an output object***).

Part 1-B- Background Material

Information Theory

Area 3



Why are we interested in flow rate?

Units of $I(x;y)$ are bits/sec

Therefore bits = $I(x;y)\,\Delta t$ where

Δt = time to complete a task.

Suppose we view bits as discrete events.

$$\Delta t = \frac{events}{I(x;y)}$$

If bits = events = fixed then:

min $I(x;y) \Rightarrow \max \Delta t$, max $I(x;y) \Rightarrow \min \Delta t$

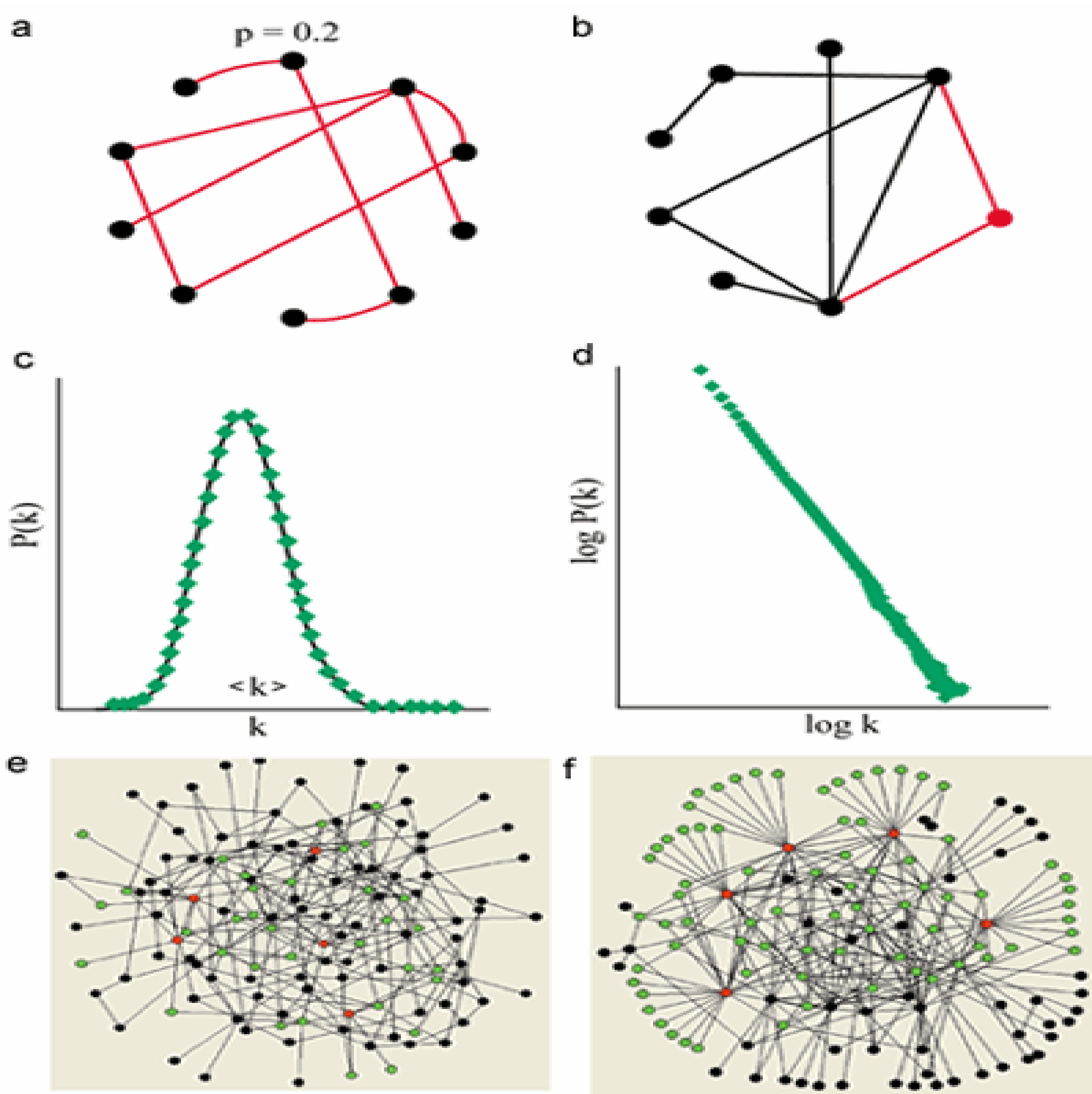
Optimal Performance ***Optimal Network Attack***

Part 1-B- Background Material-Graph Theory

Area 4

(1) Random Graphs. (Less vulnerable, uniformly connected).

(2) Scale free graphs. (Highly vulnerable, not uniformly connected).

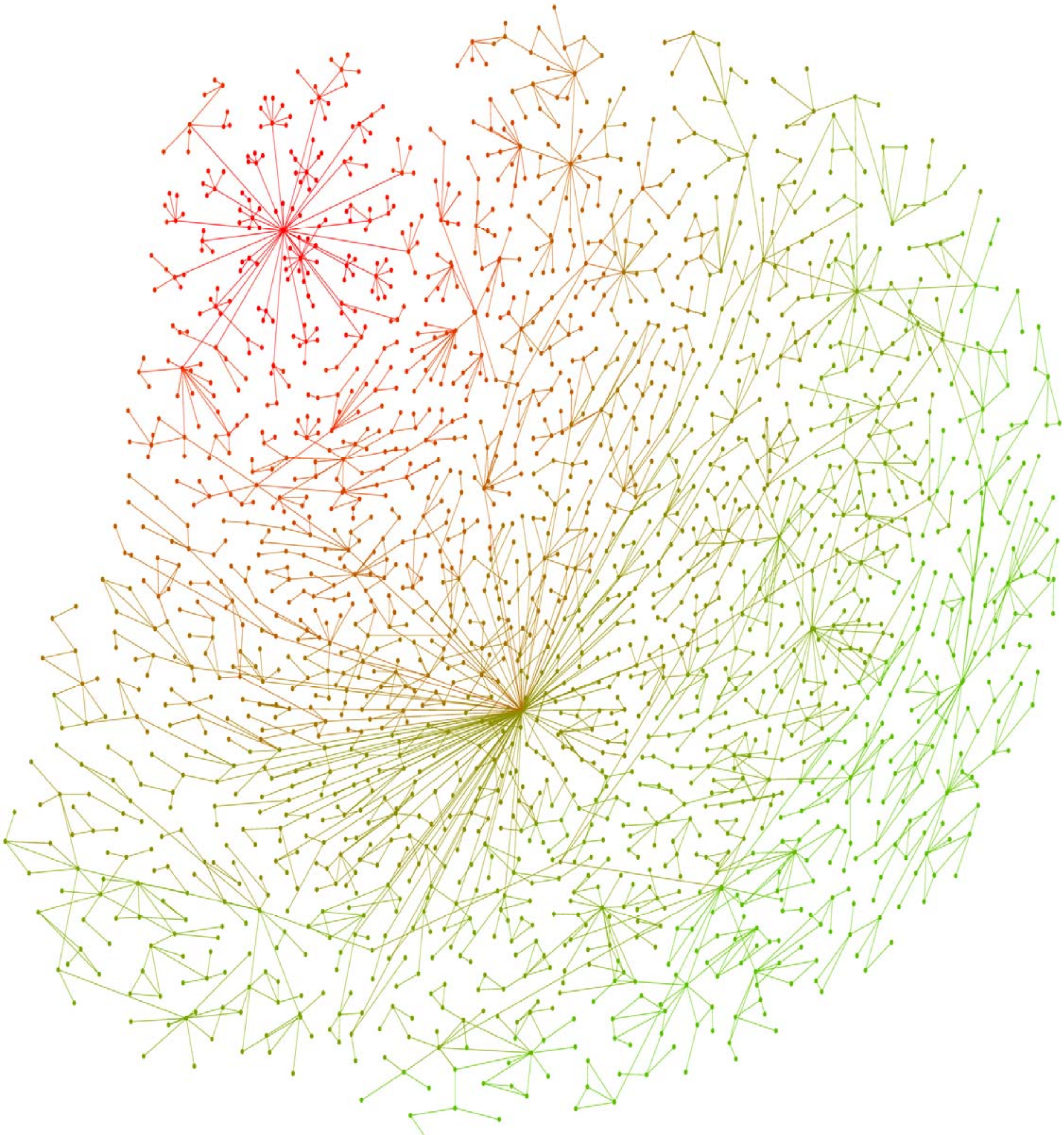


Part 1-B- Background Material

Graph Theory (Spatial Construct)

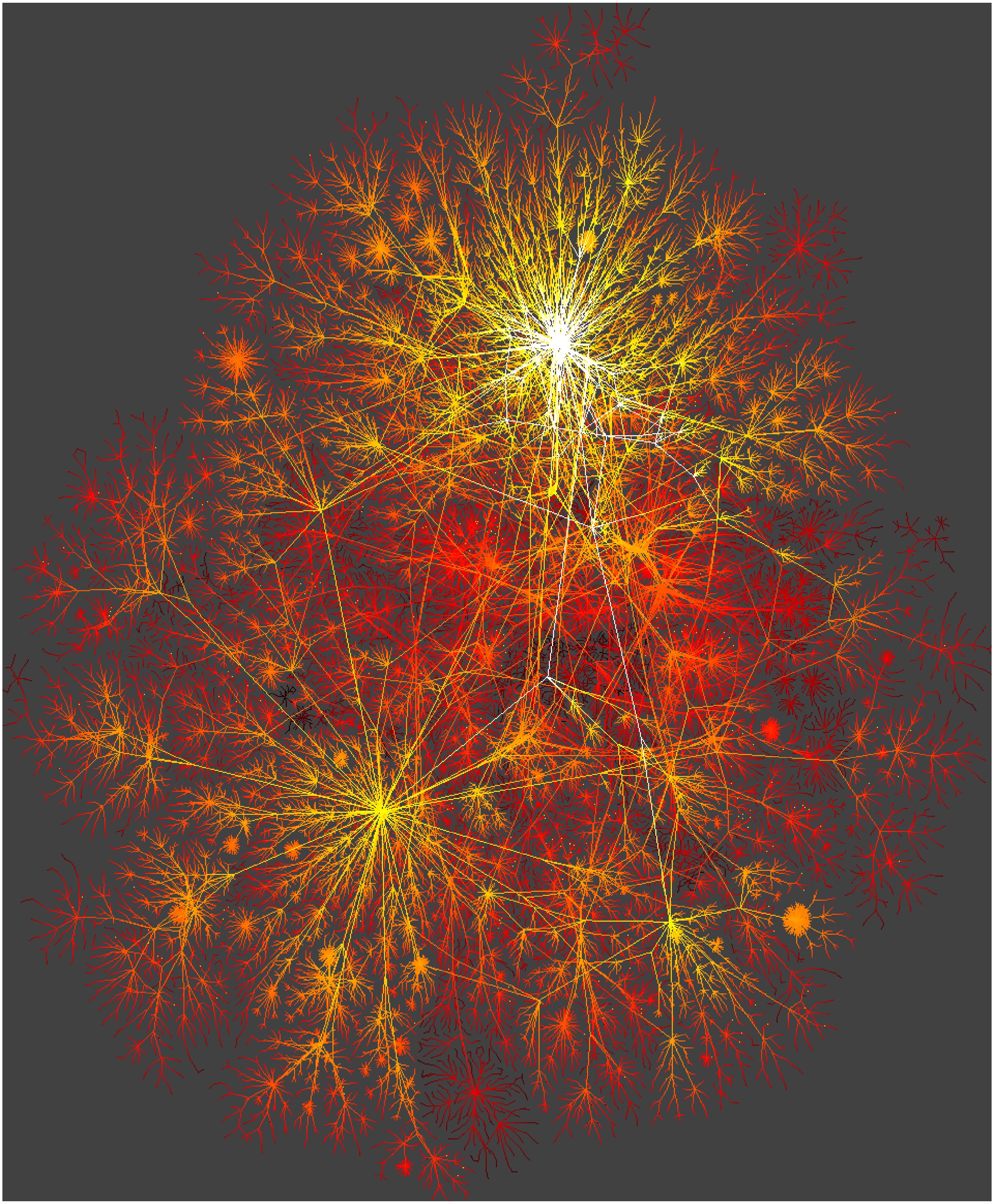
Area 4

The Internet



Internet-Map

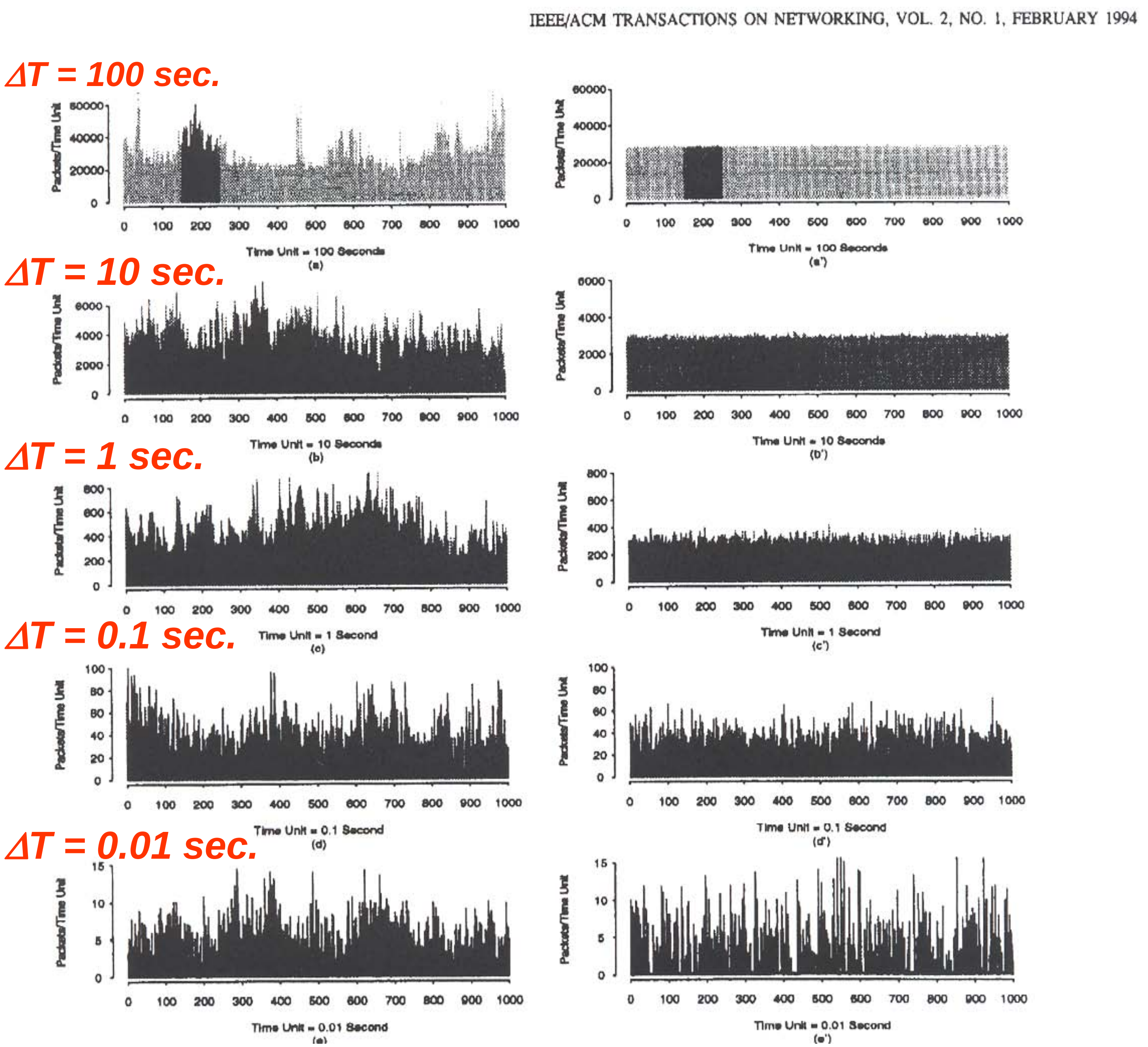
Area 4



Part 1-B - Background Material

Graph Theory *(Spatial and Temporal)* Area 4

The Internet is ***dynamically*** scale free (evidence) :



Reference: W. E. Leland, et al., *IEEE/ACM Trans. on Networking*, **vol 2**, no. 1, Feb. 1994, “On the Self-Similar Nature of Ethernet Traffic (Extended Version).”

Part 1-B - Background Material

Graph Theory

Area 4

The Internet is dynamically scale free (evidence) :

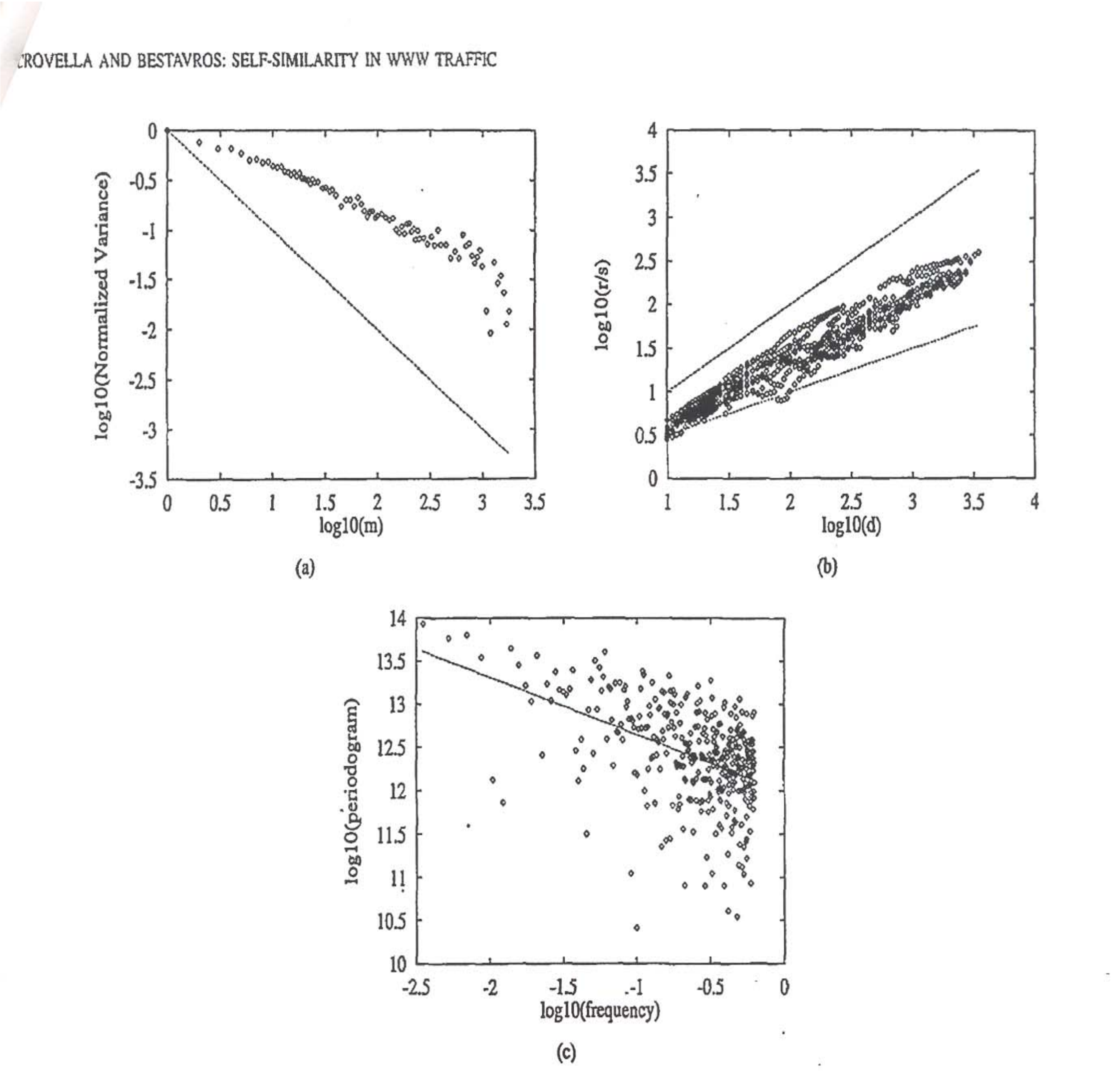
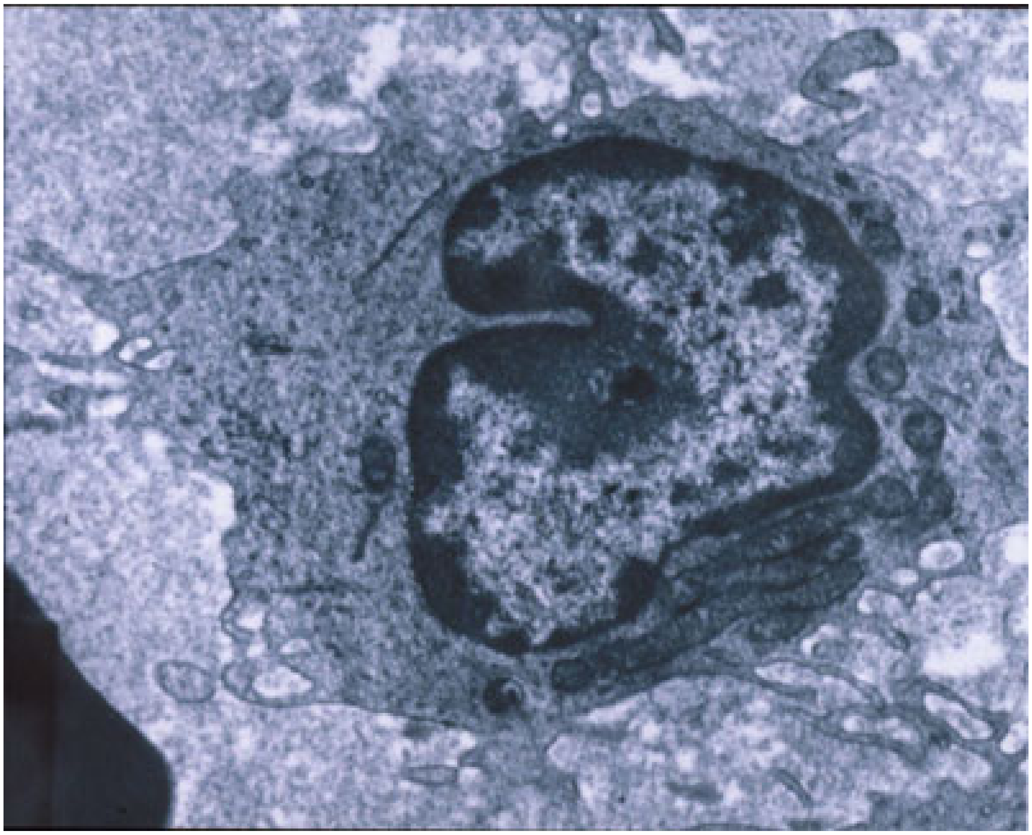


Fig. 1. Graphical analysis of a single hour.

Reference: M. Crovella and A. Bestavous, “Self-Similarity in World Wide Web Traffic: Evidence and Possible Causes,” *IEEE/ACM Trans. On Networking,* **Vol. 5**, no. 6, December, 1997.

Other Physiological Evidence

Area 4



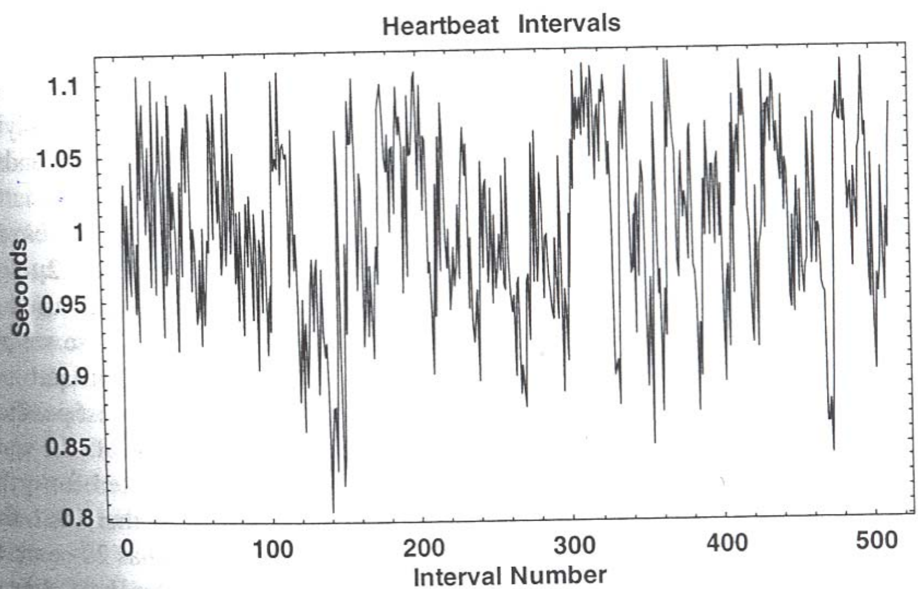


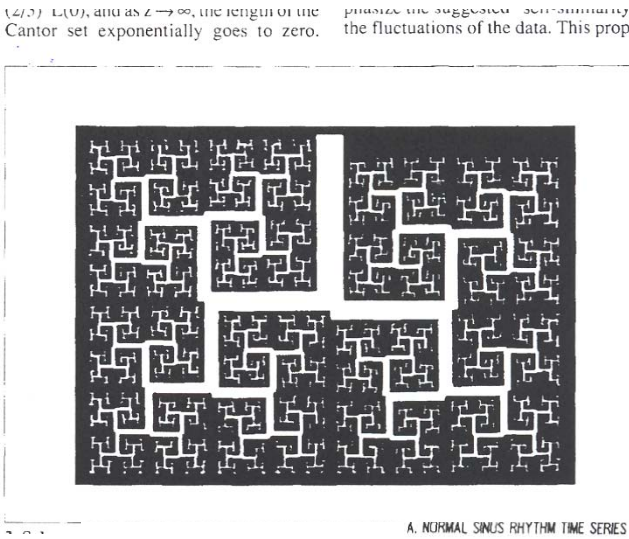
Figure 2. The time series of heartbeat intervals of a healthy young adult male is shown. It is clear that the variation in the time interval between beats is relatively modest, but certainly not negligible.

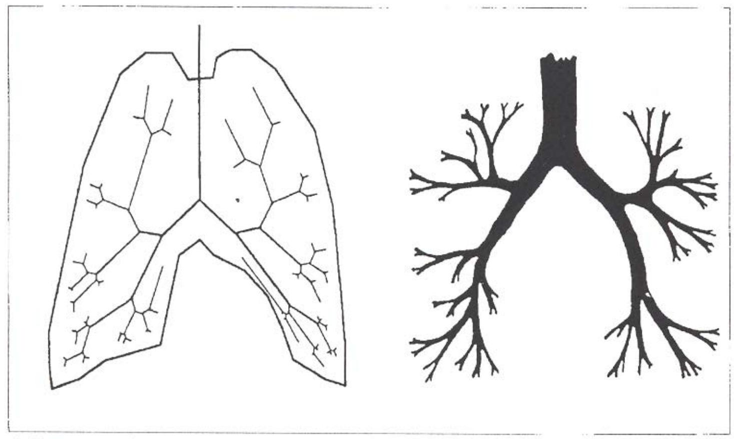
Heartbeat intervals

Reference: B. J. West, “Fractal Physiology, Complexity, and the Fractional Calculus,” Chapter 6, in “Fractals, Diffusion and Relaxation in Disordered Complex Systems,” in *Advances in Chemical Physics*, **vol 133**, part B, John Wiley, 2006, Eds. W. T. Coffey and Y. P. Kalmykov.

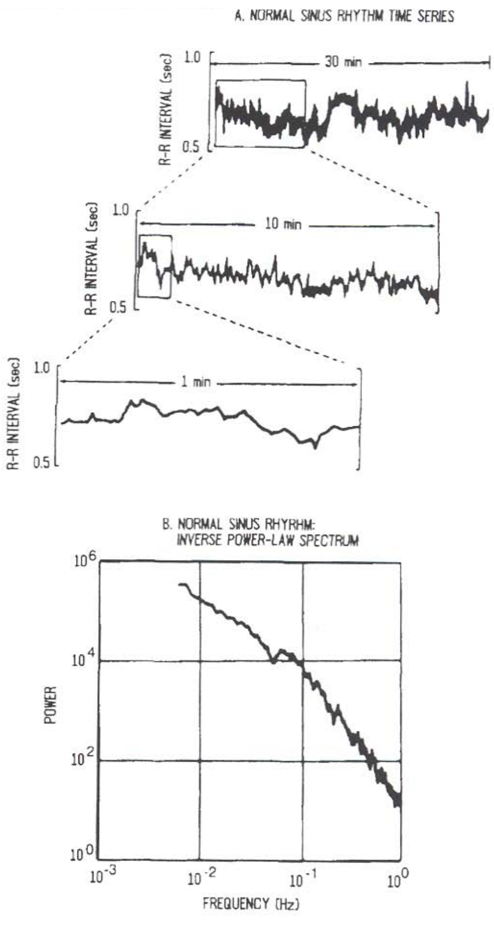
Other Physiological Evidence

Area 4





4. Computer simulation of a fractal lung, in which the boundary conditions influence morphogenesis. The boundary was derived from a chest radiograph. The model data are in good agreement with actual structural data [9].



Sinus Rhythm

Intervals

Reference:

W. Deering and B. J. West, “Fractal Physiology,” *IEEE Eng. In Medicine and Biology*, June, 1992.

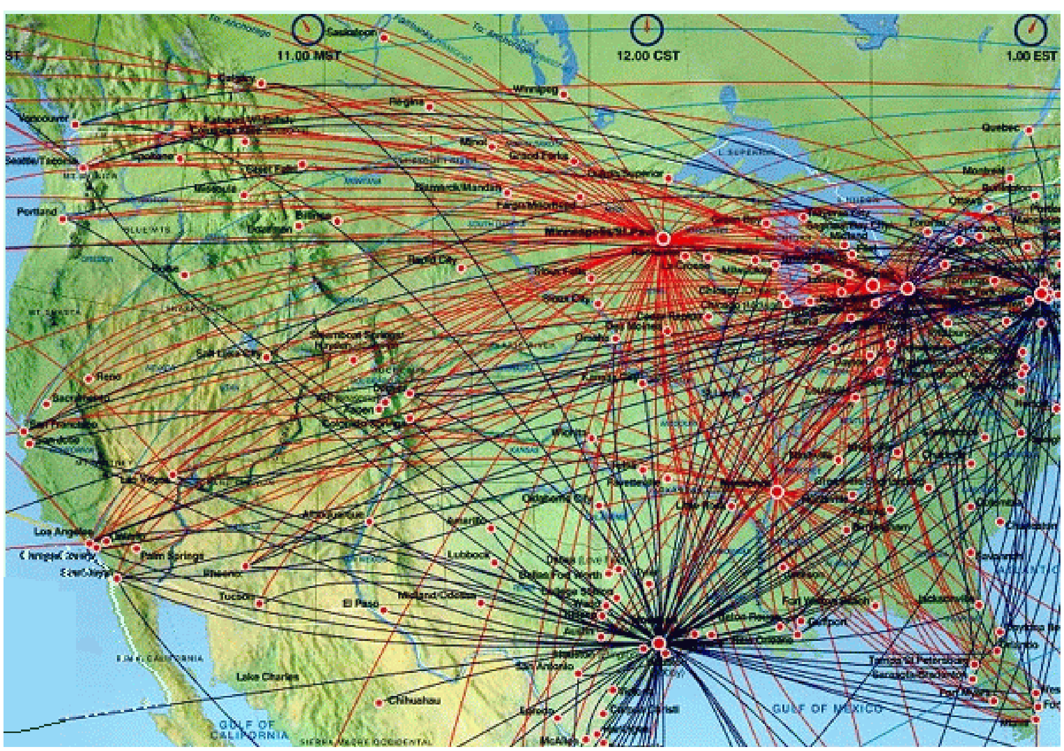
Additional Background Material – H. Jeong – Complex '07

(The difference between random and scale-free graphs)

Area 4

Highway network

Airline network

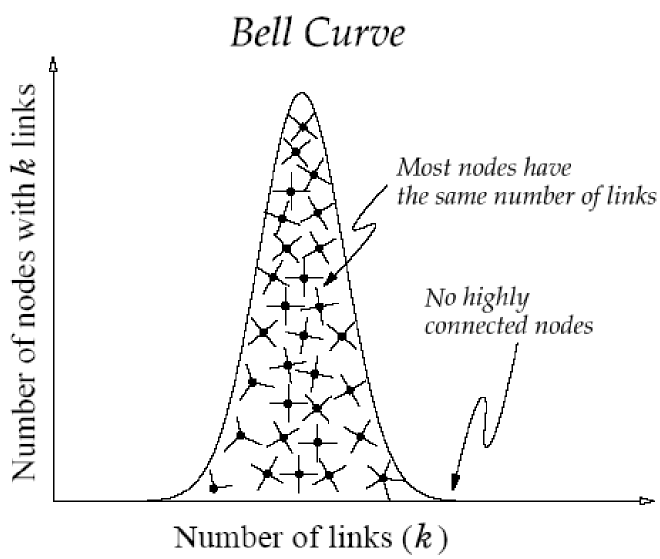


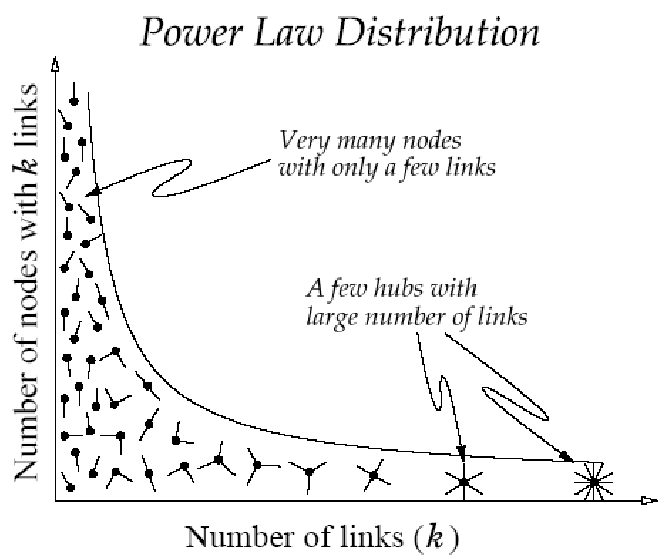
Mathematically? via Degree distribution P(k)

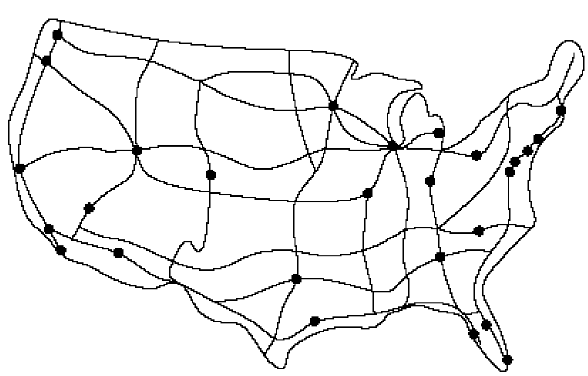
Area 4

Random

Scale Free





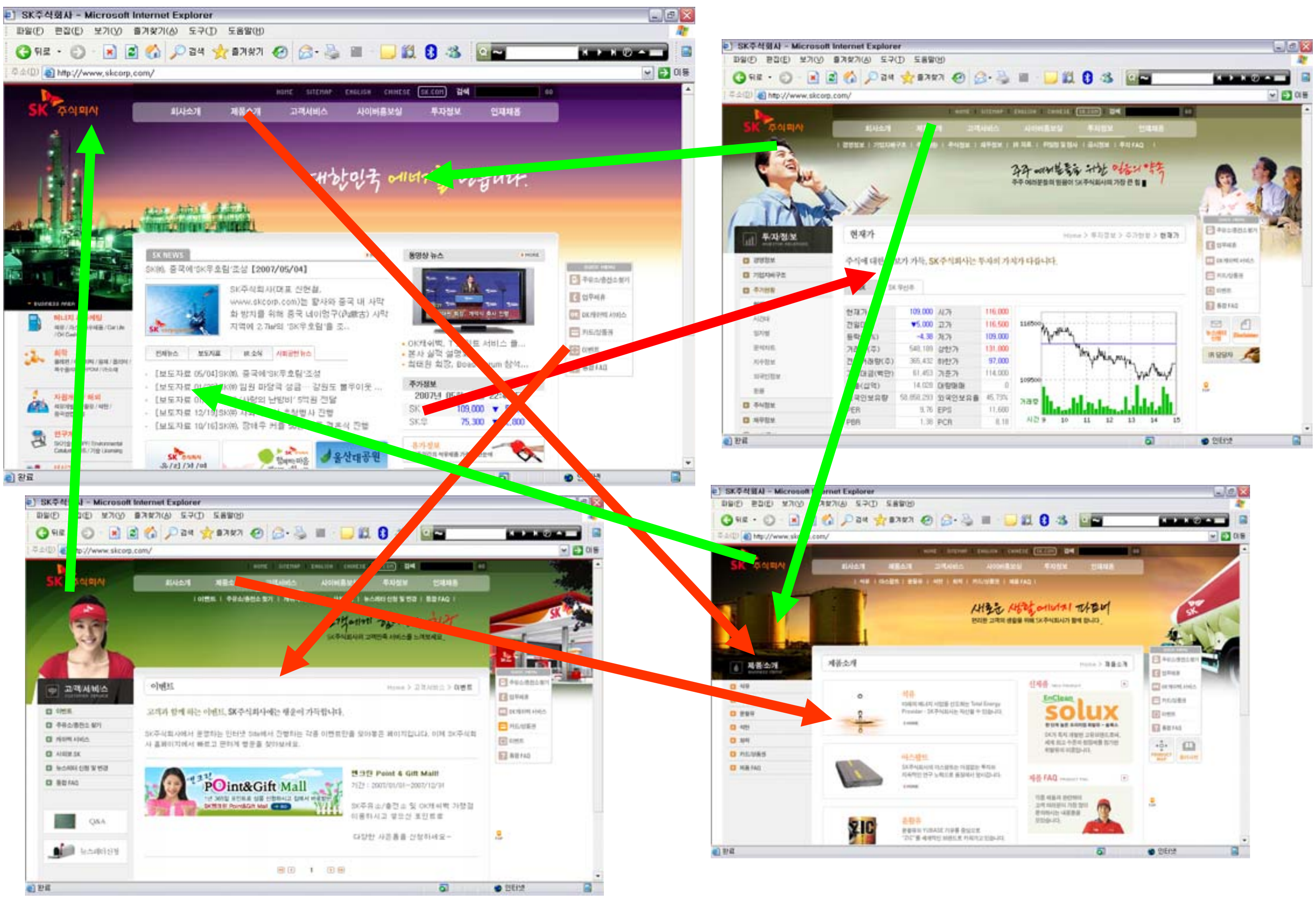


World Wide Web

Node(point): web-page

Area 4

link(line): hyper-link



The problem is to discern (for each application):

(1)What are the nodes?

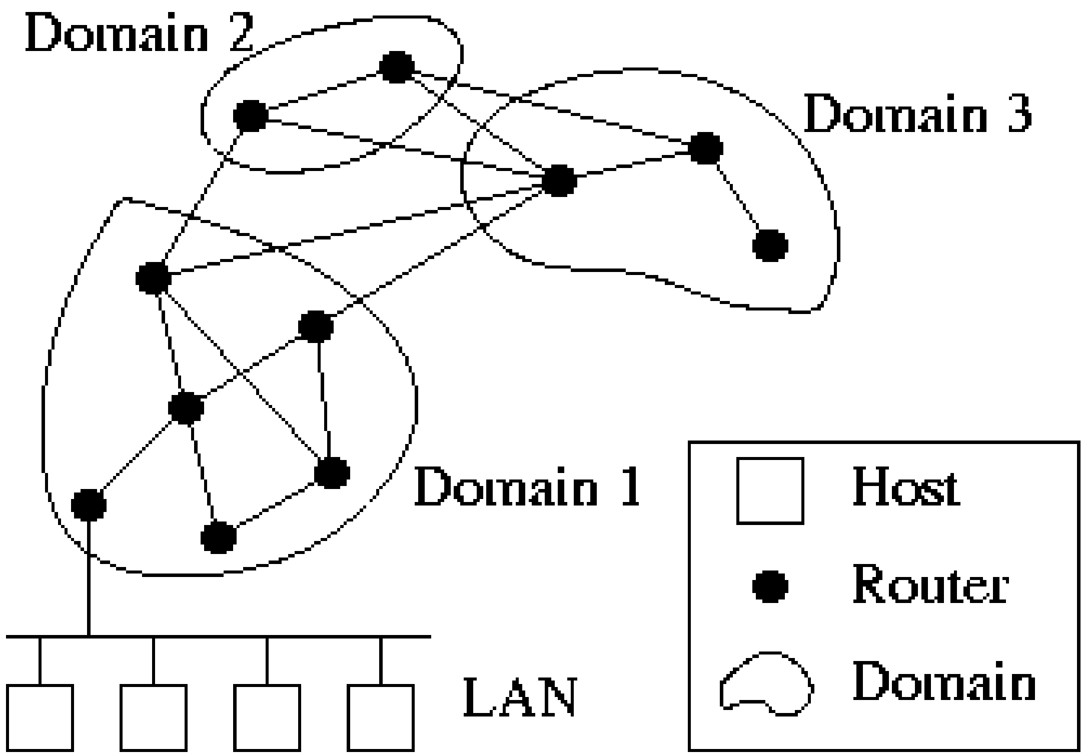
(2)What are the links?

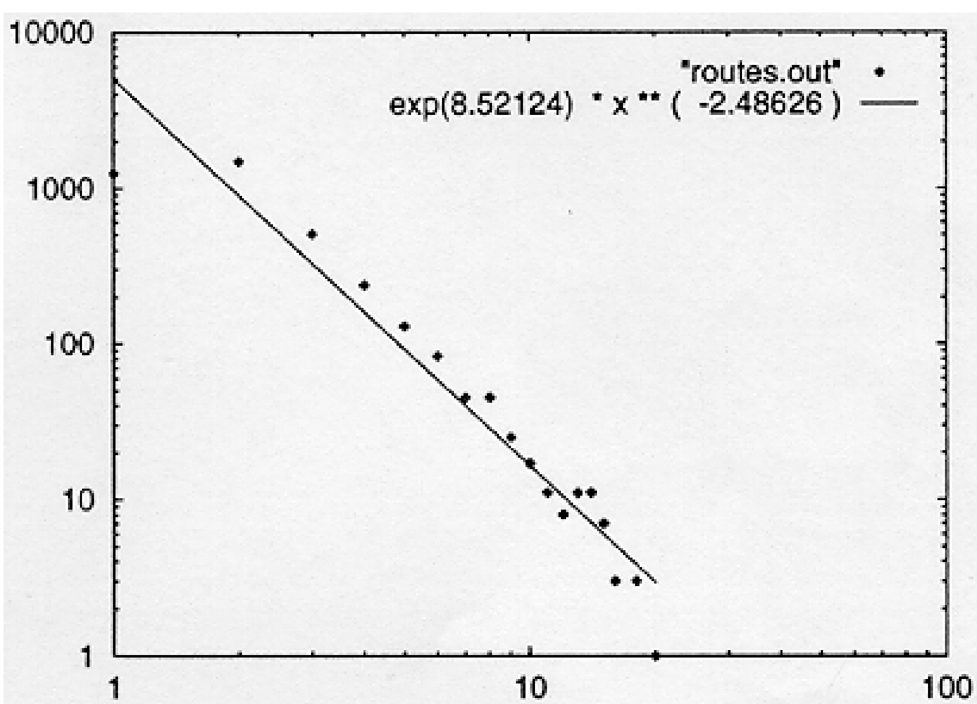
INTERNET BACKBONE

Nodes: computers, routers

Area 4

Links: physical lines





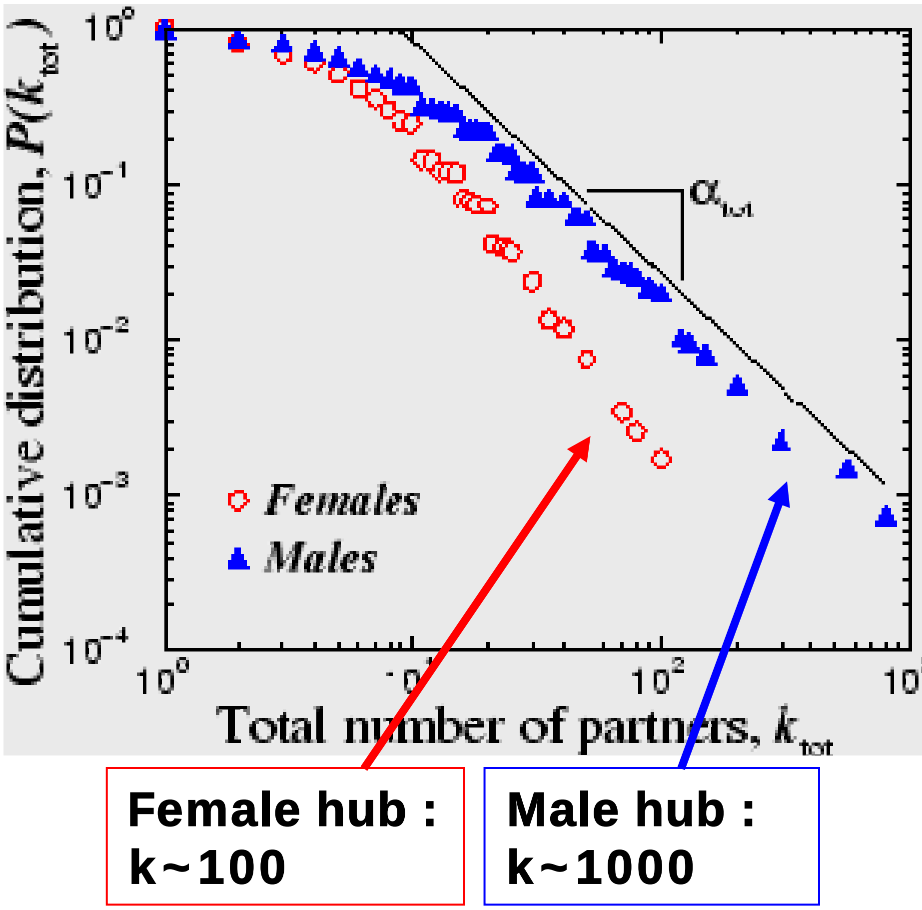
(Faloutsos, Faloutsos and Faloutsos, 1999)

SEX-Web

Nodes: people (females; males)

Links: sexual relationships

Area 4



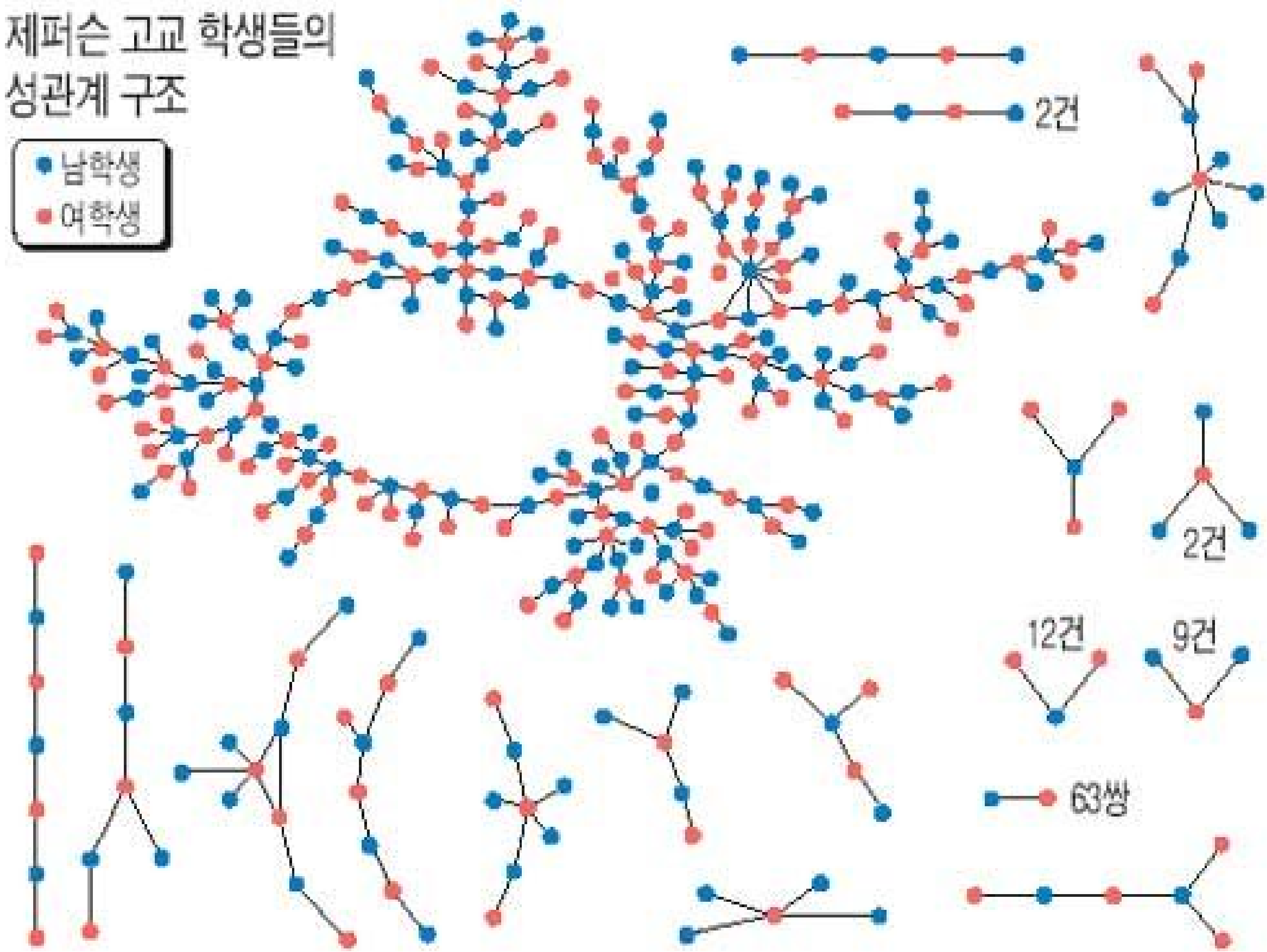
(Liljeros et al. Nature 2001)

Sexual Relationships in Jefferson High School

Area 4

Male

Female



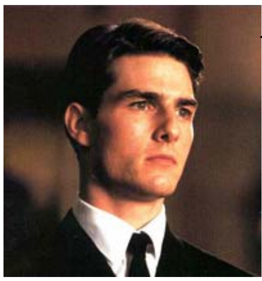
ACTOR CONNECTIVITIES

Area 4

Nodes: actors

Links: cast jointly

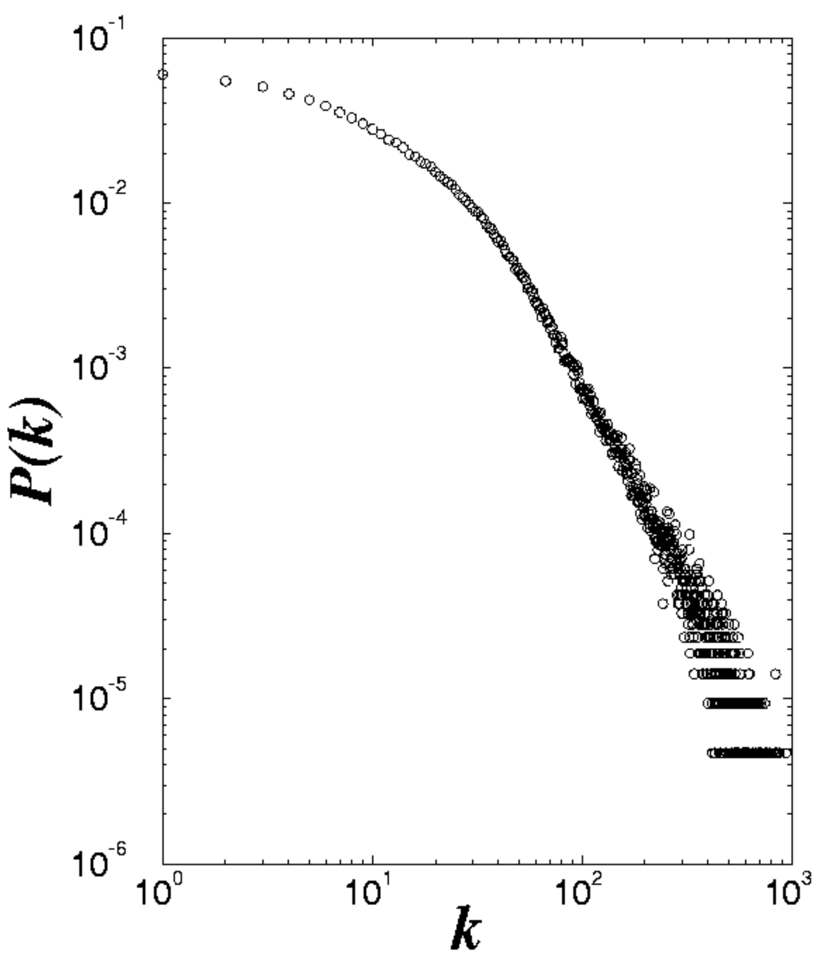
IMDb Internet Movie Database



Days of Thunder (1990)
Far and Away (1992)
Eyes Wide Shut (1999)

N = 212,250 actors **$\langle k \rangle$ = 28.78**

$P(k) \sim k^{-\gamma}$, **γ=2.3**



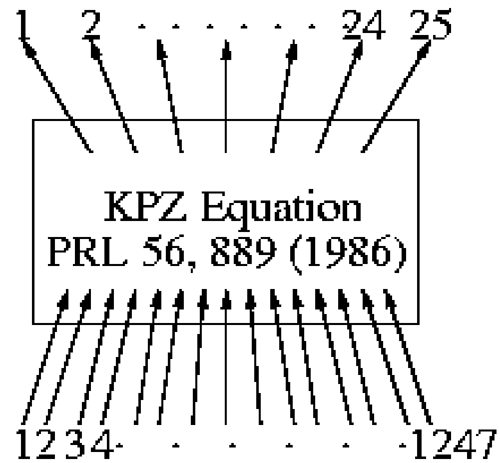
SCIENCE CITATION INDEX

Area 4

Nodes: papers

Links: citations

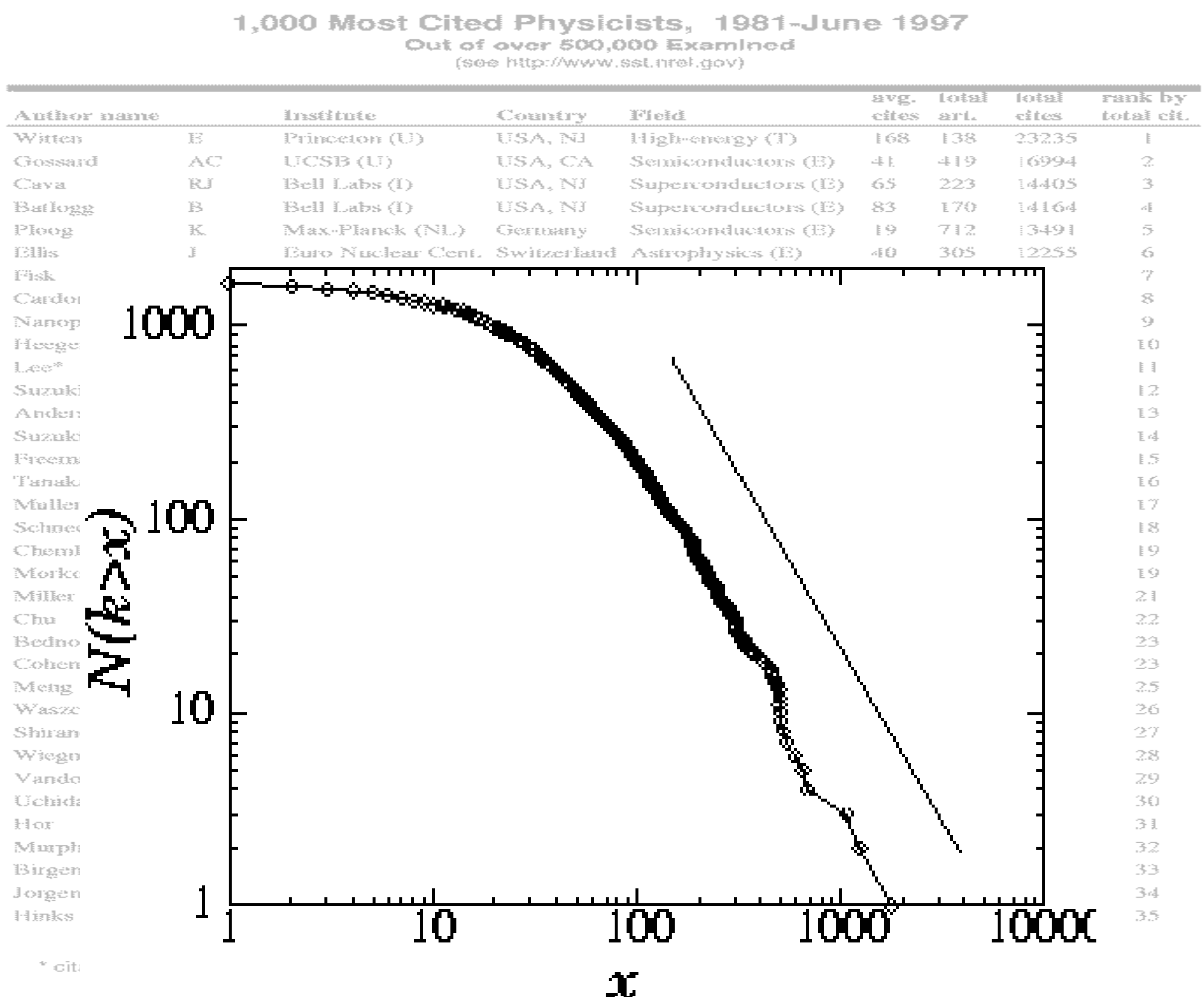
1736 PRL papers (1988)



$P(k) \sim k^{-\gamma}$

$(\gamma = 3)$

(S. Redner, 1998)



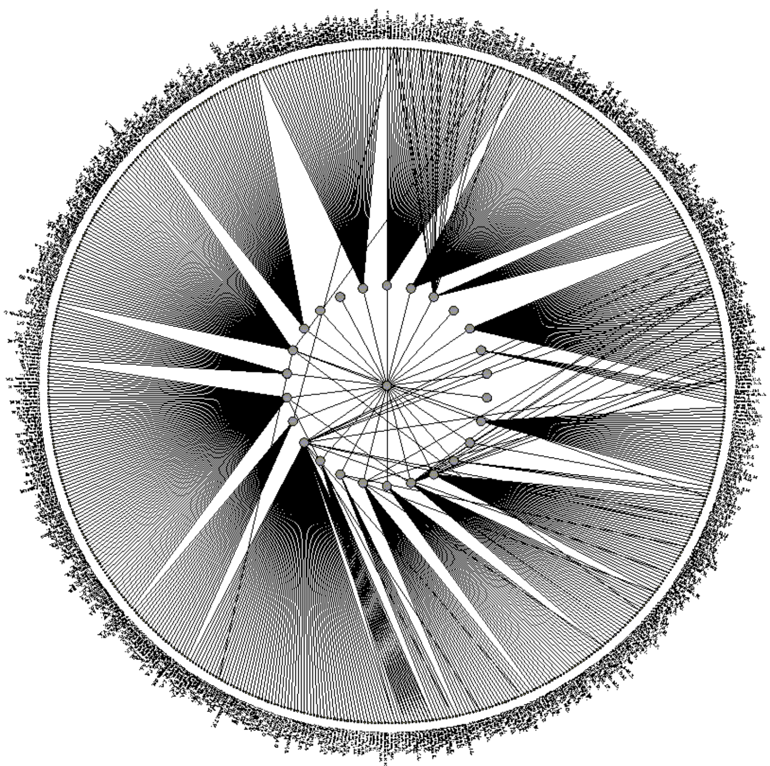
SCIENCE COAUTHORSHIP

(collaboration network)

Area 4

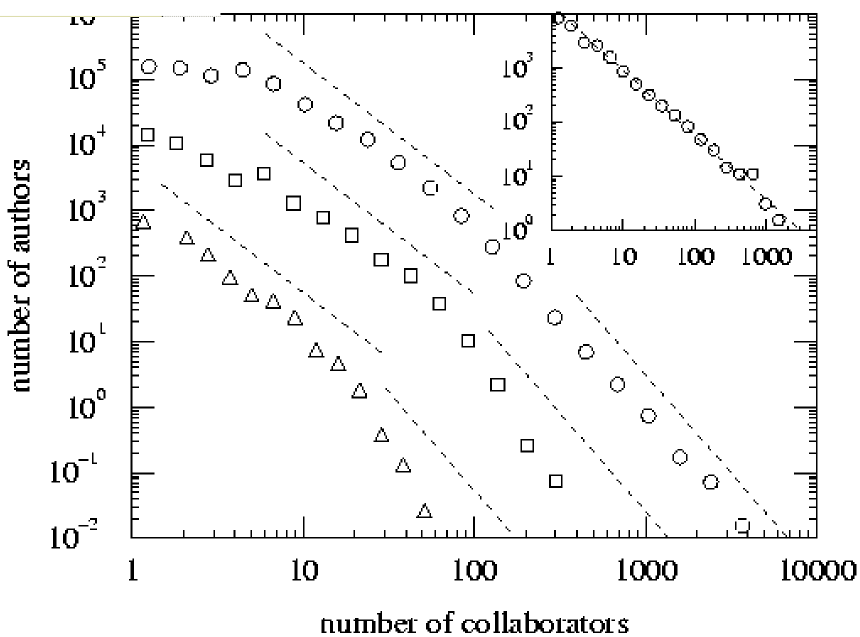
Nodes: scientist (authors)

Links: write paper together



(Newman, 2000,

H. Jeong et al 2001)



Other Examples of Scale-Free Networks

Area 4

Email network

Nodes: individual email address

Links: email communication

Phone-call networks

Nodes: phone-number

Links: completed phone call

(Abello et al, 1999)

Networks in linguistics

Nodes: words

Links: appear next or one word apart from each other

(Ferrer et al, 2001)

Networks in Electronic auction (eBay)

Nodes: agents, individuals

Links: bids for the same item

(H. Jeong et al, 2001)

THEN WHY??

Area 4

(i) Efficiency of resource usage.

Diameter (Scale-free) < Diameter (Exponential)

(* Diameter ~ average path length between two nodes)

(ii) Robustness of complex networks.

Scale-free networks are more robust under random errors, but very vulnerable under intentional attacks!

Scale-free Networks are efficient/robust.

Points:

(1)Vulnerability ([robustness]$^{-1}$) is predicated on:

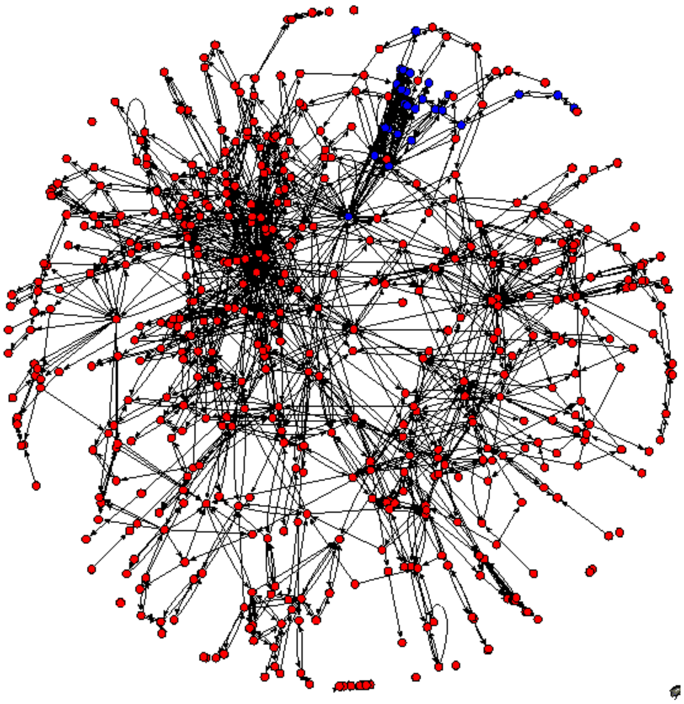
(a) Architecture of network

(b) Type of attack.

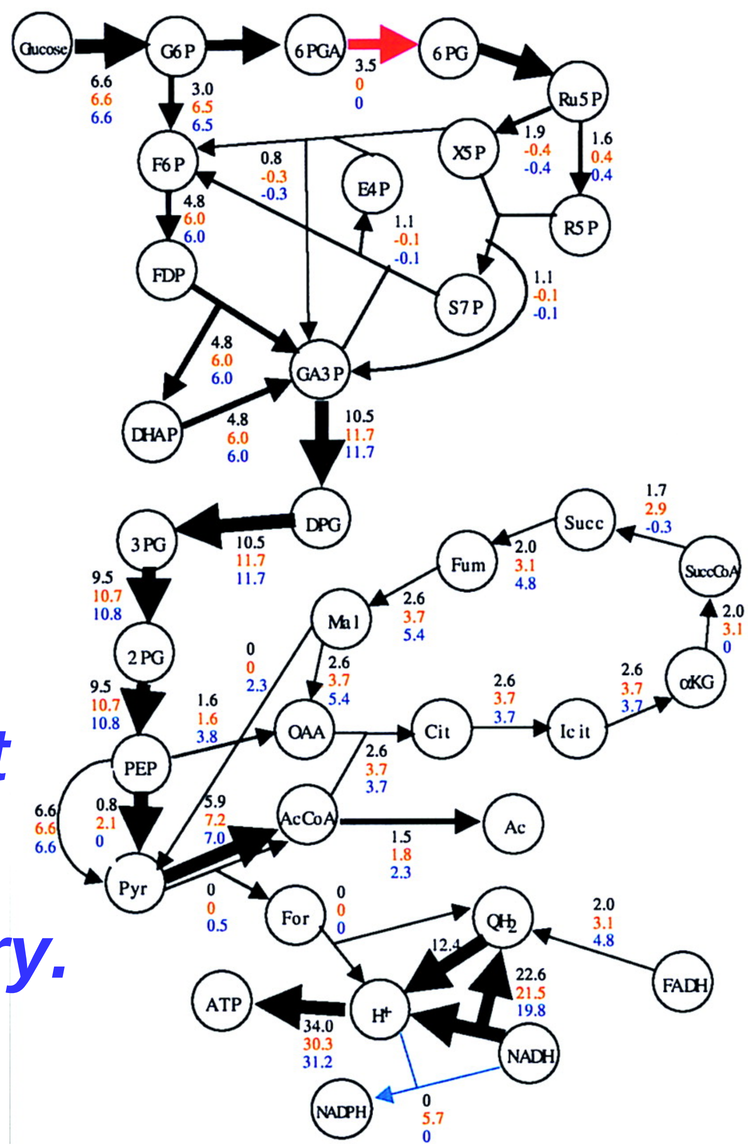
What is the Real Problem?

Area 4

Most networks are not static, they're dynamic!



e.g. real metabolic networks are DYNAMIC!!

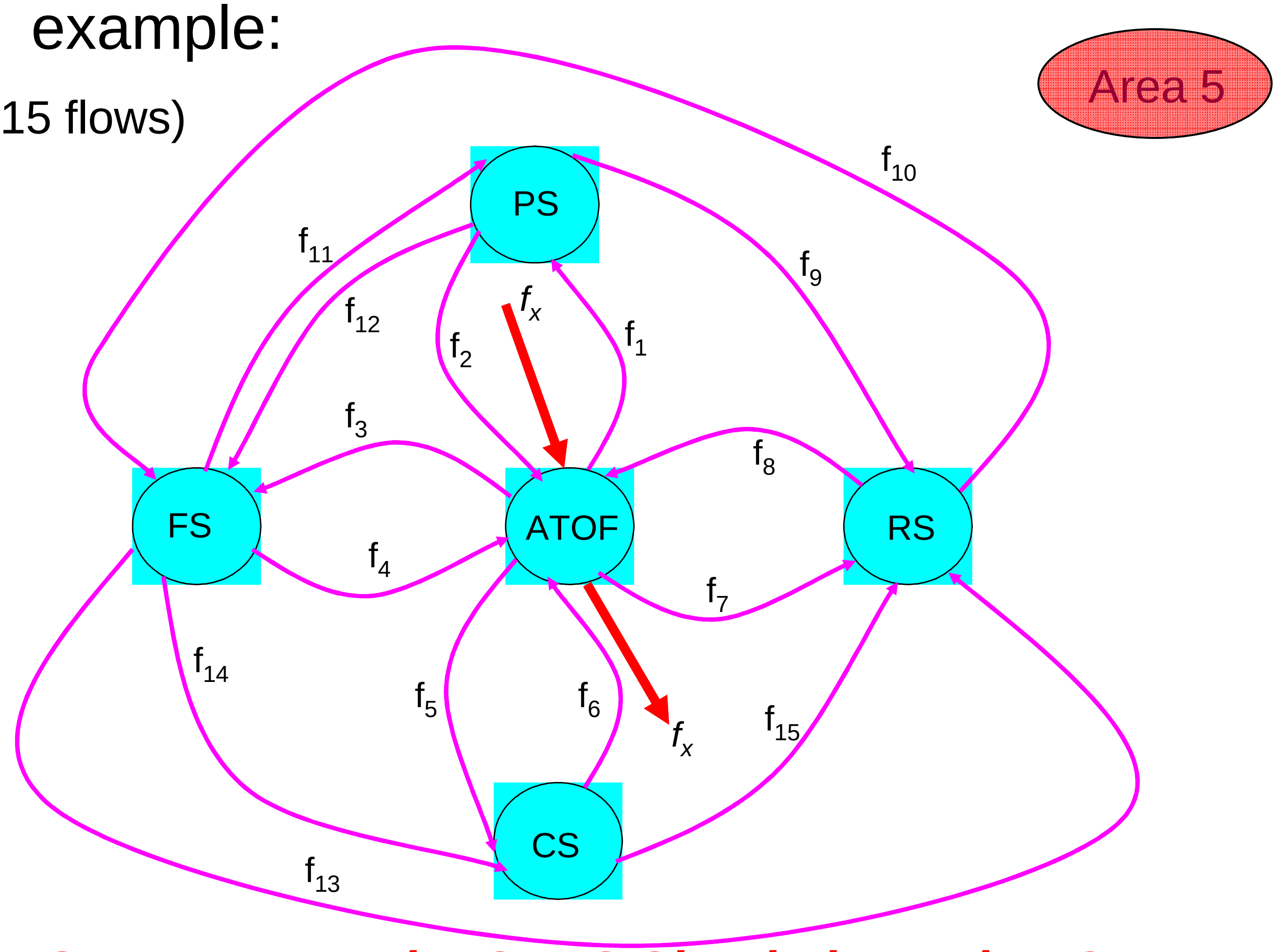


Let us stop with Graph Theory and move on to the last area

Optimization Theory.

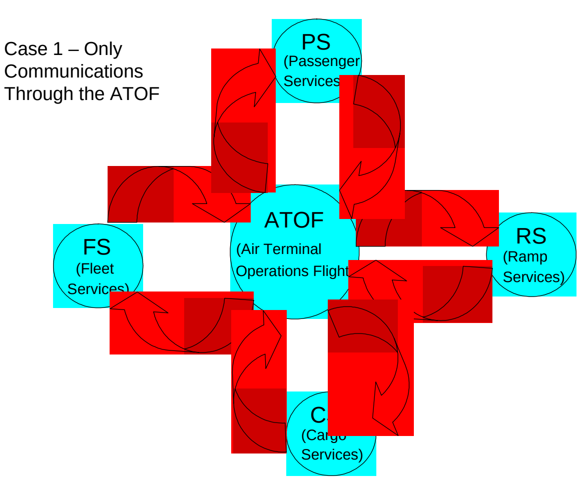
Part 1-C – Let us work a practical example:

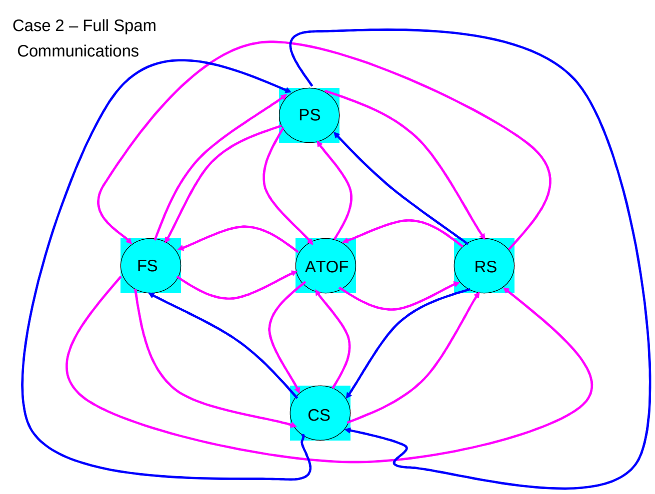
(15 flows)



Structure For the CAPS Simulation using GAs

Minimum (8 links) Maximum (20 links)



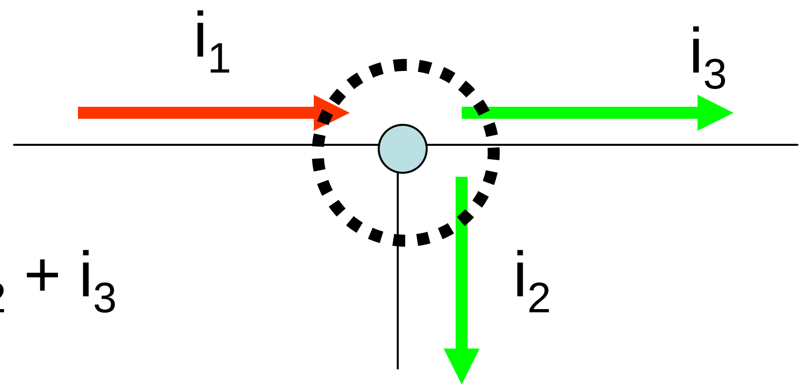


Part 1-C – Issues of Vulnerability and Performance

Area 5

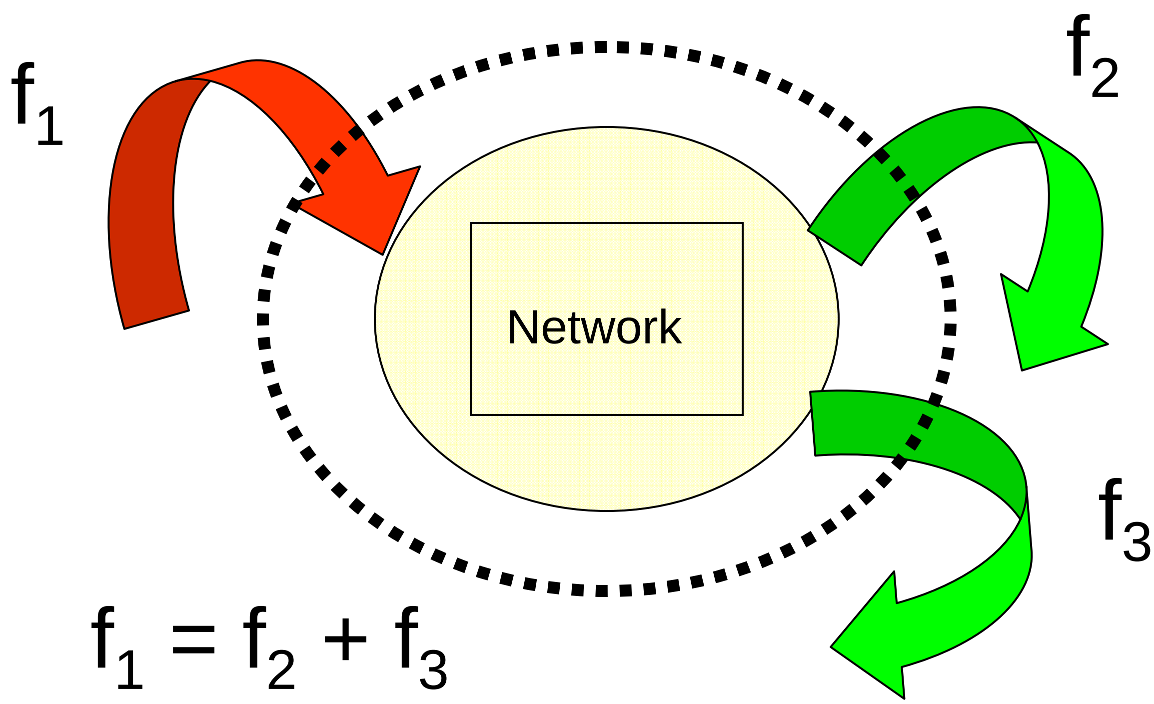
Kirchhoff’s Law and Cut sets

Σ Currents = 0 into a node.



$i_1 = i_2 + i_3$

Kirchoff’s Law also applies in Graph Theory



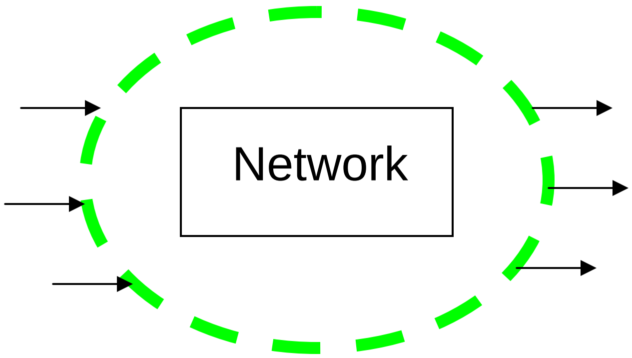
$f_1 = f_2 + f_3$

Part 1-C – Issues of Vulnerability and Performanc

Kirchhoff's Law and Cut sets

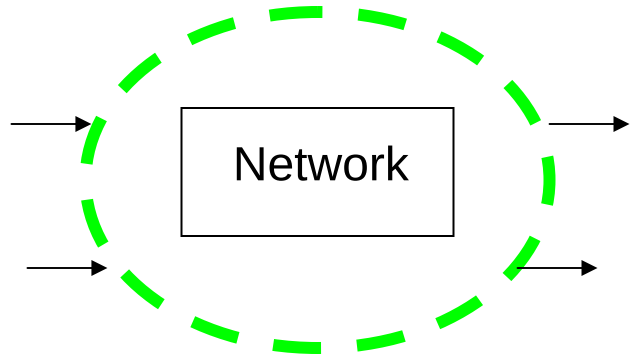
Area 5

Maximum Flow



Cut set: flows in = flows out
= 10 units

Minimal Flow



Cut set: flows in = flows out
= 1 unit

ATOF: $f_x + f_2 + f_4 + f_6 + f_8 = f_1 + f_3 + f_5 + f_7 + f_x$

PS: $f_{11} + f_1 = f_9 + f_2 + f_{12}$

RS: $f_{15} + f_7 + f_9 + f_{13} = f_{10} + f_8$

FS: $f_{10} + f_{12} + f_3 = f_{14} + f_{13} + f_4 + f_{11}$

CS: $f_5 + f_{14} = f_{15} + f_6$

Sensitivity = $$S_W^{\,T} := \lim_{\Delta W \to 0} \frac{\frac{\Delta T}{T}}{\frac{\Delta W}{W}} = \frac{\partial T}{\partial W}\frac{W}{T} \qquad (T \neq 0)$$

Let T = cut set flow, let W be the MI = I(x;y).

Area 5

$j = 1, \ldots, 11$ free chromosomes

3 bit word for each chromosome.

$j = 1$	0	1	1
$j = 2$	0	0	1

. . .

$j = 11$	1	0	1

(8^{11} possibilities, NP Hard)

Fig. 9 Configuration for the Chromosome

Area 5

How the Optimization is Conducted (Elite Pool)

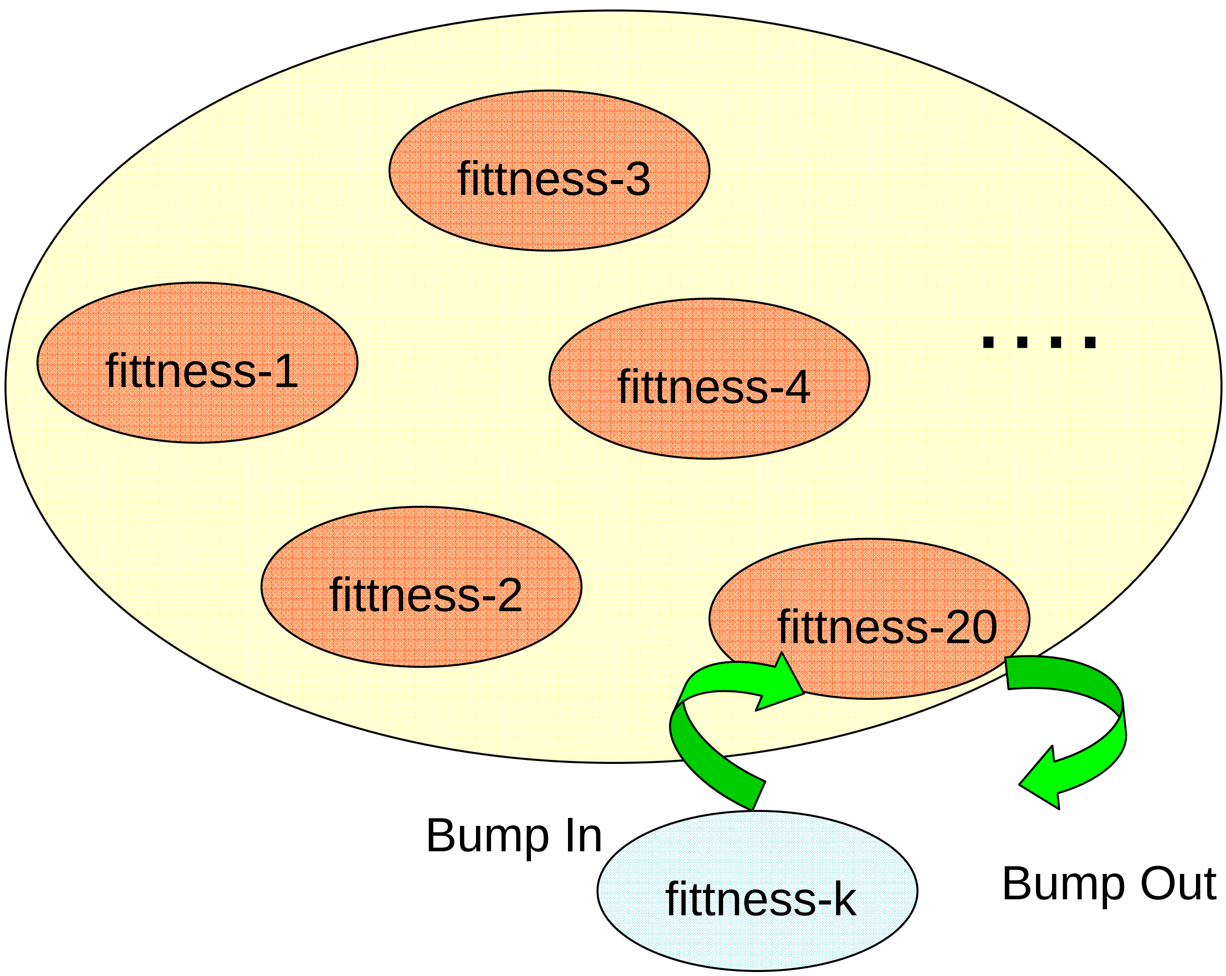
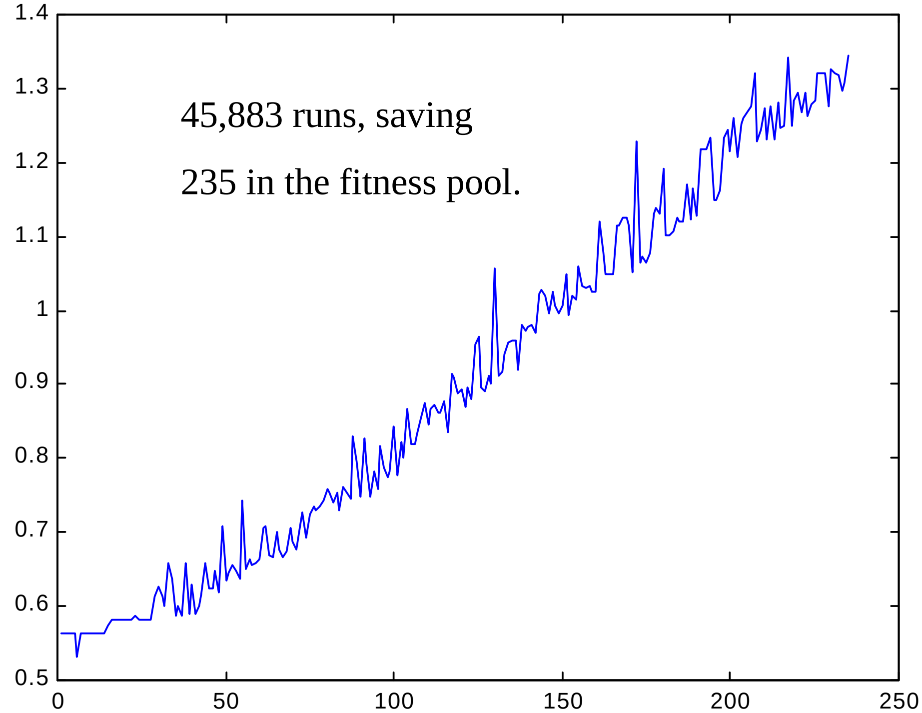
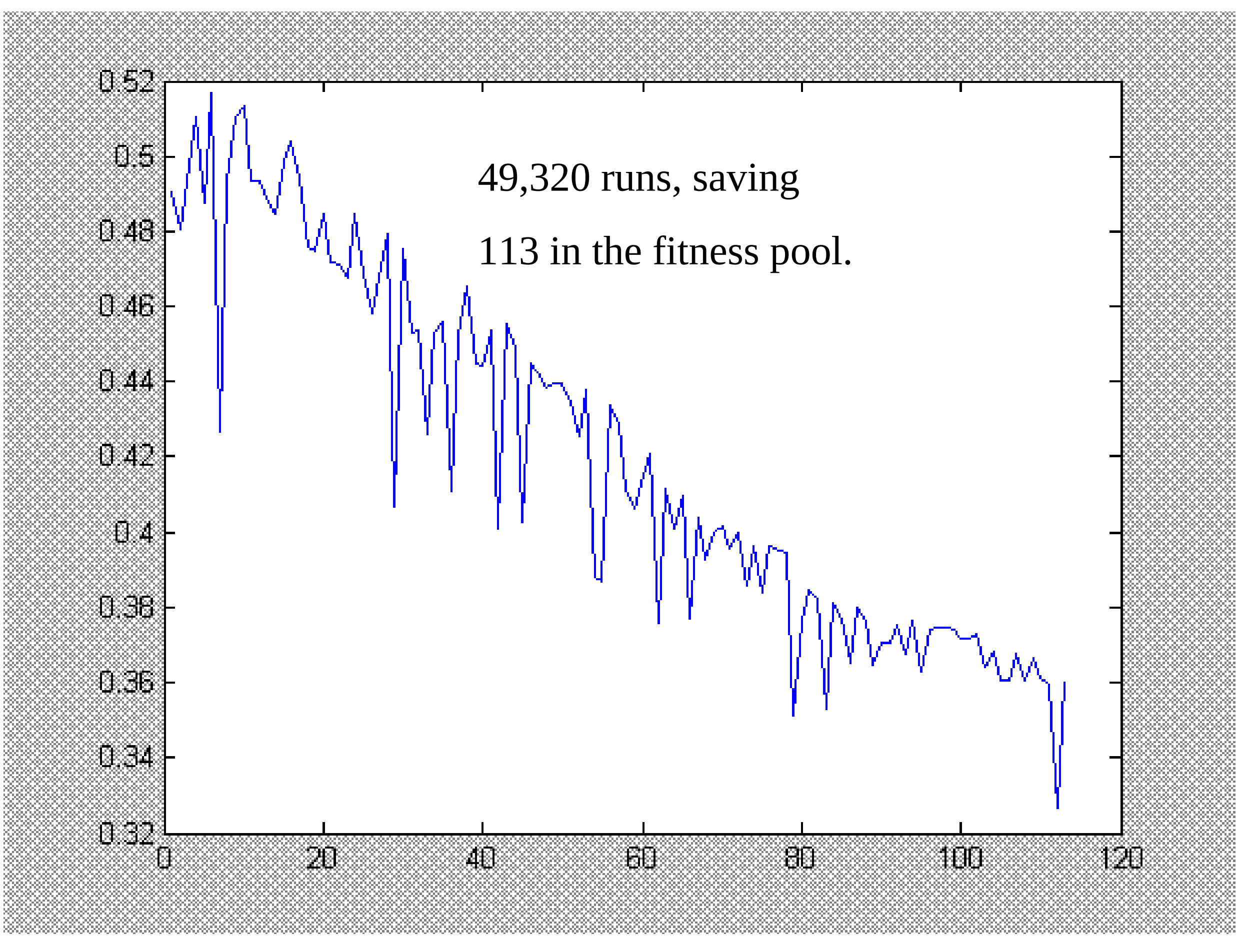


Fig. 10 – Maximizing (I(x;y)) vs. Pool Entrance Number



Area 5

Fig. 11 – I(x;y) Minimization vs. Pool Entrance Number



Sensitivity Results – Logistics Problem

Area 5

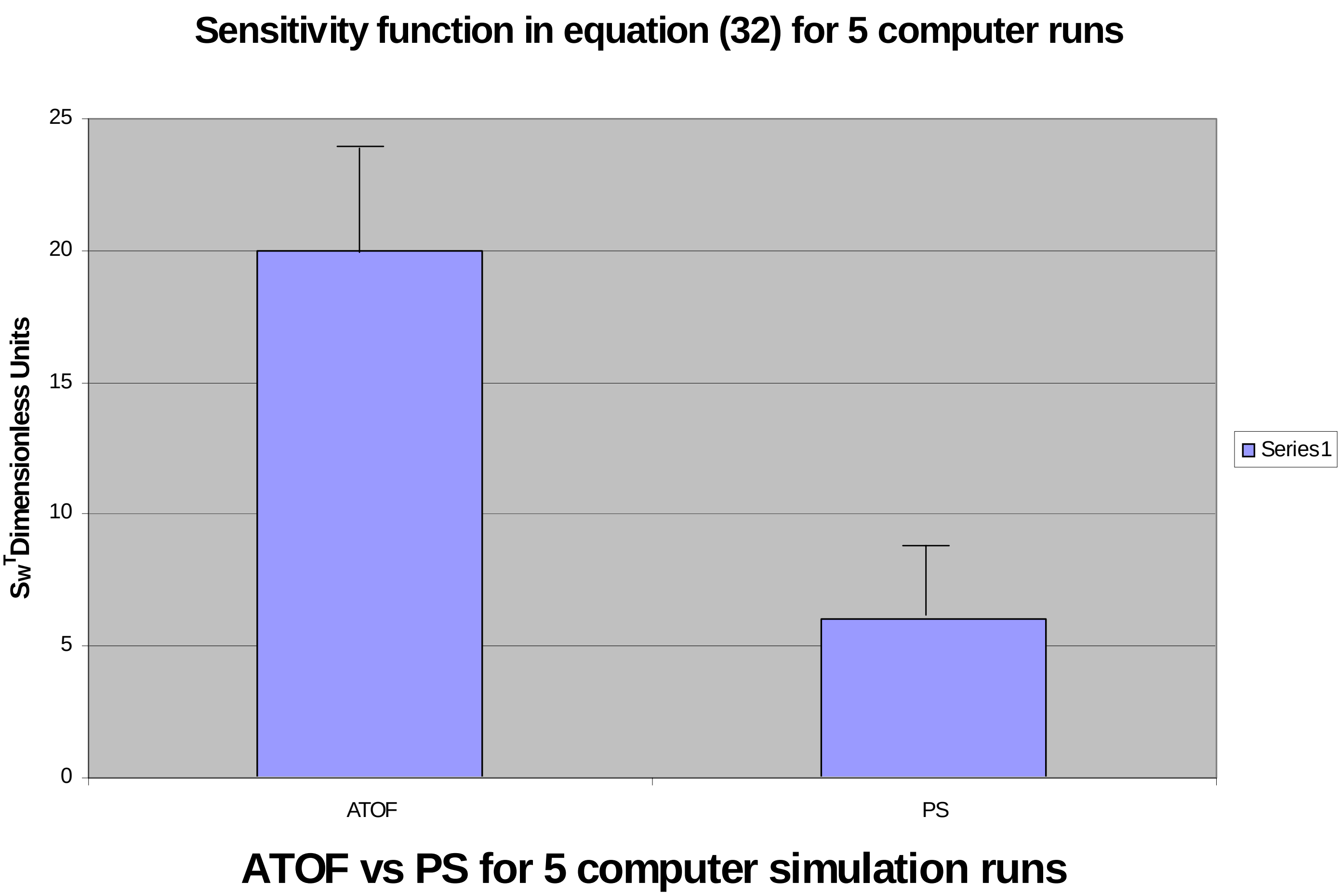
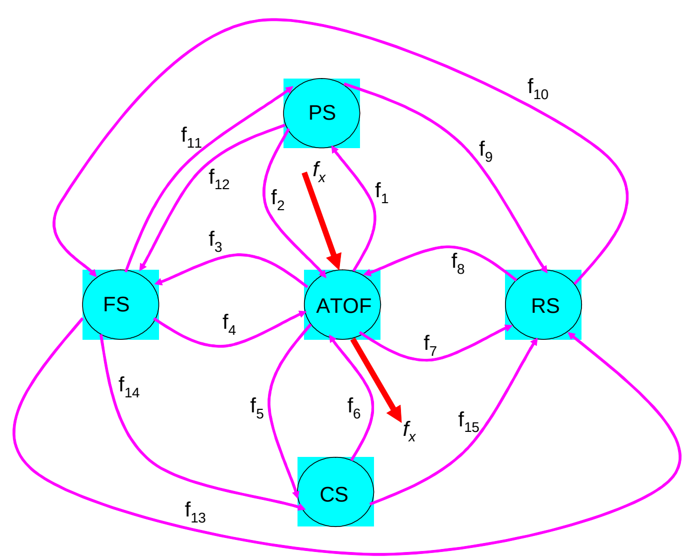


Figure (12) –The sensitivity Function defined in equation (32) for ATOF vs PS

Simulation is

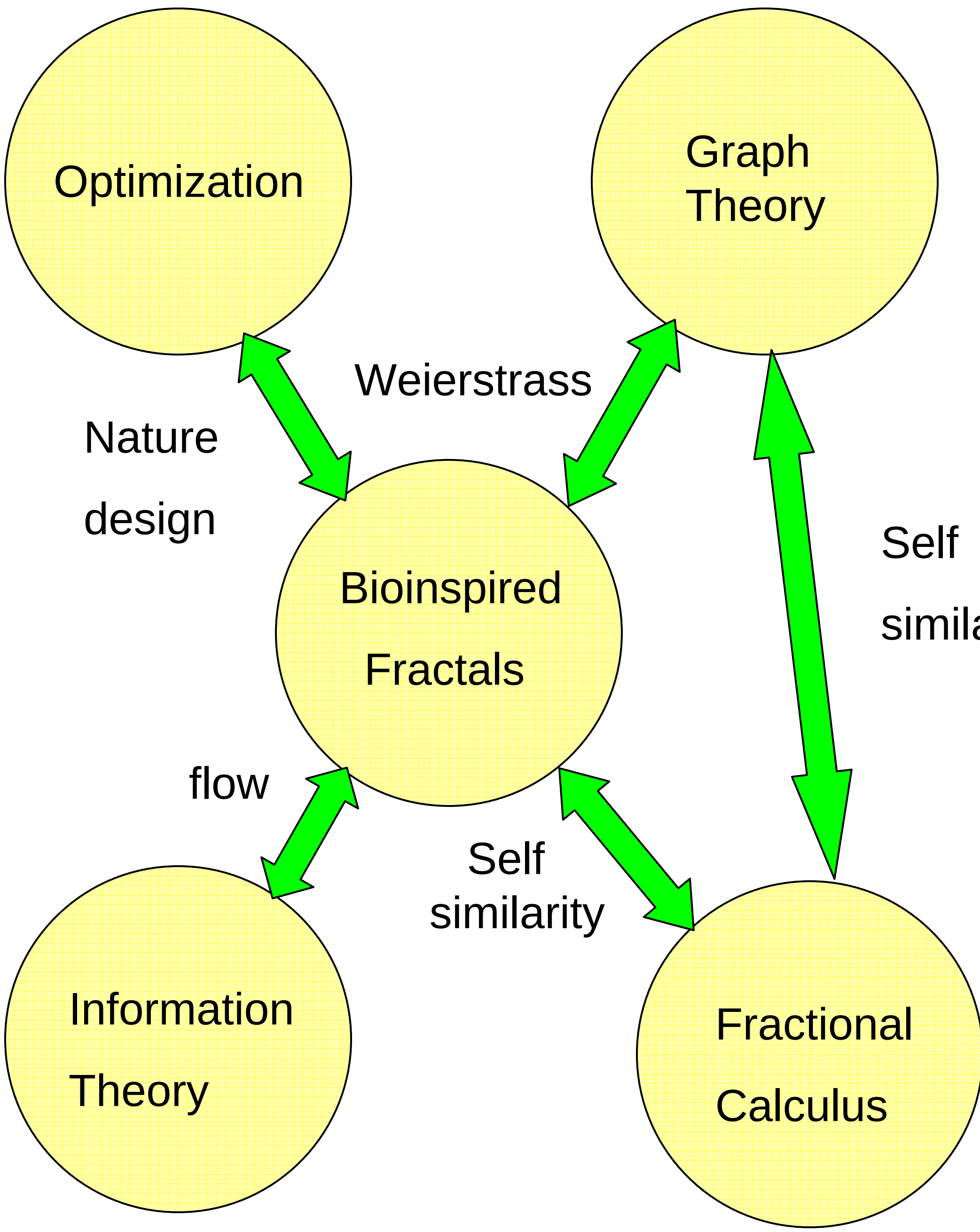
Sometimes termed

"Experimental Mathematics"



Other Common Intersections

Causality Map



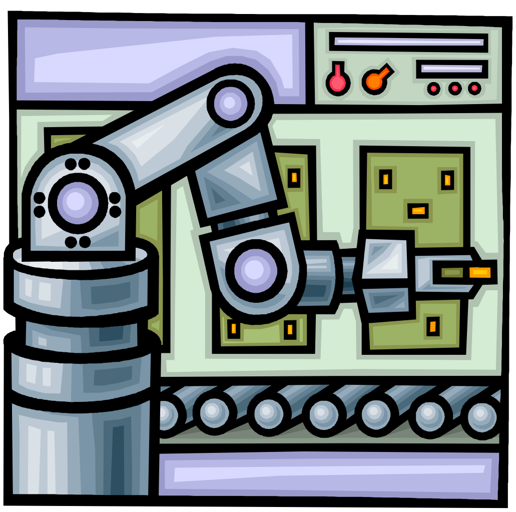
Part D -What is the solution in a theoretical sense?

. Bioinspired $\Rightarrow$ Perhaps we should not think Euclidean?

. Fractional Calculus may capture dynamics.

. Here may be a hypothesized solution?

Robotics



Network Science

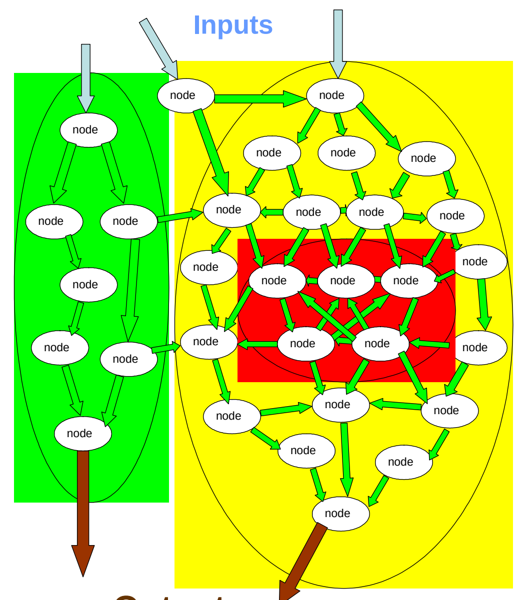


Figure 3 – The Original Network-Centric Distributed System

Minimize (J_1)

Minimize/Maximize (I(x;y))

Subject to constraints:

$$\dot{x} = J\dot{\theta}$$

Subject to Constraints:

$$\Sigma f_i = 0$$

$$\frac{d^{5/2}(\alpha y)}{d(\alpha t)^{5/2}} + \frac{d^{3/2}(\alpha y)}{d(\alpha t)^{3/2}} + \frac{d^{1/2}(\alpha y)}{d(\alpha t)^{1/2}} = \frac{d^{3/2}(\alpha u)}{d(\alpha t)^{3/2}} + \frac{d^{1/2}(\alpha u)}{d(\alpha t)^{1/2}}$$

End of Part I – Quantitative Biofractal Feedback

. Performance and vulnerability of distributed systems needs to be objectively quantified.

. We can learn from biological systems (fractals). Also the fractional calculus may offer a venue to characterize dynamics.

. There are many common connections between five different areas. For example, the diffusion equation is bioinspired.

. Computational methods allow us to synthesize a brute force approach for insight.

. Much more work needs to be accomplished.

Part II – Brief Review of QFT

. Quantitative Feedback Theory originated in the 1960's by Isaac Horowitz using frequency domain methods for efficient robust control design. In 1972 a seminal paper was published.

. QFT has been used in Flight Control, Robotics, Power Systems, unmanned air vehicles, and many other applications.

. The controller is determined by a loop shaping process employing a Nichols' Chart that displays the stability, performance and disturbance rejection bands.

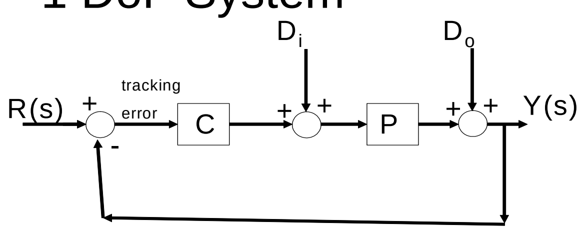
. A typical QFT Controller (synthesis) satisfies certain attributes:

(a) Robust Stability.

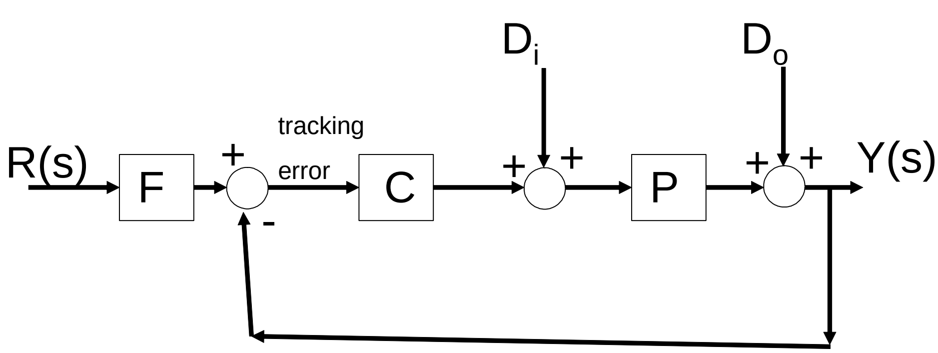
(b) Reference Tracking.

(c) Disturbance Rejection.

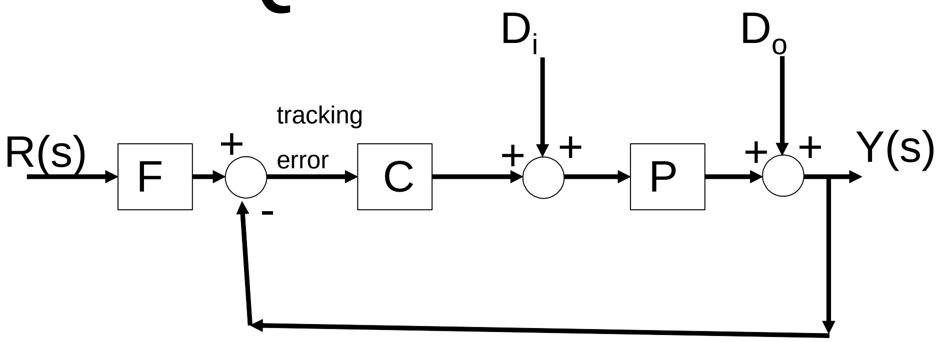
1 DoF System



2 DoF System



QFT Basics



In the Absence of Disturbances D_i and D_o:

Let: L = Loop Gain: $L = C\,P$

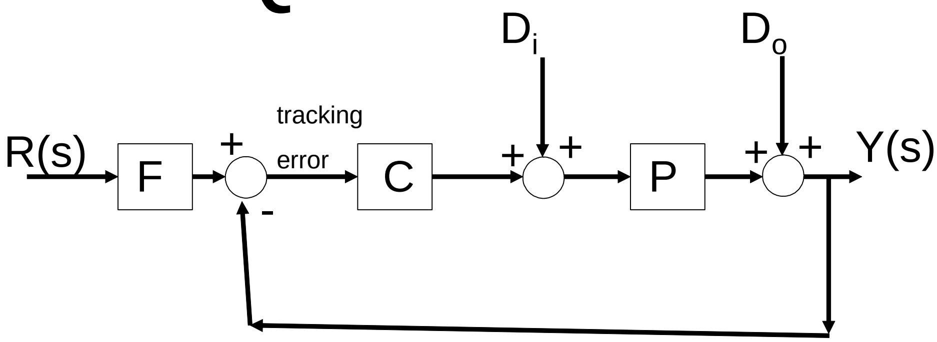
Then the closed loop transfer function between Y and R is:

$$\frac{Y(s)}{R(s)} = T(s) = \frac{F(s)L(s)}{1+L(s)} = \frac{Output}{Input}$$

The Sensitivity of The Closed Loop Transfer Function *T(s)* to plant variations *P(s)* can be specified via:

$$S(s) = \frac{\frac{\partial T}{T}}{\frac{\partial P}{P}} = \frac{1}{1+L(s)}$$

QFT Basics



For QFT Design, we have at least 3 criteria to meet:

(1) Robust Stability (closed loop Robust Stability)

$$\left|\frac{L(s)}{1+L(s)}\right| \leq \gamma$$

$\Rightarrow$ This is a constraint on the peak magnitude of the closed loop frequency response.

(2) Reference Tracking. Let T_L and T_U be the upper and lower transfer functions, then we require:

$$|T_L(j\omega)| \leq |\, T(j\,\omega)\,| \leq T_U(j\omega)|$$

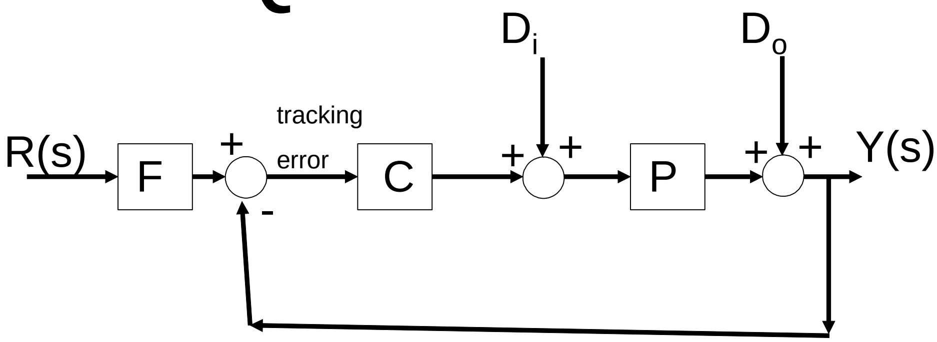
(3) Disturbance Rejection: We require:

$$\left|\frac{1}{1+L(j\omega)}\right| \leq \frac{1}{W(j\omega)}$$

Where $W(j\omega)$ is a weighting function (of frequency).

Note conditions (1-3) are for the class of plants P ε $\{P_i\}$

QFT Basics



For the Disturbances D_i and D_o

The Transfer Function between D_i and Y is given by:

$$T_{di} = \frac{Y(j\omega)}{D_i(j\omega)} = \frac{P(j\omega)}{1+L(j\omega)}$$

The Transfer Function between D_o and Y is given by:

$$T_{do} = \frac{Y(j\omega)}{D_o(j\omega)} = \frac{1}{1+L(j\omega)}$$

Then the Disturbance Rejection Can Be Specified via:

$$|T_{di}| \le B_{di} \qquad |T_{do}| \le B_{do}$$

Where the B_{di} and B_{do} are frequency dependent functions.

Some References Selected from the QFT Area

(from 164 hits in IEEE Explore, and other sources)

1. I. M. Horowitz, "Synthesis of Feedback Systems with Nonlinear Time-varying Uncertain Plants to Satisfy Quantitative Performance Specifications," *IEEE Proc*., **64,** 1976, pp.123-130.
2. I. M. Horowitz, "Feedback Systems with Nonlinear Uncertain Plants," *Int. J. Control*, **36**, pp. 155-171, 1982.
3. D. E. Bossert, G. B. Lamont, M. B. Leahy, and I. M. Horowitz, "Model-Based Control with Quantitative Feedback Theory," *Proceedings of the 29th IEEE Conference on Decision and Control*, 1990, pp. 2058-2063.
4. D. G. Wheaton, I. M. Horowitz, and C. H. Houpis, "Robust Discrete Controller Design for an Unmanned Research Vehicle (URV) Using Discrete Quantitative Feedback Theory," *1991 NAECON*, May, pp. 546-552.
5. C. H. Houpis, and P. R. Chandler, "*Quantitative Feedback Theory Symposium Proceedings*," WL-TR-92-3063, August, 1992.
6. I. M. Horowitz, *Quantitative Feedback Design Theory (QFT)*, **vol. 1**, QFT Publications, 1993.
7. S. G. Breslin and M. J. Grimble, "Longitudinal Control of an Advanced Combat Aircraft using Quantitative Feedback Theory," *Proceedings of the 1997 ACC*, pp. 113-117.
8. D. S. Desanj and M. J. Grimble, "Design of a a Marine Autopilot using Quantitative Feedback Theory," *Proceedings of the ACC*, 1998, pp. 384-388.

Some References Selected from the QFT Area

(from 164 hits in IEEE Explore, and other sources)

9. R. L. Ewing, J. W. Hines, G. D. Peterson, and M. Rubeiz, "VHDL-AMS Design for Flight Control Systems," *Proceedings of the IEEE 1998 Aerospace Conference*, March, pp. 223-229.

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QFT

Applications

Part III – The Diffusion Equation

Why?

(1) Many biological systems can be characterized in this manner.

(2) Outside biology, diffusion is a fundamental process (thermal chemical, other physical processes of all types).

(3) The diffusion equation satisfies a fractional differential equation.

(4) The diffusion equation is also a type of fractal.

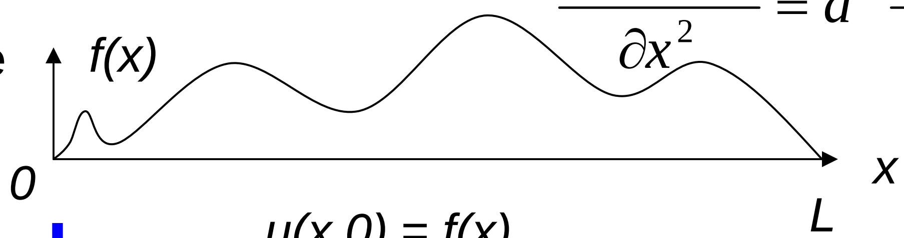
<u>Consider the following physical problem:</u>

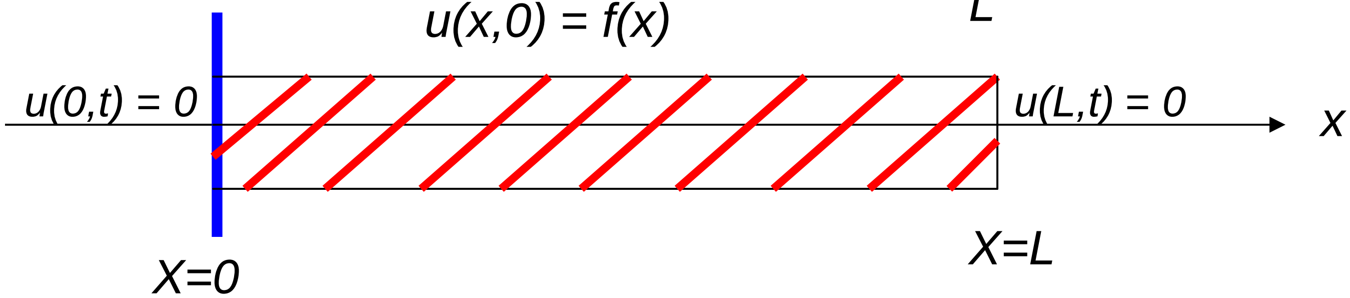
Let *u(x,t)* be the temperature distribution in a cylindrical bar of finite length L oriented along the x-axis and perfectly insulated laterally. We assume heat flow in only the x axis direction. The temperature *u(x,t)* satisfies:

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$

Where

$$a^2 = \frac{c\delta}{k}$$





and *k* is the thermal conductivity, *c* is the specific heat and δ is the linear density (mass/unit length).

The initial condition is: *u(x,0) = f(x)*

The boundary conditions are: *u(0,t) = 0 = u(L,t)* $\forall$ t

Part III – The Diffusion Equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$

u(0,t) = 0 = u(L,t) *Boundary Conditions*

u(x,0) = f(x) *Initial Condition*

Possible ways to solve the equation:

(1) Fourier Method – Separation of Variables.

(2) Laplace Transforms.

(3) Fractional Calculus.

Now examine Robustness via Quantitative Feedback Theory

Part III – The Diffusion Equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$ Initial Condition *u(x,0) = f(x)*

Boundary Conditions: *u(0,t) = 0 = u(L,t)*

(1) Fourier Method – Separation of Variables.

Assume $u(x,t) = X(x)T(t)$

$\Rightarrow$ $$a^2 X(x)\dot{T}(t) = T(t)X''(x)$$

$\Rightarrow$ $$\frac{a^2\dot{T}(t)}{T(t)} = \frac{X''(x)}{X(x)} =$$ constant = $-\lambda$

$\Rightarrow$ $\dot{T}(t) = -(\lambda / a^2)T(t)$ $\Rightarrow$ $T(t) = Ae^{-(\lambda/a^2)t}$

and $X''(x) = -\lambda X(x)$

$\Rightarrow$ $X(x) = B\sin(\sqrt{\lambda}x) + \cancel{C\cos(\sqrt{\lambda}x)}$ but *u(0,t)= 0* $\Rightarrow$ *C*=0

$\Rightarrow$ $u_i(x,t) = T_i(t)X_i(x)$

and $u(x,t) = \sum u_i(x,t)$

Note: $\sqrt{\lambda} = \frac{n\pi}{L}$

because u(L,t)=0 $\forall$ t

$\Rightarrow$ $$u(x,t) = \sum_{n=1}^{\infty} D_n(\sin(\frac{n\pi x}{L})\, e^{-\frac{(n^2\pi^2 t)}{(a^2L^2)}})$$

$$Dn = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

(Can show the infinite series converges)

Part III – The Diffusion Equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}$$ Initial Condition *u(x,0) = f(x)*

u(x,t) bounded, t > 0, -∞ < x < ∞

(2) Laplace Transforms.

Define the Laplace Transform Variable: $U(x,s) = \int_0^\infty e^{-ts} u(x,t)\,dt$

$$\Rightarrow \quad \int_0^\infty e^{-ts} \frac{\partial u}{\partial t} dt = sU(x,s) - u(x,0) \quad = s\,U(x,s) - f(x)$$

If $\frac{\partial u}{\partial x}$ and $\frac{\partial^2 u}{\partial x^2}$ are bounded and continuous

$$\int_0^\infty e^{-st} \frac{\partial^2 u}{\partial x^2} dt = \frac{\partial^2 U}{\partial x^2}$$

Now Laplace transform the partial differential equation

$$0 = L\left[\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2}\right] = sU(x,s) - f(x) - \frac{\partial^2 U}{\partial x^2}$$ Forcing Function

and solve for *U(x,s)*:

$$U(x,s) = \frac{1}{2\sqrt{s}} \int_{-\infty}^{+\infty} e^{-\sqrt{s}|x-y|} f(y)\,dy$$

To find *u(x,t)*, we need to find the inverse Laplace transform

$$u(x,t) = L^{-1}[\,U(x,s)\,]$$

or

$$u(x,t) = L^{-1}\left[\frac{1}{2\sqrt{s}} \int_{-\infty}^{+\infty} e^{-\sqrt{s}|x-y|} f(y)\,dy\right]$$

By integration in the complex plane we can show:

$$u(x,t) = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-(x-y)^2/4t} f(y)\,dy$$

Part III – The Diffusion Equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$

Initial Condition $u(x,0) = f(x)$

Boundary Conditions: $u(0,t) = 0 = u(L,t)$

(Heaviside Operational Calculus)

Consider: $$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial u}{\partial t}$$

Let $p = \frac{\partial}{\partial t}$ $\Rightarrow$ $\frac{\partial^2 u}{\partial x^2} = a^2 pu$

(treat *p* as a constant and solve for *x)*

$$\Rightarrow \quad u(x,t) = Ae^{-ap^{1/2}x} + Be^{ap^{1/2}x}$$

On physical grounds, *B* = 0

$$\Rightarrow \quad u_i(x,t) = e^{-axp^{1/2}} u_0$$

Or: $u(x,t) = u_0 + \sum_{n=1}^{\infty} \frac{(-ax)^n}{n!} p^{n/2} u_0$

(can ignore positive integral powers of p)

$$\Rightarrow \quad u(x,t) = u_0 - \frac{2u_0}{\sqrt{\pi}} \int_0^{\frac{ax}{2\sqrt{t}}} e^{-\xi^2} d\xi$$

Part III – The Diffusion Equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial u(x,t)}{\partial t}$$

Initial Condition $u(x,0) = f(x)$

Boundary Conditions: $u(0,t) = 0 = u(L,t)$

Now examine Robustness via Quantitative Feedback Theory

Step 1: Let us examine a heat control problem.

(Define units of all quantities to generalize.)

Step 2: Let us build a controller within a QFT context.

Step 3: We have now solved a heat control problem. Now generalize to flow problems as in networks.

Again look at the units of all variables.

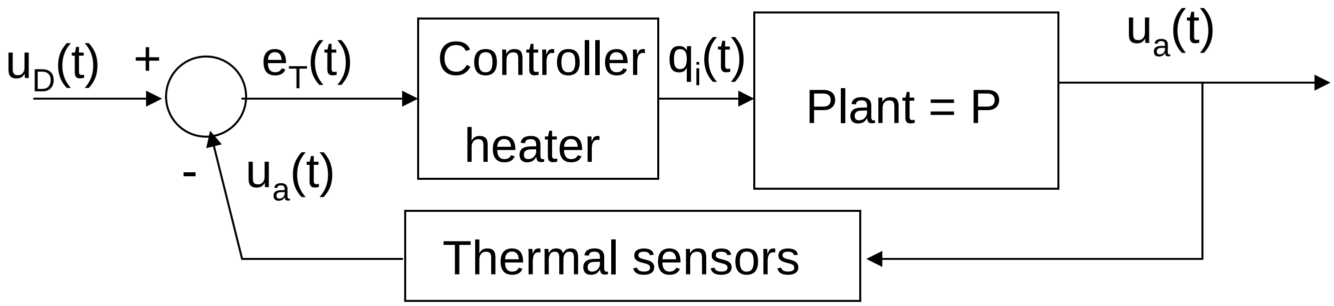
Part III

Step 1: Let us examine a heat control problem.

Let $u_{desired}(x,t)$ = desired temperature = $u_D(t)$ (assume x=const).

Let $u_{actual}(x,t)$ = actual temperature = $u_a(t)$

Temperature error $e_T(t) = u_D(t) - u_a(t)$



Units Analysis: $u_i(t)$ = temperature - C^o

C = Thermal Capacitance = kilo cal / C^o

q(t) = heat input – kilo cal / second

R_T = Thermal Resistance – C^o sec / kilo cal

Then: $C\dfrac{du_a}{dt} = q_i - q_0$ Where: $q_0 = \dfrac{u_a}{R_T}$

$$C\frac{du_a}{dt} + q_0 = q_i$$

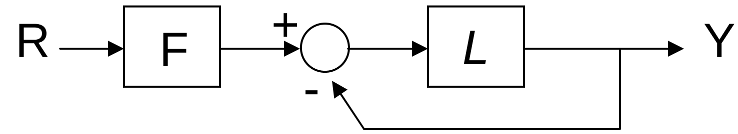
$$C\frac{du_a}{dt} + \frac{u_a}{R_T} = q_i$$

$$R_T C\frac{du_a}{dt} + u_a = R_T q_i$$

$$\frac{U_a(s)}{Q_i(s)} = \frac{R_T}{1 + R_T Cs}$$

Part III

Step 2: Let us build a controller within a QFT context



QFT Goals:

(1) Stability $T(s) = \dfrac{L}{1+L}$ is stable. L = G P

(2) Tracking Specifications

$|T_L(j\omega)| \le |F(j\omega)\, T(j\omega)| \le |T_u(j\omega)| \Rightarrow$ use F for prefilter.

(3) Disturbance Rejection

max $|T_D(j\omega)| \le |M_D(j\omega)|$

$$T_{Di} = \frac{P}{1+L}$$

$$T_{D0} = \frac{1}{1+L}$$

QFT Design Procedure:

(a) Find the plant templates $P \in \{P_i\}$ – Nichols chart.

(b) Generate Performance Bounds from Nichols chart.

$L_0(s) = P_0(s)\, G(s)$

(c) Loop Shaping: Add poles and zeros to $L_0(s)$.

(d) Design Prefilter F (keep $|T_L| < | F\, T | < |T_U|$)

(e) Finally to determine the final controller

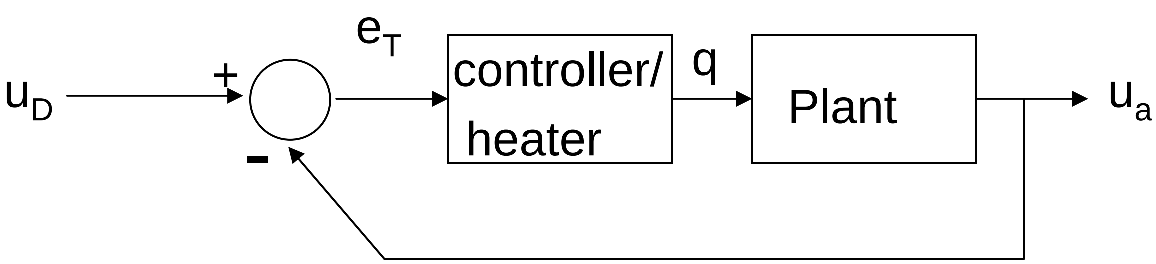
$$G(s) = \frac{\overline{L}_0(s)}{P(s)}$$

Done!

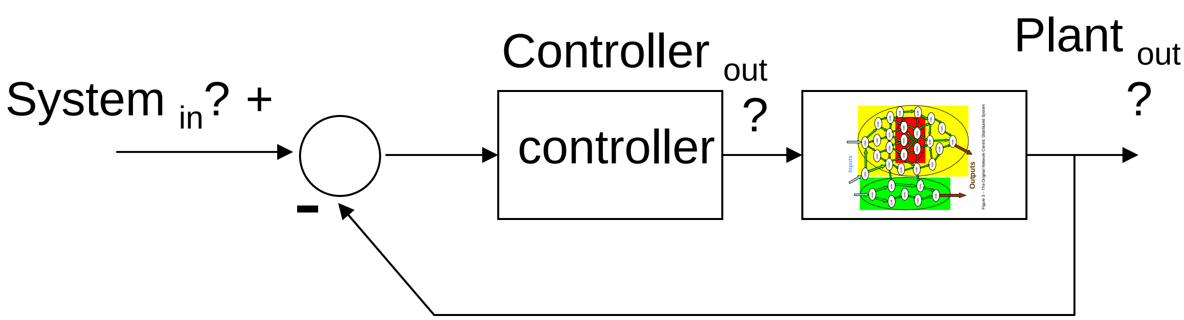
Part III

Step 3: We have now solved a ***heat control problem***. Now ***Generalize*** to flow problems as in networks.

Heat Control Problem:



The Network Flow Problem



Let us review the ***units*** of variables of interest:

Heat Control Problem

u_I units of (C^0)

q units of (kilo cal/sec)

C units of (kilo cal / C^0)

$$C\frac{du_a}{dt} = q_i - q_0$$

Network Flow

System $_{in}$?

Controller $_{out}$?

Plant $_{out}$?

Part III

Step 3: We have now solved a ***heat control problem***. Now ***Generalize*** to flow problems as in networks.

Suggestions:

Heat Control Problem – flow

q * time = kilo calories

Network Problem – flow

bits/ sec * seconds = bits

events/second * seconds = events

Equate the above variables (MI=q, events = kilo calories)

$$u = \frac{1}{C}\int q(\tau)d\tau$$

$$events = bits = \int (\text{mutual information})\, \text{dt}$$

Heat Control Problem

u_I units of (C^0)

q units of (kilo cal/sec)

C units of (kilo cal / C^0)

$$C\frac{du_a}{dt} = q_i - q_0$$

(Recall we ***modulated*** *MI* in the example)

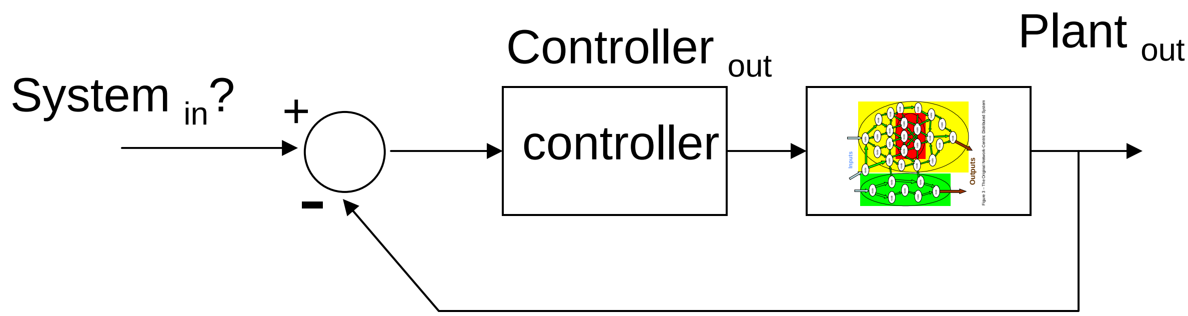
Network Flow

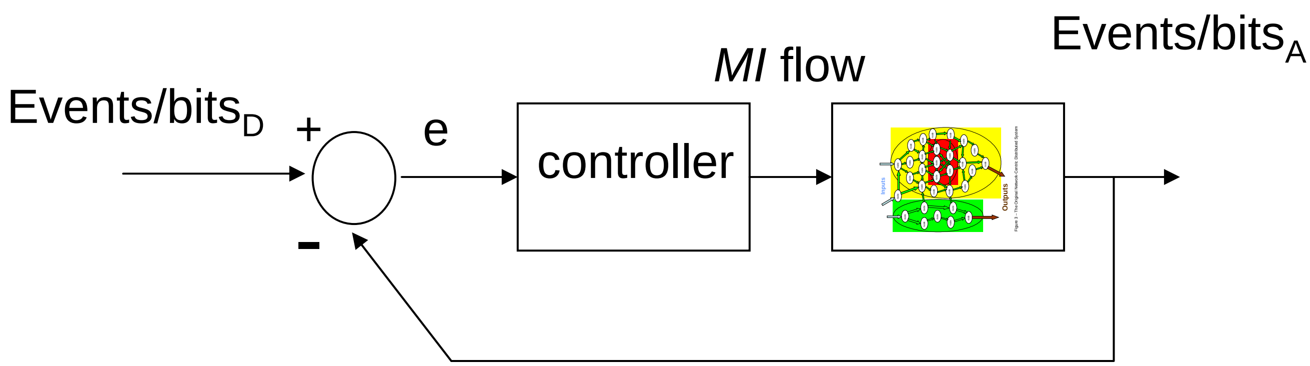
System $_{in}$? $= \int MI$

Controller $_{out}$? $= MI$

Plant $_{out}$? $= \int MI$

Part III





Network Flow

System $_{in}$? $= \int MI$

Controller $_{out}$? $= MI$

Plant $_{out}$? $= \int MI$

(Recall we ***modulated*** *MI* in the example)

Summary and Conclusions

Part I – Fractional Dimensions – non Euclidean World.

Part II – Quantitative Feedback Theory.

Part III – Diffusion Equation.

The Future - Modeling networks as control systems and applying these techniques. QFT helps because it can view robust control in terms of simple Bode/Nichols plots.