

# Beyond the Classical Performance Limitations Controlling Uncertain MIMO Systems: UAV Applications

1<sup>st</sup> Session

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| Report Documentation Page                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                    |                                     |                                                           | Form Approved<br>OMB No. 0704-0188                  |                                 |
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| 14. ABSTRACT                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     |                                                           |                                                     |                                 |
| 15. SUBJECT TERMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                    |                                     |                                                           |                                                     |                                 |
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# Outline

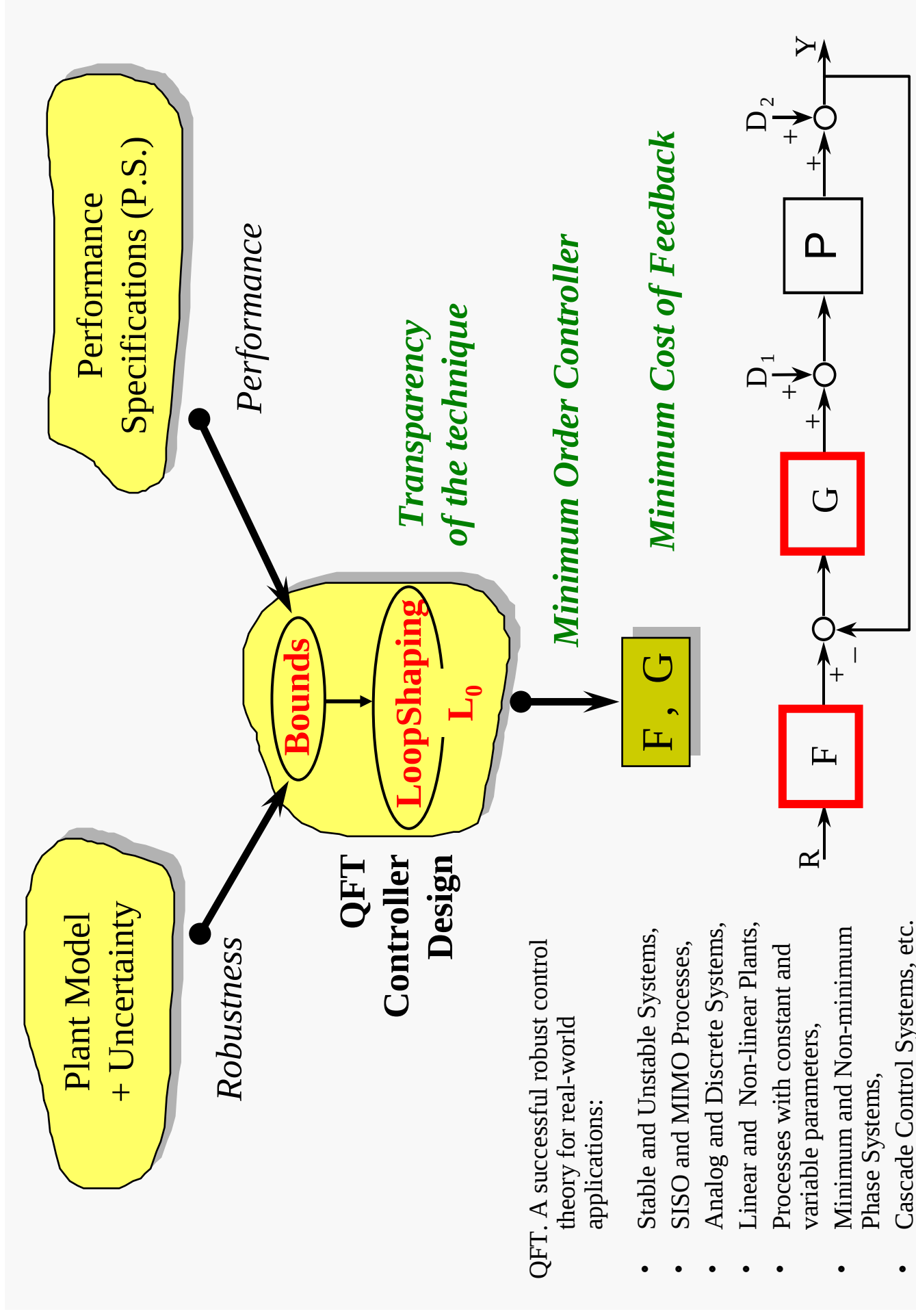
- 1.- QFT Controller Design Technique Fundamentals
- 2.- Real-world QFT control applications and examples
- 3.- Non-diagonal MIMO QFT controller design methodologies
- 4.- Application: Robust QFT control for a MIMO Spacecraft with flexible sunshield
- 5.- Switching robust control: Beyond the linear limitations.
- 6.- Example: Switching control for Unmanned Vehicles

# 1.- QFT Controller Design Technique Fundamentals

## 1.1.- Introduction

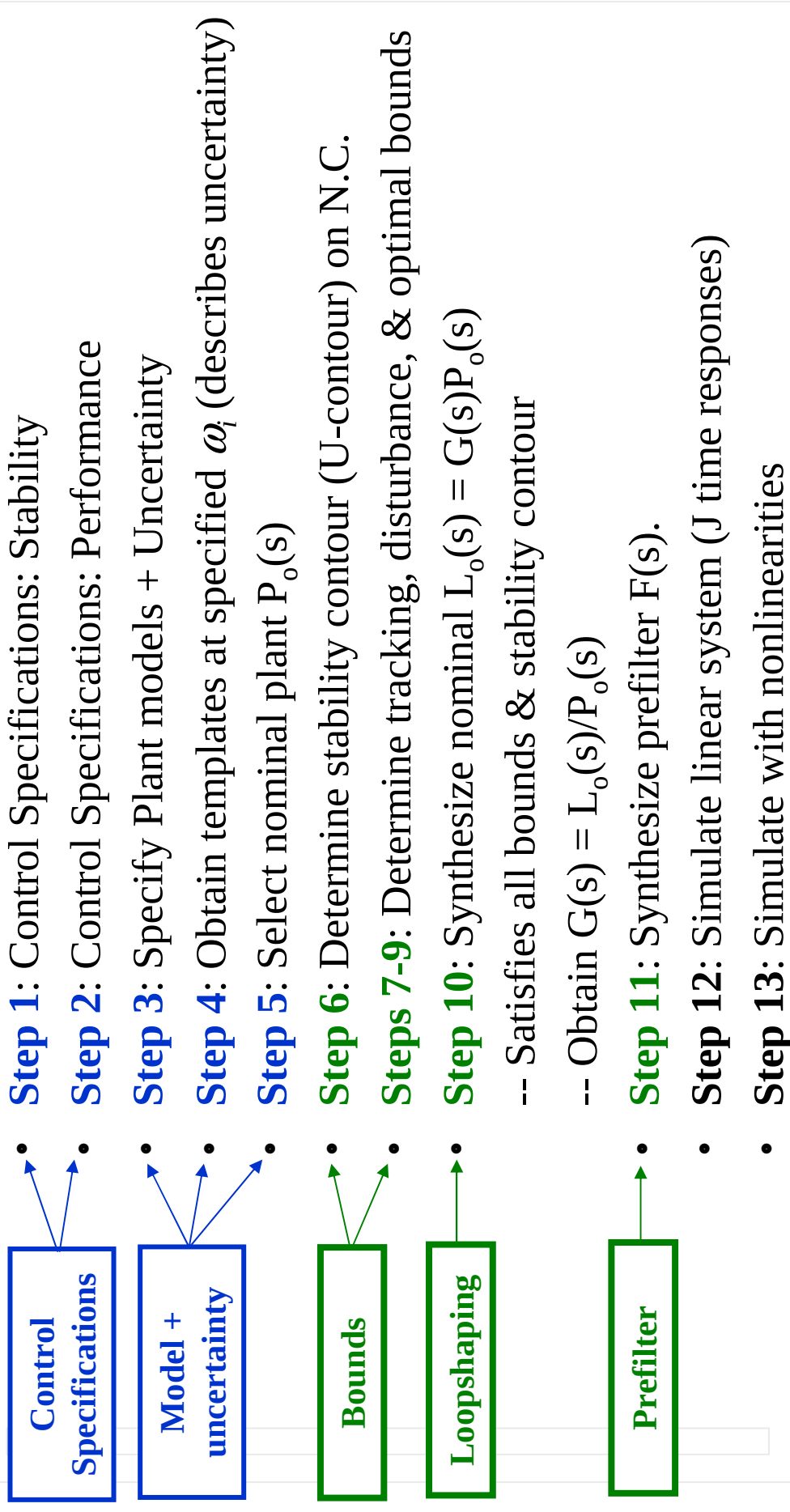
a reliable control design methodology

- Quantitative Feedback Theory (Q.F.T.). Introduced by Prof. Isaac Horowitz.
  - ➔ 1959. First ideas.
  - ➔ 1972. The name.
  - ➔ 1973. Horowitz at the Air Force Office of Scientific Research (first grant).
  - ➔ 1992. Prof. Houpis organizes the First International Symposium on QFT.
  - ➔ Until then, Int. Symposia every two years: 1995, 1997, 1999, 2001, 2003, 2005, 2007.
- The **QFT** design objective is to design and implement a robust control for a system with Uncertainty that satisfies the desired Performance Specifications.
- Frequency Domain Technique. Uses the Nichols Chart (NC).
- Looks for a design that combines:
  - ➔ Model + Parameter Uncertainty. (*Robustness*).
  - ➔ Performance Specifications.
  - ➔ Minimum Order Controller. Transparency of the Technique.
- Achieves reasonably low loop gains, i.e., avoids or minimizes: Sensor noise amplification, Saturation, High Frequency uncertainties.



## 1.2.- MISO analog control system design

### QFT Design Procedure



## Step 1, 2: Control Specifications: Stability and Performance

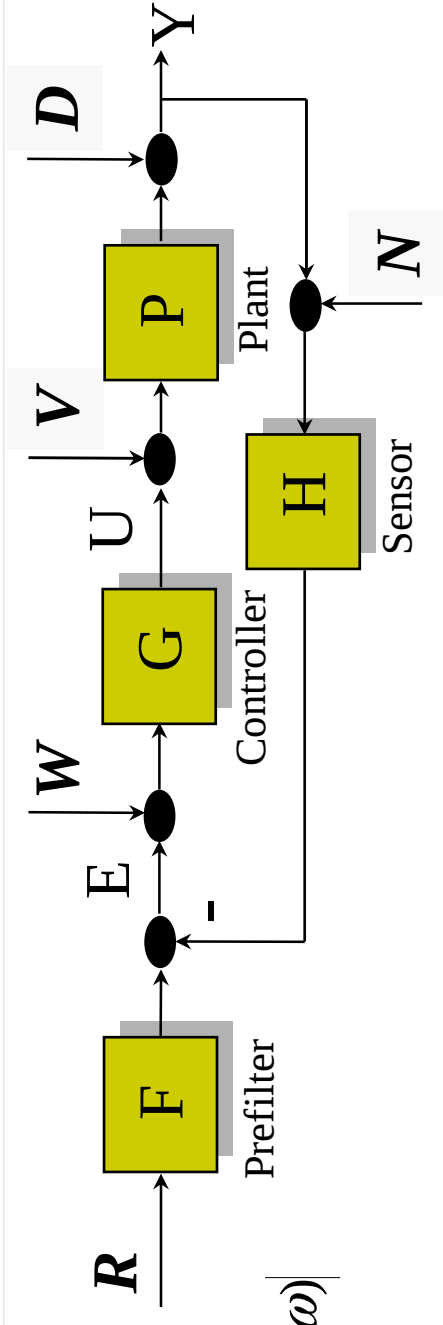
**$R$**  Reference  
 **$W$**  Controller Input Disturbance  
 **$V$**  Plant Input Disturbance  
 **$D$**  Plant Output Disturbance  
 **$N$**  Noise

Specifications

in terms of T.F.

For example:

$$\left| \frac{1}{1+PGH} \right| \leq |W_{s_1}(\omega)|$$



$$Y = \frac{1}{1+PGH} D + \frac{P}{1+PGH} V + \frac{PG}{1+PGH} (W+FR) - \frac{PGH}{1+PGH} N \quad (1)$$

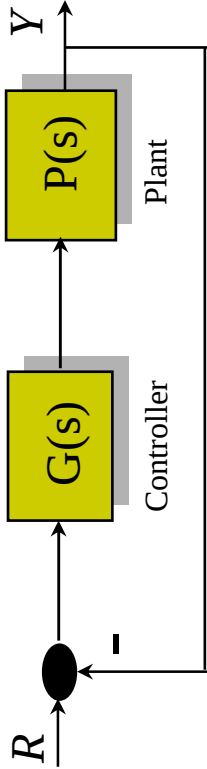
$$E = -\frac{H}{1+PGH} D + \frac{PH}{1+PGH} V + \frac{PGH}{1+PGH} W + \frac{1}{1+PGH} FR - \frac{H}{1+PGH} N \quad (2)$$

$$U = \frac{G}{1+PGH} (W+FR) - \frac{GH}{1+PGH} (N+D+PV) \quad (3)$$

# Stability

$$\left| \frac{PGH}{1+PGH} \right| \leq W_{s1}$$

The **Stability** (Gain and Phase Margins) is **related** with the Maximum closed-loop Resonance  $M_m$  specification.



Gain Margin:  $GM \geq 1 + 1/\mu$  (magnitude)

Phase Margin:  $PM \geq 180^\circ - \theta$  (deg)

where:  $\mu$  is the circle M specification in magnitude:  $M_m = 20 \log_{10}(\mu)$   
 $\theta = 2 \cos^{-1}(0.5/\mu) \in [0, 180^\circ]$

| $W_{s1}$      | GM            | PM    | Overshoot |
|---------------|---------------|-------|-----------|
| 1.1 (0.8 dB)  | 1.99 (5.9 dB) | 55°   | ~ 11 %    |
| 1.2 (1.58 dB) | 1.83 (5.2 dB) | 49°   | ~ 18 %    |
| 1.3 (2.28 dB) | 1.77 (5.0 dB) | 45°   | ~ 22 %    |
| 1.4 (2.9 dB)  | 1.71 (4.7 dB) | 41.8° | ~ 27 %    |

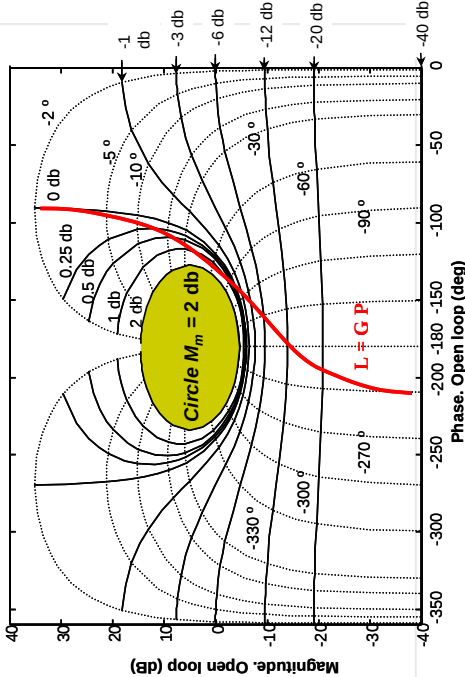
$$\frac{Y(j\omega)}{R(j\omega)} = T(j\omega) = \frac{P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)}$$

$$|T(j\omega)| \leq \mu = W_{s1}$$

$$L(j\omega) = P(j\omega)G(j\omega) = M e^{j\psi}$$

$$GM = 1/M \text{ at point } \psi = -180^\circ$$

$$PM = 180^\circ + \psi \text{ at point } |L(j\omega)| = 1$$

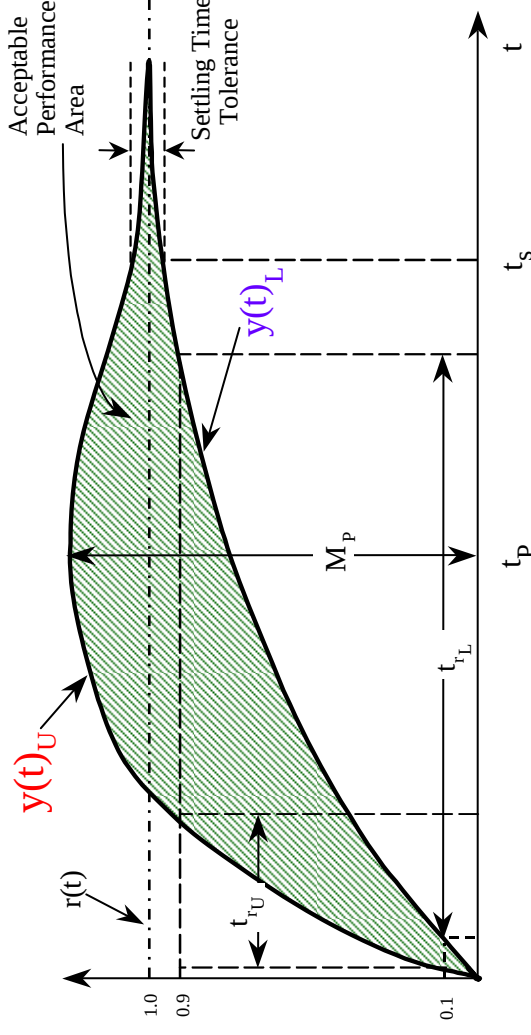


# Performance

## Tracking

$$W_{s7a} \leq \left| \frac{PG}{1+PGH} \right| \leq W_{s7b}$$

- **Time-Domain Specifications:** Desire system output  $y(t)$  to lie between specified upper and lower bounds,  $y(t)_U$  and  $y(t)_L$ , respectively.



Desired system performance specifications: time domain response specifications;

**Figures of merit** (FOM), based upon a *step input signal*  $r(t) = R_0 u_{-1}(t)$ ,

→  $M_p$  peak overshoot;  $t_r$  rise time;  $t_p$  peak time; and  $t_s$  settling time.

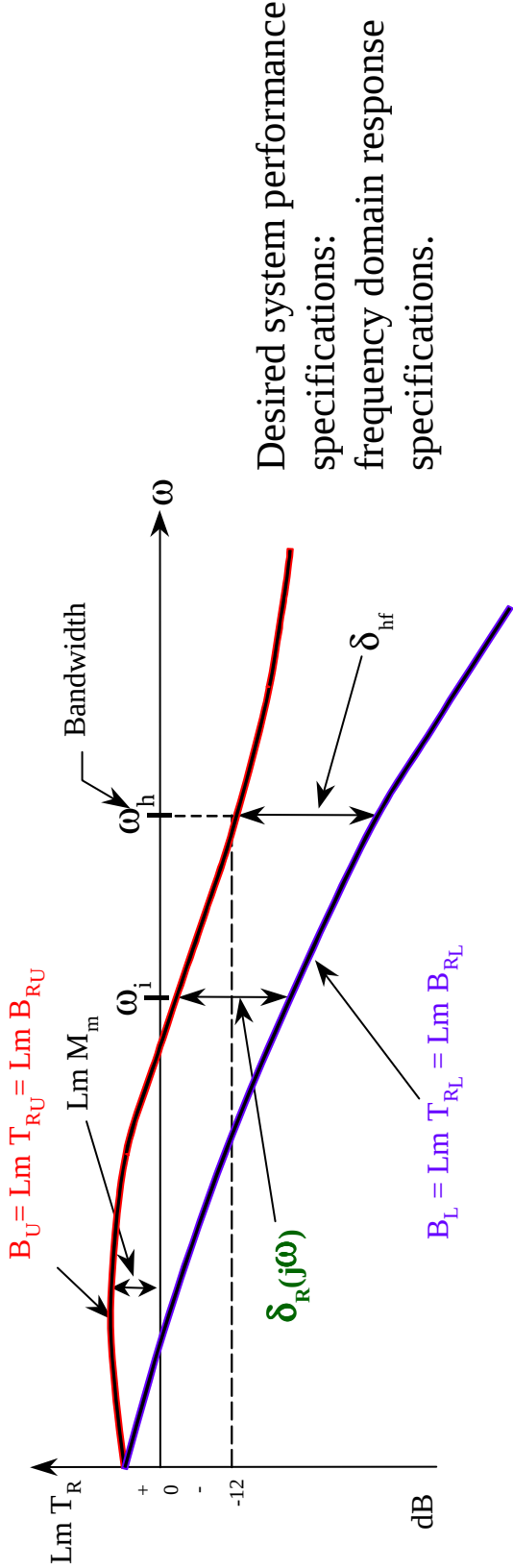
**Depends on requirements** that the designer wants for the *specific Plant*:

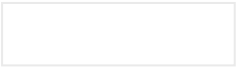
→ Airplane, Heating system, Machinery, Wind Turbine, etc...

# Tracking

$$W_{s7a} \leq \left| \frac{PG}{1+PGH} \right| \leq W_{s7b}$$

**Translated into** the frequency domain are, **B<sub>U</sub>** and **B<sub>L</sub>**, the upper and lower bounds respectively: Peak overshoot Lm M<sub>m</sub> & frequency bandwidth ω<sub>h</sub> .  
 (Note: increasing **δ<sub>R</sub>(jω)** above 0 dB crossing)





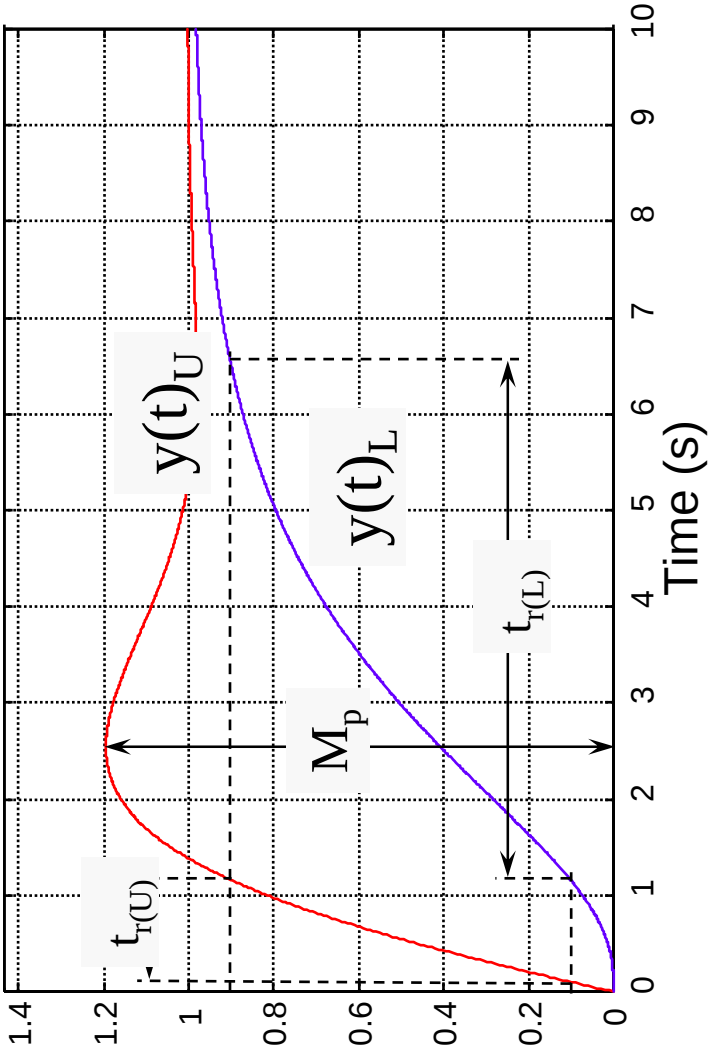
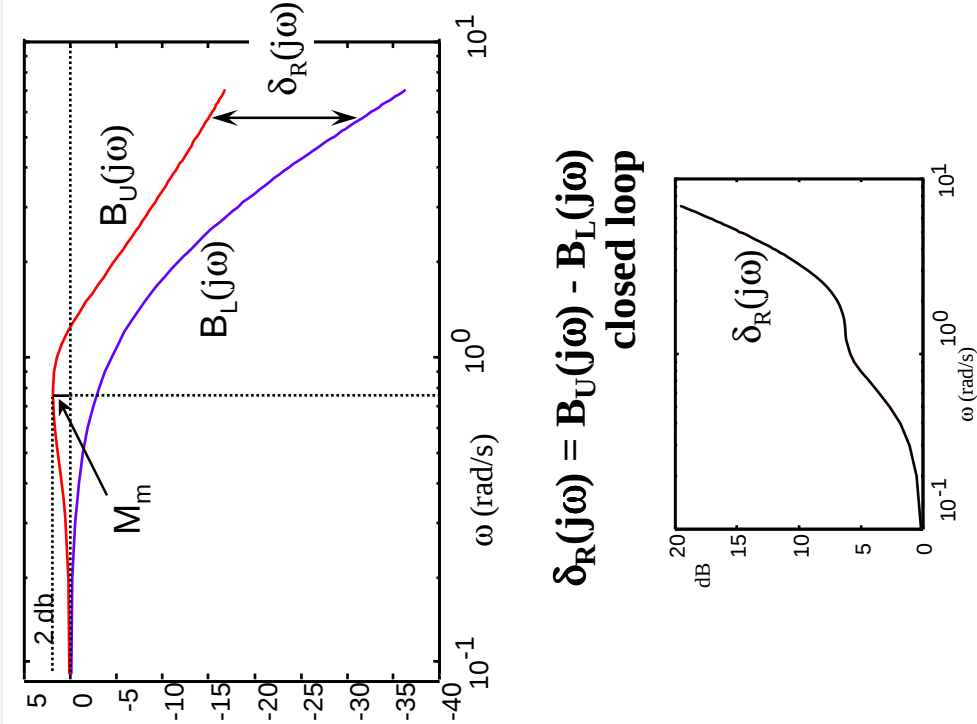
# Tracking

$W_{s7a} \leq \left| \frac{PG}{1+PGH} \right|$

$\leq W_{s7b}$

Upper function
$$W_{s7b} = T_{RU} = \frac{(\omega_n^2/a)(s+a)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

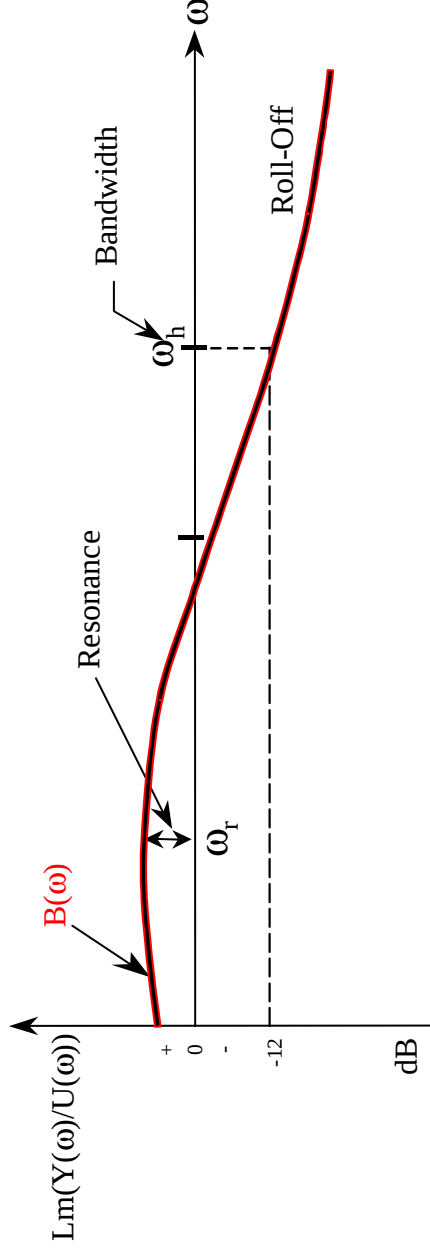
Lower function
$$W_{s7a} = T_{RL} = \frac{k}{(s+\sigma_1)\chi s + \sigma_2\chi s + \sigma_3}$$



## Disturbance rejection

$$\left| \frac{1}{1+P GH} \right| \leq W_{s2}$$

- **Frequency-Domain Specifications:** Desire TF system  $Y(\omega)/U(\omega)$  to lie under a specific bound,  $B(\omega)$



### **Figures of merit (FOM),**

→ Resonance; Bandwidth; Roll-off; Low Frequency, etc.

### **Depends on requirements** that the designer wants for the *specific Plant*:

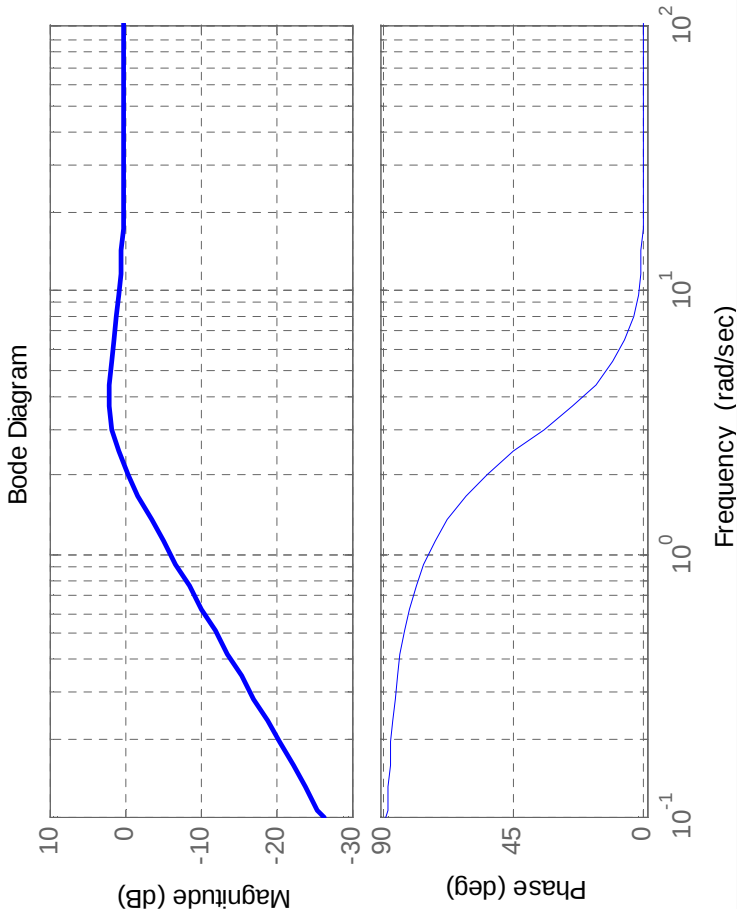
→ Structure resonance, Noise measurement, Disturbances, Steady State Errors, etc...

Disturbance rejection

$$\left| \frac{1}{1+PGH} \right| \leq W_{s2}$$

$$W_{s2} = T_d = \frac{1}{1+PGH} \leq \frac{s^2 + 2\xi \omega_n s}{s^2 + 2\xi \omega_n s + \omega_n^2}$$

Disturbance rejection



Closed loop specifications are usually described in terms of frequency functions  $\delta_k(\omega)$  that are imposed on the magnitude of the system transfer functions  $|T_k|$ ,  $k = 1, \dots, 5$

- (1) robust stability, control effort limit in the input disturbance rejection, sensor noise attenuation
- (2) output system disturbance rejection
- (3) input system disturbance rejection
- (4) control effort limit in the output disturbance rejection, noise attenuation, and tracking
- (5) signal tracking

| Transfer functions and specifications                                                                                                                                                                                                                                                                                 | Eq.No. |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|
| $ T_1(j\omega)  = \left  \frac{Y(j\omega)}{R(j\omega) \cdot F(j\omega)} \right  = \left  \frac{U(j\omega)}{D_1(j\omega)} \right  = \left  \frac{Y(j\omega)}{N(j\omega)} \right  = \left  \frac{P(j\omega) \cdot G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_1(j\omega), \omega \in \{\omega_1\}$ | (T1)   |
| $ T_2(j\omega)  = \left  \frac{Y(j\omega)}{D_2(j\omega)} \right  = \left  \frac{1}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_2(\omega), \omega \in \{\omega_2\}$                                                                                                                                           | (T2)   |
| $ T_3(j\omega)  = \left  \frac{Y(j\omega)}{D_1(j\omega)} \right  = \left  \frac{P(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_3(\omega), \omega \in \{\omega_3\}$                                                                                                                                  | (T3)   |
| $ T_4(j\omega)  = \left  \frac{U(j\omega)}{D_2(j\omega)} \right  = \left  \frac{U(j\omega)}{N(j\omega)} \right  = \left  \frac{U(j\omega)}{R(j\omega) \cdot F(j\omega)} \right  = \left  \frac{G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_4(\omega), \omega \in \{\omega_4\}$                   | (T4)   |
| $\delta_{5\text{inf}}(\omega) \leq  T_5(j\omega)  = \left  \frac{Y(j\omega)}{R(j\omega)} \right  = \left  F(j\omega) \frac{P(j\omega) \cdot G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_{5\text{sup}}(\omega), \omega \in \{\omega_5\}$                                                          | (T5)   |

## Step 3: Plant Model + Uncertainty

### Why Uncertainty?

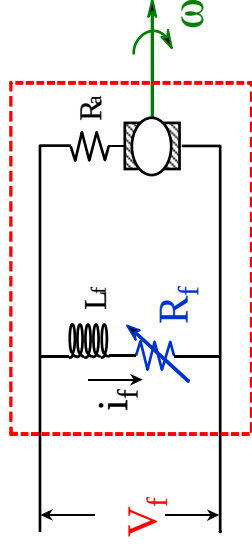
#### Test-bed for Large Multipole Generators



M.Torres

Up to four  
500 kW  
Motors  
drives a  
3000 kW  
Generator

## A Simple Mathematical Description



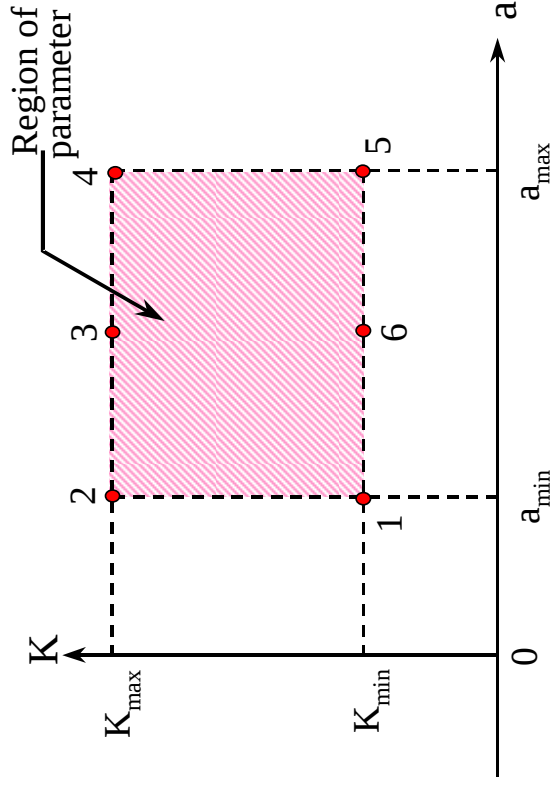
Motor transfer function is:

$$P_l(s) = \frac{\Theta_m(s)}{V_f(s)} = \frac{Ka}{s(s+a)}$$

Shaded region represents the region of plant uncertainty.

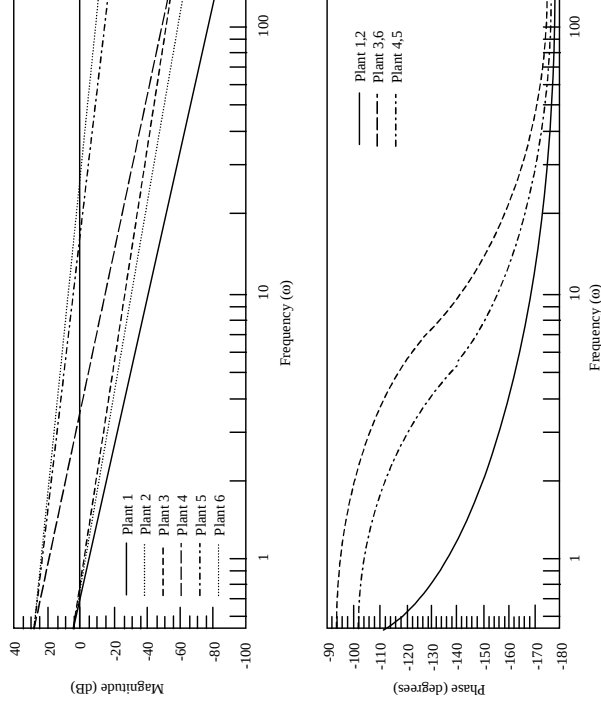
Parameters  $K$  and  $a$  vary:  $K \in (K_{\min}, K_{\max})$

and  $a \in (a_{\min}, a_{\max})$



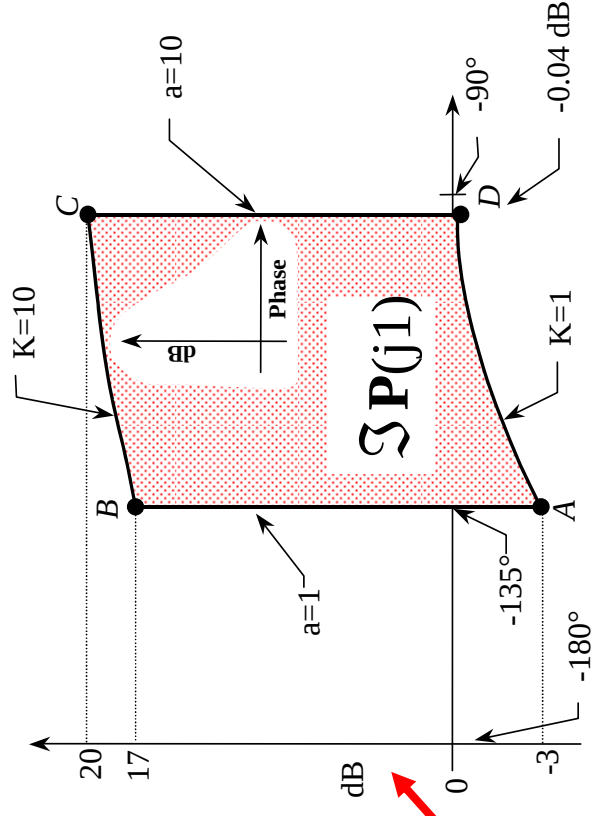
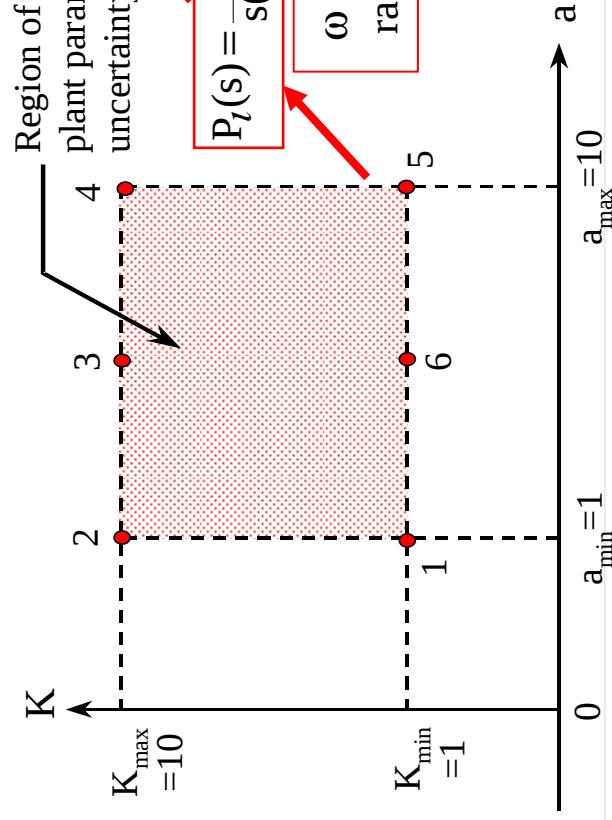
Region of plant parameter uncertainty.

Motor represented by 6 LTI transfer functions  $P_l$  ( $l = 1, 2, \dots, J$ ).



## Step 4: Templates

- Plant Template obtained by mapping  
Points of uncertainty region  
into points on the N.C.  
Curve drawn through points –  
Shaded area labeled  $\Im P(j1)$

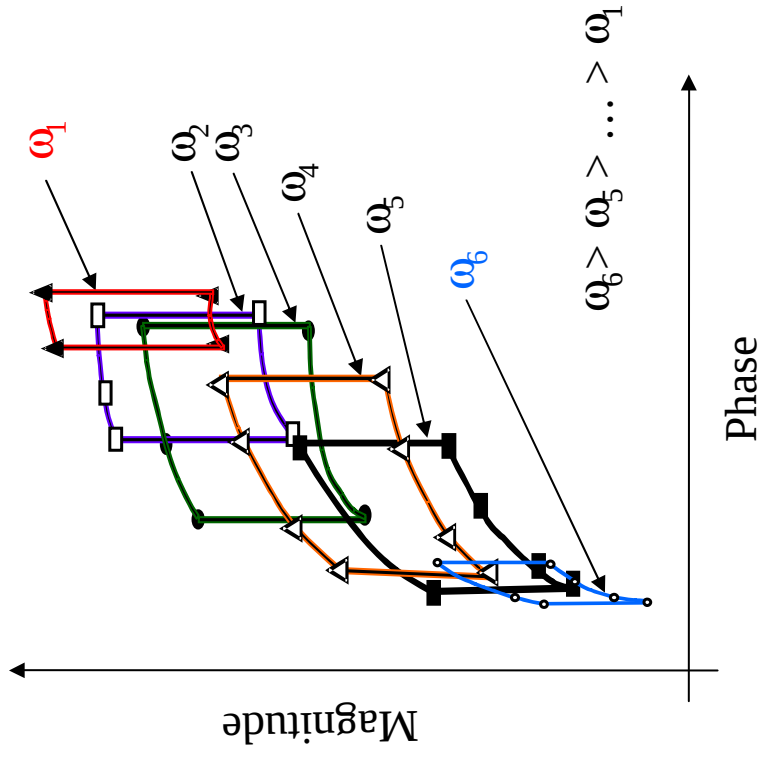


N.C. characterizing  $P(s)$  over the region of uncertainty.

- Templates for other values of  $\omega_i$  are obtained

- Characteristic of templates:

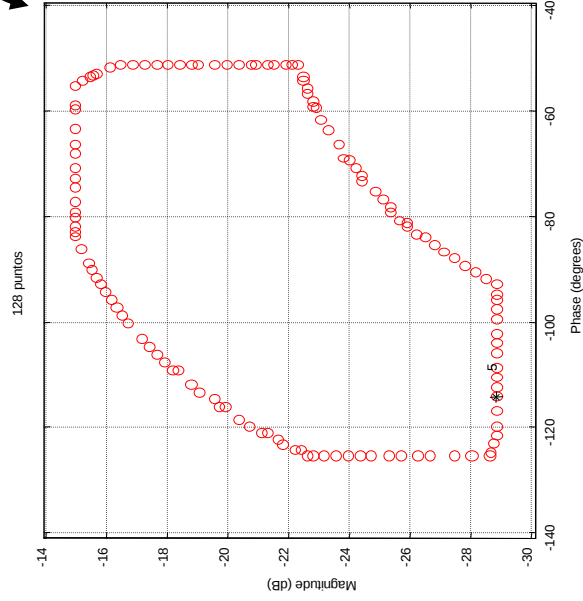
- Starting from low values of  $\omega_i$ , (narrow width), the angular width becomes larger (medium freq.)
- For increasing values of  $\omega_i$  templates become narrower again.
- Eventually approach straight line: height  $V$  dB



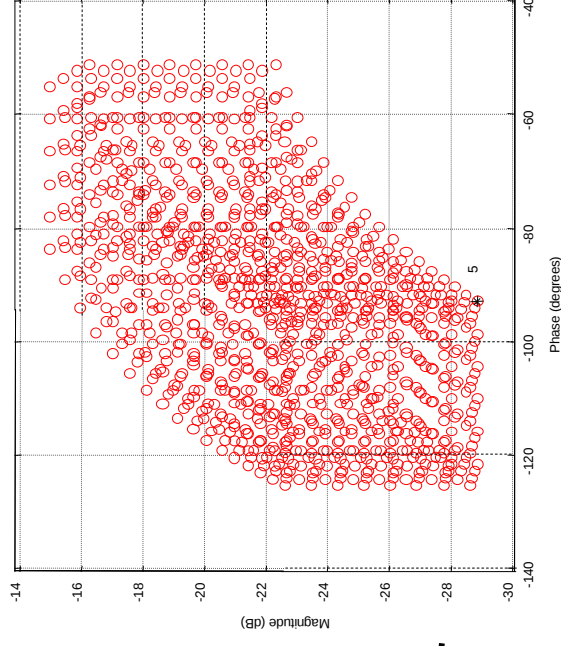
$$\Delta = \lim_{\omega \rightarrow \infty} [20 \log_{10} P_{\max} - 20 \log_{10} P_{\min}] = 20 \log_{10} K_{\max} - 20 \log_{10} K_{\min} = V \text{ dB}$$

# Example: 4-dimensional parameter space

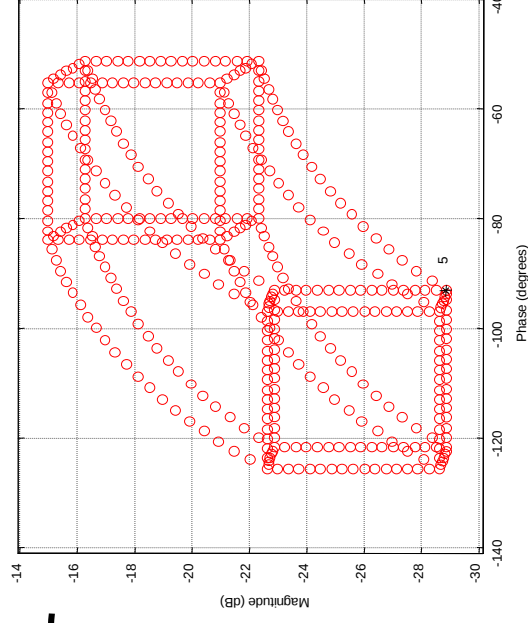
$$\mathbf{P}_1 = \left\{ \begin{array}{l} P_1(s) = \frac{k}{a s + b} \exp(-\tau s) \\ a \in [1, 5], b \in [10, 12], \\ k \in [1, 2], \tau \in [0.1, 0.2] \end{array} \right\}$$



Contour Template



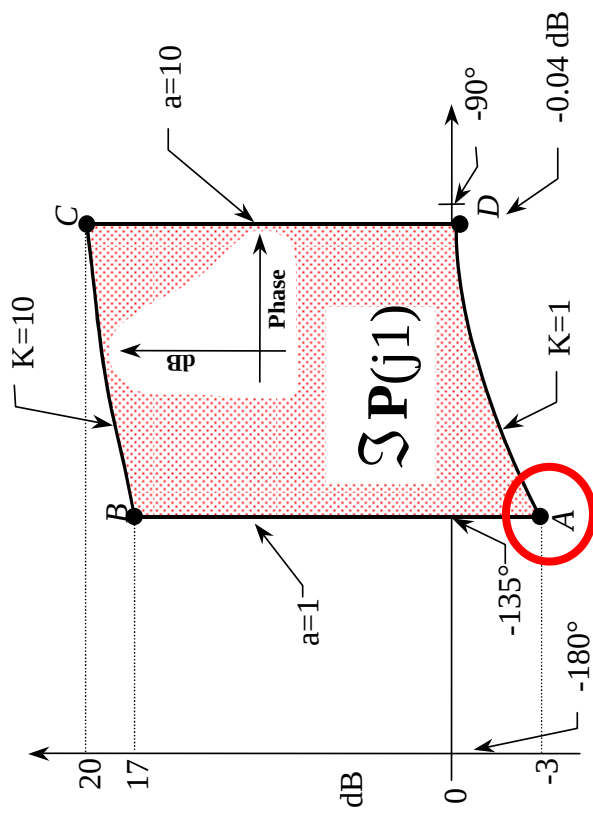
Whole Template  
 $\Omega_4$   
for  $\omega = 5$  rad/sec



Edges Template  
 $\Omega_1$

## Step 5: Nominal Plant

- Chose any plant
- Keep the same plant  
(set of parameters)  
as the nominal for all  
frequencies

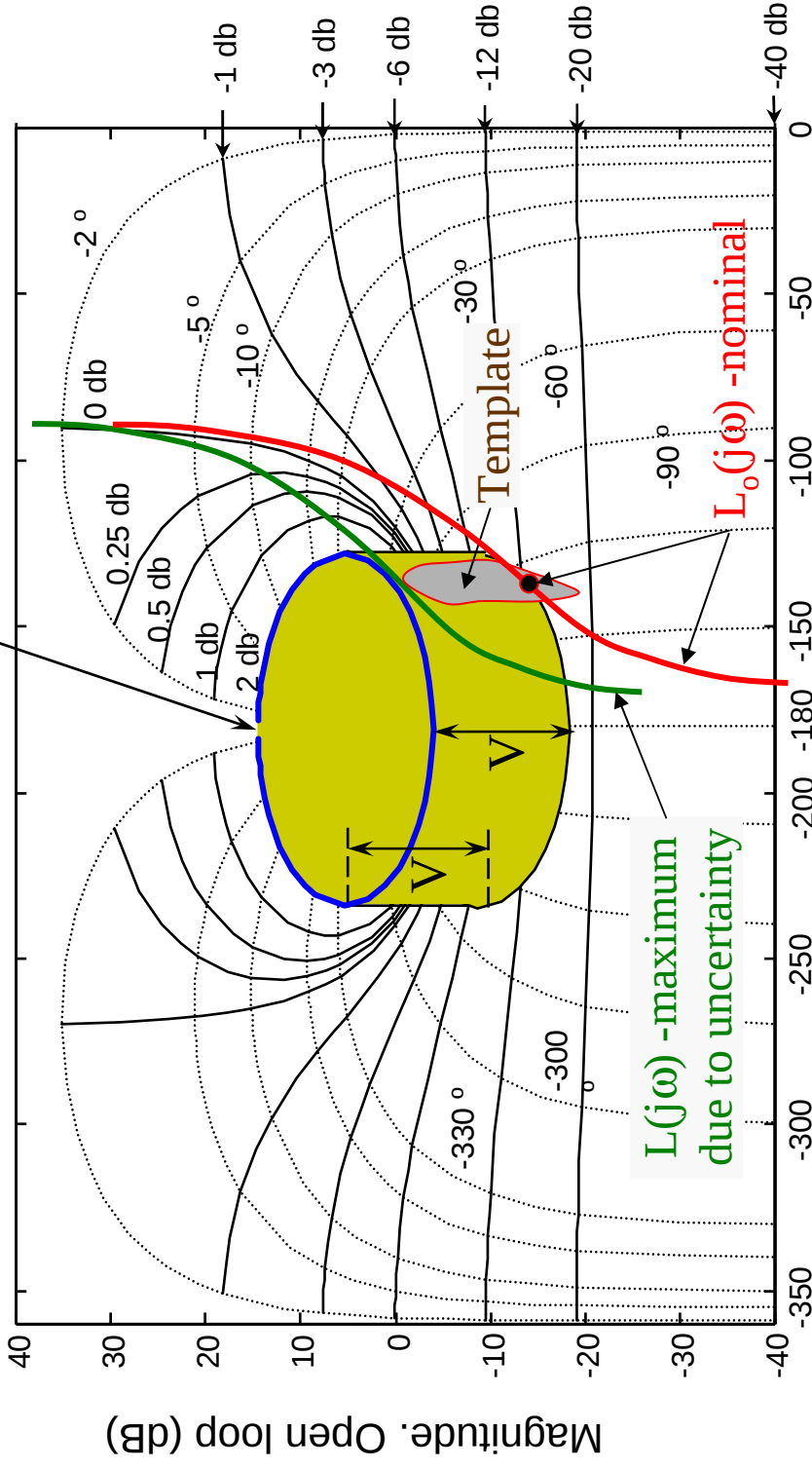


## Step 6: U-Contour (Stability bounds)

The Forbidden Region is extended by  $V$  dB.

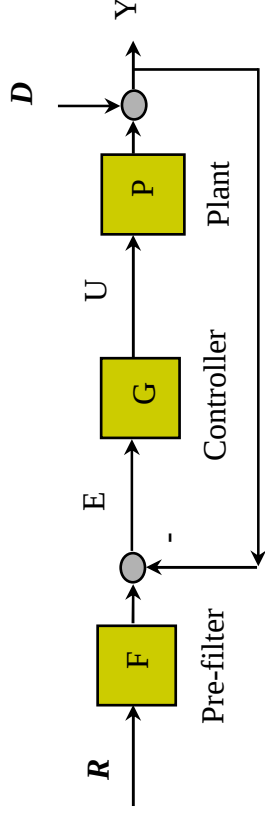
$$P(s) = K \frac{\prod_{i=1}^m (s + z_i)}{\prod_{i=1}^n (s + p_i)} \xrightarrow{\omega \rightarrow \infty} \frac{K}{s^{n-m}}$$

Universal High  
Frequency Bound (UHFb)



Phase. Open loop (deg)

## Step 7: Tracking Bounds on $L_0$

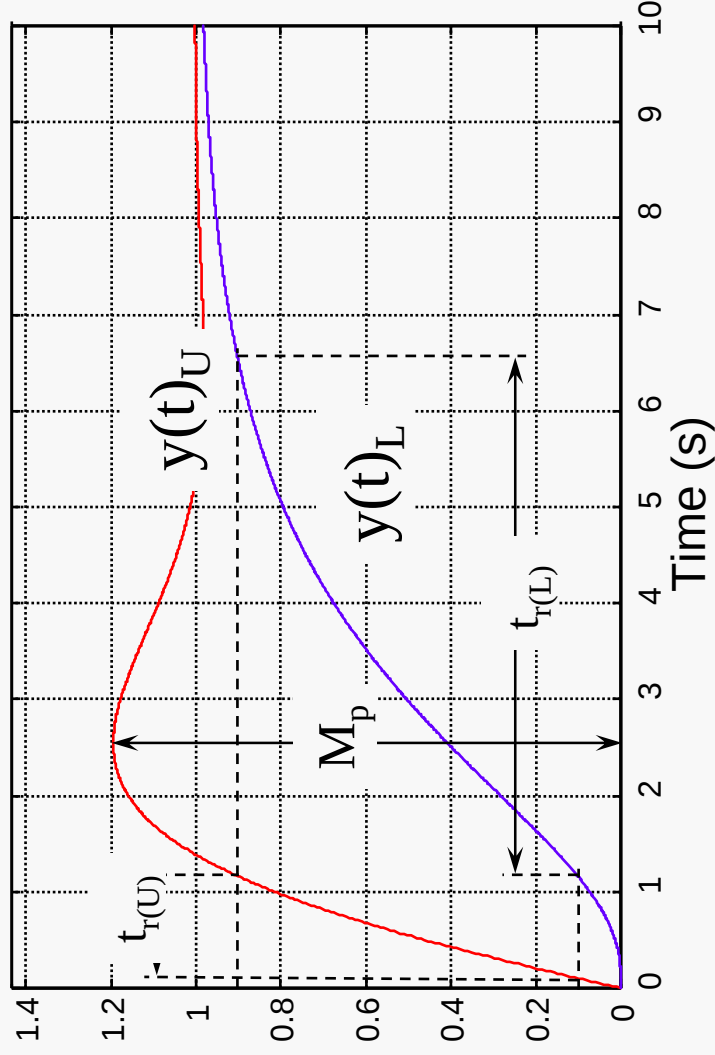


Upper function  $T_{RU} = \frac{(\omega_n^2/a)(s+a)}{s^2 + 2\xi\omega_n s + \omega_n^2}$

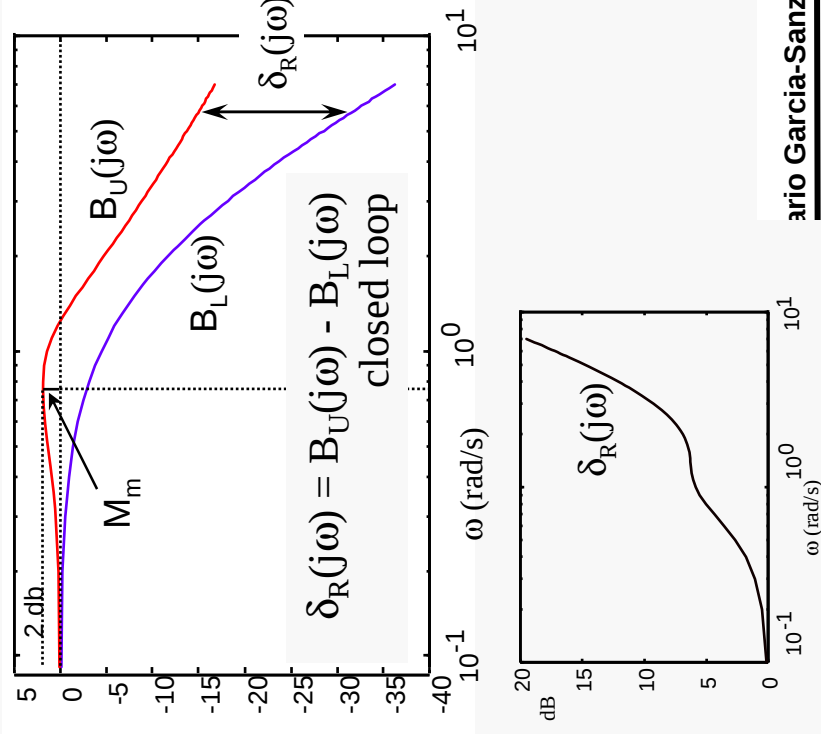
Lower function  $T_{RL} = \frac{k}{(s+\sigma_1)(s+\sigma_2)(s+\sigma_3)}$

$$y(t)_U \cdot (\omega_n = 1; a = 1; \xi = 0.6)$$

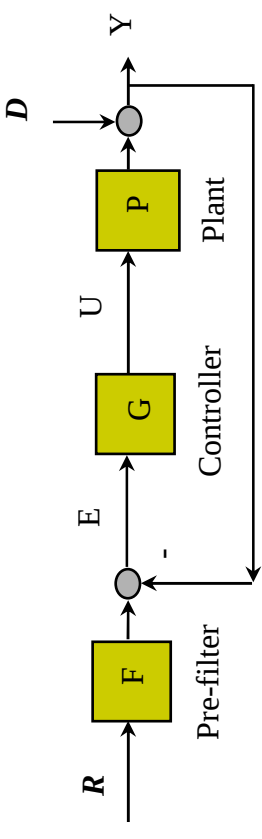
$$y(t)_L \cdot (\sigma_1 = 0.5; \sigma_2 = 1; \sigma_3 = 2; k = 1)$$



1.21

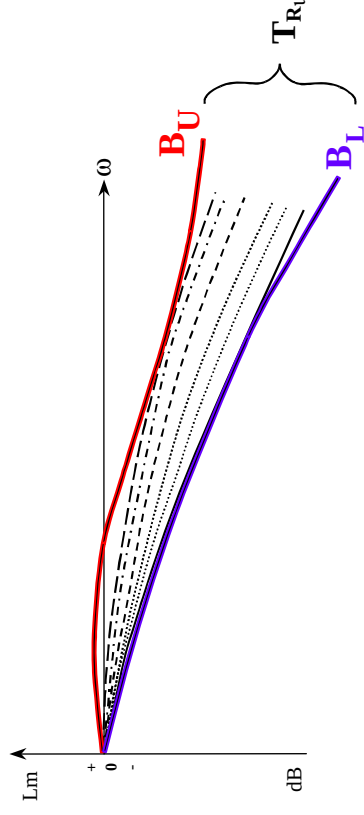


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- Solution for  $\mathbf{B}_R(j\omega_i)$  requires:  
 $\Delta T_R(j\omega_i) \leq \delta_R(j\omega_i) = B_U(j\omega_i) - B_L(j\omega_i)$  dB  
 be satisfied for all  $L_1(j\omega_i)$ .

- And  $\Delta T_R(j\omega_i)$   
 are between  $B_U$  and  $B_L$



Closed-loop responses:

LTI plants **with  $G(s)$  and  $F(s)$**

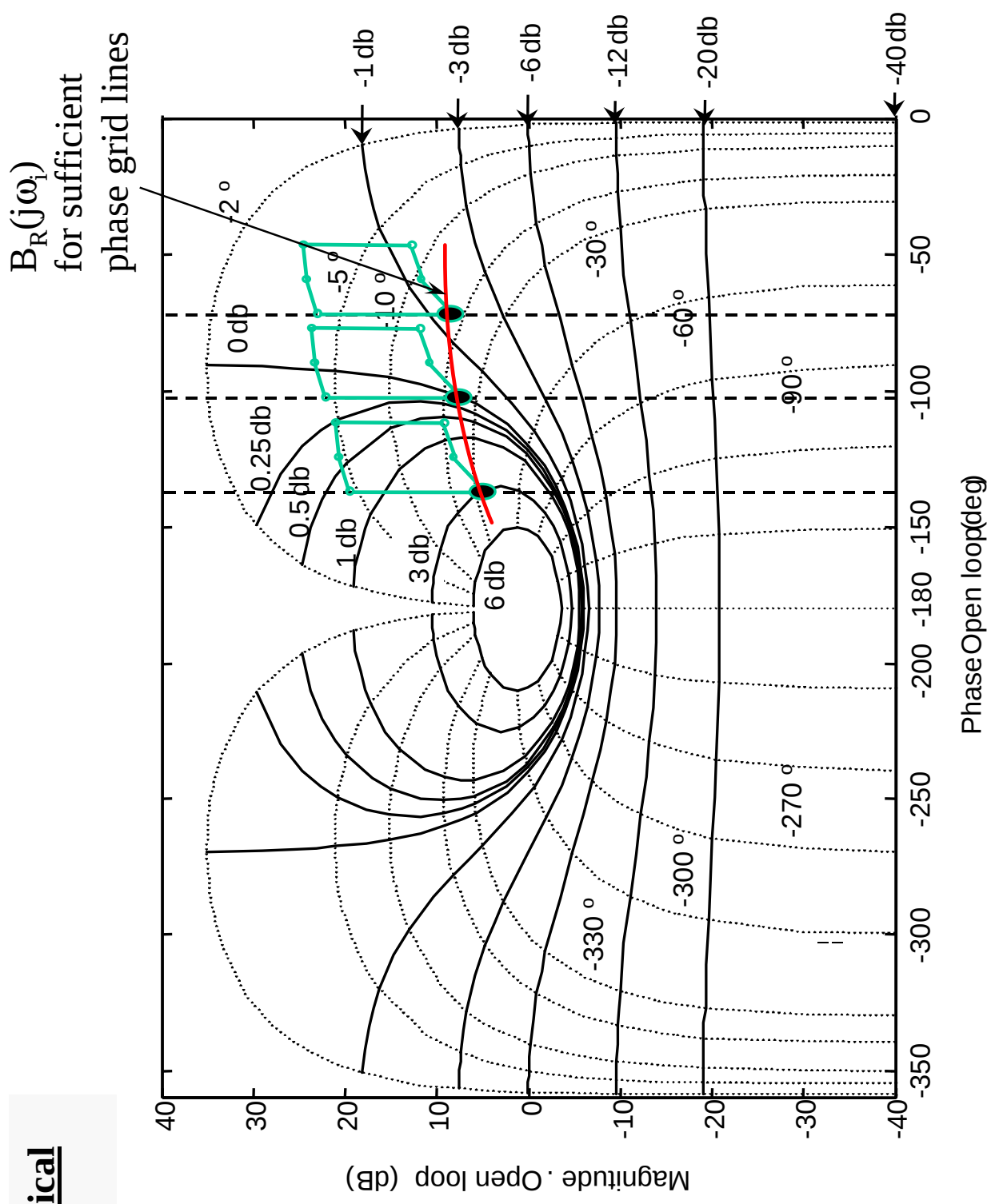
Closed-loop responses:

LTI plants with **only  $G(s)$**

## The classic graphical procedure

Based on the nominal plant  
 At specified  $\omega_i$   
 By use of templates  
 Along every NC  
 phase grid line

Repeat procedure on sufficient  
**phase grid** lines to provide enough points to draw  $B_R(\omega_i)$

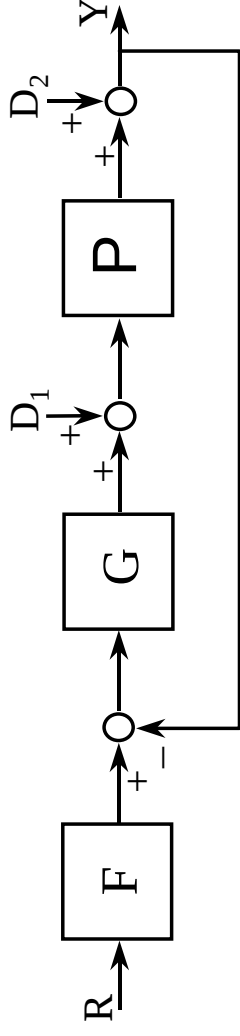


## Step 8: Disturbance Rejection Bounds

### The classic graphical procedure

- Case 1 Disturbance at Plant Output [ $d_2(t) = D_o u_{-1}(t)$ ,  $d_1(t) = 0$ ] the disturbance control ratio for input  $d_2(t)$  is,

$$T_{D2}(s) = \frac{Y(s)}{D_2(s)} = \frac{1}{1 + L}$$



A 2DOF feedback structure.

Substituting  $L = G P = 1//$   
yields

$$T_{D2}(s) = \frac{Y(s)}{D_2(s)} = \frac{\ell}{1 + \ell}$$

which has the mathematical format required to use the N.C.

is rotated  $180^\circ$

Similar that  
used for the  
tracking bounds

$$20 \log_{10} T_{D2}(j\omega) =$$

$$= 20 \log_{10} \frac{(1/L)}{1 + (1/L)} \leq \delta_{\text{D}_2}(\text{j}\omega_1) \text{ dB}$$



# Inequalities Bounds Expressions (Steps 6 to 8)

## The modern procedure

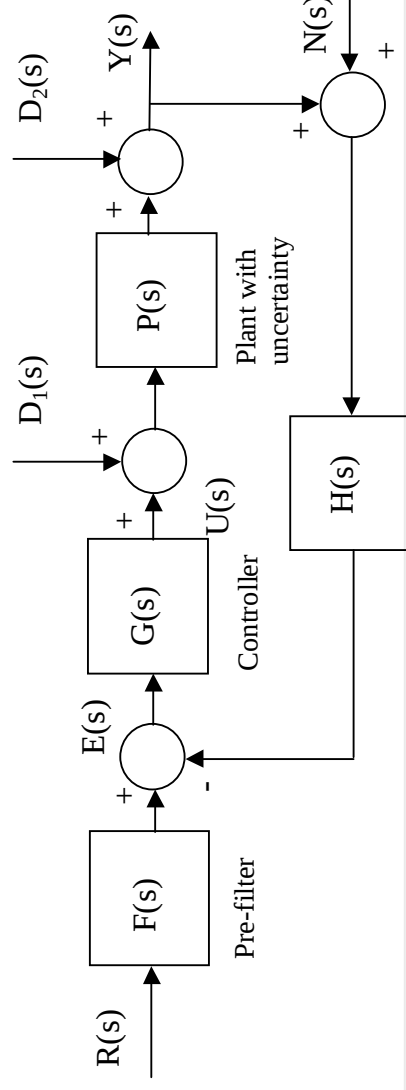
Let's consider the two-degrees-of-freedom feedback system.

In a general real-world problem  $P(s)$  will present uncertainty  $\{P\}$ .

The compensator  $G(s)$  and a the pre-filter  $F(s)$  will be designed to meet robust stability and robust performance specifications,

and to deal with references  $R(s)$ , disturbances  $D_{1,2}(s)$ , signal noise  $N(s)$  and saturable control effort  $U(s)$ ,

minimizing the 'cost of the feedback' (excessive bandwidth)



Closed loop specifications are usually described in terms of frequency functions  $\delta_k(\omega)$  that are imposed on the magnitude of the system transfer functions  $|T_k|$ ,  $k = 1, \dots, 5$

- (1) robust stability, control effort limit in the input disturbance rejection, sensor noise attenuation
- (2) output system disturbance rejection
- (3) input system disturbance rejection
- (4) control effort limit in the output disturbance rejection, noise attenuation, and tracking
- (5) signal tracking

| Transfer functions and specifications                                                                                                                                                                                                                                                                                 |  |  | Eq.No. |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--------|
| $ T_1(j\omega)  = \left  \frac{Y(j\omega)}{R(j\omega) \cdot F(j\omega)} \right  = \left  \frac{U(j\omega)}{D_1(j\omega)} \right  = \left  \frac{Y(j\omega)}{N(j\omega)} \right  = \left  \frac{P(j\omega) \cdot G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_1(j\omega), \omega \in \{\omega_1\}$ |  |  | (1)    |
| $ T_2(j\omega)  = \left  \frac{Y(j\omega)}{D_2(j\omega)} \right  = \left  \frac{1}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_2(\omega), \omega \in \{\omega_2\}$                                                                                                                                           |  |  | (2)    |
| $ T_3(j\omega)  = \left  \frac{Y(j\omega)}{D_1(j\omega)} \right  = \left  \frac{P(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_3(\omega), \omega \in \{\omega_3\}$                                                                                                                                  |  |  | (3)    |
| $ T_4(j\omega)  = \left  \frac{U(j\omega)}{D_2(j\omega)} \right  = \left  \frac{U(j\omega)}{N(j\omega)} \right  = \left  \frac{U(j\omega)}{R(j\omega) \cdot F(j\omega)} \right  = \left  \frac{G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_4(\omega), \omega \in \{\omega_4\}$                   |  |  | (4)    |
| $\delta_{5\inf}(\omega) \leq  T_5(j\omega)  = \left  \frac{Y(j\omega)}{R(j\omega)} \right  = \left  F(j\omega) \frac{P(j\omega) \cdot G(j\omega)}{1 + P(j\omega) \cdot G(j\omega)} \right  \leq \delta_{5\sup}(\omega), \omega \in \{\omega_5\}$                                                                      |  |  | (5)    |

Table 1

Each plant in the  $\omega_i$ -template and the controller can be expressed in its polar form:

$$\text{Plant} \quad P(j\omega_i) = \{P_r(j\omega_i) = p \angle \theta, r = 0, \dots, m-1\}$$

$$\text{Controller} \quad G(j\omega_i) = g \angle \phi$$

Then, substituting and rearranging the inequalities -Eq. (1) to (5) in Table 1-, they can be reduced to the quadratic inequalities  $-k\text{-problem}$  (1) to (5) in Table 2-.

Solving equalities such as  $ag^2+bg+c=0$  the set of  **$\omega_i$ -bounds** for  $\{\delta_{k=1,\dots,5}\}$  is computed.

| $k\text{-problem}$ | Bound Quadratic Inequality                                                                                                                                                                                                     |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1                  | $p^2 \cdot \left(1 - \frac{1}{\delta_1^2}\right) \cdot g^2 + 2 \cdot p \cdot \cos(\phi + \theta) \cdot g + 1 \geq 0$                                                                                                           |
| 2                  | $p^2 \cdot g^2 + 2 \cdot p \cdot \cos(\phi + \theta) \cdot g + \left(1 - \frac{1}{\delta_2^2}\right) \geq 0$                                                                                                                   |
| 3                  | $p^2 \cdot g^2 + 2 \cdot p \cdot \cos(\phi + \theta) \cdot g + \left(1 - \frac{p^2}{\delta_3^2}\right) \geq 0$                                                                                                                 |
| 4                  | $\left(p^2 - \frac{1}{\delta_4^2}\right) \cdot g^2 + 2 \cdot p \cdot \cos(\phi + \theta) \cdot g + 1 \geq 0$                                                                                                                   |
| 5                  | $p_e^2 p_d^2 \left(1 - \frac{1}{\delta_5^2}\right) \cdot g^2 + 2 p_e p_d \left(p_e \cos(\phi + \theta_d) - \frac{p_d}{\delta_5^2} \cos(\phi + \theta_e)\right) \cdot g + \left(p_e^2 - \frac{p_d^2}{\delta_5^2}\right) \geq 0$ |

Table 2

$$\delta_5 = \delta_{5\text{sup}} / \delta_{5\text{inf}}$$

# Algorithm to compute the bounds

Y. Chait, and O. Yaniv, “Multi-input/single-output computer-aided control design using the Quantitative Feedback Theory,” *Int. J. Robust & Non-linear Control*, vol.3, pp. 47-54, 1993.

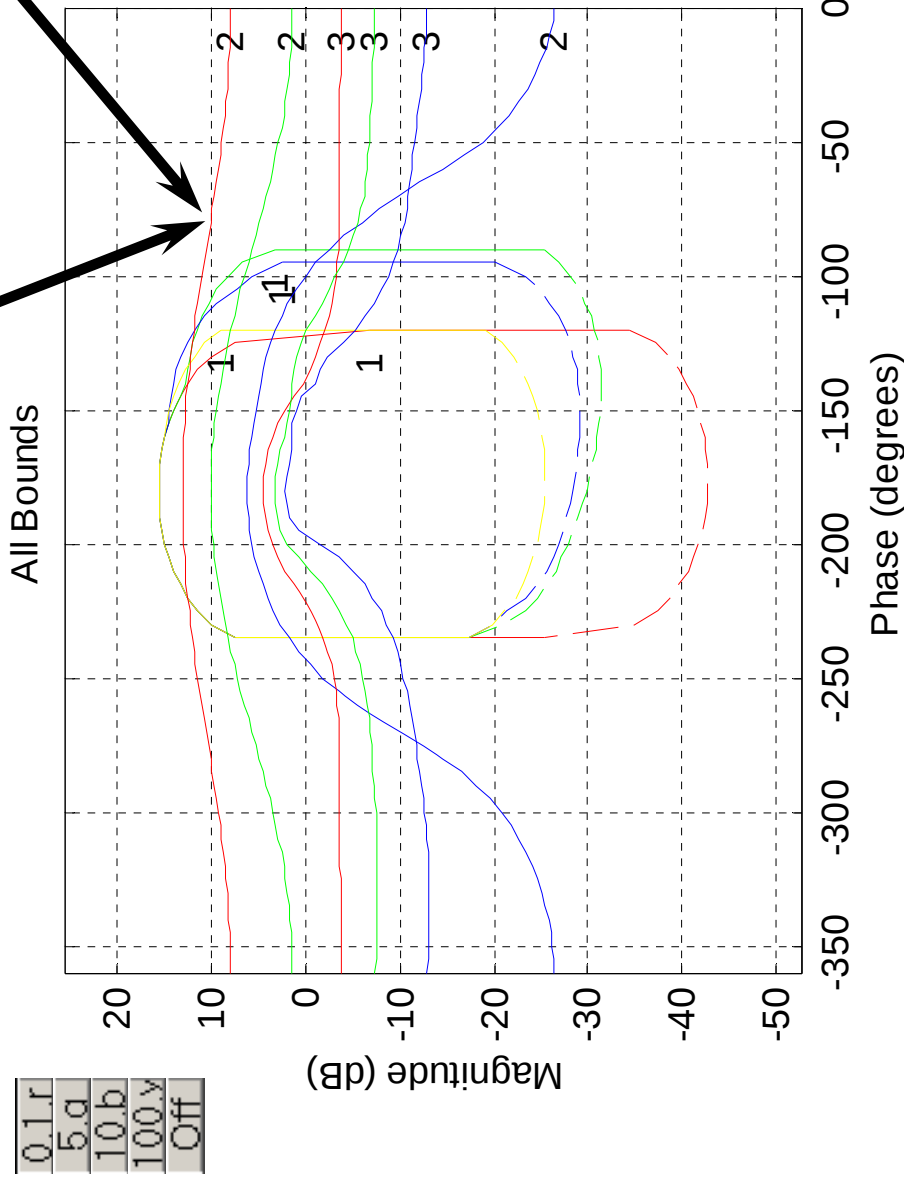
|                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Discretize the domain $\{\omega_k\}$ into a finite set $\Omega_k = \{\omega_i, i = 1, \dots, n\}_k$ .                                                                                                                                                                                                                                                                                                                                           |
| 2. Establish the uncertain LTI plant models $\mathcal{P} = \{P(j\omega)\}$ and map its boundary for each frequency $\omega_i \in \Omega_k$ on the Nichols chart. A set of $n$ templates $\{P(j\omega_i)\}$ , $i = 1, \dots, n$ is obtained. Each template $P(j\omega_i) = \{P_r(j\omega_i) = p \angle \theta, r = 0, \dots, m-1\}$ contains $m$ points or plants. Select one of them as the nominal plant $P_0(j\omega_i) = p_0 \angle \theta_0$ . |
| 3. Now, the conditions to meet by the controller $G(j\omega_i) = g \angle \phi$ have to be computed.                                                                                                                                                                                                                                                                                                                                               |
| 4. Define a range, $\Phi$ , for the compensator's phase $\phi$ , and discretize it; for example $\phi \in \Phi = [-360^\circ: 5^\circ: 0^\circ]$ .                                                                                                                                                                                                                                                                                                 |
| 5. Choose a single frequency $\omega_i \in \Omega_k$ .                                                                                                                                                                                                                                                                                                                                                                                             |
| 6. Choose a single controller's phase $\phi \in \Phi$ .                                                                                                                                                                                                                                                                                                                                                                                            |
| 7. Choose a single plant in the $\omega_i$ -template: $P_r(j\omega_i) = p \angle \theta$ .                                                                                                                                                                                                                                                                                                                                                         |
| 8. At this step, the $k$ feedback problem is reduced to solve a $k$ quadratic inequality without uncertainty. The feedback problems in equations (1) to (5) in Table I are reduced to inequalities in Table II.                                                                                                                                                                                                                                    |
| 9. Compute the maximum $g_{\max} = g_{\max}(P_r)$ and the minimum $g_{\min} = g_{\min}(P_r)$ of the two roots $g_1$ and $g_2$ that solve the $k$ quadratic inequality,.                                                                                                                                                                                                                                                                            |
| 10. Repeat Steps 6 and 7 for the $m$ plants $P_r(j\omega_i)$ , $r = 0, \dots, m-1$ in the $\omega_i$ template $P(j\omega_i)$ .                                                                                                                                                                                                                                                                                                                     |
| 11. Choose the most restrictive of the $m$ $g_{\max}(P_r)$ and the $m$ $g_{\min}(P_r)$ . Thus, $g_{\max}(P)$ and $g_{\min}(P)$ are obtained. They are the maximum and minimum bound points for the controller magnitude $g$ at a phase $\phi$ .                                                                                                                                                                                                    |
| 12. Repeat Step 5 over the range $\Phi$ . The union of $g_{\max}(P)$ and $g_{\min}(P)$ for each $\phi \in \Phi$ gives $g_{\max} \angle \phi$ and $g_{\min} \angle \phi$ , respectively.                                                                                                                                                                                                                                                            |
| 13. Now the bounds for the open loop transmission $L_0(j\omega_i) = l_0 \angle \psi_0$ are computed. Set $l_{0\max} \angle \psi_0 = p_0 \cdot g_{\max} \angle \phi$ and $l_{0\min} \angle \psi_0 = p_0 \cdot g_{\min} \angle \phi$ , being $\psi_0 = \phi + \theta_0$ , $\phi = [-360^\circ: 5^\circ: 0]$ . These bounds will be labelled $B_k(j\omega_i)$ .                                                                                       |
| 14. Repeat Step 4 over the range $\Omega_k$ . The set of bounds for the $k$ control problem $\{B_k(j\omega_i), \omega_i \in \Omega_k\}$ has just been computed.                                                                                                                                                                                                                                                                                    |

The set of  $\omega_i$ -bounds

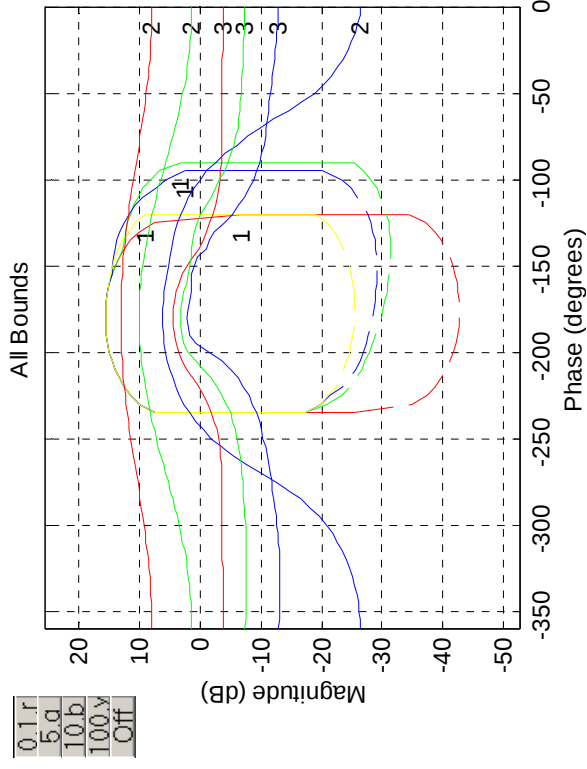
Plant Model  
+ Uncertainty

+

Performance  
Specifications (P.S.)

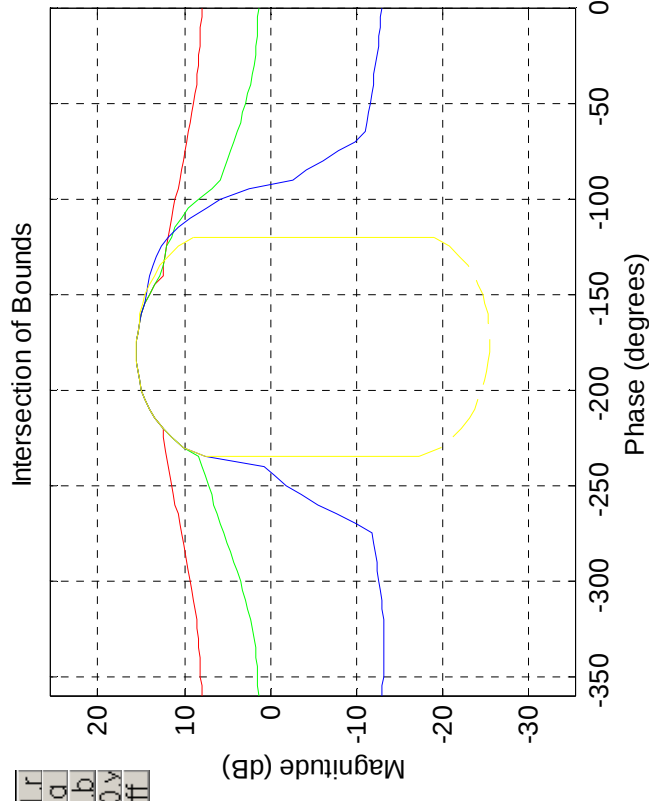


## Step 9: Optimal Bounds $B_o(j\omega_i)$



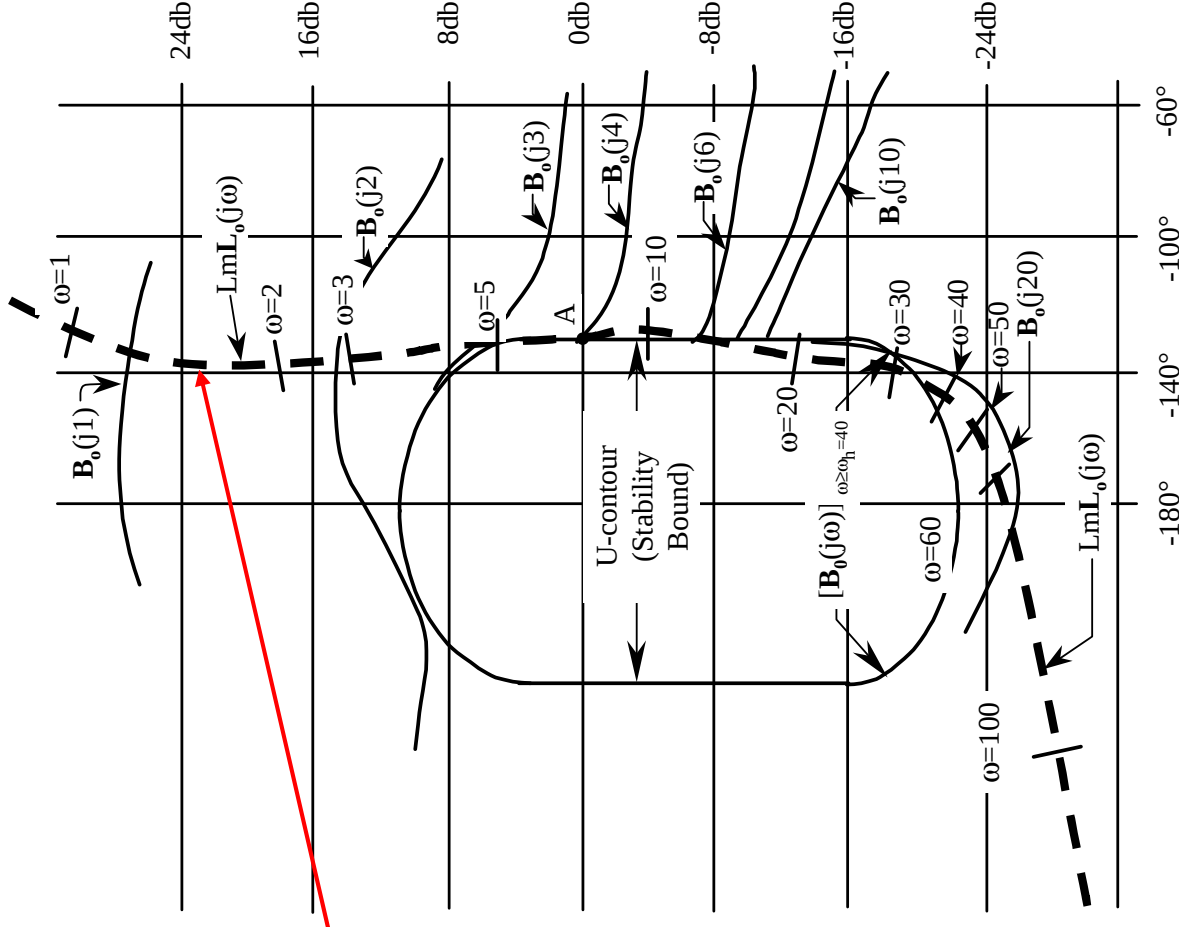
One line for each  
frequency  $\omega_i$   
The most demanding one

## Intersection of $\omega_i$ -bounds



## Step 10: Synthesizing $G(s)$ or Loop Shaping $L_o(s)$

- Shaping of  $L_o(j\omega) = P_o(j\omega) G(j\omega)$   
Only one  $L = L_o$  to be shape!!
- $L_o(j\omega_i)$  must be at every  $\omega_i$ :
  - outside the U-contour
  - above the continuous bounds  $B_o(j\omega)$
  - below the discontin. Bounds  $B_o(j\omega)$
- Synthesize rational function  $L_o(s)$   
Build up  $G(j\omega)$  term-by-term **adding** some elements like: gain, real poles and zeros, complex poles and zeros, integrators, differentiators, lead/lag networks, notch filters, second order TF, etc.
- Compensator:  $G(s) = L_o(s)/P_o(s)$
- Probably one of the most difficult steps of the methodology for the beginner.



### Procedure:

Build up  $L_O(j\omega) = P_O(j\omega) \left( K \prod_{i=0}^n [G_i(j\omega)] \right)$  term-by-term adding some elements like:

|                 |                                                                                      |                            |                                                                                                                             |
|-----------------|--------------------------------------------------------------------------------------|----------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| 1. Gain         | k                                                                                    | 6. 2° order / 2° order     | $\frac{a_1 s^2 + a_2 s + 1}{b_1 s^2 + b_2 s + 1}$                                                                           |
| 2. Real Pole    | $\frac{1}{s+1}$<br>p                                                                 | 7. Integrator              | $\frac{1}{s^n}$                                                                                                             |
| 3. Real Zero    | $\frac{s}{z} + 1$                                                                    | 8. Differentiator          | $s^n$                                                                                                                       |
| 4. Complex Pole | $\frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} \quad (\zeta < 1)$ | 9. Lead/Lag Network        | $\frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}$                                                                                   |
| 5. Complex Zero | $\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1 \quad (\zeta < 1)$           | 10. Notch Filter<br>etc... | $\frac{\frac{s^2}{\omega_n^2} + \frac{2\zeta_1 s}{\omega_n} + 1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta_2 s}{\omega_n} + 1}$ |

## The Interactive Design Environment (IDE): (Terasoft, version 2)

Interactive tool to design the controller  $G(s)$

Function:

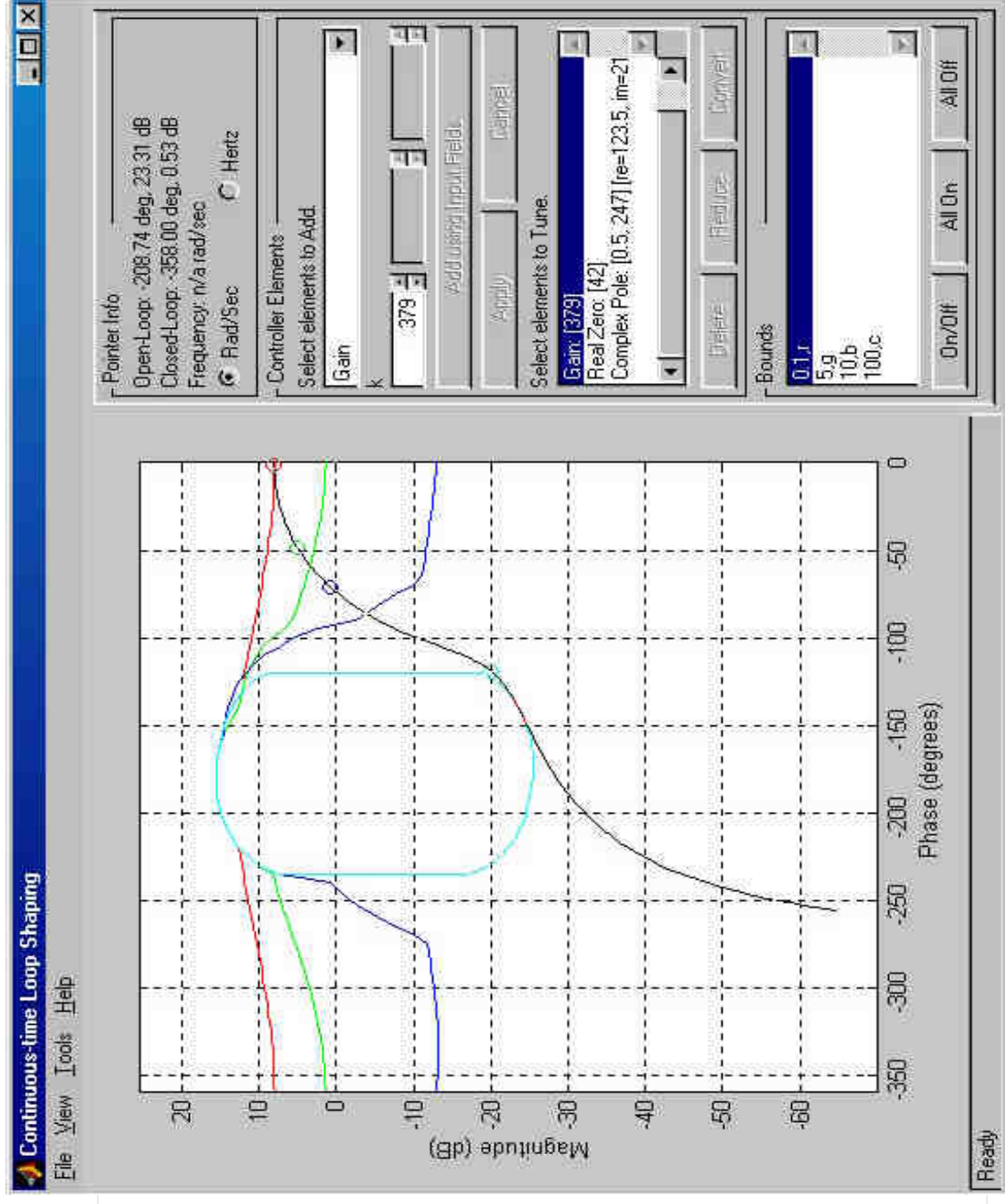
**lpshape(...)**

**Only one**  
 **$L = L_o$**   
to be shape!!

&

the controller  
is for the  
whole set of  
uncertain  
plants

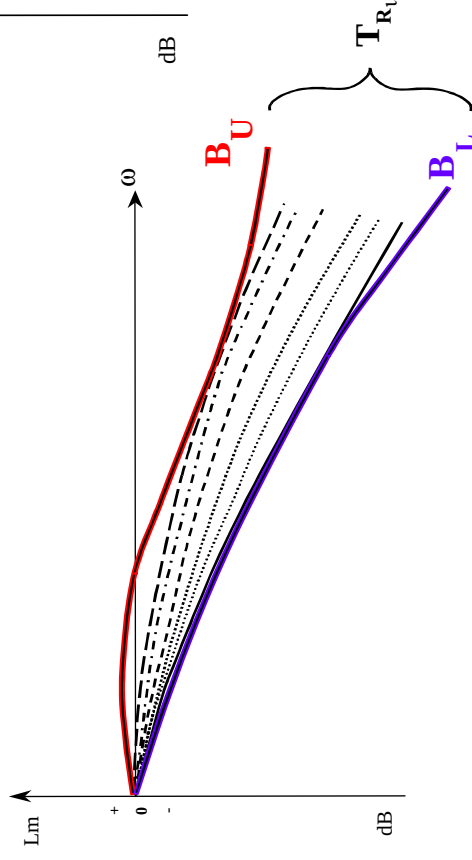
**Controller  
design**



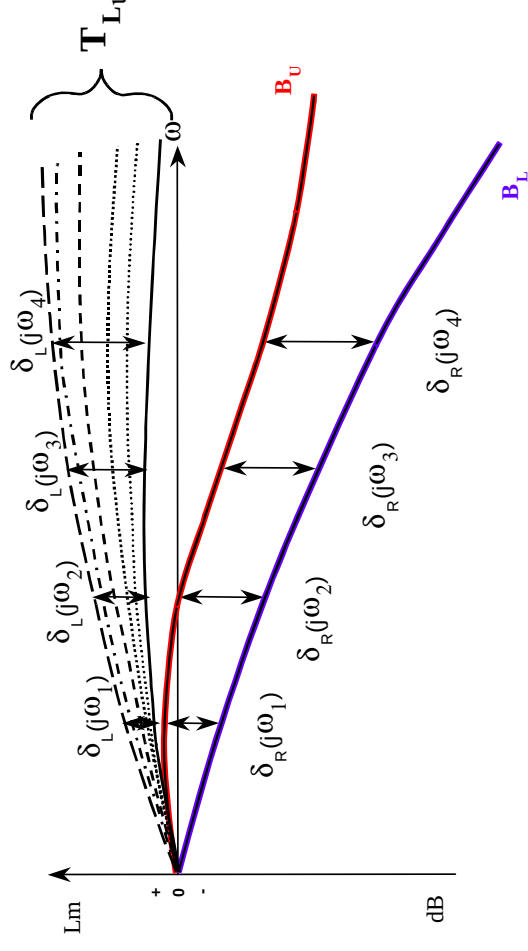
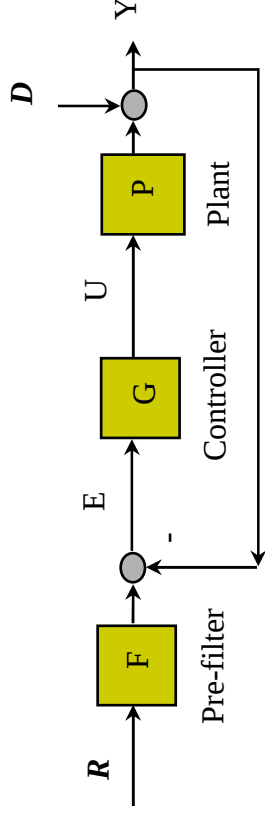
## Step 11: Prefilter Design $F(s)$

to lie between  $\mathbf{B}_U$  &  $\mathbf{B}_L$  for all  $J$  plants

$\Rightarrow$  A Prefilter  $F(s)$  is needed



**Fig. 10** Closed-loop responses: LTI plants with  $G(s)$  and  $F(s)$



**Fig. 9** Closed-loop responses: LTI plants with only  $G(s)$

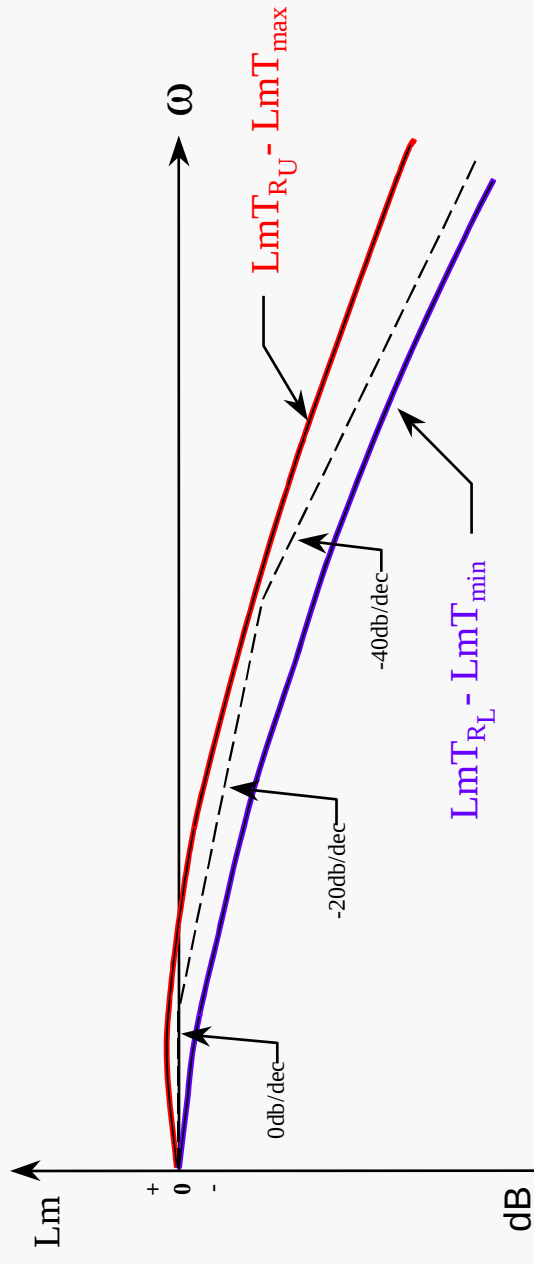
$$\begin{cases} \text{Lm } T_{RU} - \text{Lm } T_{\max} = B_U - \text{Lm } T_{\max} = \text{Lm } F_{\max} \\ \text{Lm } T_{RL} - \text{Lm } T_{\min} = B_L - \text{Lm } T_{\min} = \text{Lm } F_{\min} \end{cases}$$

We have to move down more than  $F_{\max}$  but less than  $F_{\min}$ .

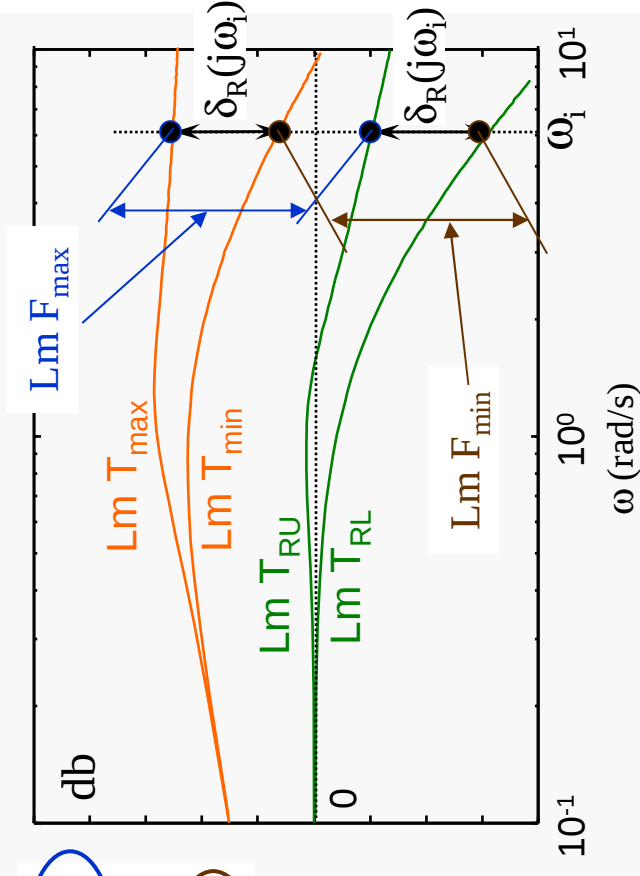
$$\text{Lm } F_{\max}(j\omega) < \text{Lm } F(j\omega) < \text{Lm } F_{\min}(j\omega)$$

$$\lim_{s \rightarrow 0} F(s) = 1$$

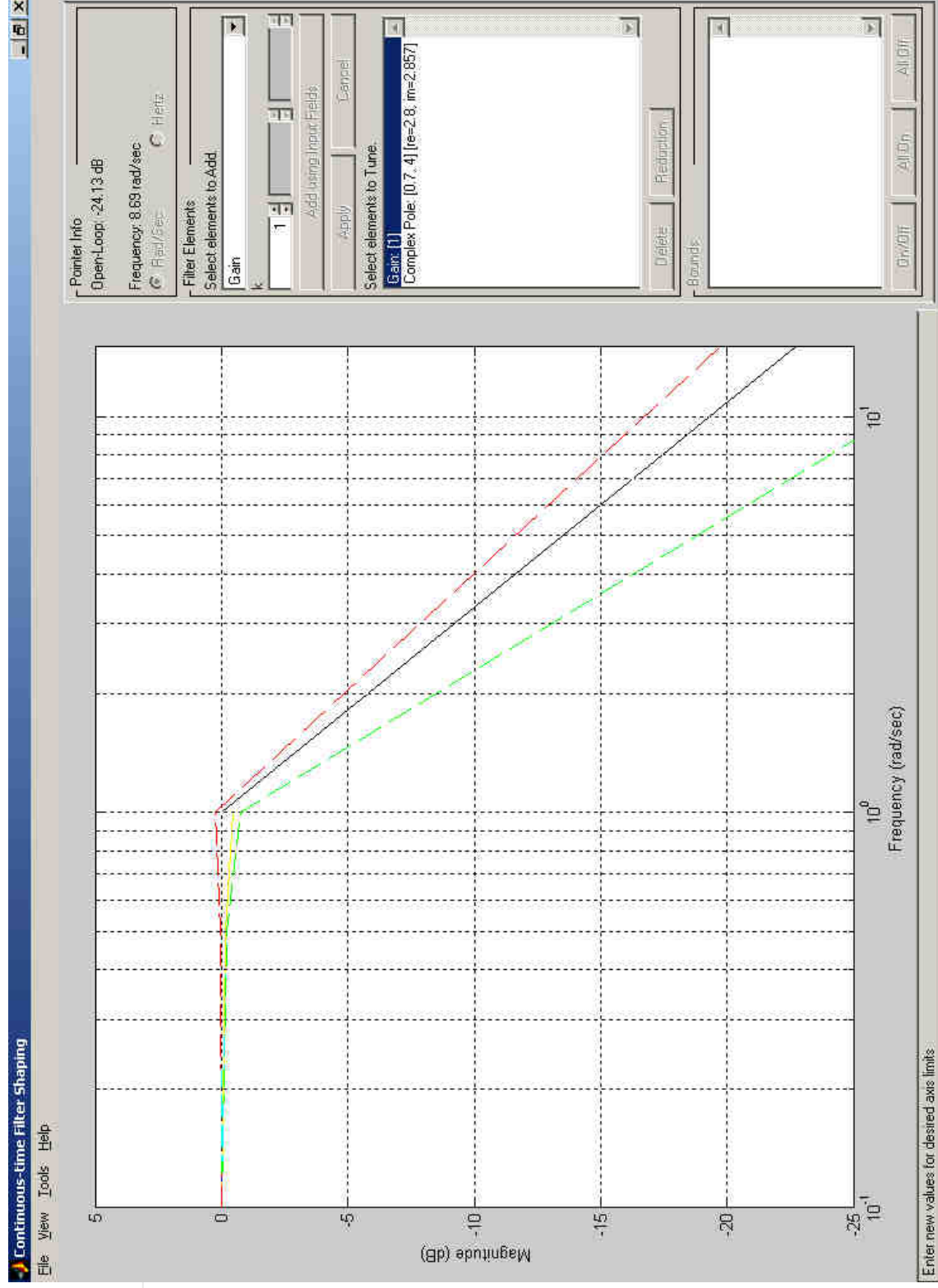
$F(s)$  is synthesized, in dashes, that lies within the upper & lower plots



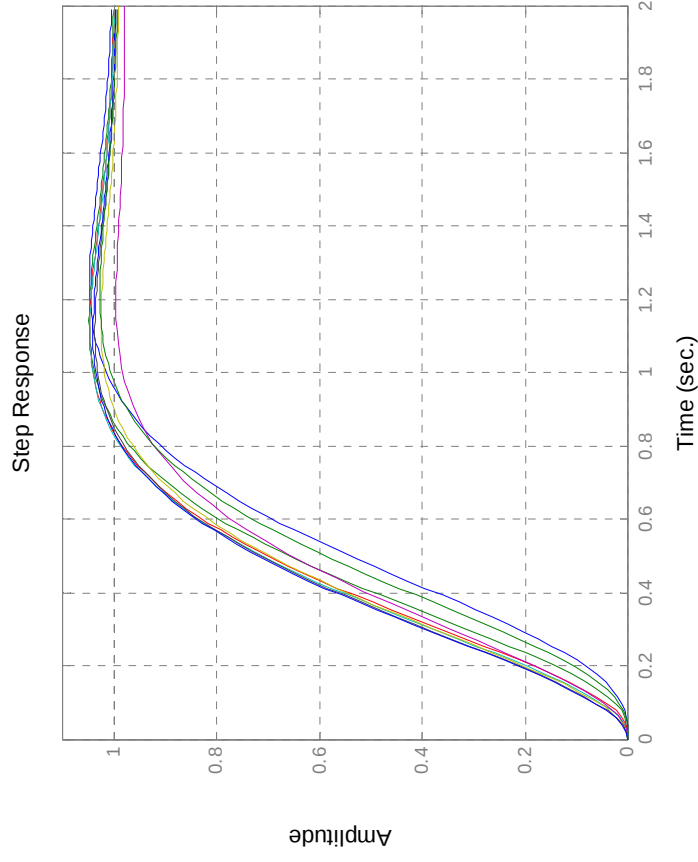
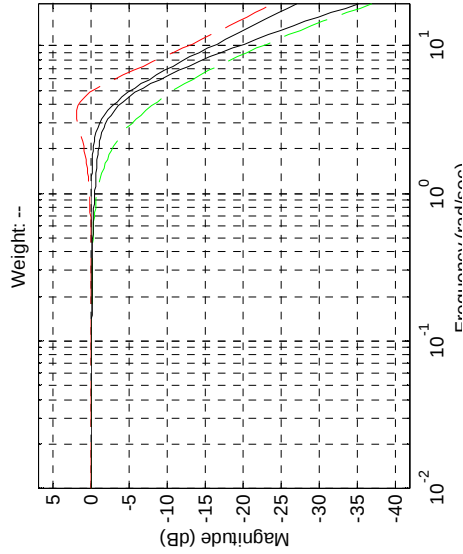
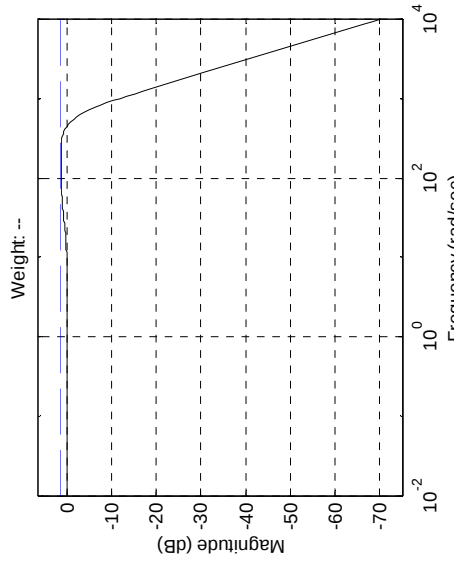
Frequency bounds on the prefilter  $F(s)$ .



## (Terasoft, version 2)



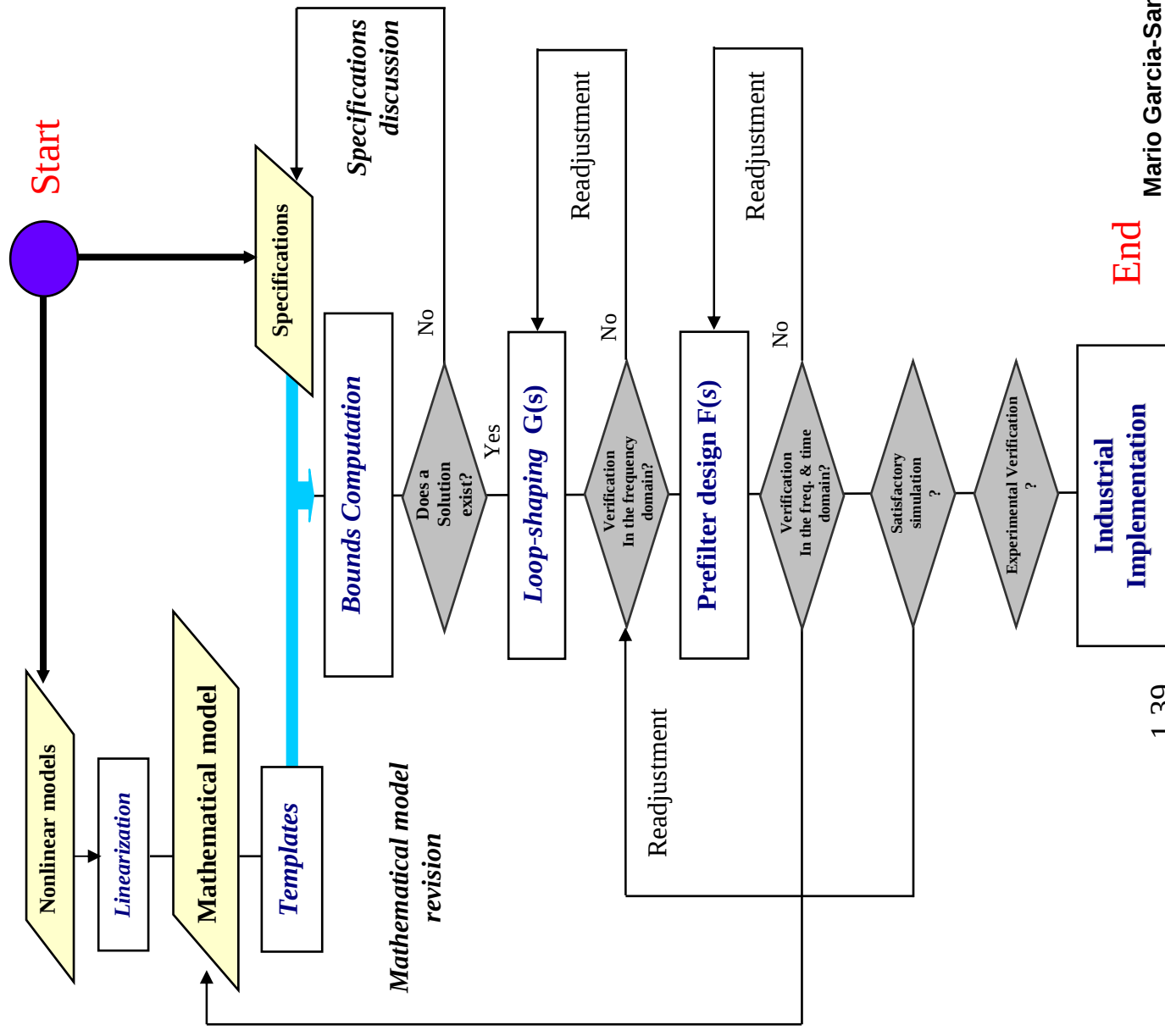
**Step 12: Simulate linear system (J time responses)**  
**Step 13: Simulate with nonlinearities**



Time Domain

Frequency Domain

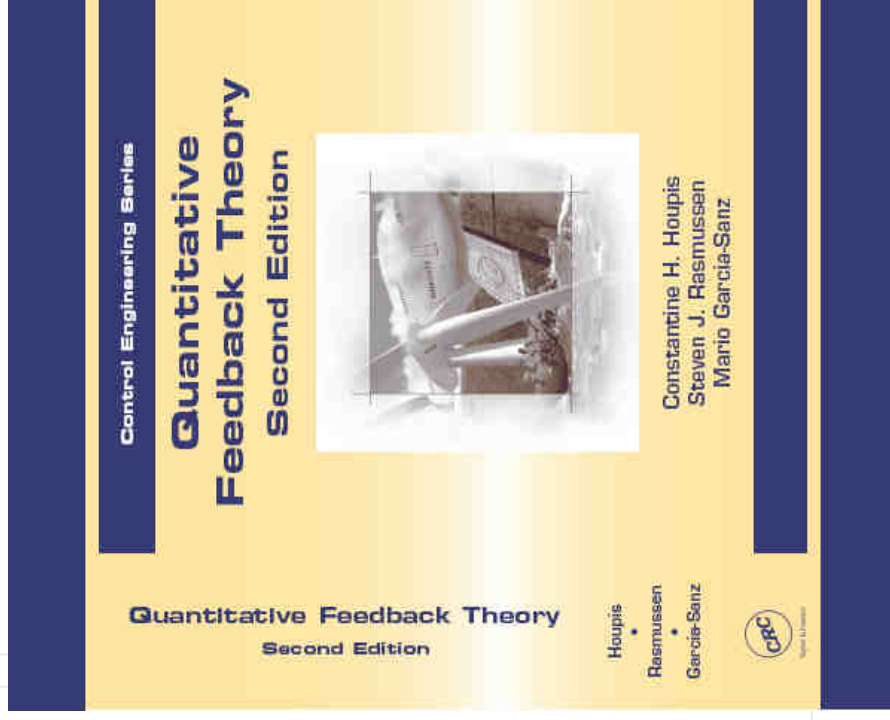
## Design procedure:



### 1.3.- References

- Books:
  - **HOROWITZ, I. M.**, 1993, *Quantitative Feedback Design Theory (QFT)*. QFT Pub., 660 South Monaco Parkway, Denver, Colorado 80224-1229.
  - **YANIV, O.**, 1999, *Quantitative Feedback Design of Linear and Non-linear Control Systems*. Kluwer Academic Pub., ISBN: 0-7923-8529-2.
  - **SIDI, M.**, 2002, *Design of Robust Control Systems: From classical to modern practical approaches*. Krieger Publishing.
  - **HOUPI, CH., RASMUSSEN, SJ., GARCIA-SANZ, M.**, 2006, *Quantitative Feedback Theory. Fundamentals and Applications*. 2<sup>nd</sup> Edition. A CRC book, Taylor and Francis.

- C.H. Houpis, S.J. Rasmussen and M. García-Sanz  
**"Quantitative Feedback Theory. Fundamentals and applications"**. 2<sup>nd</sup> edition, 624 pages, a CRC Press book, **Taylor & Francis**, Boca Ratón, Florida, USA, ISBN: 0849333709, January 2006.



Air Force Institute of Technology  
Wright-Patterson AFB,  
Dayton, Ohio, USA, 2003



- C.H. Houpis, S.J. Rasmussen and M. García-Sanz  
**"Solutions Manual to Quantitative Feedback Theory. Fundamentals and applications"**. 2<sup>nd</sup> edition, 90 pages, **Taylor & Francis**, Boca Ratón, Florida, USA, January 2006.

- International Symposia on Quantitative Feedback Theory and Robust Frequency Domain Methods

- Up to now there have been eight Int. Symp. on QFT:

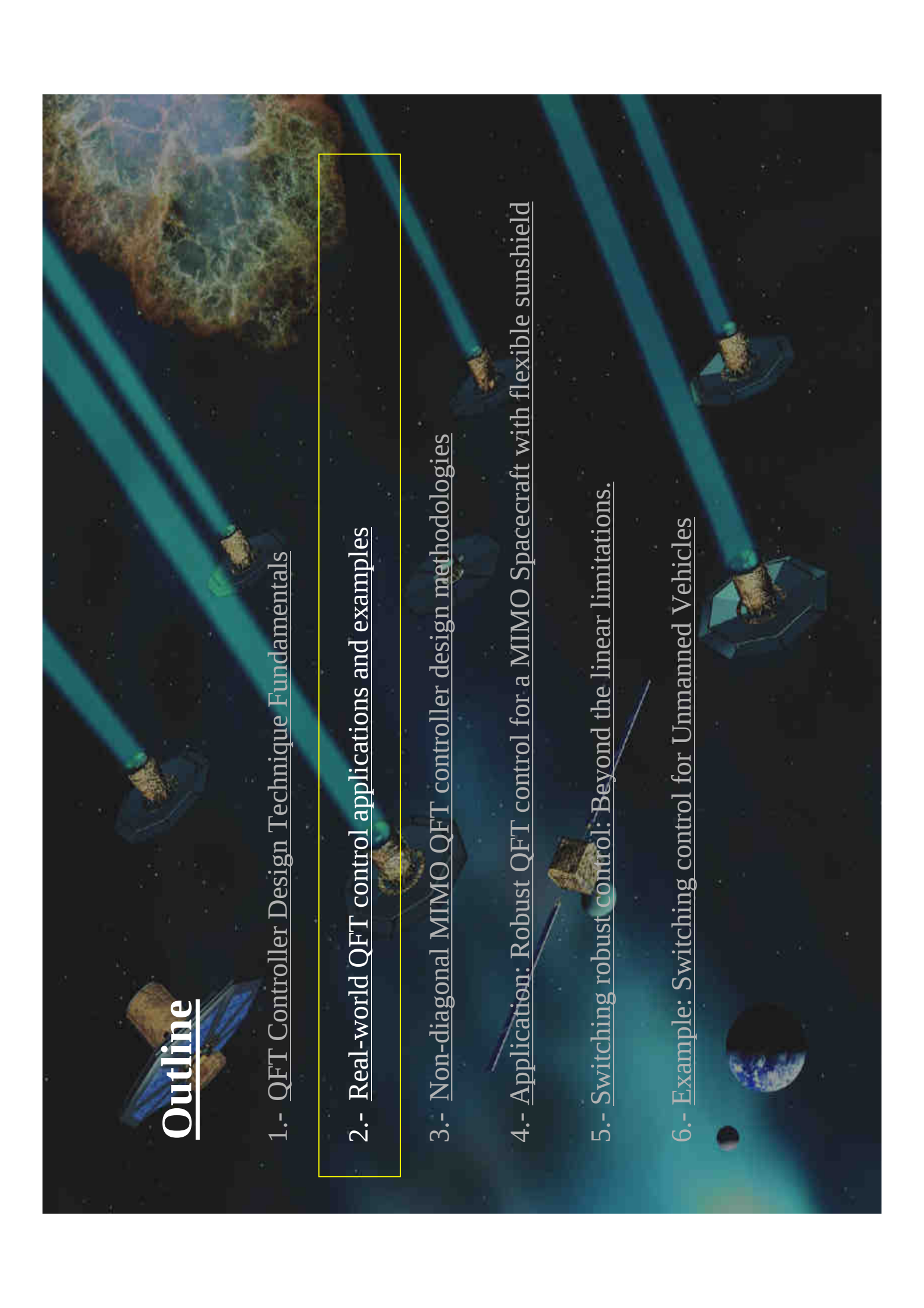
- 1.- **Houpis, C.H., Chander, P.** (Editors). Wright Patterson Airforce Base, Dayton, Ohio, USA, August 1992.
- 2.- **Nwokah, O.D.I., Chander, P.** (Editors). Purdue University, West Lafayette, Indiana, USA, August 1995.
- 3.- **Petropoulakis, L., Leithead, W.E.**(Editors). University of Strathclyde, Glasgow, Scotland, UK, August 1997.
- 4.- **Boje, E., and Eitelberg, E.** (Editors). University of Natal, Durban, South Africa, August 1999.
- 5.- **García-Sanz, M.** (Editor). Public University of Navarra, Pamplona, Spain, August 2001.
- 6.- **Boje, E., and Eitelberg, E.** (Editors). University of Cape Town, Cape Town, South Africa, December 2003.
- 7.- **Colgren, R.** (Editor). University of Kansas, Lawrence, Kansas, USA, August 2005.
- 8.- **Gutman, P-O.** (Editor). Technion, Haifa, Israel, July 2007.

- Special Issues:

- 1.- Nwokah, O.D.I. (Guest Editor). **Horowitz and QFT Design Methods. Special Issue.** *International Journal of Robust and Nonlinear Control*. Vol. 4, Num 1, January-February 1994. Wiley.
- 2.- Houpis, C.H. (Guest Editor). **Quantitative Feedback Theory. Special Issue.** *International Journal of Robust and Nonlinear Control*. Vol. 7, Num 6, June 1997. Wiley.
- 3.- Eitelberg, Eduard (Guest Editor). **Isaac Horowitz. Special Issue.** *International Journal of Robust and Nonlinear Control*. Part 1, Vol. 11, Num 10, August 2001 and Part 2, Vol. 12, Num 4, April 2002. Wiley.
- 4.- Garcia-Sanz, Mario (Guest Editor). **Robust Frequency Domain. Special Issue.** *International Journal of Robust and Nonlinear Control*. Vol. 13, Num 7, June 2003. Wiley.
- 5.- Garcia-Sanz, Mario and Houpis, Constantine (Guest Editors). **Quantitative Feedback Theory. In Memoriam of Isaac Horowitz. Special Issue.** *International Journal of Robust and Nonlinear Control*. Vol. 17, Num 2-3, January 2007. Wiley.

## 1.4.- QFT Software tools

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*Quantitative Feedback Theory Toolbox* - For use with MATLAB, 2nd Ed.  
USA  
<http://www.terasoft.com/products/qft/>
- **GUTMAN, P.O.**,  
*Qsyn*.  
Haifa, Israel.
- **HOUPI, C.H., RASMUSSEN, S., GARCIA-SANZ, M.**, 2006  
*QFT CAD Tool for MISO and MIMO systems*.  
With the book, *Quantitative Feedback Theory. Fundamentals and applications*  
Taylor and Francis, 2nd edition  
USA



# Outline

- 1.- QFT Controller Design Technique Fundamentals
- 2.- Real-world QFT control applications and examples
- 3.- Non-diagonal MIMO QFT controller design methodologies
- 4.- Application: Robust QFT control for a MIMO Spacecraft with flexible sunshield
- 5.- Switching robust control: Beyond the linear limitations.
- 6.- Example: Switching control for Unmanned Vehicles

## 2.- Real-world QFT control applications: Control of a Large Wind Turbine

Multipole,  
Variable Speed, Direct Drive  
1650 kW Wind Turbine.  
TWT1650. M.Torres (Spain).



# TORRES WIND TURBINE, TWT 1650

---

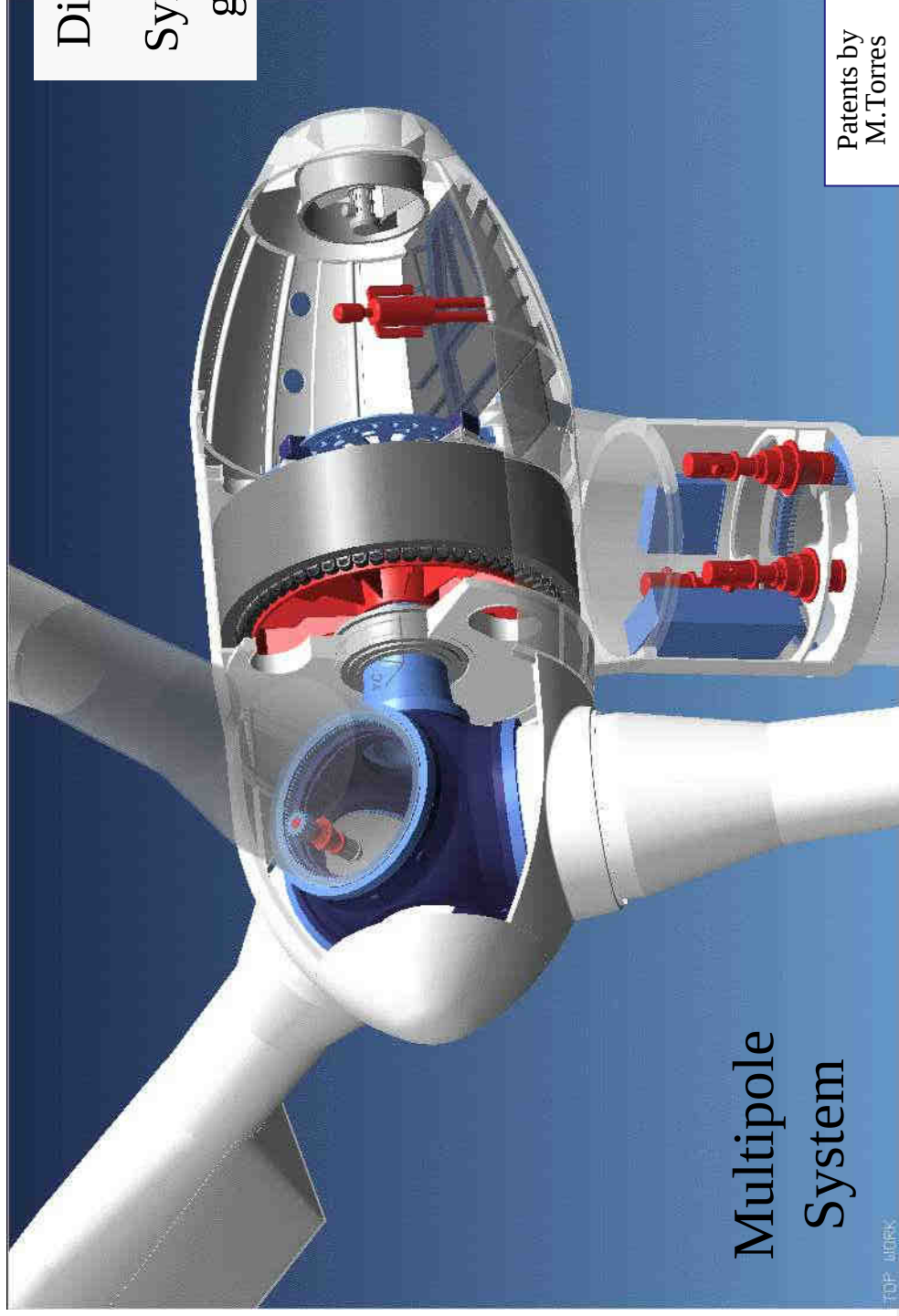
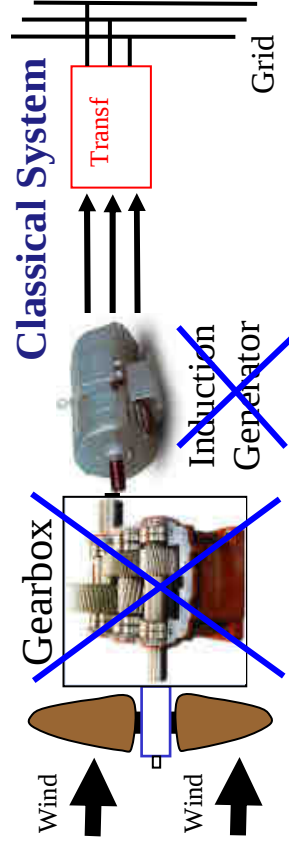
- More than 20 control loops
  - Very Non-linear models
  - MIMO plant
  - Parameter uncertainty
  - High reliability needed
  - Optimum efficiency
- 

E. Torres, M. García-Sanz

*"Experimental Results of the Variable Speed, Direct Drive Multipole Synchronous Wind Turbine: TWT1650". Wind Energy, 2004.*

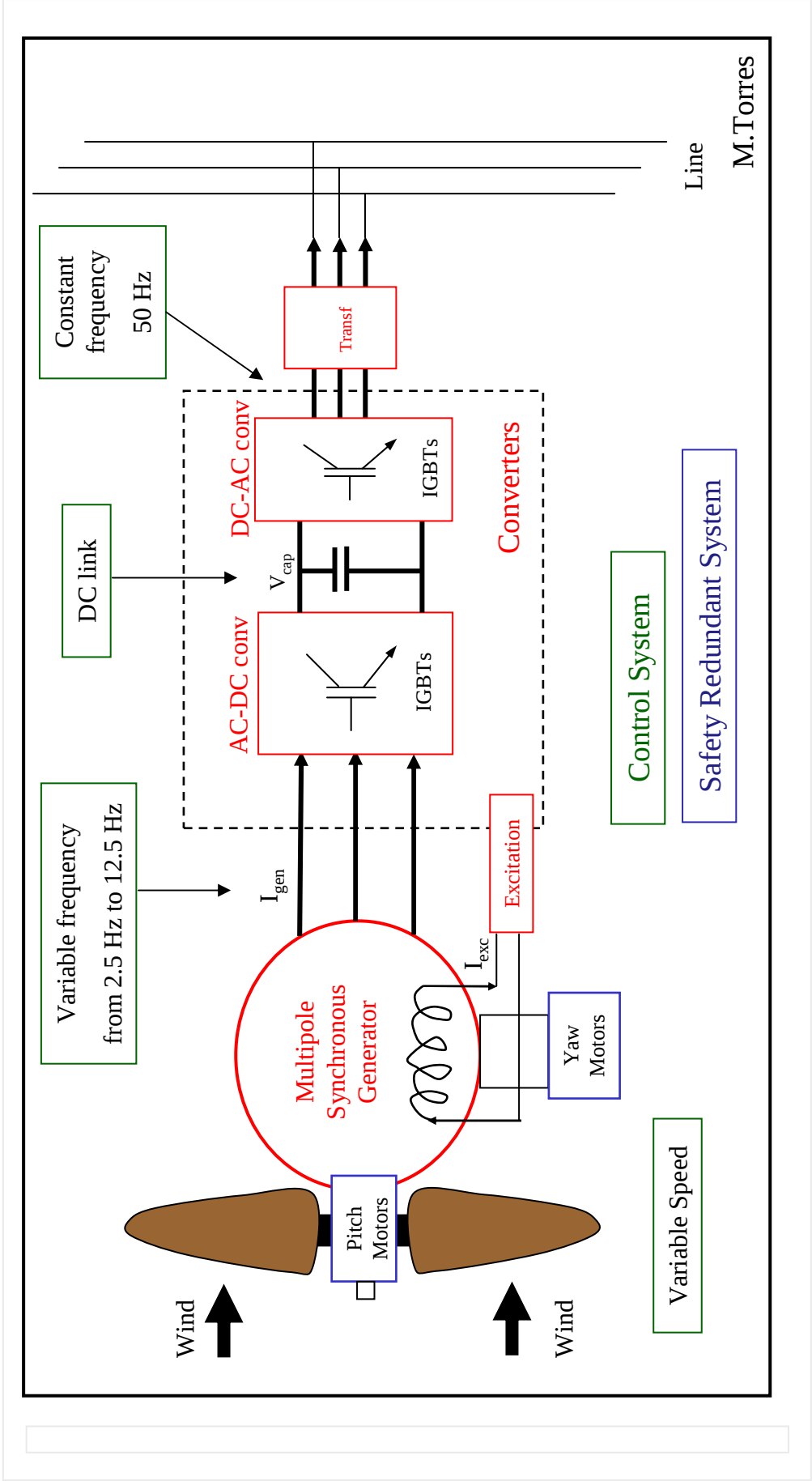
TWT1650.

## Mechanical Diagram



Direct Drive  
Synchronous  
generator

# TWT1650. Electrical Block Diagram





## TWT-1650

1.49

# TWT1650. First Prototype built in May 2001

M.Torres

First Prototype at Cabanillas Wind Farm (Spain)



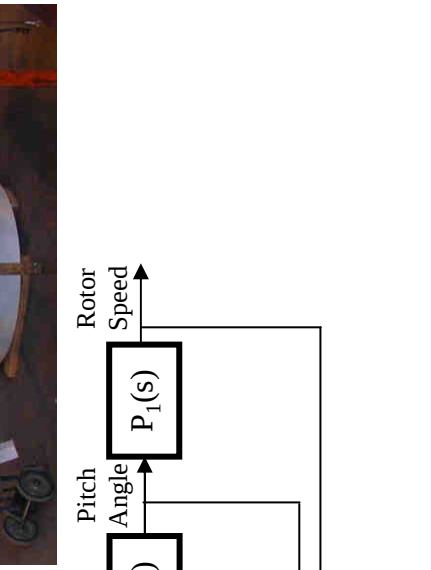
## **TWT1650. First Prototype built in May 2001**

M.Torres



Tower: 70 m ; Blade: 40 m  
Rotor: 82 m ; Inertia rotor: 5,000,000 Kg m<sup>2</sup>





# TWT1650. Actual Results.

Example of three of more than 20 loops.

06.April.2003. Cabanillas Wind Farm (Spain).

M.Torres

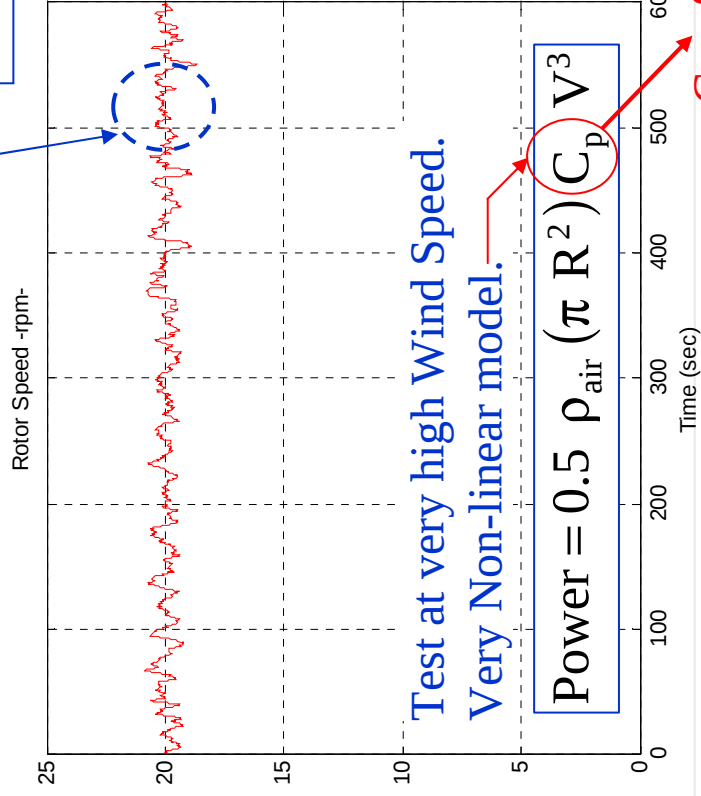
Very High Wind: Aver. 24 m/s

Target: control Rotor Speed (r.p.m.).

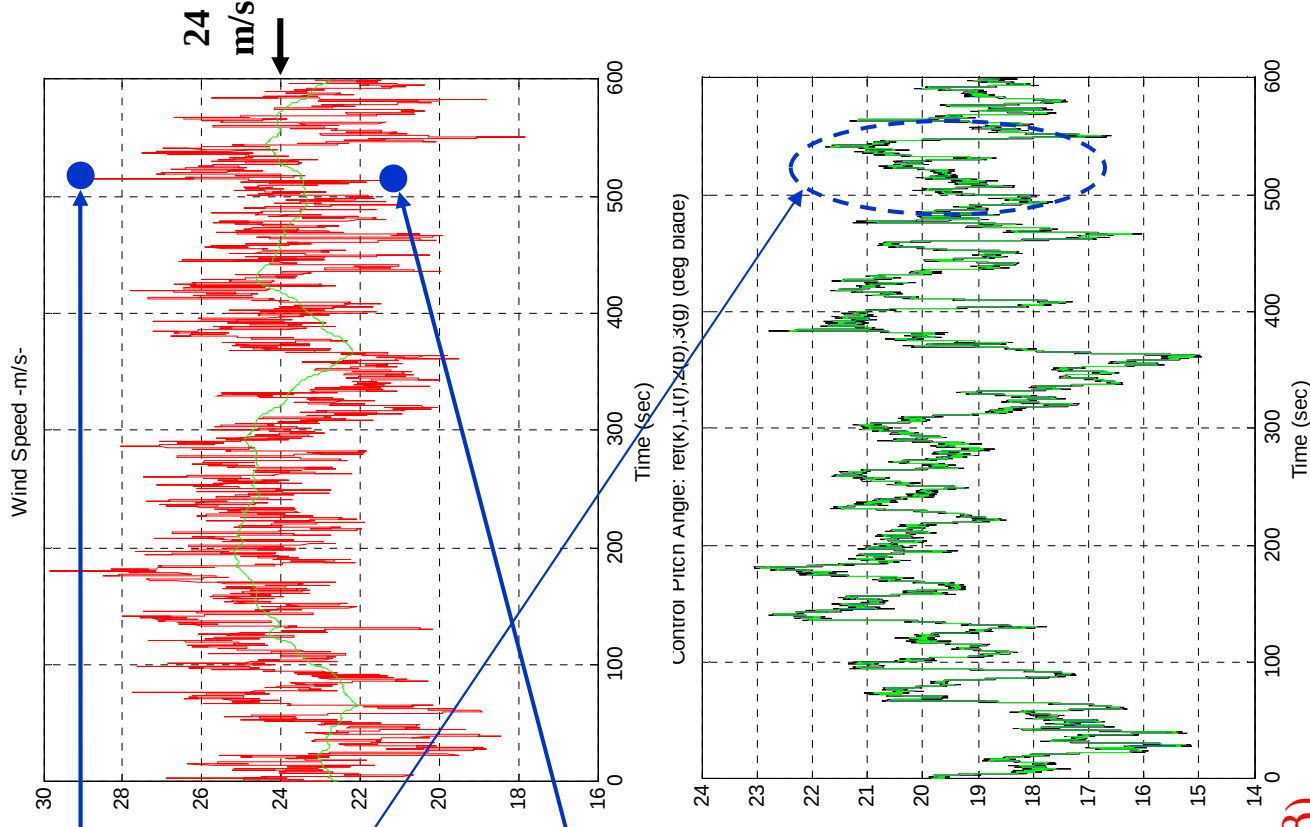
Set-point: 20 rpm

Actuators: Pitch angle movement.

3 Independent driven blades.



1.53



# TWT1650. Actual Results.

Example of three of more than 20 loops.

04.February.2003. Cabanillas Wind Farm (Spain).

M.Torres

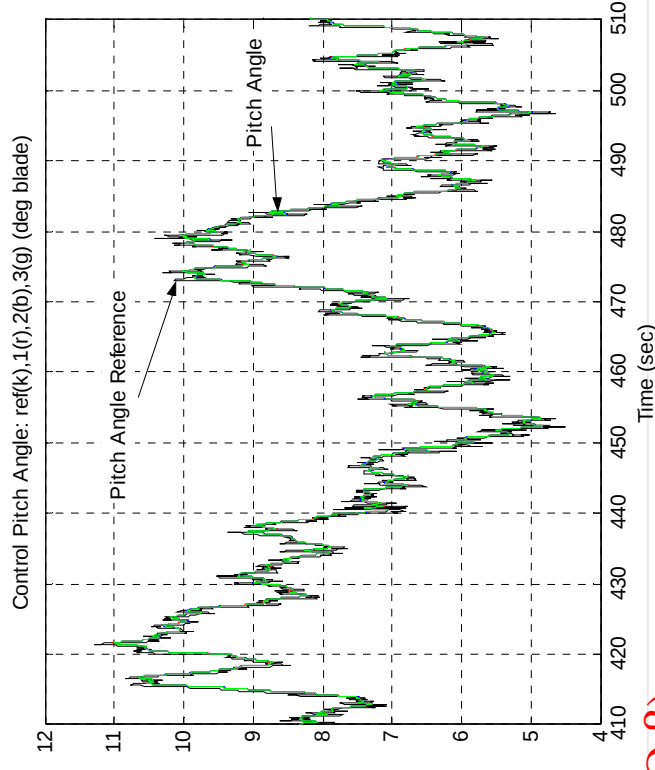
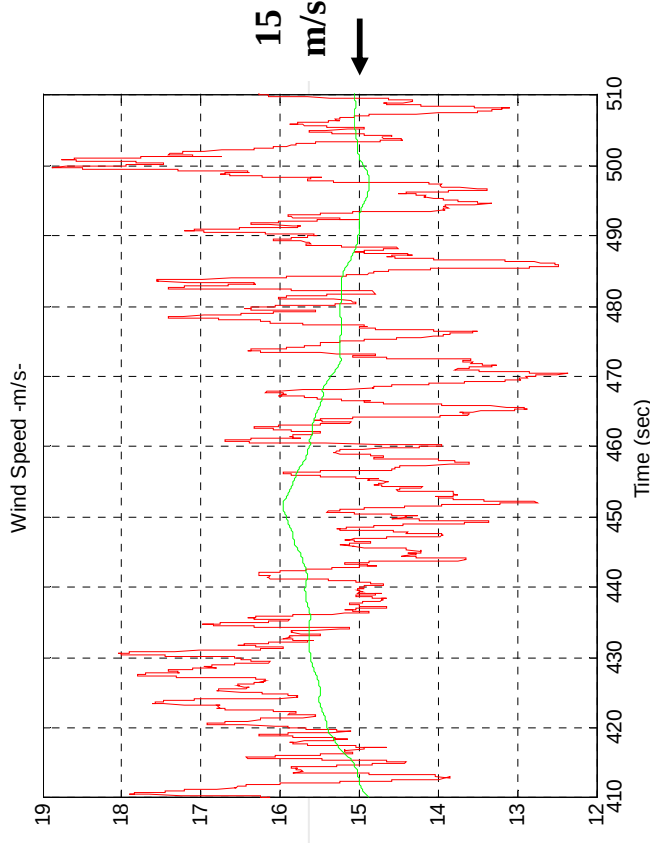
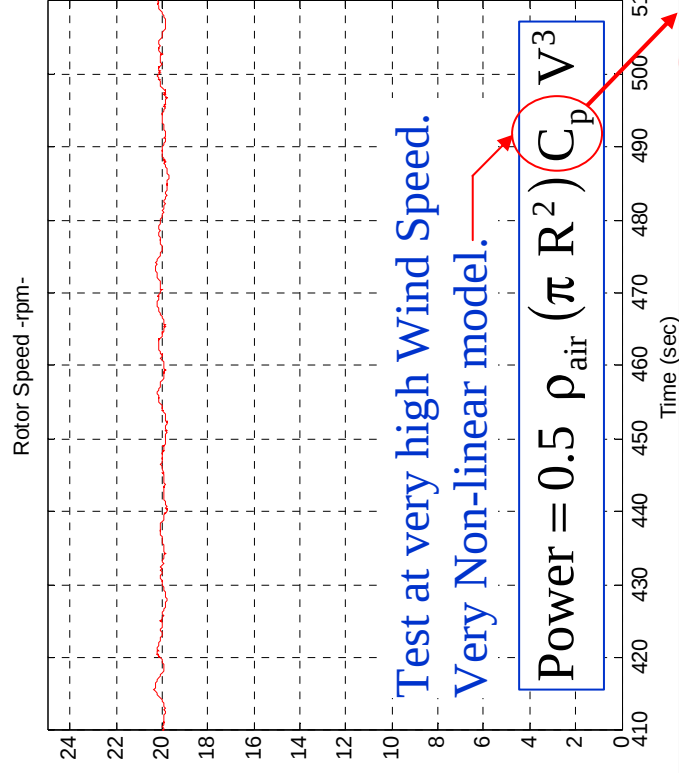
**Medium Wind: Average 15 m/s**

Target: control Rotor Speed (r.p.m.).

Set-point: 20 rpm

Actuators: Pitch angle movement.

3 Independent driven blades.





# Outline

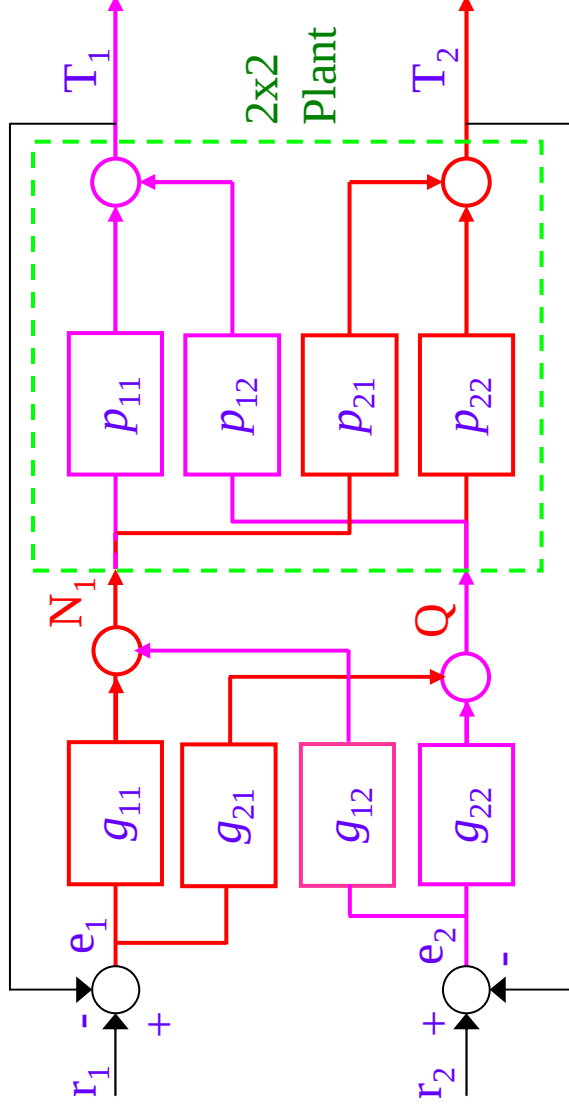
- 1.- QFT Controller Design Technique Fundamentals
- 2.- Real-world QFT control applications and examples
- 3.- Non-diagonal MIMO QFT controller design methodologies
- 4.- Application: Robust QFT control for a MIMO Spacecraft with flexible sunshield
- 5.- Switching robust control: Beyond the linear limitations.
- 6.- Example: Switching control for Unmanned Vehicles

### 3. Non-diagonal MIMO QFT Controller Design Methodologies



This section discusses how the QFT technique can be applied to the design of MIMO control systems.

#### 2x2 Example of a MIMO system (non-diagonal controller)



New Problems: **Interaction** between control loops, Input and output **Directions**, Input-output **Pairing**, **Transmission Zeros** (RHP).

New Tools: **RGA**, **SVD**, **Smith-McMillan**...

### 3.1.- Introduction

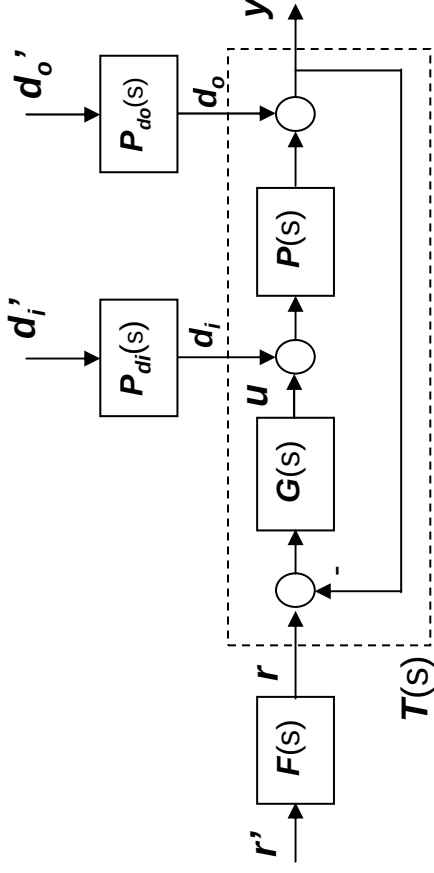
García-Sanz M., Egaña I. (2002). **Quantitative Non-diagonal Controller Design for Multivariable Systems with Uncertainty**. *Int. J. Robust Nonlinear Control*, Vol. 12, No. 4, pp. 321-333.

García-Sanz M., Egaña I., Barreras M. (2005). **Design of quantitative feedback theory non-diagonal controllers for use in uncertain multiple-input multiple-output systems**. *IEE Control Theory and Applications*. Vol. 152, N. 02, pp. 177-187.

- A fully populated (non-diagonal) matrix compensator allows the designer much more design **flexibility** to govern MIMO systems than the classical diagonal controller.
- This session **extends** the classical diagonal QFT compensator design to a fully populated matrix compensator design.
- In this session **three cases** are studied:
  - ➔ - the reference tracking,
  - ➔ - the external disturbance rejection at the plant input and
  - ➔ - the external disturbance rejection at plant output.
- The definition of three **coupling matrices** ( $C_1$ ,  $C_2$ ,  $C_3$ ) of the non-diagonal elements are used to quantify the amount of loop interaction and to design the non-diagonal compensators respectively.
- This yields a criterion to propose a **sequential design methodology** of the fully populated matrix compensator in the QFT robust control frame.

## MIMO System

- Consider an  $n \times n$  linear multivariable system (see Figure), composed of a plant  $\mathbf{P}$ , a fully populated matrix compensator  $\mathbf{G}$ , and a prefilter  $\mathbf{F}$ :



$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{12} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \dots & g_{nn} \end{bmatrix};$$

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1} & f_{n2} & \dots & f_{nn} \end{bmatrix}$$

where  $P \in \mathcal{P}$ , and  $\mathcal{P}$  is the set of possible plants due to uncertainty.

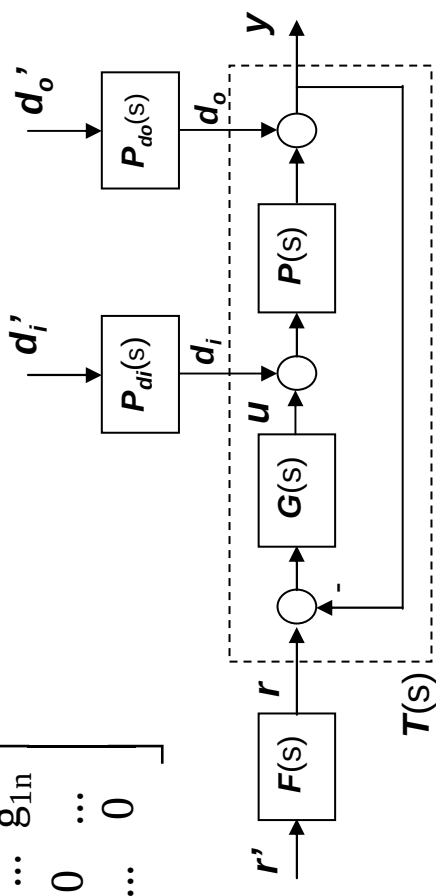
- The plant inverse, denoted by  $\mathbf{P}^*$ , is presented in the following format:

$$\mathbf{P}^{-1} = \mathbf{P}^* = [p_{ij}^*] = \mathbf{A} + \mathbf{B} = \begin{bmatrix} p_{11}^* & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & p_{nn}^* \end{bmatrix} + \begin{bmatrix} 0 & \dots & p_{1n}^* \\ \dots & 0 & \dots \\ p_{n1}^* & \dots & 0 \end{bmatrix}$$

- and where the compensator matrix is broken up into two parts as follows:

$$\mathbf{G} = \mathbf{G}_d + \mathbf{G}_b = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & g_{nn} \end{bmatrix} + \begin{bmatrix} 0 & \dots & g_{1n} \\ \dots & 0 & \dots \\ g_{n1} & \dots & 0 \end{bmatrix}$$

The following introduces a measurement index to quantify the loop interaction in the three classical cases: reference tracking, external disturbances at the plant input, and the external disturbances at the plant output.



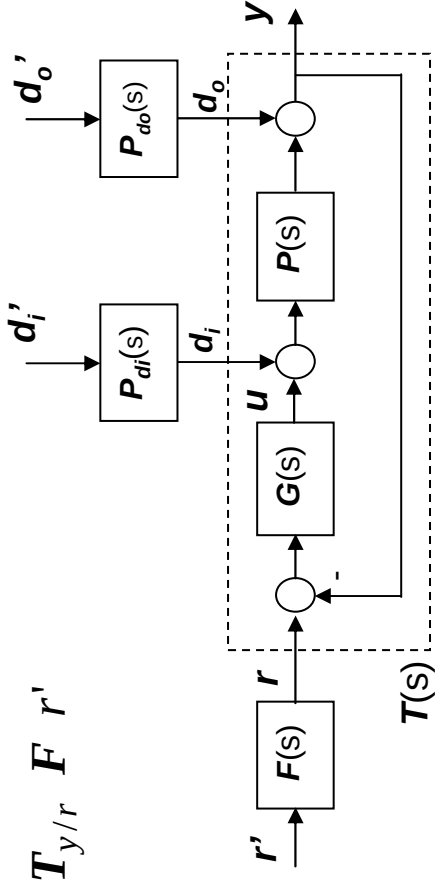
## Reference Tracking.

- The transfer function matrix of the control system for the reference tracking problem, without any external disturbance, is written as follows:

$$y = (I + P G)^{-1} P G r = T_{y/r} \quad r = T_{y/r} F r'$$

and applying the definitions of :

$$P^* = A + B \quad \text{and} \quad G = G_d + G_b$$



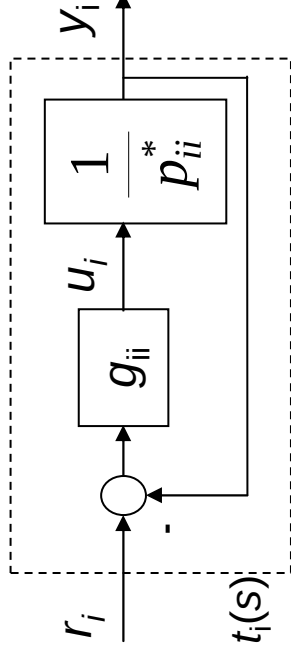
$$T_{y/r} r = \left( (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} G_d \right) r + \left( (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} [G_b \quad r - (B + G_b) T_{y/r} r] \right)$$

**A diagonal term**

**A non-diagonal term**

- A diagonal term:  $T_{y/r_d} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} G_d$

**A diagonal term**



- A non-diagonal term:

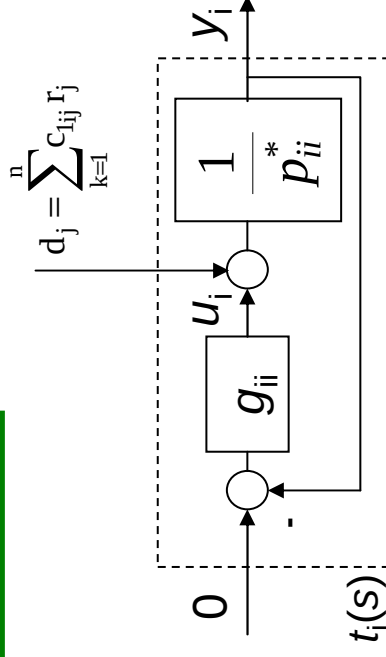
$$T_{y/r_b} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} [G_b - (B + G_b) T_{y/r}] = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} C_1$$

$$C_1 = G_b - (B + G_b) T_{y/r} = \begin{bmatrix} c_{111} & c_{112} & \dots & c_{11m} \\ c_{121} & c_{122} & \dots & c_{12m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1m1} & c_{1m2} & \dots & c_{1mm} \end{bmatrix}$$

**A non-diagonal term**

$$c_{1ij} = g_{ij} (1 - \delta_{ij}) - \sum_{k=1}^n (p_{ik}^* + g_{ik}) t_{kj} (1 - \delta_{ik})$$

$$\delta_{ki} = \begin{cases} \delta_{ki} = 1 & \Leftrightarrow k = i \\ \delta_{ki} = 0 & \Leftrightarrow k \neq i \end{cases}$$



$C_1$  represents the coupling matrix  $C$  of the equivalent system for reference tracking problems

## External disturbance rejection at plant input.

- The transfer function matrix of the control system for the external disturbance rejection at plant input problem, without any external disturbance, is written as,

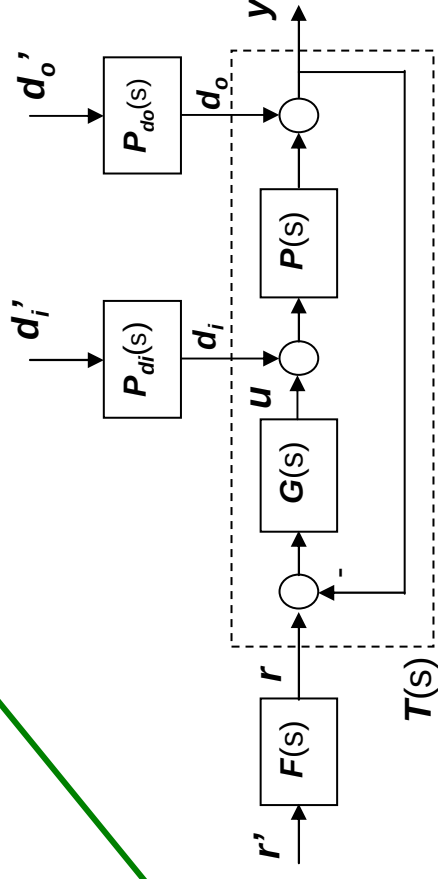
$$y = (I + P G)^{-1} P d_i = T_{y/di} P d_i \quad d_i'$$

and applying the definitions of  $P^* = A + B$  and  $G = G_d + G_b$

$$T_{y/di} d_i = \boxed{(I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} d_i} - \boxed{(I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} [(B + G_b) T_{y/di}] d_i}$$

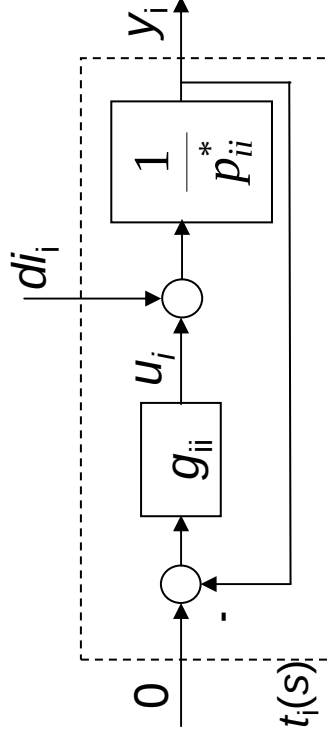
**A diagonal term**

**A non-diagonal term**



- A diagonal term:  $T_{y/di\_d} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1}$

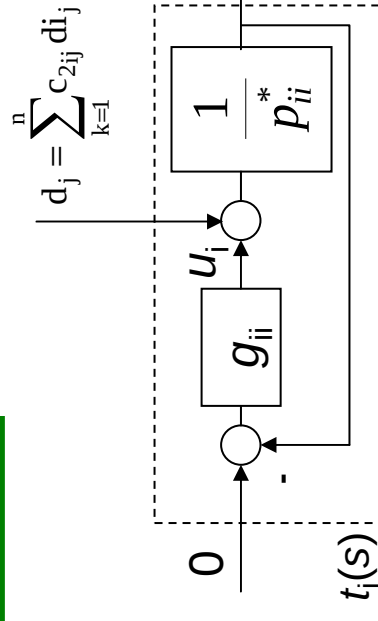
**A diagonal term**



- A non-diagonal term:

$$T_{y/di\_b} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} (B + G_b) T_{y/di} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} C_2$$

**A non-diagonal term**



$$C_2 = (B + G_b) T_{y/di}$$

$$c_{2ij} = \sum_{k=1}^n (p_{ik}^* + g_{ik}) t_{kj} (1 - \delta_{ik})$$

$$\delta_{ki} = \begin{cases} \delta_{ki} = 1 & \Leftrightarrow k = i \\ \delta_{ki} = 0 & \Leftrightarrow k \neq i \end{cases}$$

$C_2$  represents the *coupling matrix* of the equivalent system for external disturbance rejection at the plant input problems

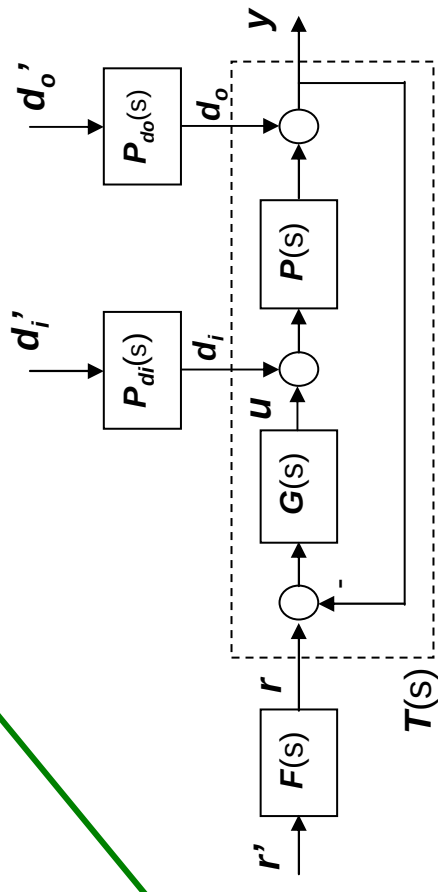
## External disturbance rejection at plant output.

- The transfer function matrix of the control system for the external disturbance rejection at plant output problem, without any external disturbance, is written as,

$$y = (I + PG)^{-1} d_o = T_{y/do} d_o = T_{y/do} P_{do} d_o'$$

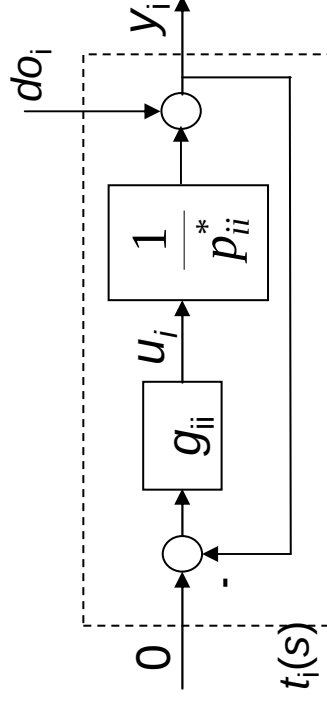
and applying the definitions of  $P^* = A + B$  and  $G = G_d + G_b$

$$T_{y/do} d_o = \underbrace{(I + \Lambda^{-1} G_d)^{-1}}_{\text{A diagonal term}} d_o + \underbrace{(I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} [B - (B + G_b) T_{y/do}]}_{\text{A non-diagonal term}} d_o$$



- A diagonal term:  $T_{y/do\_d} = (I + \Lambda^{-1} G_d)^{-1}$

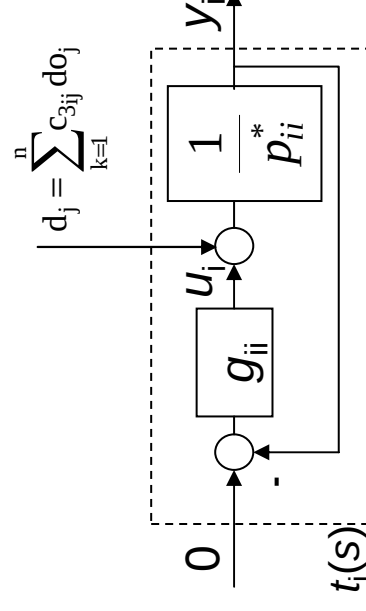
**A diagonal term**



- A non-diagonal term:

$$T_{y/do\_b} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} [B - (B + G_b) T_{y/do}] = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} C_3$$

**A non-diagonal term**



$$C_3 = B - (B + G_b) T_{y/do}$$

$$c_{3ij} = p_{ij}^* (1 - \delta_{ij}) - \sum_{k=1}^n (p_{ik}^* + g_{ik}) t_{kj} (1 - \delta_{ik})$$

$$\delta_{ki} = \begin{cases} \delta_{ki} = 1 & \Leftrightarrow k = i \\ \delta_{ki} = 0 & \Leftrightarrow k \neq i \end{cases}$$

$C_3$  represents the *coupling matrix* of the equivalent system for external disturbance rejection at the plant output problems

## The Coupling elements

- To design a MIMO compensator with a low coupling level, it is necessary to study the influence of every non-diagonal element  $g_{ij}$  on the coupling elements  $c_{1ij}$ ,  $c_{2ij}$  and  $c_{3ij}$ .

- Hypothesis

$$\left| (p_{ij}^* + g_{ij}) t_{jj} \right| \gg \left| (p_{ik}^* + g_{ik}) t_{kj} \right|, \text{ for } k \neq j, \text{ and in the bandwidth of } t_{jj}$$

- Thus,

$$\left| t_{jj} \right| \gg \left| t_{kj} \right|, \text{ for } k \neq j, \text{ and in the bandwidth of } t_{jj}$$

- Due to hypothesis, the coupling effects  $c_{1ij}$ ,  $c_{2ij}$ ,  $c_{3ij}$  are computed as ,

$$c_{1ij} = g_{ij} - \frac{g_{jj} (p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} ; \quad i \neq j \quad \text{tracking}$$

$$c_{2ij} = \frac{(p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} ; \quad i \neq j \quad \text{disturbance rejection at the plant input}$$

$$c_{3ij} = p_{ij}^* - \frac{p_{jj}^* (p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} ; \quad i \neq j \quad \text{disturbance rejection at plant output}$$

## The Optimum non-diagonal controller

- The optimum non-diagonal compensators for the three cases (tracking and disturbance rejection at plant input and output) are obtained making last three Eqs. equal to zero.

$$g_{ij}^{\text{opt}} = F_{pd} \begin{pmatrix} p_{ij}^* \\ g_{jj} \frac{p_{ij}^*}{p_{jj}^*} \end{pmatrix}, \text{ for } i \neq j \quad \text{tracking}$$

$$g_{ij}^{\text{opt}} = F_{pd} \begin{pmatrix} -p_{ij}^* \end{pmatrix}, \text{ for } i \neq j \quad \text{disturbance rejection at the plant input}$$

$$g_{ij}^{\text{opt}} = F_{pd} \begin{pmatrix} p_{ij}^* \\ g_{jj} \frac{p_{ij}^*}{p_{jj}^*} \end{pmatrix}, \text{ for } i \neq j \quad \text{disturbance rejection at plant output}$$

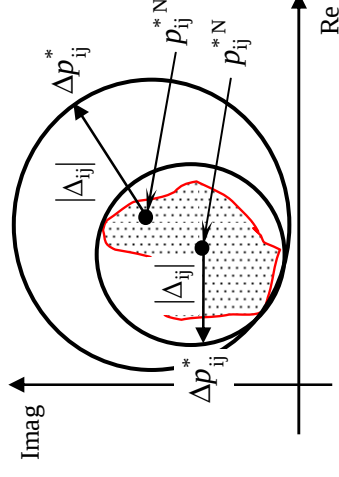
- where the function  $F_{pd}(A)$  means in every case a stable proper function made from the dominant poles and zeros of the expression  $A$ .

- Every uncertain plant can be any plant represented by the family:

$$\{p_{ij}^*\} = p_{ij}^{*N} (1 + \Delta_{ij}) \quad , \quad 0 \leq |\Delta_{ij}| \leq \Delta p_{ij}^* \quad , \quad \text{for } i, j = 1, \dots, n$$

- where  $p_{ij}^{*N}$  is the nominal plant ( $\neq P_o$ ), and  $\Delta p_{ij}^*$  is the maximum of the non-parametric uncertainty radii  $|\Delta_{ij}|$

The nominal plants  $p_{ij}^{*N}$  minimise the maximum of the non-parametric uncertainty radii  $\Delta p_{ij}^*$  and  $\Delta p_{jj}^*$  that comprise the plant templates.



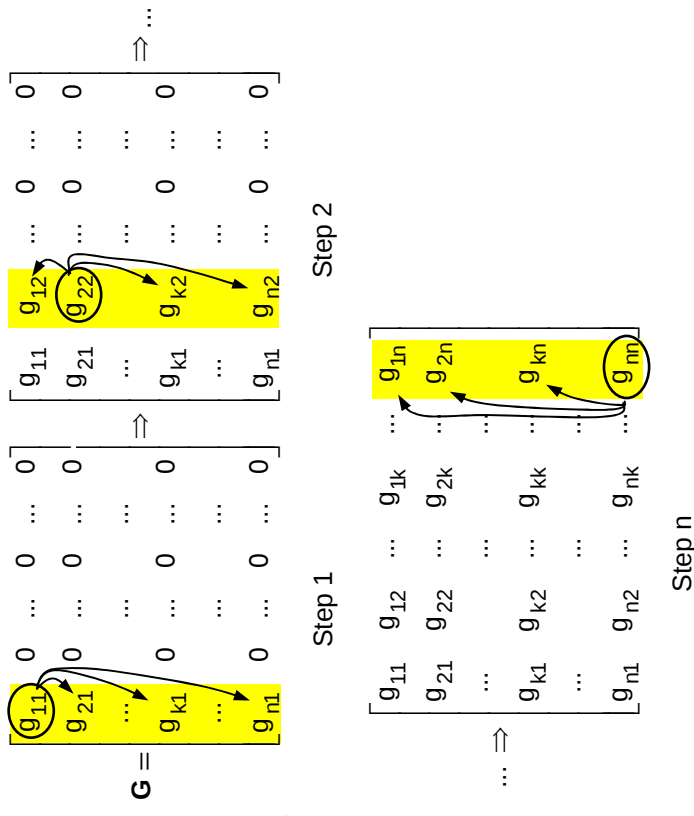
## Design Methodology

- **Step A. Controller Structure, Coupling Analysis, Input/output Pairing and loop ordering.**

First, the methodology begins **pairing** the plant inputs and outputs and selecting the **controller structure** with the Relative Gain Analysis (RGA-Bristol) technique.

This is followed by **arranging** the matrix  $\mathbf{P}^*$  so that  $(p_{11}^*)^{-1}$  has the smallest phase margin frequency,  $(p_{22}^*)^{-1}$  the next smallest phase margin frequency, and so on.

The **sequential** technique, composed of  $n$  stages ( $n$  loops), repeats steps (B and C) for every column  $k = 1$  to  $n$ .



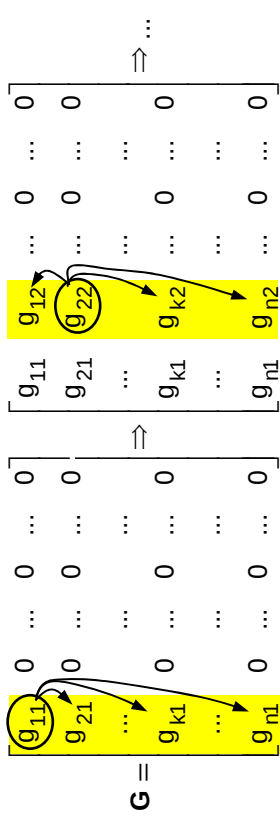
*The compensator design method is a sequential procedure by closing loops.*

- **Step B.** *Design of the diagonal compensator elements  $g_{kk}$ .*

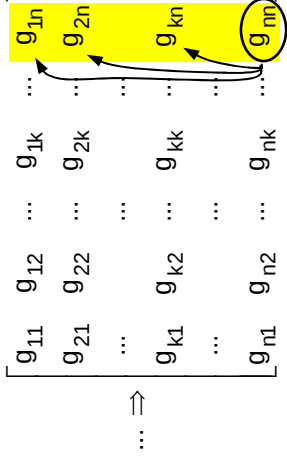
This design of the element  $g_{kk}$  is calculated using the standard QFT loop-shaping technique for the inverse of the equivalent plant  $(p_{kk}^{*e})_k^{-1}$  in order to achieve robust stability and robust performance specifications.

$$[p_{ik}^{*e}]_i = [p_{ik}^{*e}]_{i-1} - \frac{([p_{i,i-1}^{*e}]_{i-1} + g_{i,i-1})([p_{i-1,k}^{*e}]_{i-1} + g_{i-1,k})}{([p_{i-1,i-1}^{*e}]_{i-1} + g_{i-1,i-1})}$$

$$[p_{ik}^{*e}]_1 = P^{-1}$$

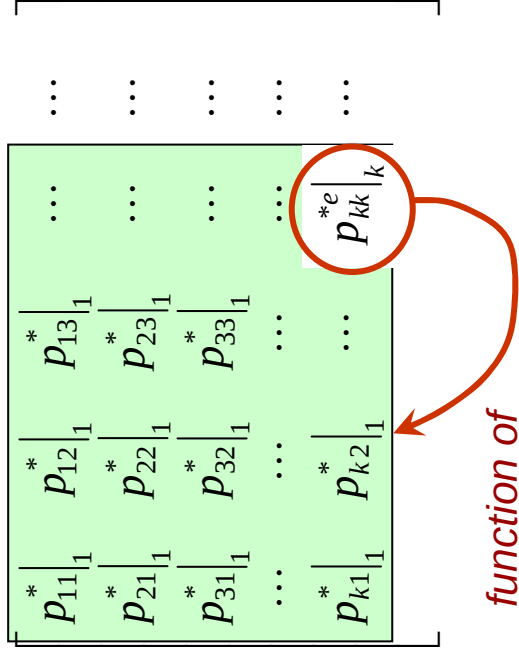


Step 1



Step 2

Step n



- **Step C.** *Design of the non-diagonal compensator elements  $g_{ij}$*

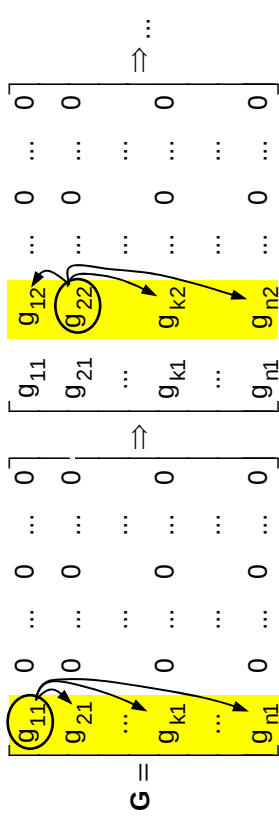
The  $(n-1)$  non-diagonal elements  $g_{ik}$  ( $i \neq k, i = 1, 2, \dots, n$ ) of the  $k$ -th compensator column are designed to, minimise the cross-coupling terms  $c_{ik}$ .

$$c_{1ij} = g_{ij} - \frac{g_{jj} (p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} = 0$$

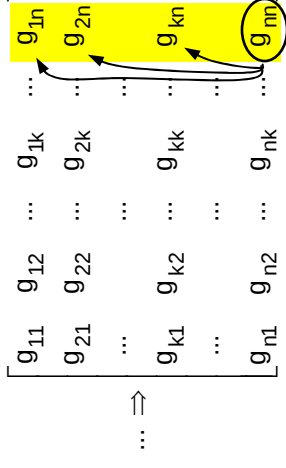
$$c_{2ij} = \frac{(p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} = 0$$

$$c_{3ij} = p_{ij}^* - \frac{p_{jj}^* (p_{ij}^* + g_{ij})}{(p_{jj}^* + g_{jj})} = 0$$

- **Step D.** The design of the prefilter  $F$  does not present any difficulty because the final  $T_{y/r}$  function shows less loop interaction. Therefore, the prefilter  $F$  can be diagonal.



Step 1



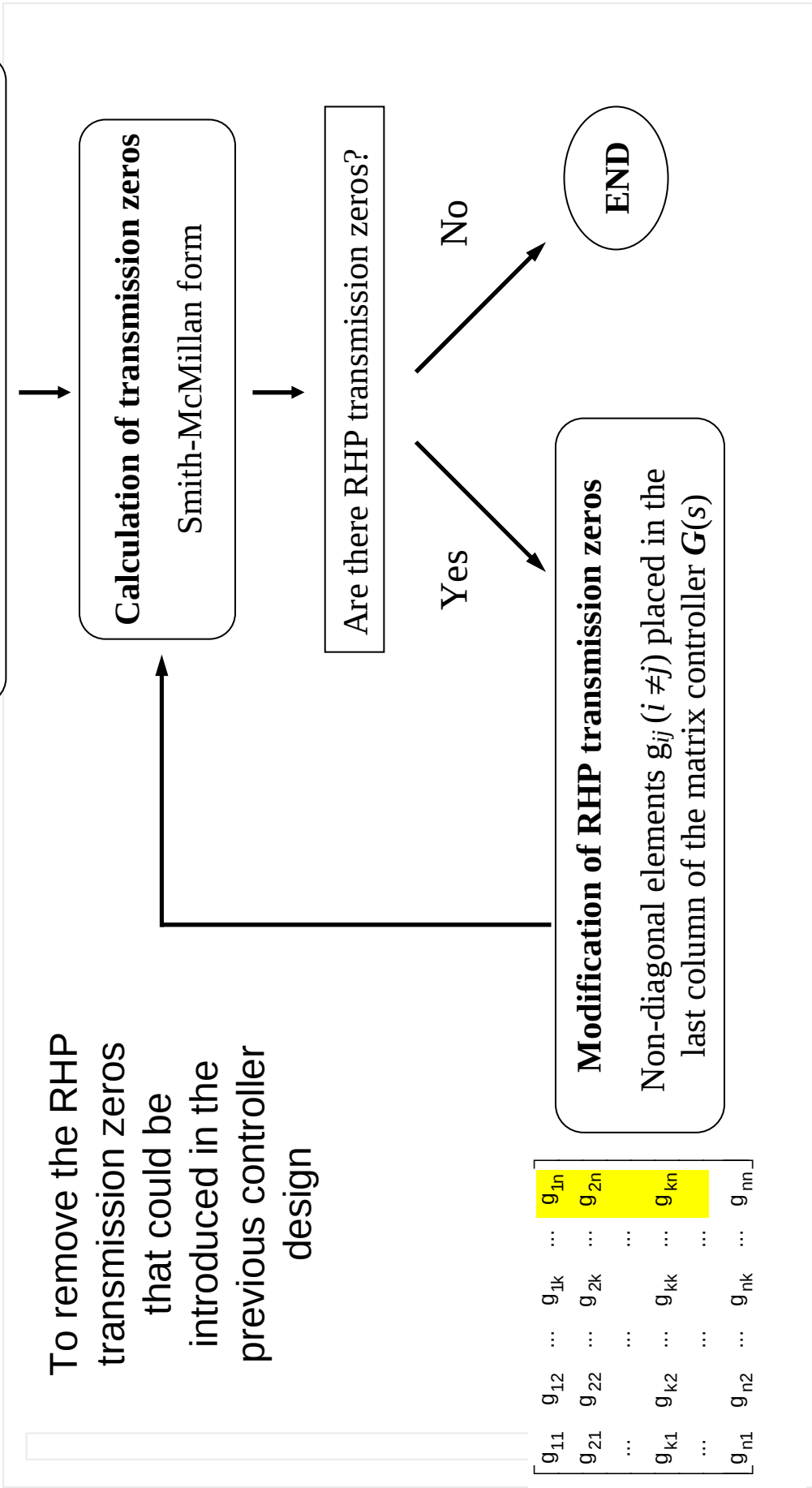
Step n

## Robust Stability of the MIMO system

- The sequential non-diagonal MIMO QFT technique introduced here arrives at a **robust stable** closed-loop system if , for each  $\mathbf{P} \in \mathbf{TP}$  ,
  - a) each  $L_i(s) = g_{ii}(s) (p_{ii}^{*e})^{-1}$ ,  $i=1, \dots, n$ , satisfies the Nyquist encirclement condition,  
*Checked at each loop*
  - b) no RHP pole-zero cancellations occur between  $g_{ii}(s)$  and  $(p_{ii}^{*e})^{-1}$ ,  $i=1, \dots, n$ ,

---
  - c) no Smith-McMillan pole-zero cancellations occur between  $\mathbf{P}(s)$  and  $\mathbf{G}(s)$ , and  
*Checked at the end*
  - d) no Smith-McMillan pole-zero cancellations occur in  $|\mathbf{P}^*(s) + \mathbf{G}(s)|$

RHP transmission zeros of the MIMO system



**End first part**

**Thanks**

**Any questions?**

