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**Final Performance Report**

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**Hearing Protection for High-Noise Environments**

Period of Performance: **Oct 01, 2007 – Nov 30, 2009**

**Attachment 2**

**MATRIX ELEMENTS OF VOLUMETRIC INTEGRAL OPERATORS  
IN ACOUSTICS AND ELASTICITY  
FOR NODE BASED BASIS FUNCTIONS ON TETRAHEDRAL SUPPORTS**

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## Abstract

A new method of evaluation of matrix elements of the monopole and dipole terms of the Green function appearing in volumetric integral equations in acoustics and elasticity is presented. The procedure offers both analytical simplicity and accuracy. It does not require the conventional singularity extraction procedure and it offers improved computational efficiency since it reduces six-dimensional volumetric integrals to four-dimensional surface integrals with nonsingular integrands.

## Summary

The purpose of this work was to derive new and efficient analytical expressions for matrix elements of the monopole and the dipole components of the Green function appearing in volumetric integral equations in acoustics. The method we present reduces *volumetric* integrals involving *singular* integrands to *surface* integrals with *nonsingular* integrands. The technique has, therefore, two main advantages:

- (i) it reduces the dimensionality of the integrals from six to four and,
- (ii) it eliminates singularities which, in general, cause difficulties in numerical integration.

The Galerkin method is used in the evaluation of matrix elements. As the result we obtained a set of semi-analytical, numerically stable expressions for all matrix elements of integral equation operators appearing in

- first order volumetric integral equations,
- second order volumetric integral equations, as well as those appearing in the
- “surface equivalent formulation” integral equations applicable to geometries composed of an arbitrary number of piecewise homogeneous material regions.

The paper on this subject is in preparation.