

**AFOSR FA9550-08-1-0341: Mathematical Foundations of Active Sensing in
Sensor Networks**

Reporting Period: June 1, 2008 to May 31, 2009

Participants

1. People who have worked on the project

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2. Organizations that have been involved as partners

Defence Science and Technology Organization (DSTO, Australia), University of Melbourne (Australia), University of Wisconsin and Colorado State University (see Contributions)

Activities and Findings

(1) Proof of concept design of pulse trains to provide coverage of position-velocity space by a simple sensor network. This includes development of algorithms for scheduling pulse trains and development of algorithms that leverage spatial diversity for distributed and joint detection.

MIMO radar involves processing signals from several transmitters, each designed to have favorable autocorrelation and low cross-correlation properties. We have investigated the use of narrowband monotonic signals, hopped in frequency over time, for use in MIMO radar. Such signals are orthogonal across transmitters at any point in time, and ambiguities sum to a delta spike over time, thus achieving the required behavior for MIMO. This simple signal model allows MIMO radar to be achieved by unsophisticated radar hardware.

Proof of concept was demonstrated during the reporting period in collaboration with DSTO Australia and the University of Melbourne. Our findings were presented in September 2009 at the 2009 Defense Applications of Signal Processing Workshop. The figures included below are for a single target and a simple network of four transmitters and four receivers.

Each transmitter sends one sinusoid per time slot, and at each receiver this signal has experienced delay (possibly fractional), Doppler shift (possibly fractional), and attenuation. As the number of time slots grows the sum of ambiguities evolves as shown below.

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14. ABSTRACT This proposal will develop the mathematical foundations of distributed active sensing in a way that is consistent with the physics of scattering. We will emphasize scheduling of sensor transmissions to minimize communications overhead and maximize energy efficiency. We will capture the fundamental tradeoff between rate and reliability that results from individual sensors either independently probing a large number of surveillance cells, or cooperating to probe a smaller number of surveillance cells more precisely. Our aim is to connect topology of the sensor network with achievable points on the rate-reliability tradeoff curve across the spatial domain.						
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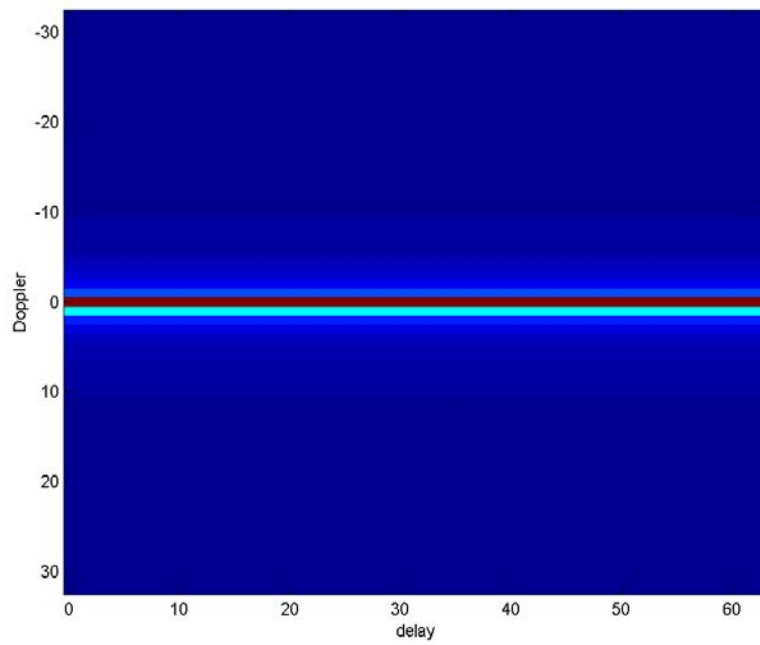


Figure 1. The ambiguity function of a single sinusoid

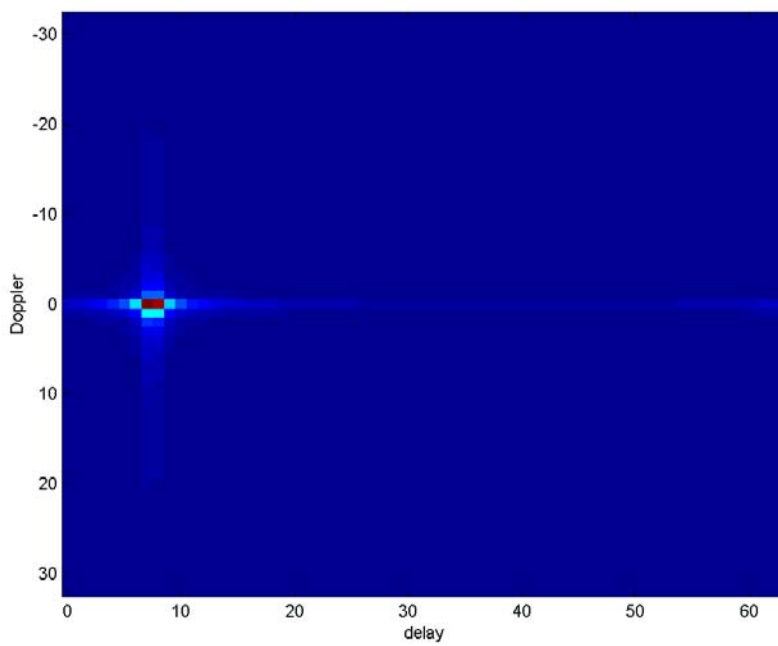


Figure 2. Sum of ambiguities over all sinusoids

The location and strength of sidelobes is a function of the transmitted frequencies. We have demonstrated that judicious choice of frequency regime can optimize sidelobe properties after relatively few time slots.

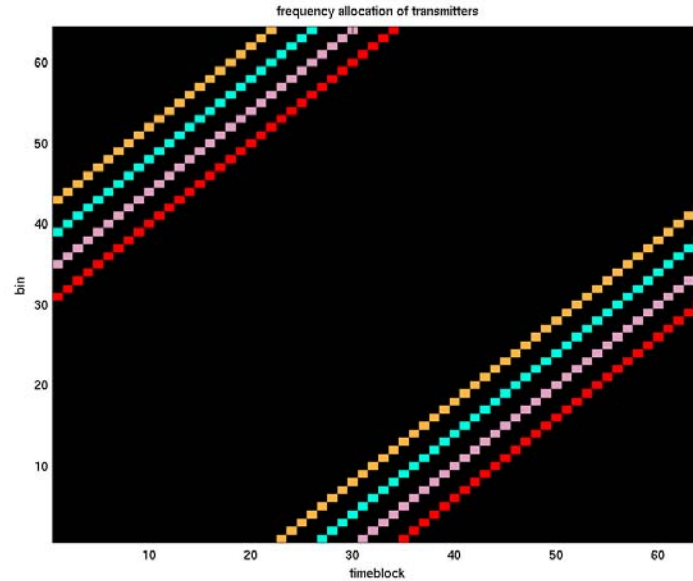


Figure 3. Linear stepped frequency assignment for 4 transmitters

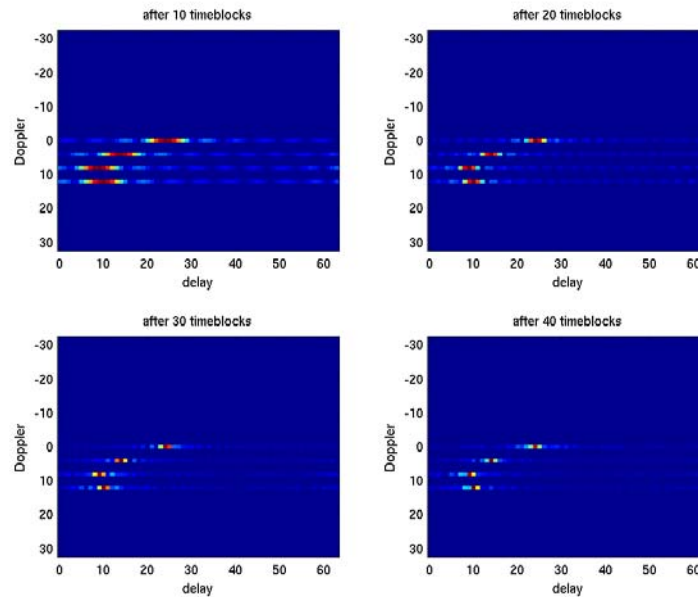


Figure 4. Ambiguity sum at an individual receiver

Results are shown below for two different frequency assignments: linear stepped frequency shown in Figure 3 where the fixed separation causes Doppler cross-transmitter lobes (see Figure 4); and randomly chosen but distinct frequencies shown in Figure 5 where the Doppler sidelobes cover more area but are less intense (see Figure 6).

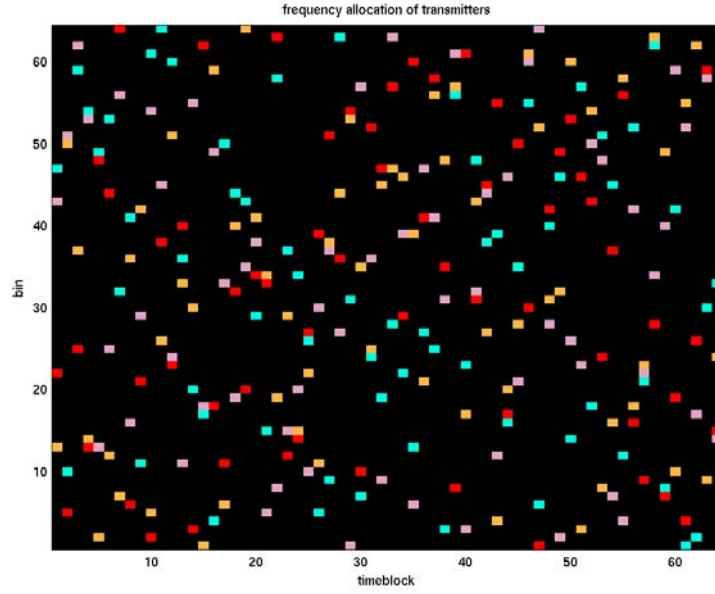


Figure 5. Random assignment of frequencies to four transmitters

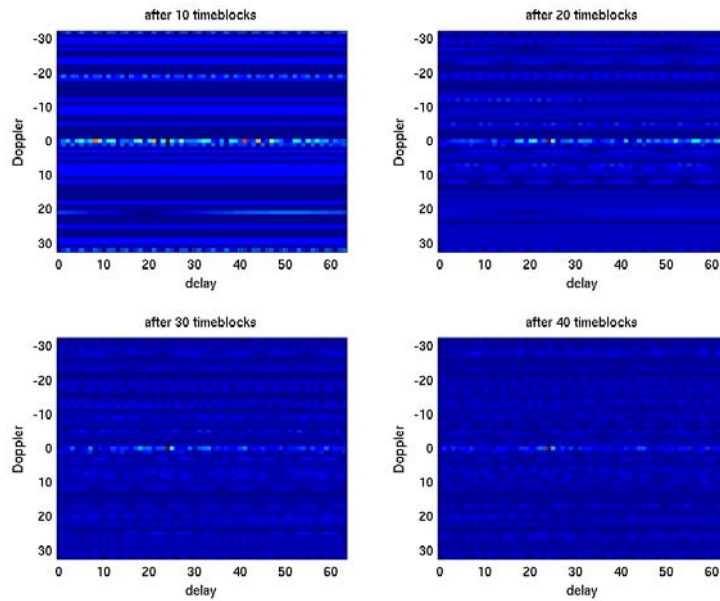


Figure 6. Ambiguity sum at a single receiver

Total ambiguity is then mapped to the 2D grid by calculating the range ambiguity for each Tx-Rx combination, converting range bins to ellipses, and accumulating the ambiguities. The two methods of frequency assignment provide similar performance as the number of time slots becomes large, but random frequency assignment narrows target position more quickly (compare Figures 7 and 8).

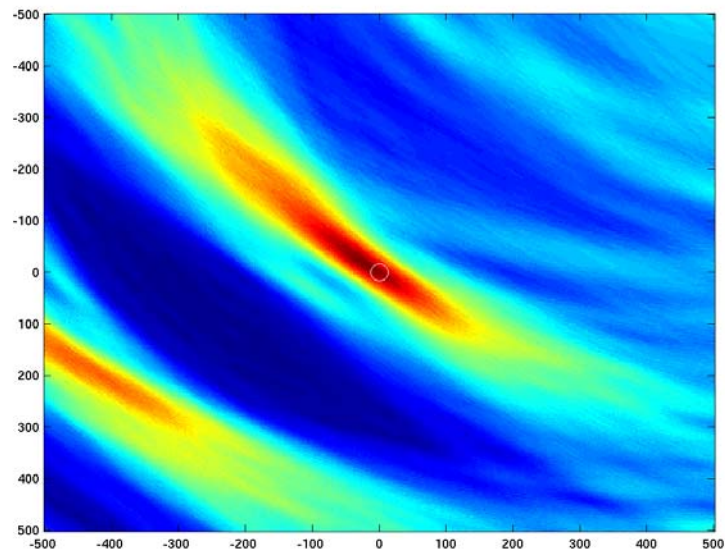


Figure 7a. Accumulated 2D ambiguity over 10 time slots for stepped frequency assignment

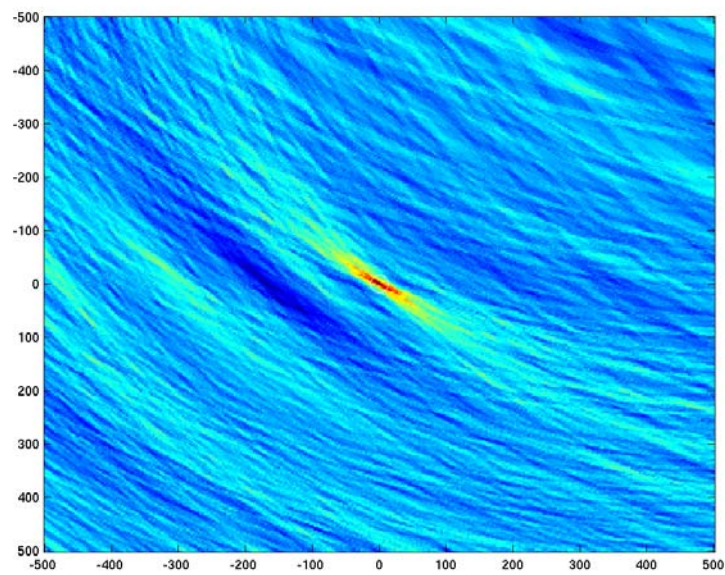


Figure 7b. Accumulated 2D ambiguity over 10 time slots for random frequency assignment

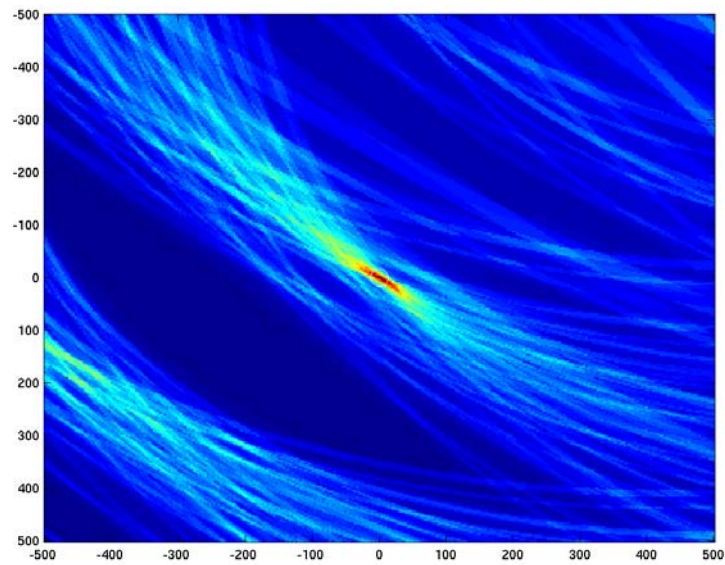


Figure 8a. Accumulated 2D ambiguity over all time slots for stepped frequency assignment

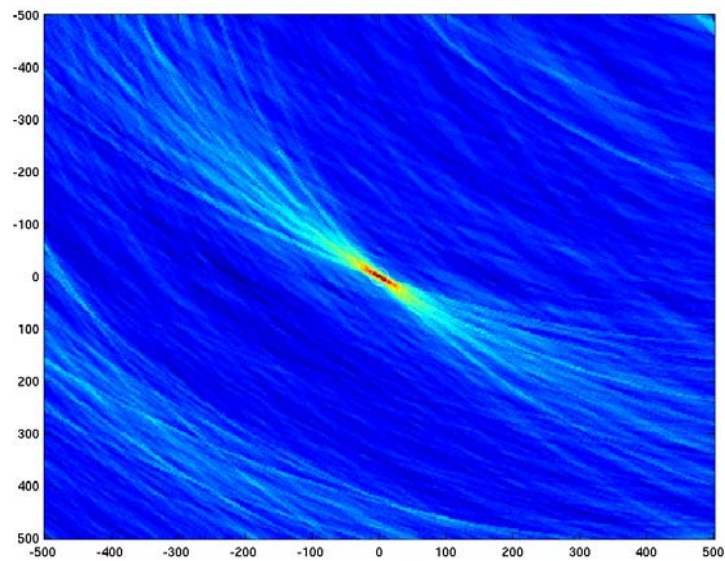


Figure 8b. Accumulated 2D ambiguity over all time slots for random frequency assignment

For more details see

A.R. Calderbank, W. Moran, S. Suvorova, S. Searle and S.D. Howard, Minimal MIMO radar with discretely spaced carrier waves, Defense Applications of Signal Processing (DASP 2009), September 27-30, Hawaii

(2) *Theoretical exploration of performance as a function of the topology of the sensor network, specifically an understanding of how fundamental limits on performance relate to the placement of sensors and the position of the target (which is not under our control).*

We have investigated feasibility of the reconstruction of a sparse scattering field by compressive sensing.

The theory of compressed sensing suggests that successful inversion of an image of the physical world (e.g., a radar/sonar return or a sensor array snapshot vector) for the source modes and amplitudes can be achieved at measurement dimensions far lower than what might be expected from the classical theories of spectrum or modal analysis, provided that the image is sparse in an *a priori known* basis. For imaging problems in passive and active radar and sonar, this basis is usually taken to be a DFT basis. The compressed sensing measurements are then inverted using an l_1 -minimization principle (basis pursuit) for the nonzero source amplitudes. This seems to make compressed sensing an ideal image inversion principle for high resolution modal analysis. .

We have developed an approach to radar imaging that exploits *sparsity in the matched filter domain* to enable high resolution imaging of targets in delay and Doppler. The starting point is a sparse representation for the vector of radar cross-ambiguity values at any fixed test delay cell in a Vandermonde frame that is obtained by discretizing the Doppler axis. The expansion coefficients are given by the auto-correlation functions of the transmitted waveforms. Orthogonal matching pursuit (OMP) algorithm is then used to identify the locations of the radar targets in delay and Doppler. Unambiguous imaging in delay is enabled by transmission of a Golay pair of phase coded waveforms to eliminate delay sidelobe effects. We have extended this work to multi-channel radar, by developing a sparse recovery approach for dually-polarimetric radar. Here sparsity is exploited in a bank of matched filters, each of which is matched to an entry of an Alamouti matrix of Golay waveforms to recover a co-polar or cross-polar polarization scattering component.

In reality no physical field is sparse in the DFT basis or in an *a priori known* basis. In fact the main goal in image inversion is to *identify* the modal structure. No matter how finely we grid the parameter space the sources may not lie in the center of the grid cells and there is always mismatch between the assumed and the actual bases for sparsity. We have studied the sensitivity of basis pursuit to mismatch between the assumed and the actual sparsity bases and compared the performance of basis pursuit with that of classical image inversion. Our mathematical analysis and numerical examples show that the performance of basis pursuit degrades considerably in the presence of mismatch, and they suggest that the use of compressed sensing as a modal analysis principle requires more consideration and refinement, at least for the problem sizes common to radar/sonar.

Results were obtained during the reporting period in collaboration with Colorado State University and developed further in a subsequent and very successful summer internship of Princeton graduate student Yuejie Chi at Colorado State University. Two conference papers are listed below and two journal papers are in preparation for submission to IEEE Transactions on Signal Processing and IEEE Transactions on Information Theory.

Y. Chi, A. Pezeshki, L. Scharf, and R. Calderbank, Golay complementary waveforms for sparse delay-Doppler radar imaging, to appear in *Proceedings of the 3rd International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP '09)*, Aruba, Dutch Antilles, December 11-13, 2009

Y. Chi, A. Pezeshki, L. Scharf, and R. Calderbank, The sensitivity to basis mismatch of compressed sensing for spectrum analysis and beamforming, to appear in *Proceedings of the 6th US/Australia Joint Workshop on Defense Applications of Signal Processing (DASP)*, Hawaii, September 27-October 1

(3) Development of simple methods for constructing radar waveforms with ambiguity functions that are free of sidelobes inside a desired range or Doppler interval.

Sixty years ago, efforts by Marcel Golay to improve the sensitivity of far infrared spectrometry led to the discovery of complementary sequences which have the property that the sum of their autocorrelation functions vanishes at all delays other than zero. Almost a decade after their invention, Welti rediscovered complementary sequences (there are his D-codes) and proposed to use them for pulsed radar. However, since then they have found very limited application in radar as it soon became evident that the perfect autocorrelation property of complementary sequences cannot be easily utilized in practice. The reason, to quote Ducoff and Tietjen, is “in a practical application, the two sequences must be separated in time, frequency, or polarization, which results in decorrelation of radar returns so that complete sidelobe cancellation may not occur. Hence they have not been widely used in pulse compression radars.” Various generalizations of complementary sequences including multiple complementary codes by Tseng and Liu, and multiphase (or polyphase) complementary sequences by Sivaswami and by Frank suffer from the same problem.

These roadblocks served as the starting point our research program. We have described in [1, 2] how to design pulse trains for which the composite ambiguity function maintains ideal shape at small Doppler shifts. We have also described new nonlinear signal processing methods that enable use of complementary waveforms in OFDM radar and provide Doppler resilience at the chip level. Looking to the future, we have proposed unitary filter banks as a new illumination paradigm that enables broad waveform adaptability across time, space, frequency and polarization.

In collaboration with Colorado State University, we have dualized (*RadarCon09*) our earlier results (*WDD 2009*) to design waveforms for which the ambiguity function is free of Doppler sidelobes across a range interval of interest. This was accomplished by exploiting the time-frequency duality between PAM and OFDM waveforms. The pulse trains designed for range sidelobe suppression employ PTM sequences of Golay complementary codes to amplitude modulate a pulse shape in time. Depending on the choice of the PTM sequence, the resulting PAM waveform annihilates range sidelobes inside some Doppler interval. The new dual waveforms stack a PTM sequence of Golay complementary codes across OFDM tones to annihilate Doppler sidelobes in a range interval of interest. Sequential application of the two designs can be used to suppress range sidelobes and Doppler sidelobes in succession. The proposed designs are simple and only require a phase code library with two components that form a complementary pair. More information can be found in the following publications:

Y. Chi, A. Pezeshki, and R. Calderbank, Complementary waveforms for sidelobe suppression and radar polarimetry, in *Applications and Methods of Waveform Diversity*, V. Amuso, S. Blunt, E. Mokole, R. Schneible, and M. Wicks, Eds. SciTech Publishing, to appear in 2009

Y. Chi, A. Pezeshki, R. Calderbank, and S.D. Howard, Range sidelobe suppression in a desired Doppler interval, *Proceedings of the 4th International IEEE Waveform Diversity and Design Conference*, Orlando, FL, February 8-13, 2009

A. Pezeshki, R. Calderbank, L. Scharf, Sidelobe suppression in a desired range/Doppler interval, *Proceedings of the IEEE Radar Conference (RadarCon09)*, Pasadena, CA, May 4-8, 2009

(4) In collaboration with the University of Wisconsin, we have developed methods for detecting and localizing anomalous network activations. These activations are supposed to be weak in strength so that they are invisible in the noise present at an individual node, and they are weak in extent so they are invisible in global averages.

We were motivated by the onset of malicious activity or congestion in the Internet, by sensor networks monitoring incipient contamination, and by patterns in social networks. We have developed a sparsifying transform adapted to (unknown) network topology that is able to amplify weak patterns of activity. We have also implemented the transform on the PlanetLab infrastructure. Here we do not have ground truth about anomalies but we have demonstrated that local activations are registered by large changes in a small number of transform coefficients instead of much smaller changes in a much larger number of coefficients for the standard basis (where every node is associated with an elementary basis vector).

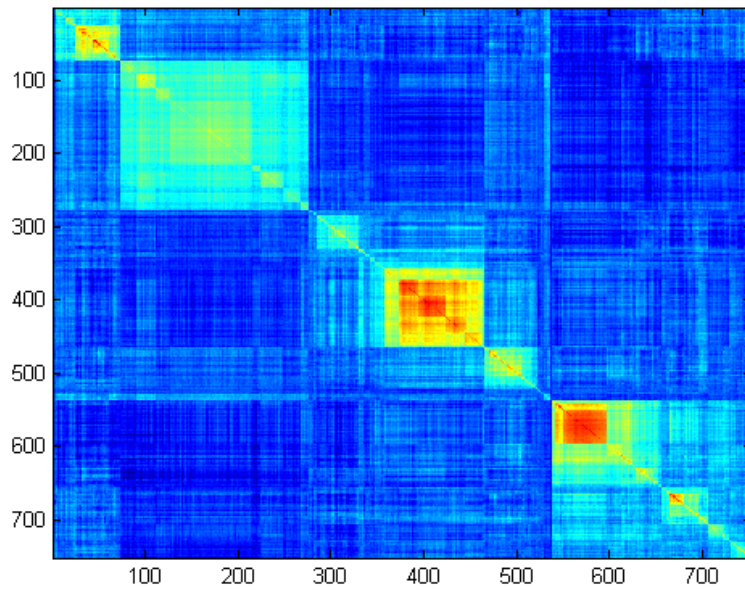


Figure 9. Multiple network monitors record packets from Internet nodes to generate a covariance matrix to which hierarchical clustering is applied. These clusters are associated with basis vectors which amplify local network activity.

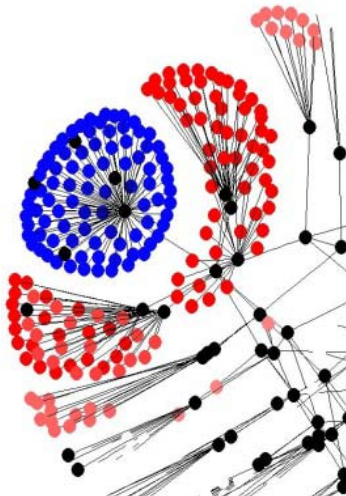


Figure 10. An example of a Haar-like basis vector produced by our hierarchical clustering. The vector is supported on red and blue nodes with the different colors representing two different values.

This work is ongoing with three principal directions:

- Minimal and Selective Probing: How to make persistent network phenomena visible with minimal sampling of a given network topology or

pairwise covariance information? How to adapt sampling to optimize a given inference task?

- Taking Advantage of Multiple Monitors: How to exploit dependencies within and between nodes and monitors to detect weak signatures?
- Incorporating Stochastics: Assuming a generative graphical model form, how to efficiently detect changes in the unknown topology using finite samples?

Appendix: Archival Publications (Published) During Reporting Period

L. Applebaum, S. D. Howard, S. Searle, and A. R. Calderbank, Chirp sensing codes: Deterministic compressed sensing measurements for fast recovery, *Applied and Computational Harmonic Analysis*, September 2008.

A. Pezeshki, A.R. Calderbank, W. Moran and S. D. Howard, Doppler resilient waveforms with perfect autocorrelation, *IEEE Transactions on Information Theory*, vol. 54, no. 9, pp. 4254-4266, September 2008

M.D. Zoltowski, T.R. Qureshi, and A.R. Calderbank, Complementary Codes based Channel Estimation for MIMO-OFDM Systems, *Proceedings of 46th Allerton Conference on Communication, Control, and Computing*, Monticello, IL, September 24-26, 2008.

M.D. Zoltowski, T.R. Qureshi, A.R. Calderbank, and W. Moran, Unitary Design of Radar Waveform Diversity Sets, *Proceedings of Asilomar Conference on Signals, Systems, and Computers*, October 2008.

A.R. Calderbank, S.D. Howard, and W. Moran, Waveform diversity in radar signal processing, *IEEE Signal Processing Magazine*, Vol. 26 (1), pp. 32-41, January 2009.

Y. Chi, A. Pezeshki, R. Calderbank, and S.D. Howard, Range sidelobe suppression in a desired Doppler interval, *Proceedings of the 4th International IEEE Waveform Diversity and Design Conference*, Orlando, FL, February 8-13, 2009

A. Pezeshki, R. Calderbank, L. Scharf, Sidelobe suppression in a desired range/Doppler interval, *Proceedings of the IEEE Radar Conference (RadarCon09)*, Pasadena, CA, May 4-8, 2009