# Aiding GPS With Additional Satellite Navigation Services

THESIS

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AFIT/GSS/ENG/10-01

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THESIS

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# *Abstract*

In modern warfare navigation services are very important. GPS is currently providing service for accurate navigation, except in some areas, especially urban areas, where GPS signals cannot always be tracked by users. In these cases some additional navigation support could be provided by other global navigation satellite systems. If GPS is combined with other navigation systems than the navigation gap will be minor.

In this thesis, the effect of combining GPS with other satellite navigation systems, specifically GLONASS, Galileo and Compass, is evaluated in terms of availability and position dilution of precision (PDOP) values. First, satellite constellations are simulated in Satellite Tool Kit (STK) to generate ephemeris data. A street scenario is then established for simulating different elevation mask angles to represent urban and mountainous areas. The performance of the combined system is also evaluated as a function of the uncertainty in the time offset between systems.

Combined GPS/GLONASS and GPS/Compass solutions showed little improvement for low elevation mask angles, however they provided some enhancement for higher elevation angles. Combined GPS/Galileo performance was improved for all elevation angles compared to only GPS, GPS/GLONASS, and GPS/Compass. The best results for availability and PDOP were obtained from combining all four systems. Although using satellites from other constellations enhances availability and decreases errors. It also brings dependency on other systems other than GPS. Adding two satellites from only the Galileo constellation to GPS is shown to be a configuration with a good compromise between dependency and performance.

# *Acknowledgements*

First and foremost, I would like to thank my lovely wife and my family for supporting me in all cases. I would also like to express my appreciation to Lt Col Michael Stepaniak for his ideas and teachings. He not only taught the knowledge but also showed the proper way of applying that knowledge. His advice about the research helped to bring out this valuable study.

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Yasin Ali MUTLU

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| GNSS | Global Navigation Satellite System | 1 |
| GLONASS | GLObal NAvigation Satellite System | 1 |
| JPO | Joint Program Office | 5 |
| SA | Selective Availability | 5 |
| WGS-84 | World Geodetic System 1984 | 5 |
| UTC | Coordinated Universal Time | 5 |
| USNO | US Naval Observatory | 5 |
| TAI | International Atomic Time | 5 |
| SPS | Standard Positioning Service | 5 |
| PPS | Precise Positioning Service | 5 |
| PRN | Pseudorandom Noise | 6 |
| CDMA | Code Division Multiple Access | 6 |
| NGA | National Geospatial-Intelligence Agency | 8 |
| PVT | Position, Velocity, Time | 9 |
| PZ-90 | Earth Parameter System 1990 | 9 |
| HPS | High Precision Service | 10 |
| FDMA | Frequency Division Multiple Access | 10 |
| GBCC | Ground Based Control Complex | 11 |
| ESA | European Space Agency | 12 |
| GTRF | Galileo Terrestrial Reference Frame | 13 |
| ITRS | International Terrestrial Reference System | 13 |
| GST | Galileo System Time | 13 |
| GIOVE-A | Galileo in Orbit Validation Element-A | 14 |
| GCS | Ground Control Segment | 15 |

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|---|---|---|
| GMS | Ground Mission Segment | 15 |
| TTC | Telemetry, Tracking and Command | 15 |
| MEO | Medium Earth Orbit | 17 |
| GEO | Geostationary Orbit | 17 |
| GGTO | Galileo System Time Offset | 19 |
| UAV | Unmanned Aerial Vehicle | 22 |
| TOA | Time of Arrival | 26 |
| DOP | Dilution of Precision | 32 |
| DCM | Direction-Cosine-Matrix | 35 |
| GDOP | Geometric Dilution of Precision | 37 |
| UERE | User Equivalent Range Error | 37 |
| HDOP | Horizontal Dilution of Precision | 37 |
| VDOP | Vertical Dilution of Precision | 37 |
| TDOP | Time Dilution of Precision | 37 |
| PDOP | Position Dilution of Precision | 1 |

# AIDING GPS WITH ADDITIONAL SATELLITE NAVIGATION SERVICES

# I. Introduction

### *1.1 Motivation*

It is very important to know your position if you are travelling from one point to another. There have been many techniques used throughout time for navigation, such as comparing sightings of fixed stars to their apparent positions as in nautical almanacs. Today, the Global Positioning System (GPS) is used instead, and users can determine their positions with the help of the GPS satellites and a small receiver. Users can determine position with the help of GPS satellites and a receiver. Unlike the stars, GPS can be used for all weather conditions and it does not matter if it is day or night. GPS was designed for primarily for the military; however, civilians can use the GPS satellites with their receivers without any charge. GPS also provides accurate timing and velocity information and is available on a continuous basis woldwide [1].

However navigation with GPS can be very difficult in urban and mountainous areas due to lack of enough visible satellites in view. If there are not at least four visible satellites in the sky according to your position then a position solution is not possible. In addition, if the satellite geometry, the distribution of satellites in the sky, is poor then the error in positioning can grow very large, even if four or more satellites are available. As other global navigation satellite systems (GNSS) become available, like GLONASS, Galileo, and Compass, there will be more satellites that can be used in the calculation of position to increase availability or improve satellite geometry.

Dr. Bradford Parkinson, the first GPS program director, mentioned the "Big Three" requirements that the world deserves of GPS in particular, and all GNSS in general. Parkinson's first requirement is signal availability for both military and civilians, "with the obvious implication that we should strive for a 30-slot GPS constellation." He talks about the war in Afghanistan and critical civil applications as reasons for this. The second requirement is that all GNSS be "totally interchangeable... the key phrase is 'any four will do'-we can use just four satellites [from any of the four systems] to achieve full operational accuracies." The third requirement that Parkinson mentions is "to recognize that interference is a very real possibility" and he suggests that a major effort will be required to address this issue and reduce the impact on user equipment [2].

Interoperability is a significant problem for combining other GNSSs with each other, interoperability can be defined as ability to use different satellites from different constellations to calculate position. Interoperability depends on exchanging information between different constellations and using this information for calculating precise position of user. Because constellations have different characteristics such as the time frame and the coordinate frame then receiver muat obtain this information to combine these systems. In addition, the signals from satellites have to be combined in a way that they do not interference each other [3].

### *1.2 Problem Statement*

There are civilian receivers that are compatible with multiple GNSS providers such as Javad (Sigma Q-G3D, GISmore), Topcon (GR-3, Net G3A), NovAtel (Europak-15a), Leica (GRX1200+GNSS, GS10) etc. [4]. They are not widely used and they mostly process the signals on an all-in-view basis, which maximizes availability and geometry, but is appropriate for civilian users due to the in use of Standard Positioning Service (SPS) instead of Precise Positioning Service (PPS). US and allied military users restrict themselves to GPS PPS, which offers improved accuracy, anti-jam, and interference rejection. As a result, military users are limited to the 30 active GPS satellites when there may be 111 satellites available in the near future. In some cases, PPS users could benefit from including signals from other satellite navigation services in certain situations, such as urban and mountainous areas, to overcome a lack of availability or to mitigate large errors due to poor geometry.

This thesis investigates possible advantages and drawbacks of combining other satellite navigation systems in terms of PDOP and availability with GPS as view from a PPS user.

### *1.3 Thesis Overview*

In Chapter 2, presents a background for GPS, GLONASS, Galileo and Compass and also contains related research about GNSSs. In Chapter 3, the structure of the simulation, assumptions, scenario, a background of position estimation, dilution of precision and weighted least squares method are described. In Chapter 4, the data generated by the simulation is analyzed. Finally in Chapter 5, the results and the possible benefits to study are discussed.

# II. Background

## *2.1 Overview*

The purpose of this chapter is to provide background information to the reader about this research. In Section 2.2, a brief explanation of Satellite Navigation is covered. Section 2.3 through Section 2.6 gives information about GPS, GLONASS, Galileo and Compass satellite navigation systems respectively. In Section 2.7 related research about this study is given.

## *2.2 Satellite Navigation*

GPS satellites are located in Medium Earth Orbits (MEO) at the altitude of about 20,000 km above the earth. The Satellites send signals to the receivers which travel at the speed of light. Satellites have very accurate atomic clocks. Receivers also have clocks, but they are less accurate compared to the satellites' atomic clocks. Receiver can calculate distance from satellite to the user by multiplying signal travel time with the speed of light.

If the receiver is getting distance measurements from satellites position can be calculated with technique called "trilareteration". If the receiver is getting measurement from one satellite then the position will be somewhere on a circle that has the radii of distance measurement between satellite and receiver. If the receiver is getting measurements from two satellites then the position will be on one of the intersecting points of two circles, so receiver can be located on these two points. If the receiver is getting measurements from three satellites then receiver can get its exact position, because one of the points will be eliminated and so there will be only one point left. However there will be a time offset between satellite clock and receivers clock due to their different accuracies. A fourth satellite is needed to solve this problem. By adding a range measurement to a fourth satellite, the receiver can determine the time offset and can calculate the exact position of user. Also, when the signal travels from satellite to the receiver there will be frequency change which helps to get doppler shift. The receiver can calculate its velocity by measuring the doppler shift.

GPS is currently the dominate system that provides three dimensional position, velocity, and time for users around the world. However there are several global navigation satellite systems coming in theater and they will be compatible with GPS in varying degrees. The next sections will give information about GPS and the upcoming systems.

## *2.3 GPS Overview*

GPS was designed primarily for military usage, but now it can be used by both military and civilian users. GPS was declared as fully operational capability on July 1995 and the Joint Program Office (JPO) at Los Angeles Air Force Base, California, is responsible for whole system [1].

Selective availability (SA) is the purposeful degradation of the signal to limit the accuracy available to civilian users and it was turned off on May 1, 2000 by the order of president. After this event accuracy for civilian users got better and civilian GPS receivers increased significantly [1, 5].

*2.3.1 Reference and Timing Frames.* GPS uses the World Geodetic System 1984 (WGS-84) for its coordinate system. WGS-84 has been updated three times since 1984 and is converging with the International Terrestrial Reference Frame (ITRF). The difference between the two reference systems is currently at the centimeter-level. GPS uses an atomic time system and is referenced to coordinated universal time (UTC) which is maintained by US Naval Observatory (USNO). UTC (USNO) is similar to International Atomic Time (Temps Atomique International-TAI) in that it does not include leap seconds [1, 6, 7].

*2.3.2 Signals and Performance.* GPS has two levels of services; standard positioning service (SPS) for civilian users and precise positioning service (PPS) for military and authorized users. SPS has about 10m horizontal accuracy and PPS has

Table 2.1: Comparison between C/A and P codes of GPS

| Parameter | C/A code | P code |
|---|---|---|
| Chipping rate (chips/sec) | $1.023 \times 10^6$ | $10.23 \times 10^6$ |
| Chipping period (nsec) | 977.5 nsec | 97.75 nsec |
| Range of one chip | 293 m | 29.3 m |
| Code repeat interval | 1 msec | 1 week |

about 2m horizontal accuracy. Even better performance is possible using differential techniques [1].

GPS satellites have atomic clocks (rubidium or cesium) and the accuracy of system depends on these clocks. Atomic clock stability is about $10^{-13}$ to $10^{-14}$ over one day. The GPS satellites fundamental frequency is 10.23 MHz and has two carrier frequencies derived from this fundamental frequency. These frequencies are:

$$L1 = 154 \times 10.23MHz = 1575.42MHz$$

$$L2 = 120 \times 10.23MHz = 1227.60MHz$$

Wavelengths of carriers are 19.03 cm and 24.42 cm for L1 and L2 respectively. GPS is also modulated with two codes that use pseudorandom noise PRN codes. A PRN code is a binary sequence that appears to be random. The PRN code sequence is generated in hardware using a tapped feedback shift register. GPS uses two classes of codes. These are coarse-acquisition (C/A) code and precise (P) code. Coarse Acquisition (C/A) code is modulated on only L1 and it is for civilian usage. The reason for not having a L2 component is for eliminating civilian users from full accuracy of system. C/A code has approximately a 300m wavelength. Precision (P) code is the second code reserved for military and authorized users. P code is encrypted today and named P(Y) code. It is modulated on both L1 and L2 frequencies.

GPS uses Code Division Multiple Access (CDMA) on common carrier frequency. For military user, C/A code is intended for initial acquisition of the GPS signal and P code has a higher chipping rate, so it provides better performance [1,8]. It is more difficult to lock onto P code because of the length of the code. It requires accurate

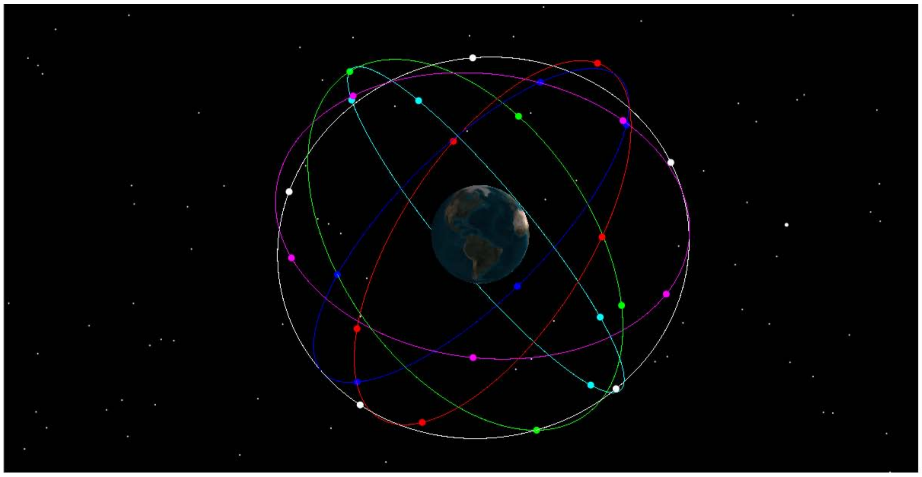

Figure 2.1: GPS 30 Satellite-Constellation from STK

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knowledge of time. Normally C/A code is locked first because there is only 1 ms to search over. After locking onto C/A code receiver has accurate time information for locking onto P code. P code is unclassified and defined in ICD-GPS-200C. Satellites do not normally transmit P code, however P code is encrypted by an encryption code and after that it is called P(Y) code. Y code is classified, so unauthorized users cannot directly lock onto Y code.

In addition to C/A or P(Y) codes the signal is also modulated with 50 bit/sec navigation message. One frame is 1500 bits (30 sec) and broken into 5x300 bit subframes (6 sec each). The 50 bps navigation message is modulated on both L1 and L2 signals [8].

*2.3.3 Space Segment.* GPS has a nominal constellation of 24 satellites in 6 orbital planes. As of 17 February 2010 satellite number is 32 with spare satellites. Orbital inclination is 55°, semi major axis is about 26,560 km, orbital plane separation is 60°, the phase within planes is irregular (it is for having a better coverage at northern hemisphere), orbital period is about 11 h 57.96 min, ground track repeat period is about 1 sidereal day, and ephemeris data consists of Kepler elements [5, 9].

*2.3.4 Control Segment.* The control segment (CS) is responsible for monitoring commanding and controlling the GPS satellite constellation. Functionally, the CS monitors downlink L-band navigation signals, updates the navigation messages and resolves satellite anomalies. Additionally the CS monitors each satellite's state of health and manages tasks associated with satellite payloads, as required. The master control station is located at Schiever Air Force Base (AFB), Colorado Springs, Colorado. It provides continuous service for all GPS satellites. Currently there are six permanent monitoring stations throughout the world. These are located at Hawaii, Schiever AFB, Cape Canaveral, Ascension, Diego Garcia, and Kwajalein. Monitoring of the GPS constellation is also supported by 11 stations maintained by the National Geospatial-Intelligence Agency (NGA). There are also four ground antenna located around the world which are located at Cape Canaveral, Ascension, Diego Garcia, Kwajalein. The CS can communicate with satellites by the help of the these ground antennas using S-band data link. At the time of communication control station uploads navigation message to the satellites and corrects attitude of satellites [1,8].

*2.3.5 User Segment.* The user segment consists of all GPS receivers including space, air, ground and marine. Early receivers were mainly analog and especially designed for military operations. They were bulky, heavy and large compared to recent receivers. A typical GPS receiver consists of mainly five components: antenna, receiver, processor, input/output (I/O), and power supply [1]. Receiver's cost and dimensions can change dramatically according to the intended application. As an example, a receiver may be embedded in a cell phone like an integrated chip or it can be integrated in an aircraft as a big box. Of course accuracy is the main driver in this situation and aircraft's receiver is more robust than cell phone's receiver.

*2.3.6 Modernization.* GPS was designed in early 70s and extensive civilian usage was not anticipated. Today there are so many civilian users compared to military users. Civilian users can only use one signal from GPS which is not very accurate compared to military signals. So civilian users' demand for better accuracy is increasing as the applications of GPS is spreading in many different areas. In the future GPS satellites will transmit additional civilian signal which will provide better accuracy compared to today [1]. Modernization goals for GPS are, as civilian goals: providing redundancy, improving positioning accuracy, improving signal availability and integrity, improving continuity of service, improving resistance to RF interference. Military goals include: protection of military service in theater of operation, prevention of adversarial exploitation of GPS services, preservation of civil service outside of operations. Present signals with Block II/IIA/IIR satellites are L1 and L2 and civilian users can only use L1 signal. Next generation Block IIR-M satellites will provide an additional military signal, the M code, on both L1 and L2 and there will be an additional signal on L2 named L2C as a second civilian signal. After Block IIF/GPS III satellites L5 signal will be added to current signals [5, 7].

## *2.4 GLONASS Overview*

GLONASS is the Russian competitor for GPS which started in mid 70s. It was declared operational in September 1993. GLONASS is designed to provide position, velocity, time (PVT) information to suitably equipped civil and military users. The Soviet Union was not able to sustain the satellite constellation at full strength, and, therefore users could only navigate with GLONASS in a limited time. After the Soviet Union dissolved, the Russian Federation is now developing new satellites in order to obtain nominal satellite constellation but it is expected not to happen until 2011-2012 [1, 5, 8].

*2.4.1 Reference and Timing Frames.* Before August 1993, GLONASS was referenced to the Soviet Geodetic System 1985 (SGS-85). GLONASS now uses Earth Parameter System 1990 (PZ-90). The relationship between WGS-84 and PZ-90 is not precisely defined. Coordinate transformation between PZ-90 and WGS-84 has been empirically derived. As PZ-90 converges with the ITRF, errors between PZ-90 and WGS-84 will diminish. GLONASS uses an atomic time system similar to

GPS which is maintained by All Union Institute of Physical Technical and Radio-Technical Measurements in Mendeelevo, Moscow. The two systems are independently maintained and they are not synchronized together [1,6,10].

*2.4.2 Signals and Performance.* GLONASS has two levels of services similar to GPS: Standard Precision Service (SPS) and High Precision Service (HPS), which is like PPS in GPS. GLONASS uses Frequency Division Multiple Access (FDMA) unlike the CDMA used in GPS. Every GLONASS satellite in the constellation has a unique frequency for L1 and L2 signals [5]. GLONASS satellites were using very wide frequency spectrum due to each satellite's unique frequency assigned to itself. After the assignment of same frequency to the satellites at the opposite sides of earth frequency spectrum decreased. The new-designed GLONASS-M satellites started to transmit civilian L2 signal at 2003 and, GLONASS became the first system that has two frequencies for civilian users. L1 includes 0.511 MHz C/A code and 5.11 MHz P code, L2 includes 5.11 MHz P code only. Carrier frequencies for satellites are :

$$f_{L1} = 1602 + 0.5625KMHz$$

$$f_{L2} = 1246 + 0.4375KMHz$$

K is a changing integer between -7 to 12 that makes different frequencies for different satellites [1,11].

*2.4.3 Space Segment.* Like GPS, GLONASS has a nominal constellation of 24 satellites but they are, in three orbital planes unlike GPS. Satellite altitude is about 19,100 km and inclination is 64.8° which gives a good coverage over northern regions of Russia [5]. The orbital period is about 11h 15 min. Orbital planes are placed 120° apart from each other and satellites are evenly spaced every 45° in each plane. The ground track of a satellite repeats every 8 sidereal days. Satellites had a design life of 2 years but now for new modernized satellites it is going to be seven years [1,11].

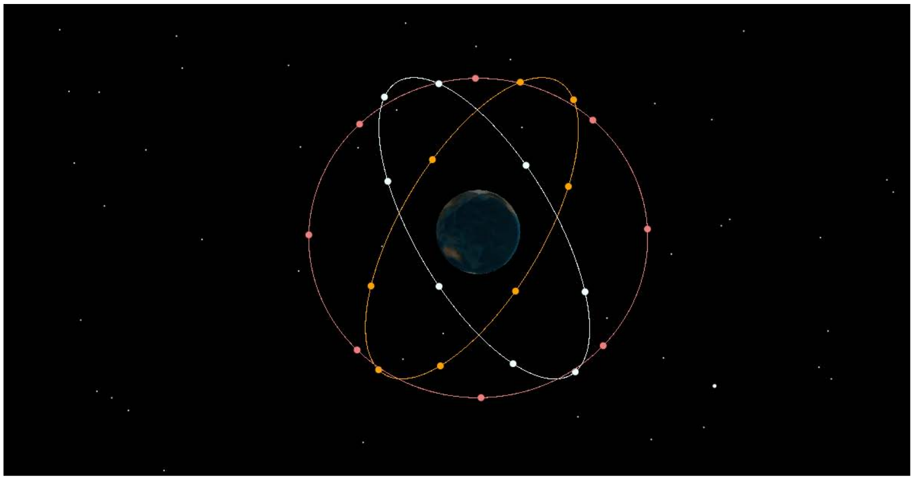

Figure 2.2: GLONASS 24 Satellite-Constellation from STK

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*2.4.4 Control Segment.* The main control segment for GLONASS constellation is Ground based control complex (GBCC). It is responsible for tracking satellites, uploading navigation message, synchronization of atomic clocks and controlling satellite attitudes. Command tracking stations make trajectory tracking by using laser radar (2-3 m error). These stations make calibration using laser and optical ranging devices and calculate clock error by differencing radar range with range from clock [11, 12].

*2.4.5 User Segment.* Because GLONASS uses FDMA, its receiver design is more costly compared to GPS. The GLONASS user segment is small and located primarily in Russia. There are some GLONASS and GLONASS-GPS receivers manufactured by Russia. Outside of Russia only a few type of receivers designed and manufactured, these were especially for high-end geodetic applications [1, 13]. Russia is going to develop user segment in the future for having more civilian users.

Each GLONASS satellite transmits the same PRN code pair on a different frequency unlike GPS, where each satellite transmits a unique PRN code pair C/A and P(Y) codes on the same frequency in a CDMA format. Choosing between FDMA or CDMA is a design tradeoff. Using FDMA results in larger and expensive receivers be-

Table 2.2: Comparison between C/A and P codes of GLONASS

| Parameter | C/A code | P code |
|---|---|---|
| Chipping rate (chips/sec) | $0.511 \times 10^6$ | $5.11 \times 10^6$ |
| Code repeat interval | 1 msec | 1 sec |

cause of more front-end components for processing multiple frequencies. With CDMA one can process signals with same front-end components [7]. FDMA has a better performance for interference rejection. If there is a narrow band interference source then this will affect only one FDMA signal, but will impact all CDMA signals. GLONASS satellites transmit signals centered on two discrete L band carrier frequencies. Carrier frequencies modulated by the modulo-2 summation of either a 0.511 MHz or 5.11 MHz PRN ranging code sequence or a 50 bps data signal [5].

*2.4.6 Modernization.* The Russian government declared that constellation of GLONASS will be completed at 2011-2012. For that reason 18 newly designed GLONASS-M satellites are going to be launched. The first of these satellites launched in December 2003. After these satellites, a new generation GLONASS-K satellites will be launched after development. GLONASS-M satellites have a longer design life, about 7 years, and additional complementary data bits for getting system time difference between GPS and GLONASS. In the future, an L3 signal which is going to be a third civilian signal, will be added [6, 7].

## *2.5 Galileo Overview*

The European Union and European Space Agency (ESA) is responsible for building and maintaining Galileo constellation. Unlike GPS, Galileo is operated by civilian authorities. After long negotiations an agreement was signed between United States and European Union (EU) in June 2004 regarding the usage of GPS and Galileo together. In fact Galileo system was designed to be able to interoperable with GPS and GLONASS. According to agreement between EU countries Galileo system is expected to be fully operational by 2013 [1].

### *2.5.1 Reference and Timing Frames.*

Galileo will use the Galileo Terrestrial Reference Frame (GTRF) which is an independent implementation of International Terrestrial Reference System (ITRS). GTRF will be very similar to WGS-84 as used by GPS. Error between these two reference frames will be on the order of a few centimeters. Galileo System Time (GST) is specified to be kept to within 50 ns 95 percent of TAI over any 1 year time interval. There are two options for getting Galileo-GPS time offset. The accuracy of this offset modulo-1 second is specified to be less than 5 ns with 2-sigma confidence interval over any 24 hour period [3,5,7].

### *2.5.2 Signals and Performance.*

Galileo is going to have six navigation signals. These are the E5 band 1164-1215 MHz, E6 band 1260-1300 MHz and E2-L1-E1 (or as known as just L1) 1559-1592 MHz. The Galileo signals have advantages of interoperability with GPS, better multipath mitigation, decreased ionospheric errors by using dual frequency. Galileo and GPS have E5a(L5) 1176.45 MHz and L1 1575.42 MHz signals in common [1,14].

Galileo has four services for users and an additional fifth service for search and rescue missions. These services are;

- Open Source Service (OS): This service is designed for civilian users and free of charge. OS will include E5a, E5b and L1 signals.
- Safety of Life Service (SOL): The difference between OS and SOL is integrity data placed in SOL. This service will be open to everyone especially designed for the situations where lives can be in danger. It can be used for aviation and maritime. E5b and L1 frequencies will be used for this service.
- Commercial Service (CS): This service provides more accurate positioning information to users who pays for this service. It has a higher data rate and there will be two additional signals.

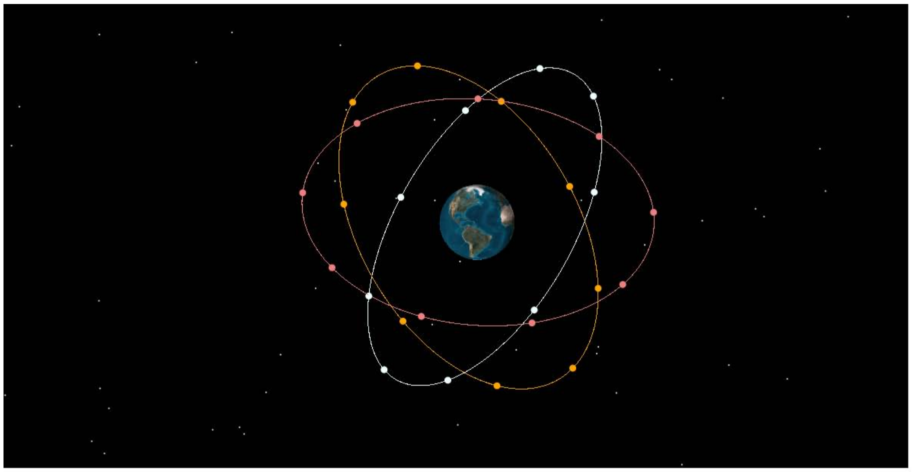

Figure 2.3: Galileo 27 Satellite-Constellation from STK

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- Public Regulated Service (PRS): This service is an encrypted service like GPS P(Y) code. Only government approved users could use this with appropriate receivers.
- Search and Rescue Service (SAR): SAR organizations could get distress signals worldwide with the help of Galileo satellites' ability to relay of alarms [6, 7, 15].

There is an agreement between European Union and United States for using Open Service (OS) for Galileo and future GPS on L1 [14].

*2.5.3 Space Segment.* The nominal Galileo constellation consists of 27 satellites at an altitude of 29,601 km . There are also three spare satellites in the constellation and so the total satellite number becomes 30 for Galileo. Satellites have an orbital period of 14h. There are three orbital planes like GLONASS which each composed of ten satellites. Orbit plane separation is 120° and phase within planes is 40°. Inclination of each orbital plane is 56°, similar to GPS but at higher altitude, which leads to a better coverage than GPS for polar regions, where European countries still have population and territories [1, 9, 16]. GIOVE-A(Galileo in Orbit Validation Element-A) satellite was launched in 2005 and it is transmitting signals that is allocated to Galileo system [5].

*2.5.4 Control Segment.* The ground segment of Galileo will be consist of two components. These are the ground control segment (GCS) responsible for control functions like spacecraft housekeeping and constellation maintenance and the ground mission segment (GMS) responsible for mission functions like navigation system control. Nominally five telemetry, tracking and command (TTC) stations will be used by GCS to communicate with each satellite [1,6].

*2.5.5 User Segment.* Galileo receiver development studies are going on still. There are some receivers in the market that can receive both GPS and Galileo signals such as Javad receivers. Because the Galileo constellation is not completed yet, they can not take full advantage of using all satellites from Galileo constellation, but eventually these, receivers will use all available satellites from GPS and Galileo constellations [17].

## 2.6 Compass Overview

China is developing its own global navigation satellite system that is called Compass or Beidou-2. China has developed a regional navigation system called Beidou-1 which gives service for only Asian region. However Compass is not a follow-on system to Beidou-1, but an independent global navigation system which can be competitor to other global navigation systems like GPS and GLONASS. It is expected to be fully operational between 2015-2020. There is little information about specifications of this system [18].

*2.6.1 Reference and Time Frames.* Chinese officials has not declared exact specifications of the Compass coordinate frame. Beidou System Time (BDT) is the time reference for Compass and atomic clocks are used for keeping time like other global navigation satellite systems. BDT does not include leap seconds similar to GPS time. National Time Service Center (NTSC) keeps the tracking of BDT time according to Chinese national official time, UTC(NTSC). In 2006, UTC(NTSC) was kept to within ±20ns of UTC. The navigation message of Compass satellites will in-

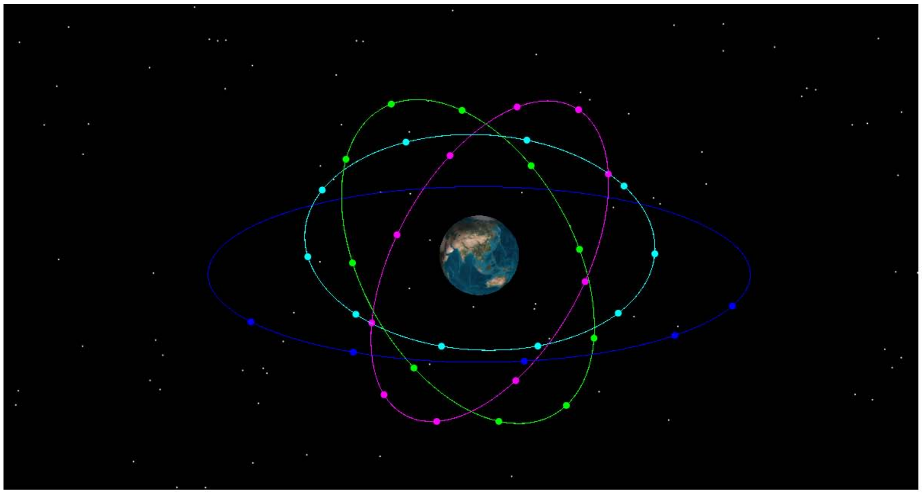

Figure 2.4: Compass 30-MEO and 5-GEO Satellite-Constellation from STK

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clude time offset information between Compass-GPS and Compass-Galileo [19]. By knowing time offsets between systems, receivers which can receive signals from Compass, GPS and Galileo can calculate positions by using more available satellites. So the user could get the benefit of more reliable and accurate navigation [20, 21].

*2.6.2 Signals and Performance.* Compass will provide two kinds of services. These are the open service which is free of charge designed for especially civilian usage and the authorized service, similar to GPS P(Y) code, which will provide more accurate positioning, velocity, timing services and integrity information for government authorized users (e.g. military, police etc.) [6]. Compass will be broadcasting on four frequencies. These are 1195-1219 MHz, 1256.52-1280.52 MHz, 1559.05-1563.15 MHz and 1587.69-1591.79 MHz. However there are some problems about usage of these frequencies due to overlay of some of the parts of Galileo and GPS frequencies. Assuming that all constellations are fully operational, if these frequencies are used, users may face some interference problem. There had been some negotiations to avoid this problem for future, but it can be said that there is not very much progress for now. The Compass navigation signals are Code Division Multiple Access (CDMA) similar to GPS and Galileo and different than GLONASS [5].

*2.6.3 Space Segment.* The Compass constellation is going to consist of 30 MEO satellites and 5 GEO satellites. MEO satellites will have an altitude of about 21,490 km and GEO satellites about 35,786 km (in geostationary orbit). MEO satellites will be in 3 orbital planes and will have 56° inclination. GEO satellites will be placed in 55°E, 80°E, 110°E, 140°E, 160°E longtitudes [1,6].

*2.6.4 Control and User Segment.* The Compass ground segment consists of a master station and upload station. Because the constellation is not completed yet there are no Compass receivers in market. Compass/Beidou-2 receivers are intended to be compatible with GPS, GLONASS and Galileo signals according to Chinese officials [18].

## 2.7 Literature Review

*2.7.1 Combined Operations.* Changsheng Cai studied a solution for combined GPS/GLONASS navigation in conditions of limited satellite visibility. This research investigated if it is possible to get a position information with only combined total four GPS/GLONASS satellites. They obtained the time offset between GPS and GLONASS when there are more than four satellites are available and then used this information when there are only four satellites are available or satellite geometry is bad. They have found that combining GPS/GLONASS has some advantages especially for urban areas [10].

Kang, Lee and Park investigated "Application of GPS/GLONASS Combination to the Revision of Digital Map". They examined the usage of combined GPS/GLONASS system for the situations when only GPS is not adequate to get three dimensional positioning like urban areas. They stated that if there are more than four GLONASS satellites then combined DGPS/DGLONASS has a better accuracy compared to only DGPS for urban and mountainous areas [13].

Zinoviev provided the current status of using combined GPS/GLONASS receivers. He indicated that GLONASS has a better coverage over high latitudes and combination of GPS/GLONASS can give more accurate position results compared to only GPS especially for these areas. He also mentioned that combining two systems increases availability and decreases anomalies. [12].

O'Keefe, Ryan and Lachapella studied about global availability and reliability assessment of the GPS and Galileo global navigation satellite systems. Results showed that simulated Galileo does not have a significant improvement to GPS for low elevation mask angles. However, for extreme mask angles such as elevation angles greater than 30° up to 40° combined GPS/Galileo has good overall availability compared to only GPS constellation. These elevation mask angles simulated urban and mountainous areas where GPS availability degrades significantly [16].

Chao, Chen and Ding investigated the performance of GPS-based vehicle positioning in very dense urban areas (Hong Kong). They have collected data for both GPS and combined GPS/GLONASS in Hong Kong streets. They have found that GPS/GLONASS has a better availability compared to GPS, however combined system also does not provide continuous availability for that kind of very dense urban areas [22].

Alkan, Karaman and Şahin investigated GPS, Galileo and GLONASS Satellite Navigation Systems and GPS Modernization. They stated that GPS availability is decreasing significantly for urban, mountainous areas. So combining satellites with GPS from other constellations like GLONASS and Galileo will provide better coverage and accurate positioning [23].

O'Donnell, et al. researched GPS interoperability and discriminators for urban and indoor environments. They stated that combined GPS/Galileo will provide better accuracy compared to only GPS. They mentioned that there will also be costs for designing and combining these two systems. However benefits of combined system were shown to be very good especially when satellite visibility is very limited [24].

Yanming studied the technical perspective of combined Galileo and GPS. He stated that combined GPS/Galileo system will bring some new applications to real time kinematics (RTK). RTK means that a correction signal is transmitted in real time from a known point to receivers to get centimeter-level accuracy. However he also states that the affects of combined system should be verified with real data when Galileo constellation is fully operational [25].

Hegarty and Chatre studied the evolution of the Global Navigation Satellite System (GNSS). They stated that there will be more available global navigation satellites for users when GLONASS, Galileo, and Compass constellations become fully operational. These will provide users more accuracy and availability [6].

Langley provided a GLONASS update. He mentioned that with an increased number of satellites when GLONASS is fully operational, combined GPS/GLONASS will provide increased accuracy and better availability. Also he stated that although GLONASS is not fully operational yet, it can be used with GPS to get better availability for urban and mountainous areas [11].

Hein and Rodrigez studied combining Galileo PRS and GPS M-Code. They have stated that performance of only Galileo PRS or only GPS-M code has a very similar characteristics in terms of positioning error in urban areas. However usage of these two signals together can decrease the errors very significantly especially for limited satellite visibility cases [14].

*2.7.2 Time Transfer.* Moudrak, Konovaltsev and Furthner studied timing aspects of GPS-Galileo interoperability. The GPS and Galileo system time offset (GGTO) will be introduced if these two systems are combined. They found that GGTO will result in position errors when receiver tries to calculate exact position. They mentioned that if the time offset is kept to 5ns, 95% uncertainty then GPS/-Galileo system will have better accuracy compared to only Galileo constellation [3].

Vanschoenbeek, Bonhoure and Boschetti investigated GNSS time offset and effects on GPS-Galileo interoperability performance. In their simulation they have used GPS and Galileo constellations. They stated that when combining these two systems there will be a time offset between systems due to their different time frames.

They also showed that this time offset does not have a significant effect on position accuracy for urban areas that have low elevation mask angles. However their results indicate that time offset causes big errors for very dense urban areas [26].

Shaowu, Xiaohui and Haitao investigated Compass time and its coordination with other GNSSs. They stated that time offsets between Compass and GPS or Galileo will be calculated and navigation message will include this information. So receivers capable of receiving Compass, GPS and Galileo signals could calculate position of user more accurately. They also mentioned that using more satellites from three constellations will improve availability [19].

#### *2.7.3 PDOP/GDOP Analysis.*

Eissfeller, Ameres compared the performances of GPS, GLONASS and Galileo. They have stated that combinations of satellites will provide better position dilution of precision (PDOP) values for worldwide and users also will get more accurate positioning by the help of more available satellites [7].

Burian, Brown and Srinivas studied building height characteristics in three U.S. Cities. These cities were Los Angles, Phoenix, and Salt Lake City. The results indicate that the Phoenix area contains much shorter buildings on average than Los Angeles. It is important for understanding that PDOP so position solution will be degraded much more in Los Angles rather than Phoenix. And results also shows that for example if you are in a city like Salt Lake City, you will have a bad satellite signal reception in some areas due to 26% residential area [27].

#### *2.7.4 GNSS Toolkits.*

Tetewsky and Soltz have introduced the GPS Matlab Toolbox. It is useful for GPS and GLONASS but it does not include Galileo and Compass constellations and it does not account for different UEREs and time biases between systems [28].

Tolman, Harris, Gaussiran, Munton described the GPS Toolkit-Open Source GPS Software and Fajt introduced GPS Simulation Package for Matlab. They are good toolkits for GPS but they do not have flexibility for other constellations [29,30].

D'Angelo, Fernández and Diez mentioned performance and visibility analysis for different Galileo/GPS receivers with the GRANADA environment and navigation simulator. GRANADA is stands for Galileo Receiver ANAlysis and Design Application. Their simulator results showed that combined GPS/Galileo has good PDOP values beginning from low elevation mask angle. They mentioned that real contribution of combined system can be seen for especially for high elevation mask angles which represents very dense urban or mountainous areas. However, it still does not include GLONASS and Compass constellations [31].

Previous research efforts include some combinations of GNSSs like GPS/Galileo or GPS/GLONASS, but they do not include the combination of all four GNSS systems. Also they are all about civilian applications of navigation. For that reason a new simulation created in Matlab for calculations. In this thesis an urban area scenario will be created to investigate how other GNSS systems could aid GPS. Ephemeris data will be obtained from simulation for all constellations and for combining these systems the weighted least squares method will be used for more realistic results when calculating the PDOP. For certain cases the benefits or drawbacks will be shown graphically from PDOP calculations.

## *2.8 Summary*

A brief overview of the Satellite Navigation, GPS, GLONASS, Galileo and Compass was given in the beginning of this chapter. A literature review was introduced after that. In Chapter 3, the methodology of the thesis is explained.

# III. Methodology

## *3.1 Overview*

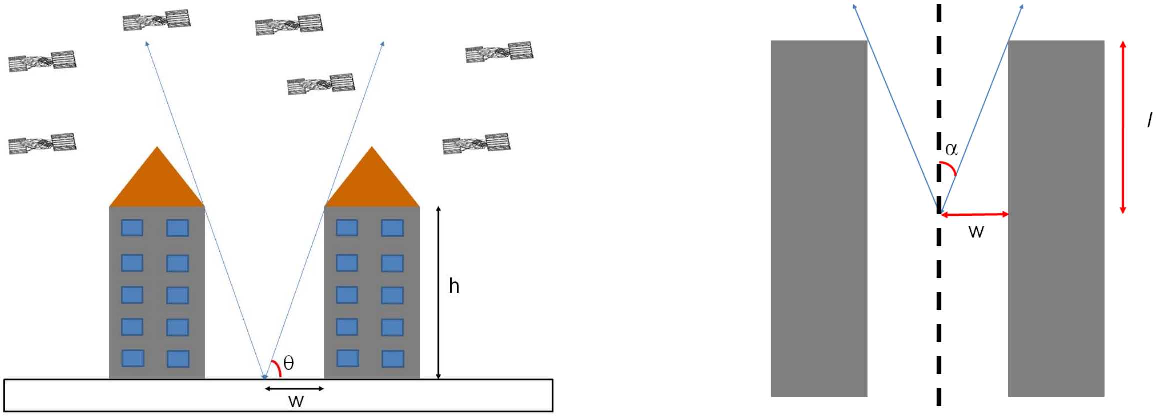
The purpose of this chapter to describe the devolopment of the Matlab® simulation for this study. In Section 3.2, the scenario of the simulation is provided to give a big picture of process. Next the assumptions that have been made for the simulation is introduced and the simulation steps are described. In Section 3.5, how position, velocity and time can be obtained from satellite measurements is provided. In Section 3.6, the Dilution of Precision (DOP) calculation is explained. Finally, Section 3.7 introduces the Weighted Least Squares Method and explains how it is used in this study.

## *3.2 Scenario*

In this scenario it is assumed that a military PPS user is performing operations in a city. The user finds that he does not have adequate coverage due to availability or PDOP limitations. Availability means that there are at least four visible satellite at the time of signal reception and PDOP value would be equal to or smaller than 6. In present operations the soldier would have to continue in a degraded effectiveness without precise navigation. Other types of users, for example an unmanned aerial vehicle (UAV) flying in a city, which perhaps could not continue at all without a navigation solution, so the operation will be failed due to insufficient navigation data.

For simulating this situation a scenario is created, assuming that the user with receiver is in the downtown area of a city standing on a street surrounded with buildings. The street is assumed to have dimensions typical of street in the U.S. Figure 3.1 shows a street view from Dayton's downtown. Average building height is changed gradually according to obtain different elevation mask angles. For each case PDOP values are calculated for the following: GPS, GPS/GLONASS, GPS/Galileo, GPS/Compass, GPS/GLONASS/Galileo and GPS/GLONASS/Galileo/Compass visible satellites. Figure 3.2 represents the scenario and angles that has been used in calculations. In Figure 3.2 **w** and **h** represent the width of the street and the height of the

Figure 3.1: Example street from Dayton Downtown for scenario



(a)

(b)

Figure 3.2: (a) Street scenario for Elevation Mask calculations
(b) Street scenario looking at God's view for Azimuth Angle calculations

Table 3.1: Building Heights, Number of Stories, and Corresponding Elevation Mask Angles for 30m wide Street

| Building Heights (m) | 5.46 | 8.67 | 12.59 | 17.9 | 26 | 41.3 |
|---|---|---|---|---|---|---|
| Number of Stories | 1.47 | 2.34 | 3.4 | 4.8 | 7 | 11.1 |
| Elevation Mask Angle(Degree) | 20° | 30° | 40° | 50° | 60° | 70° |

buildings respectively. Elevation mask can be defined as angle $\theta$ and $\theta = atan(\frac{h}{w})$. So the satellites that have equal or higher elevation angles will be tracked to by user. Azimuth angle can be defined as angle $\alpha$ and $\alpha = atan(\frac{w}{l})$, but we are assuming that our receiver has a 360° coverage so "$\alpha$" will be accounted for in both the forward and backward directions. In this azimuth lane elevation mask is set to 5° for simulating normal receiver specification and satellite over this elevation angle will be tracked by user.

Table 3.1 shows the corresponding elevation mask angles for a narrow street that is 30 meters wide. It is assumed 12ft (3.7m) per floor information taken from [32].

### *3.3 Assumptions*

There are four GNSS systems that will be considered in this thesis. These are GPS, GLONASS, Galileo and Compass. Because all four of them have different characteristics such as orbital parameters, signal specifications, time and coordinate frames etc., combining these systems at the receiver level is beyond the scope of this effort. Instead, this thesis will focus on the calculation of PDOP and some assumptions that have to be made. In this thesis study it is assumed that:

- Exact positions of satellites from all constellations are known.
- All errors within a system are accounted for using a single "User Equivalent Range Error" ($\sigma_{UERE}$). The user equivalent range error will vary between systems.

- GPS constellation consists of 30 satellites and evenly spaced in orbits. Orbital inclination and semimajor axis are 55° and 26,560 km, respectively.
- GLONASS constellation consists of nominal 24 satellites. Orbital inclination and semimajor axis are 64.8° and 25508 km, respectively.
- Galileo constellation consists of nominal 27 satellites. Orbital inclination and semimajor axis are 56° and 29,601 km, respectively.
- Compass constellation consists of 30 MEO and 5 GEO satellites at the altitude of 21,490 km and 35,780 km respectively. Orbital inclination of the MEO satellites is 56°.
- All GNSSs' reference frames are matched.
- Time offsets are known by tracking or navigation message. The accuracy of the offset, and its impact will be evaluated.
- The simulation orbits for calculating PDOP values is based on two-body equations of motion and J2 perturbation, which accounts for the changes in the orbit of a satellite due to the oblateness of the earth, is included to make it more realistic.

### *3.4 Simulation Steps*

GPS is currently fully operational and ephemeris data can be collected easily for this constellation. GLONASS is partially operational and ephemeris can be collected just for current satellites. Due to Galileo and Compass not being currently operational, it is impossible to get ephemeris data for these constellations. For this research all satellite positions will be determined from simulation of Walker constellations.

Walker constellations are used to establish satellite constellations that have the same altitude and inclination. Satellites in Walker constellation have circular orbits and they are equally placed within orbital planes. For building a Walker constellation a $T/P/F$ term is used. In this term $T$ represents the total number of satellites in the constellation, $P$ represents the number of orbital planes, and $F$ represents the phase offset factor that gives the phasing between adjacent orbital planes. Total number of satellites can be easily found as multiplying number of satellites per plane with total number of orbital planes. $360^\circ \times F/P$ gives the offset in mean anomaly between the first satellite in each adjacent orbital plane. As an example GLONASS is a Walker Delta 64.8° :24/3/1. In this example it means that constellation has 24 satellites in 3 orbital planes inclined at 64.8° and 1 represents the phasing between planes. GPS satellites do not have equal spaces within orbital planes due to increase the availability for the Northern hemisphere. However, GPS constellation still can be considered as manipulated Walker constellation [1].

The Walker constellation is used, in Satellite Tool Kit (STK), to establish GNSS constellations including GPS. This technique allows for the best approximation of satellite ephemeris data. This ephemeris data consists of longitude, latitude and altitude informations of satellites, and these saved in Excel files. A Matlab® script is written for reading this ephemeris files. They are converted into Earth-Centered-Earth-Fixed (ECEF) by the *lla2ecef* function in Matlab®. After getting ECEF coordinates for both receiver and satellites, the elevation angle of the Line-of-Sight (LOS) vectors from receiver to satellites are determined by using built in Matlab® function called *elevation*. The elevation mask angle is set, and, according to this mask angle if the satellite's elevation angle is bigger than elevation mask than it is considered as visible to receiver for that time.

### *3.5 PVT Calculation*

A GNSS receiver has to receive signals from at least four satellites to estimate its current position. The receivers calculate range to visible satellites by using the method of time-of-arrival (TOA). To be able to get the TOA correctly, the receiver must have the information of the speed of signal propagation (assumed to be the speed of light), the time of signal transmission, and the time of signal reception. GNSS satellite signals include time of transmission and their position in space. In addition, satellites have very precise atomic clocks but they also have some small clock error, and the correction for this error is also acquired from the satellites' data signals. Receivers have less stable quartz clocks for determining reception time, but if there are four visible satellites we can calculate this clock error. The vectors from receiver to satellites in Cartesian coordinate system, where $(x_r, y_r, z_r)$ represents receiver position vector and $x_s, y_s, z_s$ represents the position vector of $s^{th}$ satellite for s=1,2,3,4...,n [1,8].

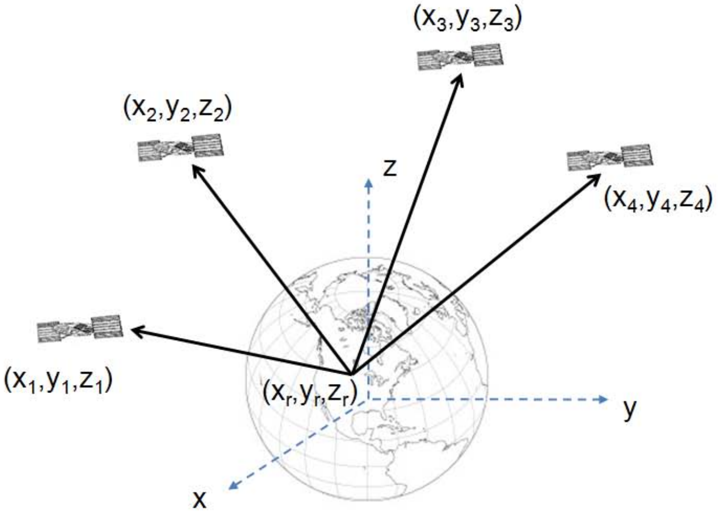


Figure 3.3: Receiver to satellite vectors

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Pseudorange measurements could be made to four satellites to determine receiver position in three dimensions $(x_r, y_r, z_r)$ and the clock offset $t_r$ by assuming the only error is receiver clock error;

$$\rho_s = \| x_s - x_r \| + ct_r \tag{3.1}$$

where $\rho_s$ is the pseudorange between the receiver and the $s^{th}$ satellite, $x_k$ is the $s^{th}$ satellite position vector, and $x_r$ is the receiver position vector. Equation (3.1) can be expanded as:

$$\begin{aligned}\rho_s &= \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2} + ct_r \\ &= f(x_r, y_r, z_r, t_r)\end{aligned} \tag{3.2}$$

Iterative techniques based on linearization can solve these nonlinear equations. If the approximate receiver position $(\hat{x}_r, \hat{y}_r, \hat{z}_r)$ is known, then true position $(x_r, y_r, z_r)$ can be calculated from approximate receiver position by a small displacement $(\Delta x_r, \Delta y_r, \Delta z_r)$.

The receiver clock offset and position is then written as the sum of the approximate position and the displacement:

$$\begin{aligned}x_r &= \hat{x}_r + \Delta x_r \\ y_r &= \hat{y}_r + \Delta y_r \\ z_r &= \hat{z}_r + \Delta z_r \\ t_r &= \hat{t}_r + \Delta t_r\end{aligned} \tag{3.3}$$

So, the pseudorange can be written as:

$$f(x_r, y_r, z_r, t_r) = f(\hat{x}_r + \Delta x_r, \hat{y}_r + \Delta y_r, \hat{z}_r + \Delta z_r, \hat{t}_r + \Delta t_r) \tag{3.4}$$

And the approximate pseudorange can be calculated by using the approximate location $(\hat{x}_r, \hat{y}_r, \hat{z}_r)$ and time bias estimate $\hat{t}_r$;

$$\begin{aligned}\hat{\rho}_s &= \sqrt{(x_s - \hat{x}_r)^2 + (y_s - \hat{y}_r)^2 + (z_s - \hat{z}_r)^2} + c\hat{t}_r \\ &= f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)\end{aligned} \tag{3.5}$$

If it is expanded about the approximate point using a Taylor series expansion:

$$\begin{gathered} f(\hat{x}_r + \Delta x_r, \hat{y}_r + \Delta y_r, \hat{z}_r + \Delta z_r, \hat{t}_r + \Delta t_r) = f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r) + \\ \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{x}_r}\Delta x_r + \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{y}_r}\Delta y_r + \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{z}_r}\Delta z_r + \\ \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{t}_r}\Delta t_r + \; Higher\, Order\, Terms \end{gathered} \tag{3.6}$$

For eliminating nonlinear terms, the higher order terms are neglected. The partial derivatives are as follows:

$$\begin{aligned} \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{x}_r} &= -\frac{x_s - \hat{x}_r}{\hat{r}_s} \\ \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{y}_r} &= -\frac{y_s - \hat{y}_r}{\hat{r}_s} \\ \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{z}_r} &= -\frac{z_s - \hat{z}_r}{\hat{r}_s} \\ \frac{\partial f(\hat{x}_r, \hat{y}_r, \hat{z}_r, \hat{t}_r)}{\partial \hat{t}_r} &= c \end{aligned} \tag{3.7}$$

where $\hat{r}_s$ is the magnitude of the vector from the approximate user position to the $s^{th}$ satellite and can be written as:

$$\hat{r}_s = \sqrt{(x_s - \hat{x}_r)^2 + (y_s - \hat{y}_r)^2 + (z_s - \hat{z}_r)^2} \tag{3.8}$$

Substituting Equations (3.3) and (3.7) into Equation (3.6) gives:

$$\rho_s = \hat{\rho}_s - \frac{x_s - \hat{x}_r}{\hat{r}_s}\Delta x_r - \frac{y_s - \hat{y}_r}{\hat{r}_s}\Delta y_r - \frac{z_s - \hat{z}_r}{\hat{r}_s}\Delta z_r + c\Delta t_r \tag{3.9}$$

Arranging equation with the known quantities, we get:

$$\hat{\rho_s} - \rho_s = \frac{x_s - \hat{x}_r}{\hat{r}_s}\Delta x_r + \frac{y_s - \hat{y}_r}{\hat{r}_s}\Delta y_r + \frac{z_s - \hat{z}_r}{\hat{r}_s}\Delta z_r - c\Delta t_r \tag{3.10}$$

Introducing new variables, Equation (3.10) can be written in a much simple form:

$$\begin{aligned}\Delta\rho &= \hat{\rho}_s - \rho_s \\ a_{xs} &= \frac{x_s - \hat{x}_r}{\hat{r}_s} \\ a_{ys} &= \frac{y_s - \hat{y}_r}{\hat{r}_s} \\ a_{zs} &= \frac{z_s - \hat{z}_r}{\hat{r}_s}\end{aligned} \tag{3.11}$$

where variables $(a_{xs}, a_{ys}, a_{zs})$ represent the unit vector pointing from approximate receiver position to the $s^{th}$ satellite. Substituting these variables into Equation (3.10):

$$\Delta\rho_s = a_{xs}\Delta x_r + a_{ys}\Delta y_r + a_{zs}\Delta z_r - c\Delta t_r \tag{3.12}$$

The four unknowns $\Delta x_r$, $\Delta y_r$, $\Delta z_r$ and $\Delta t_r$ can be solved by making ranging measurements to at least four satellites, and the following linear equations can be solved:

$$\begin{aligned}\Delta\rho_1 &= a_{x1}\Delta x_r + a_{y1}\Delta y_r + a_{z1}\Delta z_r - c\Delta t_r \\ \Delta\rho_2 &= a_{x2}\Delta x_r + a_{y2}\Delta y_r + a_{z2}\Delta z_r - c\Delta t_r \\ \Delta\rho_3 &= a_{x3}\Delta x_r + a_{y3}\Delta y_r + a_{z3}\Delta z_r - c\Delta t_r \\ \vdots &= \vdots \\ \Delta\rho_n &= a_{xn}\Delta x_r + a_{yn}\Delta y_r + a_{zn}\Delta z_r - c\Delta t_r\end{aligned} \tag{3.13}$$

The matrix form of the parameters can be written as:

$$\boldsymbol{\Delta}\rho = \mathbf{H}\boldsymbol{\Delta}\mathbf{x} \tag{3.14}$$

$$\text{where } \boldsymbol{\Delta}\rho = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \vdots \\ \Delta\rho_n \end{bmatrix}, \mathbf{H} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & -1 \\ a_{x2} & a_{y2} & a_{z2} & -1 \\ a_{x3} & a_{y3} & a_{z3} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xn} & a_{yn} & a_{zn} & -1 \end{bmatrix}, \text{ and } \boldsymbol{\Delta}\mathbf{x} = \begin{bmatrix} \Delta x_r \\ \Delta y_r \\ \Delta z_r \\ c\Delta t_r \end{bmatrix}$$

In these matrices $\boldsymbol{\Delta}\rho$ is pseudorange difference vector, $\mathbf{H}$ is measurement matrix and $\boldsymbol{\Delta}\mathbf{x}$ is user position displacement vector.

For the solution of these equation there are three cases that can be encountered. These cases are:

1. $n < 4$ :Under-determined case
    - Cannot solve for $\boldsymbol{\Delta}\mathbf{x}$
2. $n = 4$ :Uniquely determined case
    - Only one valid solution for $\boldsymbol{\Delta}\mathbf{x}$
    - Can be solved by calculating $\mathbf{H^{-1}}$ so that, $\boldsymbol{\Delta}\mathbf{x} = \mathbf{H^{-1}}\boldsymbol{\Delta}\rho$
3. $n > 4$ : Overdetermined case
    - There is no perfect solution that solves equation
    - An estimate can be found using least-squares techniques can be used

The more common case is the third case which is a scenario with more than four satellites and can be solved by least-squares method. Note that the least squares method assumes independent measurements of equal variance. The solution is:

$$\boldsymbol{\Delta}\mathbf{x} = (\mathbf{H^T H})^{-1}\mathbf{H^T}\boldsymbol{\Delta}\rho \tag{3.15}$$

After the $\boldsymbol{\Delta}\mathbf{x}$ is calculated, using Equation (3.3) the receiver's coordinates $x_r, y_r, z_r$ and the receiver clock offset are calculated. If the position displacement $(\Delta x_r, \Delta y_r, \Delta z_r)$ is in between desired values then this linearization is acceptable as the higher order terms are negligible. If it is not very close, an iteration can be made to get better estimates. We can continue the iteration until getting to the desired accuracy [1, 5].

## *3.6 Dilution Of Precision*

The satellite geometry according to receiver at the time of signal reception has a big effect on position solution. This geometry is defined by Dilution of Precision (DOP). If the satellites are close to each other that means a bad geometry and if satellites are well distributed in the sky that means a good geometry for position solution calculations [1, 33]. Figure 3.4 shows these two situations. Sometimes, although satellite geometry is good, DOP can be bad, due to some obstacles like buildings, mountains etc. Figure 3.5 represents this geometry.

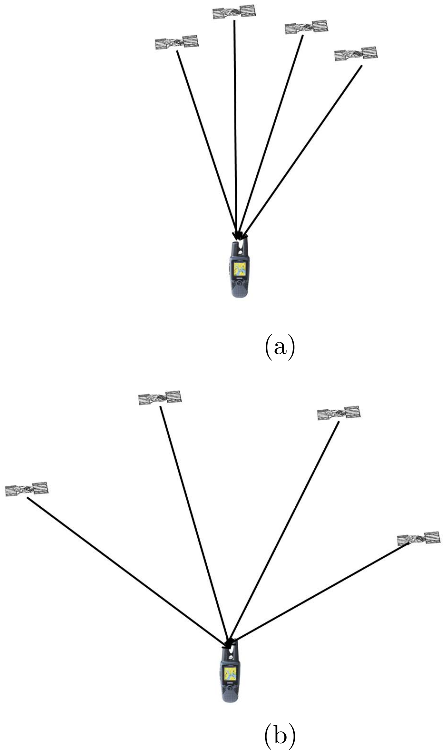

Figure 3.4: (a) Bad satellite geometry for DOP calculations
(b) Good satellite geometry for DOP calculations

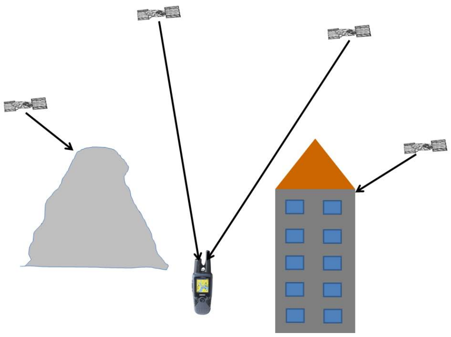

Figure 3.5: Good geometry and obstacles

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Let the covariance matrix of measurements be defined as $\mathbf{C}_\rho$ :

$$\mathbf{C}_\rho = \begin{bmatrix} \sigma^2_{\rho_1} & \sigma_{\rho_1\rho_2} & \cdots & \sigma_{\rho_1\rho_n} \\ \sigma_{\rho_2\rho_1} & \sigma^2_{\rho_2} & \cdots & \sigma_{\rho_2\rho_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\rho_n\rho_1} & \sigma_{\rho_n\rho_2} & \cdots & \sigma^2_{\rho_n} \end{bmatrix} \tag{3.16}$$

In Equation (3.16) the diagonal terms are the variances of the individual measurements. Also, the off-diagonal terms represent the covariance between measurements. If the following assumptions are made:

1. Measurement errors are zero mean and Gaussian distribution
2. All measurements have the same variance
3. Measurement errors are uncorrelated

Then the covariance of the measurements takes the form of:

$$\mathbf{C}_\rho = \mathbf{I}\sigma^{\mathbf{2}}_\rho \tag{3.17}$$

The covariance matrix of calculated position and clock error $\mathbf{C_x}$ is found by taking the covariance of Equation (3.15).

$$\mathrm{Cov}\{\mathbf{\Delta x}\} = \mathrm{Cov}\{(\mathbf{H^T H})^{-\mathbf{1}}\mathbf{H^T}\mathbf{\Delta}\rho\} \tag{3.18}$$

Since $\mathrm{Cov}\{\mathbf{A\Delta}\rho\} = \mathbf{A}\,\mathrm{Cov}\{\mathbf{\Delta}\rho\}\mathbf{A^T}$ then;

$$\begin{aligned}\mathrm{Cov}\{\mathbf{\Delta x}\} &= (\mathbf{H^T H})^{-\mathbf{1}}\mathbf{H^T}\,\mathrm{Cov}\{\mathbf{\Delta}\rho\}\mathbf{H}(\mathbf{H^T H})^{-\mathbf{1}} \\ &= (\mathbf{H^T H})^{-\mathbf{1}}\mathbf{H^T H}(\mathbf{H^T H})^{-\mathbf{1}}\sigma_\rho^{\mathbf{2}} \\ \mathbf{C_x} &= (\mathbf{H^T H})^{-\mathbf{1}}\sigma_\rho^{\mathbf{2}}\end{aligned} \tag{3.19}$$

Therefore DOP the matrix is defined as $(\mathbf{H^T H})^{-\mathbf{1}}$ and it relates directly the measurement errors to position errors. The DOP matrix can be written as :

$$(\mathbf{H^T H})^{-\mathbf{1}} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \tag{3.20}$$

Note that the DOP matrix is solely a function of satellite geometry. Normally, DOPs describe errors in a geodetic (local-level) coordinate frame (east, north, up) rather than the ECEF frame. Therefore it is necessary to modify the H matrix so that the errors refer to the local-level frame. To convert ECEF coordinates $[X, Y, Z]^T$ to local-level coordinates $p^G = [p_e, p_n, p_u]$, define the origin of local-level frame as:

$$p_0 = [X_0, Y_0, Z_0]^T \tag{3.21}$$

$$p_0 = [\lambda_0, \phi_0, h_0]^T \tag{3.22}$$

where Equation (3.21) represents ECEF and Equation (3.22) represents geodetic coordinates of origin. Then create the position vector relative to the new origin, but still expressed in the ECEF frame :

$$\mathbf{p^E = [X - X_0, Y - Y_0, Z - Z_0]} \tag{3.23}$$

After that, direction cosine matrix is used to rotate from the ECEF frame to the local-level frame:

$$\mathbf{p^E = C^G_E p^E} \tag{3.24}$$

In Equation (3.24), $C^G_E$ is called direction cosine matrix (DCM) and it can be written as :

$$\mathbf{C^G_E} = \begin{bmatrix} -sin\lambda_0 & cos\lambda_0 & 0 \\ -sin\phi_0 cos\lambda_0 & -sin\phi_0 sin\lambda_0 & cos\phi_0 \\ cos\phi_0 cos\lambda_0 & cos\phi_0 sin\lambda_0 & sin\phi_0 \end{bmatrix} \tag{3.25}$$

In Equation (3.25) $\lambda$ and $\phi$ represents geodetic longitude and latitude respectively. Local level "a" vectors can be calculated in geodetic (local-level) coordinate frame (East, North, Up) using direction cosine matrix as:

$$\mathbf{a^G = C^G_E a^E} \tag{3.26}$$

A new H matrix for DOP calculations including "a" vectors, which are now unit line-of-sight vectors between receiver and satellite in geodetic (ENU) frame, is:

$$\mathbf{H^G} = \begin{bmatrix} a_1^{G^T} & 1 \\ a_2^{G^T} & 1 \\ \vdots & \vdots \\ a_n^{G^T} & 1 \end{bmatrix} \tag{3.27}$$

The covariance matrix of calculated position and clock error $\mathbf{C_x}$ is defined as:

$$\mathbf{C_x} = \begin{bmatrix} \sigma_{x_r}^2 & \sigma_{x_r y_r} & \sigma_{x_r z_r} & \sigma_{x_r \delta t_r} \\ \\ \sigma_{y_r x_r} & \sigma_{y_r}^2 & \sigma_{y_r z_r} & \sigma_{y_r \delta t_r} \\ \\ \sigma_{z_r x_r} & \sigma_{z_r y_r} & \sigma_{z_r}^2 & \sigma_{z_r \delta t_r} \\ \\ \sigma_{\delta t_r x_r} & \sigma_{\delta t_r y_r} & \sigma_{\delta t_r z_r} & \sigma_{t_r}^2 \end{bmatrix} \tag{3.28}$$

When $\mathbf{H^G}$is used to calculate the covariance:

$$\mathbf{C_x} = (\mathbf{H^{G^T} H^G})^{\mathbf{-1}} \sigma_\rho^{\mathbf{2}} \tag{3.29}$$

So $\mathbf{C_x}$ matrix becomes:

$$\mathbf{C_x} = \begin{bmatrix} \sigma_e^2 & \sigma_{ne} & \sigma_{eu} & \sigma_{e\delta t_r} \\ \\ \sigma_{en} & \sigma_n^2 & \sigma_{nu} & \sigma_{n\delta t_r} \\ \\ \sigma_{eu} & \sigma_{nu} & \sigma_u^2 & \sigma_{u\delta t_r} \\ \\ \sigma_{e\delta t_r} & \sigma_{n\delta t_r} & \sigma_{u\delta t_r} & \sigma_{\delta_r}^2 \end{bmatrix} \tag{3.30}$$

Equation (3.30) represents the desired matrix to describe DOP. Rewriting the DOP matrix for convenience:

$$(\mathbf{H^T H})^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ \\ D_{21} & D_{22} & D_{23} & D_{24} \\ \\ D_{31} & D_{32} & D_{33} & D_{34} \\ \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \tag{3.31}$$

The geometric Dilution of Precision (GDOP) can be calculated from the DOP matrix :

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}} \tag{3.32}$$

The component errors are related to User Equivalent Range Error (UERE) and can be related by GDOP as:

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2 + \sigma_{\delta t_r}^2} = GDOP \times \sigma_{UERE} \tag{3.33}$$

There are four more DOP parameters for different situations. These are Position Dilution of Precision (PDOP), Horizontal Dilution of Precision (HDOP), Vertical Dilution of Precision (VDOP) and Time Dilution of Precision (TDOP) [1, 26]. We can define these DOP values as:

$$\begin{aligned} PDOP &= \sqrt{D_{11}+D_{22}+D_{33}} \\ HDOP &= \sqrt{D_{11}+D_{22}} \\ VDOP &= \sqrt{D_{33}} \\ TDOP &= \sqrt{D_{44}} \end{aligned} \tag{3.34}$$

And we can relate these DOP values with $\sigma_{UERE}$ as:

$$\begin{aligned} \sqrt{\sigma_e^2+\sigma_n^2+\sigma_u^2} &= PDOP \times \sigma_{UERE} \\ \sqrt{\sigma_e^2+\sigma_n^2} &= HDOP \times \sigma_{UERE} \\ \sigma_u &= VDOP \times \sigma_{UERE} \\ \sigma_{\delta t_r} &= TDOP \times \sigma_{UERE} \end{aligned} \tag{3.35}$$

For this research, PDOP was chosen as the metric for comparison since it best characterized the 3 dimensional positioning error. Because all measurements have the same variance, these calculations will not be sufficient for combining two or more systems which have different variances. So to solve this problem weighted least squares method will be introduced in Section 3.7.

## *3.7 Weighted Least Squares Method*

If two different GNSS is wanted to be combined such as GPS and GLONASS, a weight matrix, W, should be inserted into Equation (3.15) because of their non equal different error variances. So Equation (3.15) becomes:

$$\mathbf{\Delta\hat{x}} = (\mathbf{H^TWH})^{-1}\mathbf{H^TW\Delta}\rho \tag{3.36}$$

For each system, different weight values should be used in calculations to provide more realistic results. And also if some satellites have significant errors this will affect the whole solution at very minimal level. The covariance matrix $R$ and the weighting matrix $W$ can be written as [10, 26]:

$$\mathbf{W} = \mathbf{R^{-1}} \tag{3.37}$$

### 3.8 Combining Systems

The methodology for combining systems is the technique presented by Vanschoenbeck, et al in Reference [26]. For two systems like GPS and GLONASS pseudoranges can be written as in Equation (3.44):

$$\begin{aligned}
\Delta\rho_{1GPS} &= a_{x1GPS}\Delta x_r + a_{y1GPS}\Delta y_r + a_{z1GPS}\Delta z_r - c\Delta t_{rGPS} \qquad (3.38)\\
\Delta\rho_{2GPS} &= a_{x2GPS}\Delta x_r + a_{y2GPS}\Delta y_r + a_{z2GPS}\Delta z_r - c\Delta t_{rGPS}\\
\Delta\rho_{3GPS} &= a_{x3GPS}\Delta x_r + a_{y3GPS}\Delta y_r + a_{z3GPS}\Delta z_r - c\Delta t_{rGPS}\\
\vdots &= \vdots\\
\Delta\rho_{nGPS} &= a_{xnGPS}\Delta x_r + a_{ynGPS}\Delta y_r + a_{znGPS}\Delta z_r - c\Delta t_{rGPS}\\
\\
\Delta\rho_{1GLN} &= a_{x1GLN}\Delta x_r + a_{y1GLN}\Delta y_r + a_{z1GLN}\Delta z_r - c\Delta t_{rGLN}\\
\Delta\rho_{2GLN} &= a_{x2GLN}\Delta x_r + a_{y2GLN}\Delta y_r + a_{z2GLN}\Delta z_r - c\Delta t_{rGLN}\\
\Delta\rho_{3GLN} &= a_{x3GLN}\Delta x_r + a_{y3GLN}\Delta y_r + a_{z3GLN}\Delta z_r - c\Delta t_{rGLN}\\
\vdots &= \vdots\\
\Delta\rho_{nGLN} &= a_{xnGLN}\Delta x_r + a_{ynGLN}\Delta y_r + a_{znGLN}\Delta z_r - c\Delta t_{rGLN}
\end{aligned}$$

Assuming that the pseudorange measurements from different satellites are independent from each other and have a Gaussian distribution, the weight matrix for “n” GPS and “m” GLONASS satellites can be written as:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & & \vdots \\ \vdots & 0 & 1 & 0 & & \\ 0 & & 0 & \frac{\sigma^2_{1GLN}}{\sigma^2_{GPS}} & 0 & 0 \\ \vdots & & \cdots & 0 & \ddots & \\ 0 & \cdots & 0 & \cdots & 0 & \frac{\sigma^2_{mGLN}}{\sigma^2_{GPS}} \end{bmatrix} \sigma^2_{GPS} \tag{3.39}$$

If the measurements are assumed to be of equal variance, then $\mathbf{R}$ is simply the identity matrix multiplied by "$\sigma^2_{GPS}$" and covariance of $\hat{\mathbf{x}}$ is the familiar $(\mathbf{H^T H})^{-1}$ used for typical PDOP calculations.

So $\boldsymbol{\Delta}\rho$, $\mathbf{H}$, and $\boldsymbol{\Delta}\mathbf{x}$ matrices for two systems can be written as:

$$\begin{gathered} \boldsymbol{\Delta}\rho = \begin{bmatrix} \Delta\rho_{1GPS}, \cdots, \Delta\rho_{nGPS}, \Delta\rho_{1GLN}, \cdots, \Delta\rho_{mGLN} \end{bmatrix}^{\mathbf{T}} \\ \mathbf{H} = \begin{bmatrix} a_{x1GPS} & a_{y1GPS} & a_{z1GPS} & -1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xnGPS} & a_{ynGPS} & a_{znGPS} & -1 & 0 \\ a_{x1GLN} & a_{y1GLN} & a_{z1GLN} & 0 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xmGLN} & a_{ymGLN} & a_{zmGLN} & 0 & -1 \end{bmatrix} \\ \boldsymbol{\Delta}\mathbf{x} = \begin{bmatrix} \Delta x_r, \Delta y_r, \Delta z_r, c\Delta t_{rGPS}, c\Delta t_{rGLN} \end{bmatrix}^{\mathbf{T}} \end{gathered} \tag{3.40}$$

In Equation (3.47) $a_i$ represents the direction cosines of unit vectors pointing from the receiver to the satellites. Index of the unit vectors stands for the constellation that they belong to such as $GPS$ and $GLN$. Number of measurement from different constellations are marked with different indexes and $n$, $m$ represents measurements for GPS and GLONASS respectively. If the time offset between systems is known from navigation data, as assumed for this study, the receivers only have to determine the 3D position and the time biases between the receiver and the satellites. The navigation solution can now be calculated from at least four pseudorange measurements, as if only one navigation system is used. If the time offset between GPS and GLONASS defined as:

$$\Delta t_{GPS-GLN} = c\Delta t_{GPS} - c\Delta t_{GLN} \tag{3.41}$$

In Equation 3.41 $c$ is the speed of light. So the new $\mathbf{\Delta}\rho$, $\mathbf{H}_{\text{SYST}}$, and $\mathbf{\Delta x}$ matrices taking into account the time offset between GPS and GLONASS ($\Delta t_{GPS-GLN}$) can be written as :

$$\mathbf{\Delta}\rho = \begin{bmatrix} \Delta\rho_{1GPS}, \cdots, \Delta\rho_{nGPS}, \Delta\rho_{1GLN}, \cdots, \Delta\rho_{mGLN}, \Delta t_{GPS-GLN} \end{bmatrix}^{\mathbf{T}}$$

$$\mathbf{H}_{\text{SYST}} = \begin{bmatrix} a_{x1GPS} & a_{y1GPS} & a_{z1GPS} & -1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xnGPS} & a_{ynGPS} & a_{znGPS} & -1 & 0 \\ a_{x1GLN} & a_{y1GLN} & a_{z1GLN} & 0 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xmGLN} & a_{ymGLN} & a_{zmGLN} & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{3.42}$$

$$\mathbf{\Delta x} = \begin{bmatrix} \Delta x_r, \Delta y_r, \Delta z_r, c\Delta t_{rGPS}, c\Delta t_{rGLN} \end{bmatrix}^{\mathbf{T}}$$

The last row of the $\mathbf{H}_{\text{SYST}}$ matrix takes the GPS-GLONASS time bias into account, and $\mathbf{\Delta x}$ contains the position and time evolution of the system where $c$ is the speed of light.

After these changes are made $R$ matrix also needs to be rearranged for accounting GPS-GLONASS time bias. Then the $R$ matrix becomes:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & & & \vdots \\ \vdots & 0 & 1 & 0 & & & \\ \vdots & & 0 & \frac{\sigma^2_{1GLN}}{\sigma^2_{GPS}} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & \ddots & 0 & \vdots \\ \vdots & & & \cdots & 0 & \frac{\sigma^2_{mGLN}}{\sigma^2_{GPS}} & 0 \\ 0 & \cdots & \cdots & 0 & \cdots & 0 & \frac{\sigma^2_{GPS-GLN}}{\sigma^2_{GPS}} \end{bmatrix} \sigma^{\mathbf{2}}_{\mathbf{GPS}} \tag{3.43}$$

For this case measurements from different satellites free of a bias and they do not have dependency to eachother.

For combining three systems like GPS/GLONASS/Galileo, additional Galileo pseudoranges to Equation (3.8) can be written as:

$$\begin{aligned} \Delta\rho_{1Gal} &= a_{x1Gal}\Delta x_r + a_{y1Gal}\Delta y_r + a_{z1Gal}\Delta z_r - c\Delta t_{rGal} \\ \Delta\rho_{2Gal} &= a_{x2Gal}\Delta x_r + a_{y2Gal}\Delta y_r + a_{z2Gal}\Delta z_r - c\Delta t_{rGal} \\ \Delta\rho_{3Gal} &= a_{x3Gal}\Delta x_r + a_{y3Gal}\Delta y_r + a_{z3Gal}\Delta z_r - c\Delta t_{rGal} \\ \vdots &= \vdots \\ \Delta\rho_{pGal} &= a_{xpGal}\Delta x_r + a_{ypGal}\Delta y_r + a_{zpGal}\Delta z_r - c\Delta t_{rGal} \end{aligned} \tag{3.44}$$

So $\boldsymbol{\Delta}\rho$, $\mathbf{H}$ and $\boldsymbol{\Delta}\mathbf{x}$ matrices for three systems can be written as:

$$\boldsymbol{\Delta}\rho = \begin{bmatrix} \Delta\rho_{1GPS}, \cdots, \Delta\rho_{nGPS}, \Delta\rho_{1GLN}, \cdots, \Delta\rho_{mGLN}, \Delta\rho_{1Gal}, \cdots, \Delta\rho_{pGal} \end{bmatrix}^{\mathbf{T}}$$

$$\mathbf{H} = \begin{bmatrix} a_{x1GPS} & a_{y1GPS} & a_{z1GPS} & -1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xnGPS} & a_{ynGPS} & a_{znGPS} & -1 & 0 & 0 \\ a_{x1GLN} & a_{y1GLN} & a_{z1GLN} & 0 & -1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xmGLN} & a_{ymGLN} & a_{zmGLN} & 0 & -1 & 0 \\ a_{x1Gal} & a_{y1Gal} & a_{z1Gal} & 0 & 0 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{xpGal} & a_{ypGal} & a_{zpGal} & 0 & 0 & -1 \end{bmatrix} \tag{3.45}$$

$$\mathbf{\Delta x} = \left[ \; \Delta x_r, \Delta y_r, \Delta z_r, c\Delta t_{rGPS}, c\Delta t_{rGLN}, c\Delta t_{rGal} \; \right]^{\mathbf{T}}$$

In Equation (3.45) $a_i$ represents the direction cosines of unit vectors pointing from the receiver to the satellites. Index of the unit vectors stands for the constellation that they belong to such as *GPS*, *GLN* and *Gal*. Number of measurement from different constellations are marked with different indexes and $n$, $m$, $p$ represents measurements for GPS, GLONASS and Galileo respectively. If the time offset between GPS and Galileo defined as:

$$\Delta t_{GPS-Gal} = c\Delta t_{GPS} - c\Delta t_{Gal} \tag{3.46}$$

The new $\mathbf{\Delta}\rho$ and $\mathbf{H}_{\mathrm{SYST}}$ matrices taking into account the time offsets between GPS-GLONASS ($\Delta t_{GPS-GLN}$) and GPS-Galileo ($\Delta t_{GPS-Gal}$) can be written as:

$$\boldsymbol{\Delta}\rho = \begin{bmatrix} \Delta\rho_{1GPS} \\ \dots \\ \Delta\rho_{nGPS} \\ \Delta\rho_{1GLN} \\ \dots \\ \Delta\rho_{mGLN} \\ \Delta\rho_{1Gal} \\ \dots \\ \Delta\rho_{pGal} \\ \Delta t_{GPS-GLN} \\ \Delta t_{GPS-Gal} \end{bmatrix} \mathbf{H}_{\mathrm{SYST}} = \begin{bmatrix} a_{x1GPS} & a_{y1GPS} & a_{z1GPS} & -1 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{xnGPS} & a_{ynGPS} & a_{znGPS} & -1 & 0 & 0 \\ a_{x1GLN} & a_{y1GLN} & a_{z1GLN} & 0 & -1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{xmGLN} & a_{ymGLN} & a_{zmGLN} & 0 & -1 & 0 \\ a_{x1Gal} & a_{y1Gal} & a_{z1Gal} & 0 & 0 & -1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{xpGal} & a_{ypGal} & a_{zpGal} & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \tag{3.47}$$

New $\boldsymbol{\Delta}\mathbf{x}$ matrix becomes:

$$\Delta x = \begin{bmatrix} \Delta x_r \\ \Delta y_r \\ \Delta z_r \\ c\Delta t_{rGPS} \\ c\Delta t_{rGLN} \\ c\Delta t_{rGal} \end{bmatrix} \tag{3.48}$$

The last two rows of the matrix take the GPS-GLONASS and GPS-Galileo time bias into account.

The covariance matrix of the measurements becomes:

$$
\mathbf{R} = \begin{bmatrix}
1 & 0 & \cdots & 0 & \cdots & \cdots & & & 0 & 0 & 0 \\
0 & \ddots & 0 & \cdots & & & & & \vdots & \vdots & \vdots \\
\vdots & 0 & 1 & 0 & \cdots & & & & & & \\
 & & 0 & \frac{\sigma^2_{1GLN}}{\sigma^2_{GPS}} & 0 & \cdots & & & & & \\
0 & \cdots & \cdots & 0 & \ddots & 0 & \cdots & & & & \\
0 & \cdots & & & 0 & \frac{\sigma^2_{mGLN}}{\sigma^2_{GPS}} & 0 & \cdots & & & 0 \\
0 & \cdots & \cdots & & 0 & 0 & \frac{\sigma^2_{1Gal}}{\sigma^2_{GPS}} & 0 & \cdots & & 0 \\
0 & \cdots & \cdots & & & & & \ddots & 0 & 0 & 0 \\
0 & 0 & \cdots & \cdots & & & & & \frac{\sigma^2_{pGal}}{\sigma^2_{GPS}} & 0 & 0 \\
0 & \cdots & \cdots & & & & & & & \frac{\sigma^2_{GPS-GLN}}{\sigma^2_{GPS}} & 0 \\
0 & \cdots & \cdots & & & & & 0 & 0 & 0 & \frac{\sigma^2_{GPS-Gal}}{\sigma^2_{GPS}}
\end{bmatrix} \sigma^2_{GPS} \tag{3.49}
$$

This derivation is an example for combining GPS, GLONASS and Galileo with different variances and time biases. It can be applied to combine four systems like GPS/Galileo/Compass. For combining four systems the $H$, $\Delta x$ and $R$ matrices will be changed, additional terms will be added to these matrices. For this study, the uncertainty time offset between systems will be analyzed. Values considered are: 5ns, 10ns, 20ns, 30ns, 40ns, and 50ns, however, perfect knowledge of the time offset situation is not included this search, because it is very difficult to maintain no time offset between systems for real life applications.

### *3.9 Summary*

In this chapter, the structure of the simulation for this study is covered. The equations of PVT and DOP calculations are given. The adopted approach for combining satellite measurements is explained. Finally, Weighted Least Squares Method and its usage in this study is covered. In Chapter 4, the obtained data and analysis is given.

# IV. Data and Analysis

## *4.1 Overview*

In this chapter, PDOPs taken from the Matlab Simulation will be analyzed and results will be discussed. Because there are four different GNSS constellations and our baseline system is GPS, results will be divided into sections that contain GPS only, Combined GPS/GLONASS, Combined GPS/Galileo, Combined GPS/Compass, Combined GPS/GLONASS/Galileo and GPS/GLONASS/Galileo/Compass. These sections will show how increasing elevation mask affects GPS availability and PDOP values and also discusses benefits and the drawbacks of adding available satellites from other constellations to GPS. This study and results are unique because previous research only considered combinations of two systems like GPS/GLONASS.

## *4.2 World Wide PDOP Distribution for Constellations*

A simulation is run to see the performance of GPS for elevation mask angles 10°, 20°, and 30° as showed in Figures 4.1, 4.2, and 4.3. GPS PDOP values are getting higher as elevation mask angle increases. For this simulation Earth was divided into $10^\circ \times 10^\circ$ bins and a single point was evaluated within each bin at 5 minutes intervals. Red solid lines indicates $10^\circ \times 10^\circ$ squares have PDOP values bigger than 6. However, combining GPS with Galileo provide better PDOP values for elevation angle 30° compared to only GPS as shown in Figure 4.4. The rest of the thesis will investigate if the same improvement is seen for the urban canyon scenario, when applied to a single location: Dayton, OHIO.

## *4.3 GPS Results*

GPS is the main navigation system for US military and allied countries' military systems. It generally provides reliable positioning, navigation, and timing services to users on a continuous worldwide basis, but that is not always true. Results show that GPS has a good service for only low elevation masks. Good service or availability means PDOP values are lower than 6 and available satellite number is bigger than

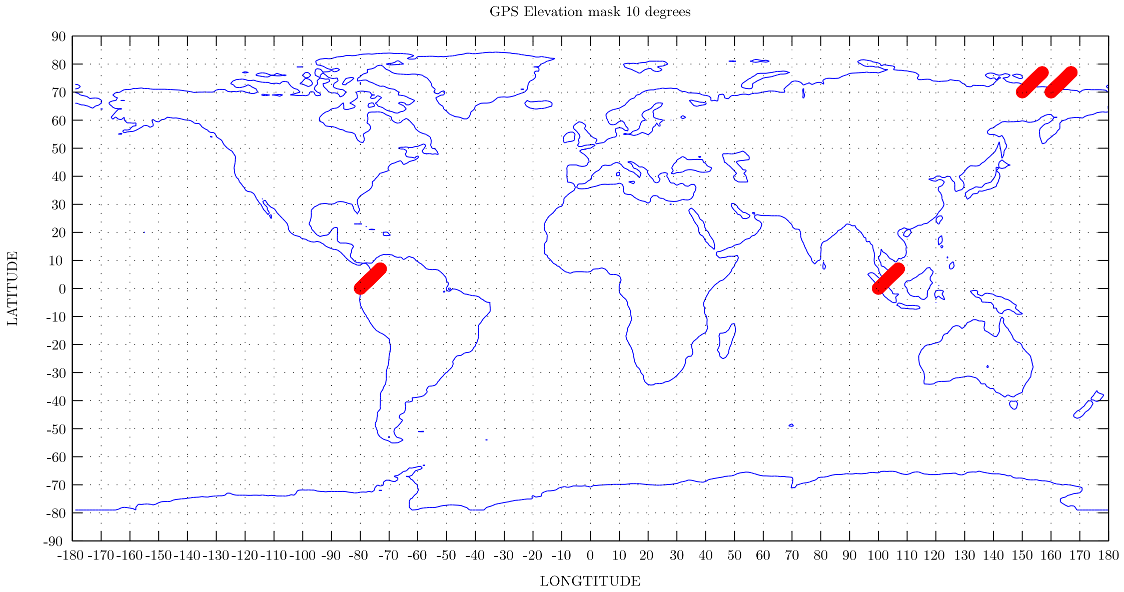


Figure 4.1: World Wide GPS PDOP Values for 10 Degrees Elevation Mask: Red solid lines indicates the areas that have PDOP values bigger than 6

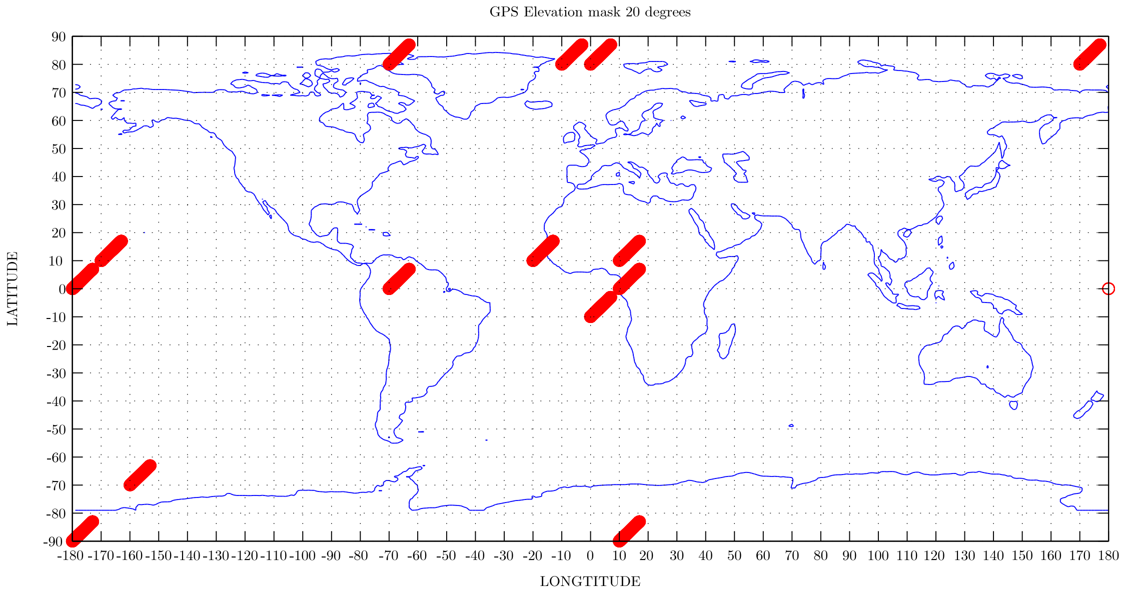


Figure 4.2: World Wide GPS PDOP Values for 20 Degrees Elevation Mask: Red solid lines indicates the areas that have PDOP values bigger than 6

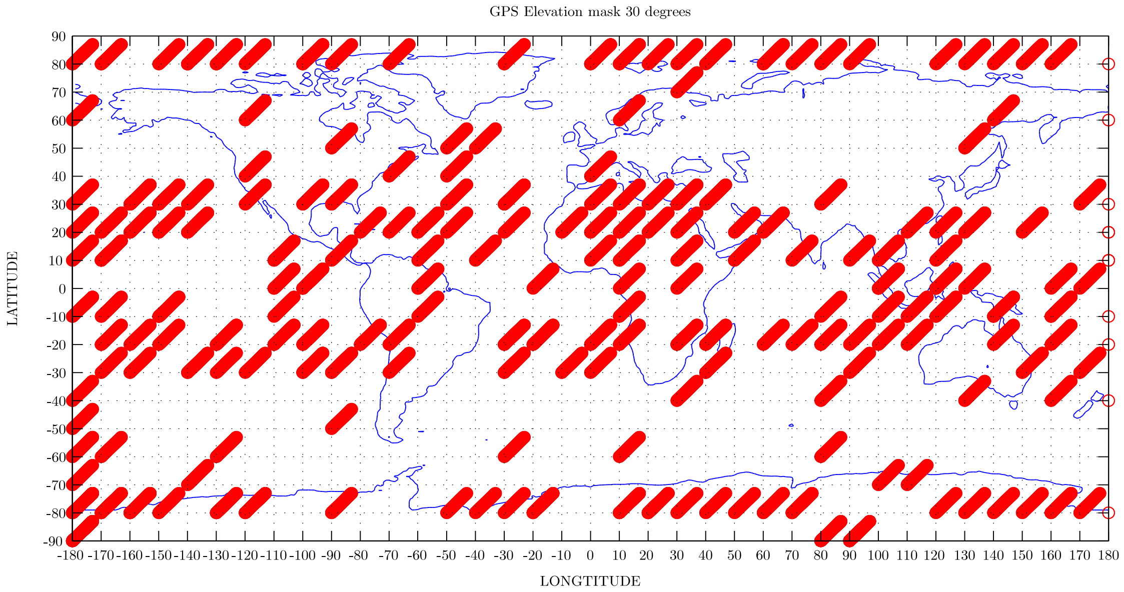


Figure 4.3: World Wide GPS PDOP Values for 30 Degrees Elevation Mask: Red solid lines indicates the areas that have PDOP values bigger than 6

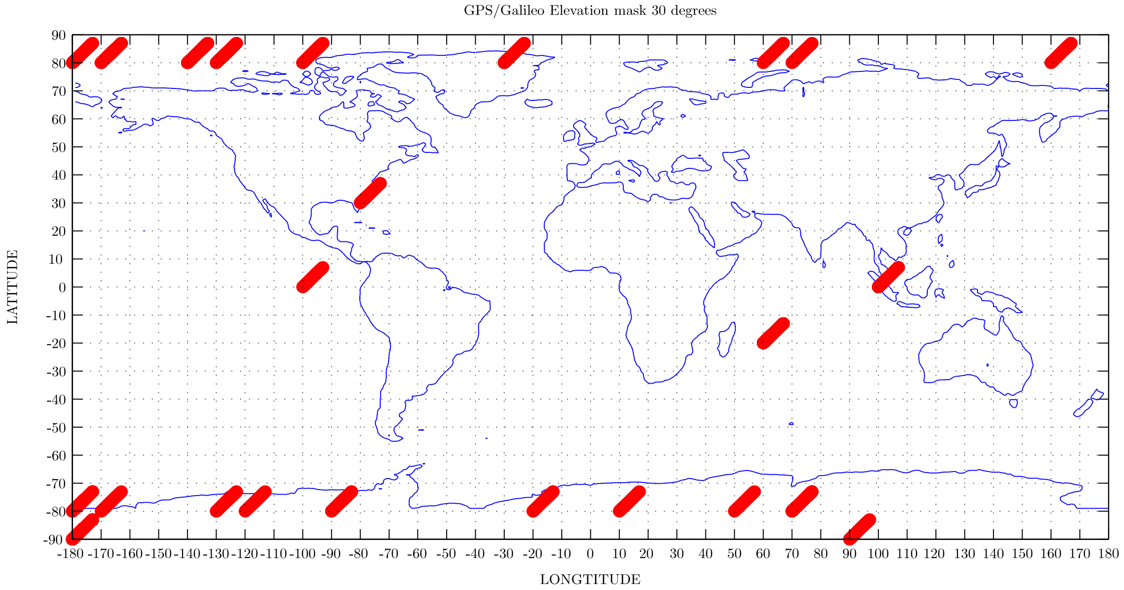
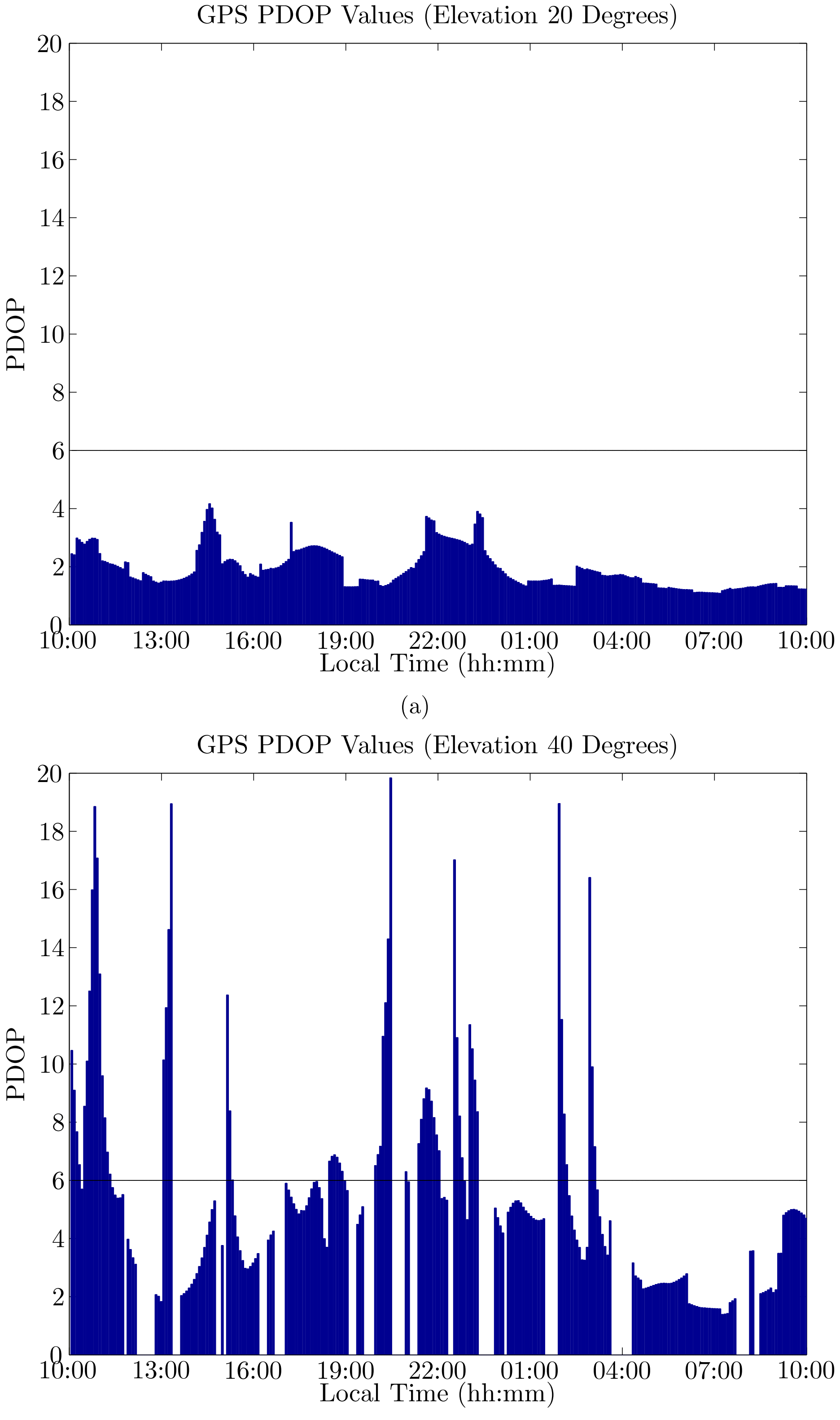


Figure 4.4: World Wide GPS/Galileo PDOP Values for 30 Degrees Elevation Mask: Red solid lines indicates the areas that have PDOP values bigger than 6

Table 4.1: Elevation Mask Angles and GPS Availability

| Elevation Angle | Availability (%) |
|---|---|
| 20° | 100 |
| 25° | 99.3 |
| 30° | 96.4 |
| 35° | 82 |
| 40° | 56.74 |
| 45° | 34.6 |
| 50° | 21.79 |
| 55° | 17.3 |
| 60° | 12.45 |
| 65° | 0 |
| 70° | 0 |
| 75° | 0 |

4. As an example, Figure 4.5(a) for a 20° elevation mask shows GPS has good over all PDOP values and availability. As elevation mask increases GPS PDOP values increase and availability decreases, which degrades navigation. Figure 4.5(b) shows GPS PDOP values for 40° elevation angle. Blank areas on the plot means that the PDOP value for this time is bigger than 20 or there are less than 4 satellites available. Acceptable service is only available in those areas where PDOP is below the indicated threshold. If the elevation mask increases to 50°, as shown in Figure 4.6, GPS information is not reliable any more due to very high PDOP values and limited availability. So if the military is operating in an area with this level of elevation mask then GPS will not help for navigation. For that reason other GNSS satellites can be used to improve PDOP values and availability so that navigation information will still be available for the troops over a full range of elevation angles. Appendix A includes GPS PDOP figures for other elevation mask angles. Table 4.1 shows the elevation mask angles and corresponding availability values for GPS for elevation angles between 20° and 75°.



(a)

(b)

Figure 4.5: GPS PDOP Values for Elevation Masks 20° and 40°

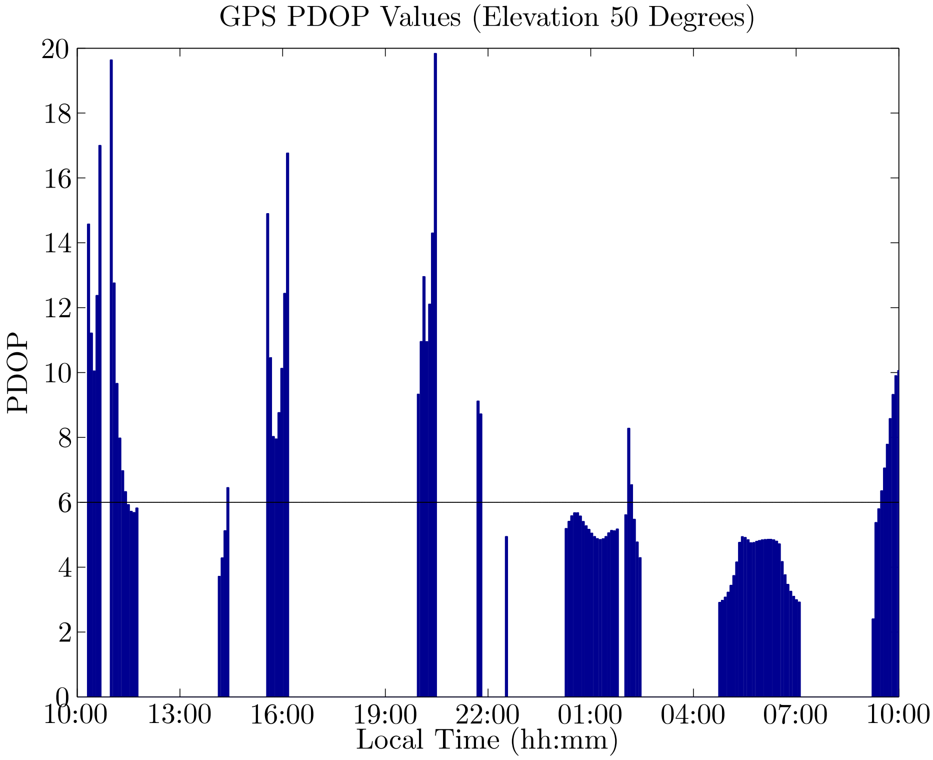


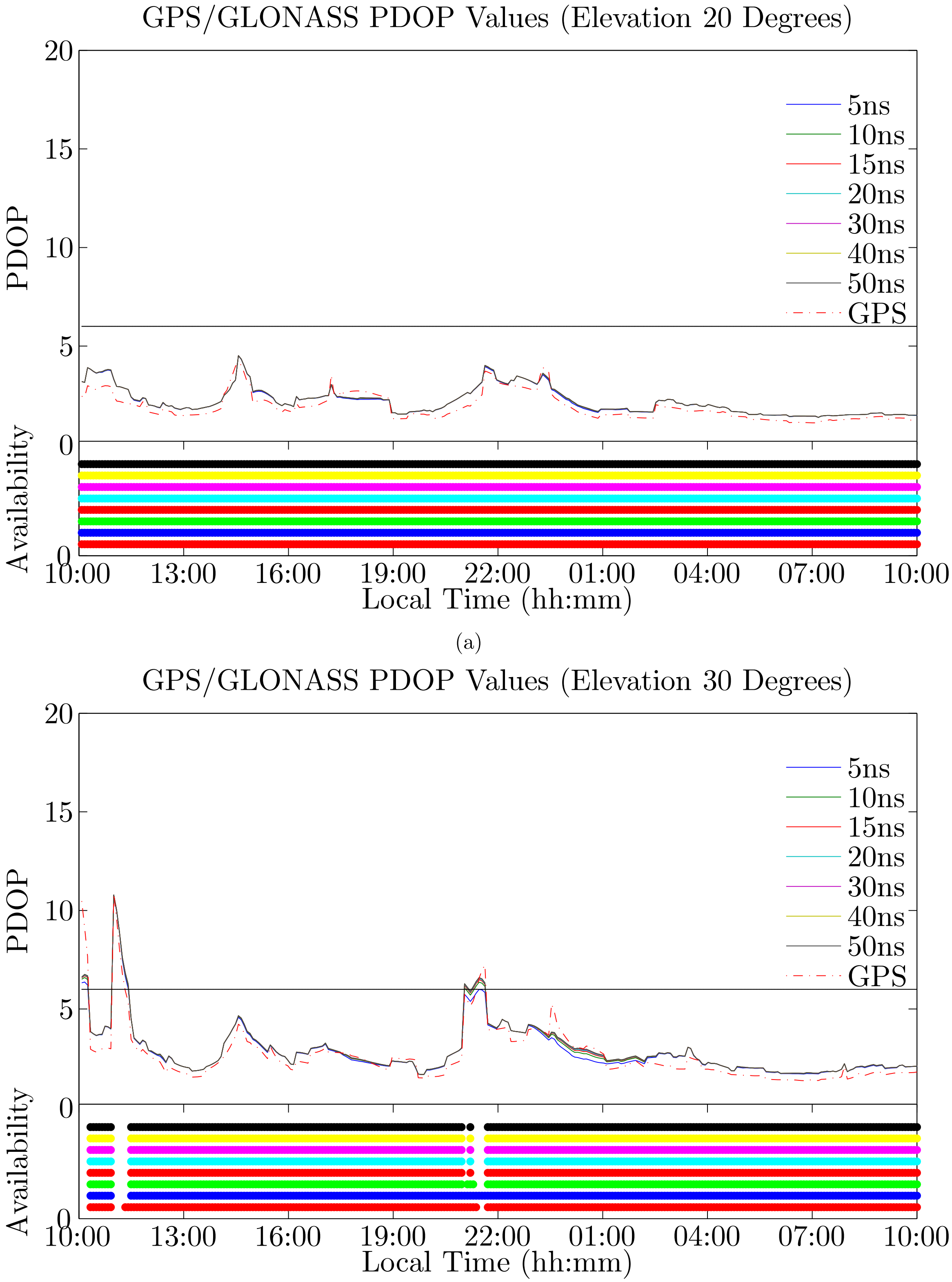
Figure 4.6: GPS PDOP Values for 50° Elevation Mask

## *4.4 Combined GPS/GLONASS Results*

GLONASS will have 24 satellites for its nominal constellation. Therefore, a GPS/ GLONASS combined constellation will have more satellites compared to the GPS constellation and availability is going to be better than GPS only availability. It can be expected that PDOP values also will be better than GPS only, but this statement is not always valid and can be change due to some different situations. Because these are two different systems and they have different time and UERE values, PDOP sometimes can be worse. This is especially true for low elevation masks when GPS has a good PDOP and availability. Adding GLONASS satellites to PDOP calculations will then degrade PDOP values because of GLONASS's higher UERE value and time bias between systems. Figure 4.7(a) shows 20° elevation mask GPS PDOP values as dotted red line and GPS/GLONASS PDOP values with different colors for different time biases between GPS and GLONASS. It can be seen that even if there is a small time bias, such as 5ns, then GPS PDOP will be better or some times equal to GPS/GLONASS PDOP values. For 30° elevation mask, shown in Figure 4.7(b), GPS/GLONASS PDOP values and GPS PDOP values with 50ns time bias is equal or very close to each other most of the time. So using additional GLONASS satellites will contribute to PDOP values but it will not be so significant. As elevation mask increases the contribution of GLONASS satellites become significant. In Figure 4.8(a) for 40° elevation mask, although GPS PDOP values are bigger than 6 most of the time, GPS/GLONASS PDOP values are still acceptable most of the time and availability is 80.97% with 50ns time bias compared to 56.74% GPS availability. For 50° elevation mask GPS PDOP values are degraded significantly as showed in Figure 4.8(b). However, GPS/GLONASS has PDOPs that can be used 49.83% of the time if time bias uncertainty is not bigger than 5ns. After 65° elevation mask, GPS/-GLONASS PDOP are bigger than 6 or not available most of the time and even the combined system is not useful for navigation over this elevation angle. This situation is shown in Figure 4.9. Table 4.2 and Figure 4.10 show the GPS and GPS/GLONASS availability according to elevation mask angles. It is worth noting that there is only about 5% average difference between 5ns and 50ns time bias availability values. So availability will be better than only GPS, even if the time bias is 50ns between GPS and GLONASS systems.

### *4.5 Combined GPS/Galileo Results*

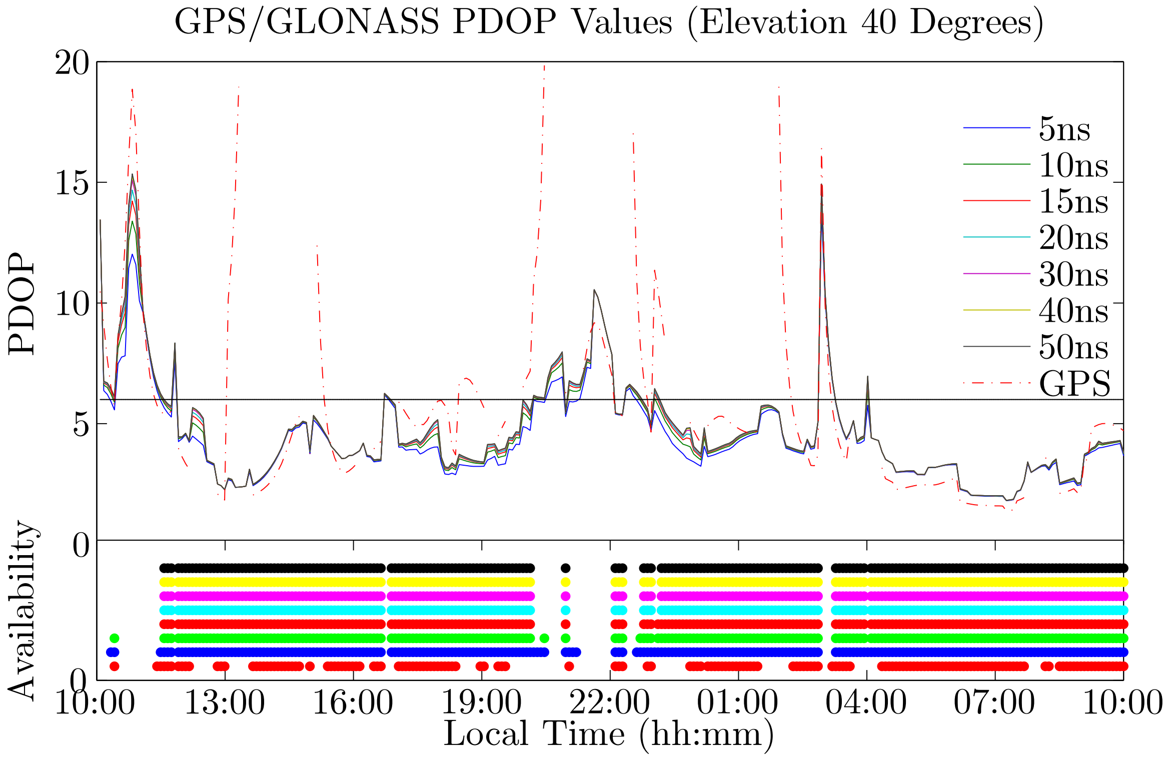
The Galileo constellation will have 27 satellites in its nominal constellation. The satellites' orbits are higher than GPS satellites but the inclination is similar at 56°. Because Galileo satellites have a higher orbit they will provide better coverage than the GPS and geometric distribution of satellites for PDOP is better. A combination of GPS/Galileo has better PDOP values compared to only GPS, especially for low elevation masks like even when time bias is equal to 50ns. Figure 4.11(a) shows the difference between GPS only and combined GPS/Galileo PDOP values for 30°. As elevation mask increases GPS/Galileo maintains continuous PDOP values and availability until 35°. Figure 4.11(b) shows PDOP and availability for GPS and



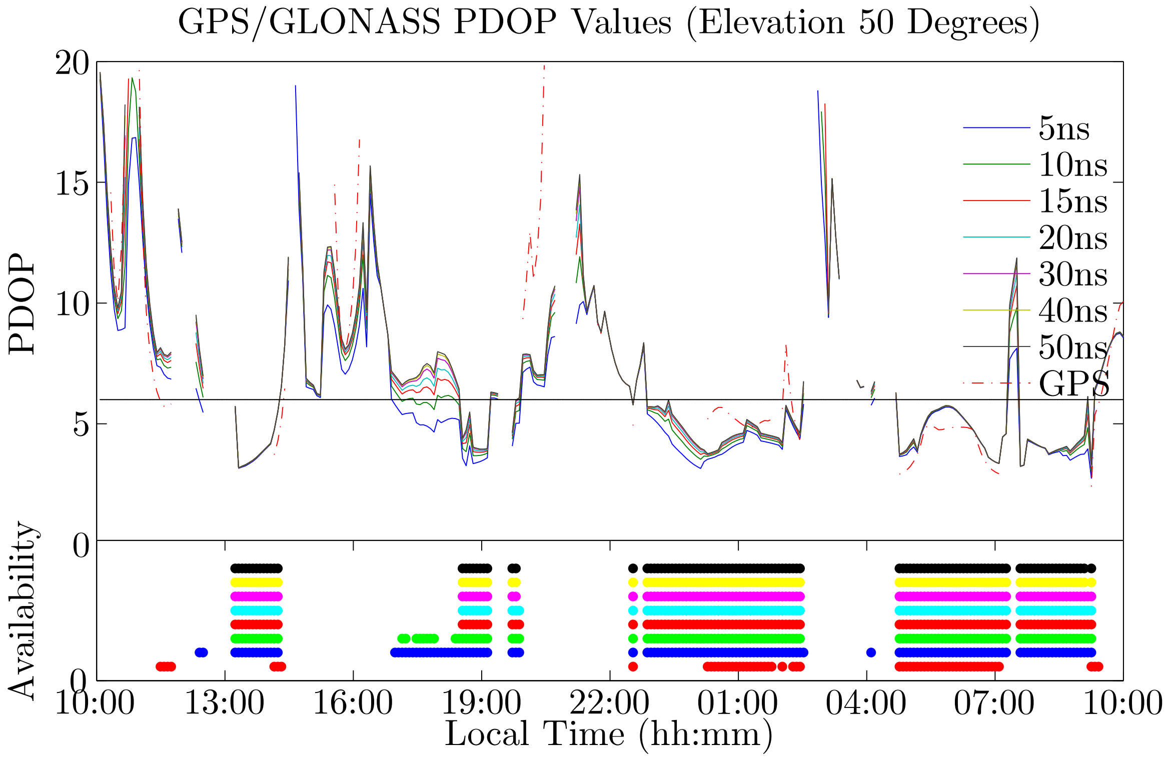
(a)

(b)

Figure 4.7: GPS/GLONASS PDOP Values for Elevation Masks 20° and 30°



(a)



(b)

Figure 4.8: GPS/GLONASS PDOP Values for Elevation Masks 40° and 50°

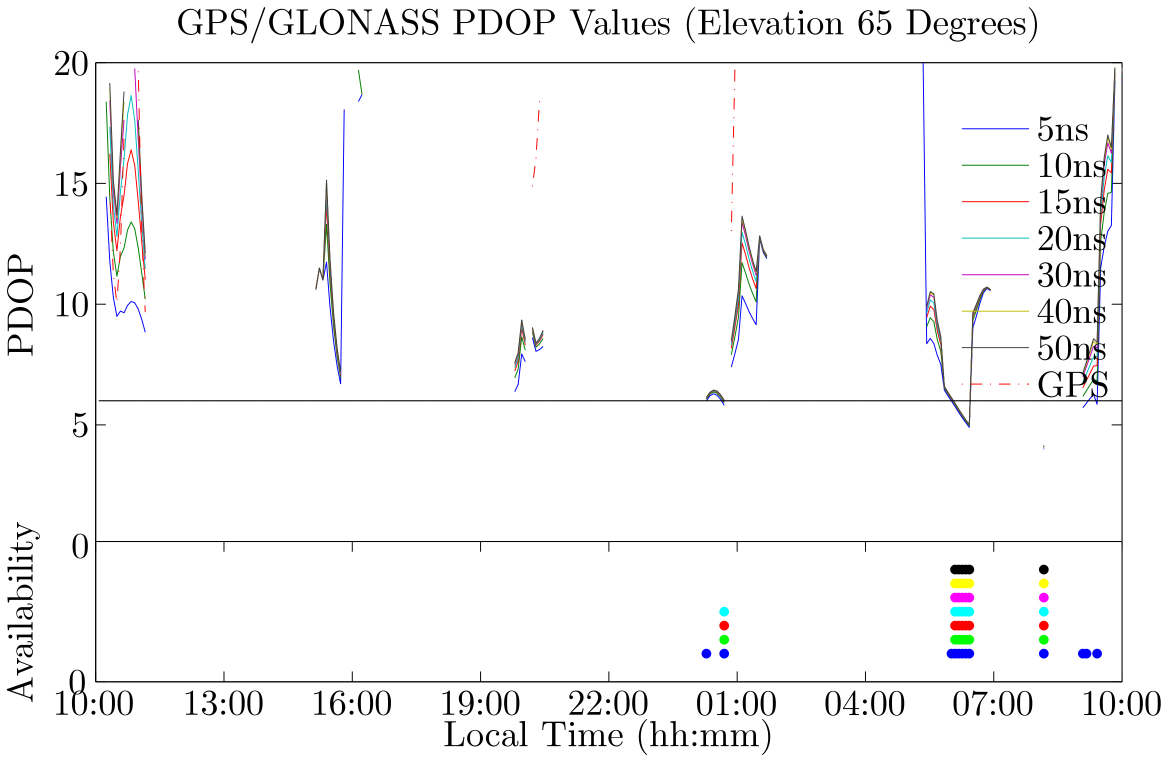


Figure 4.9: GPS/GLONASS PDOP Values for 65° Elevation Mask

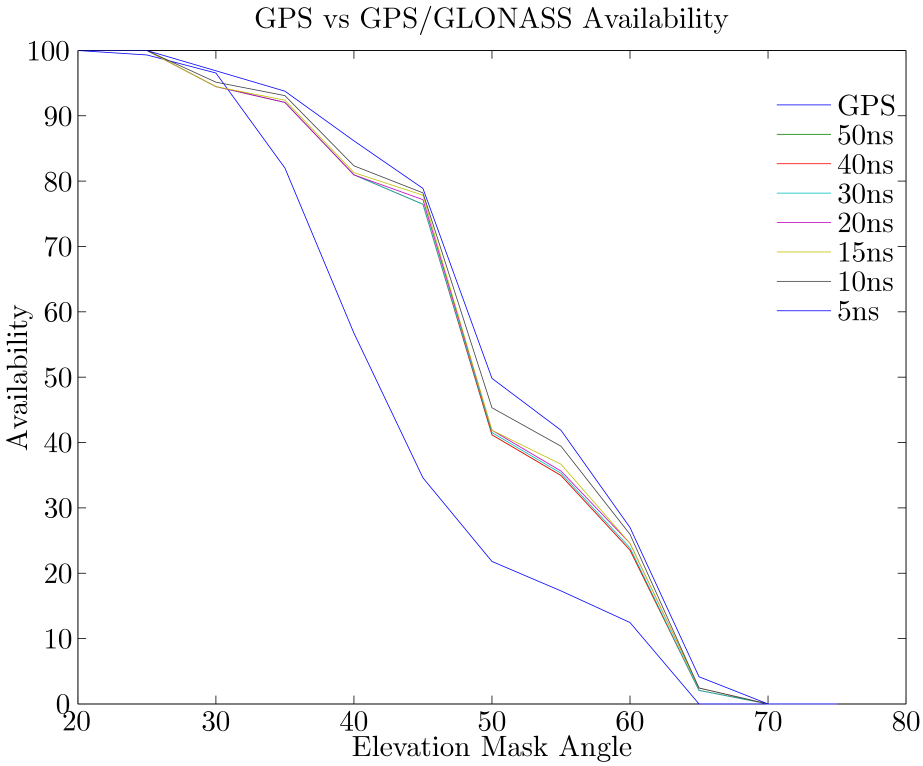
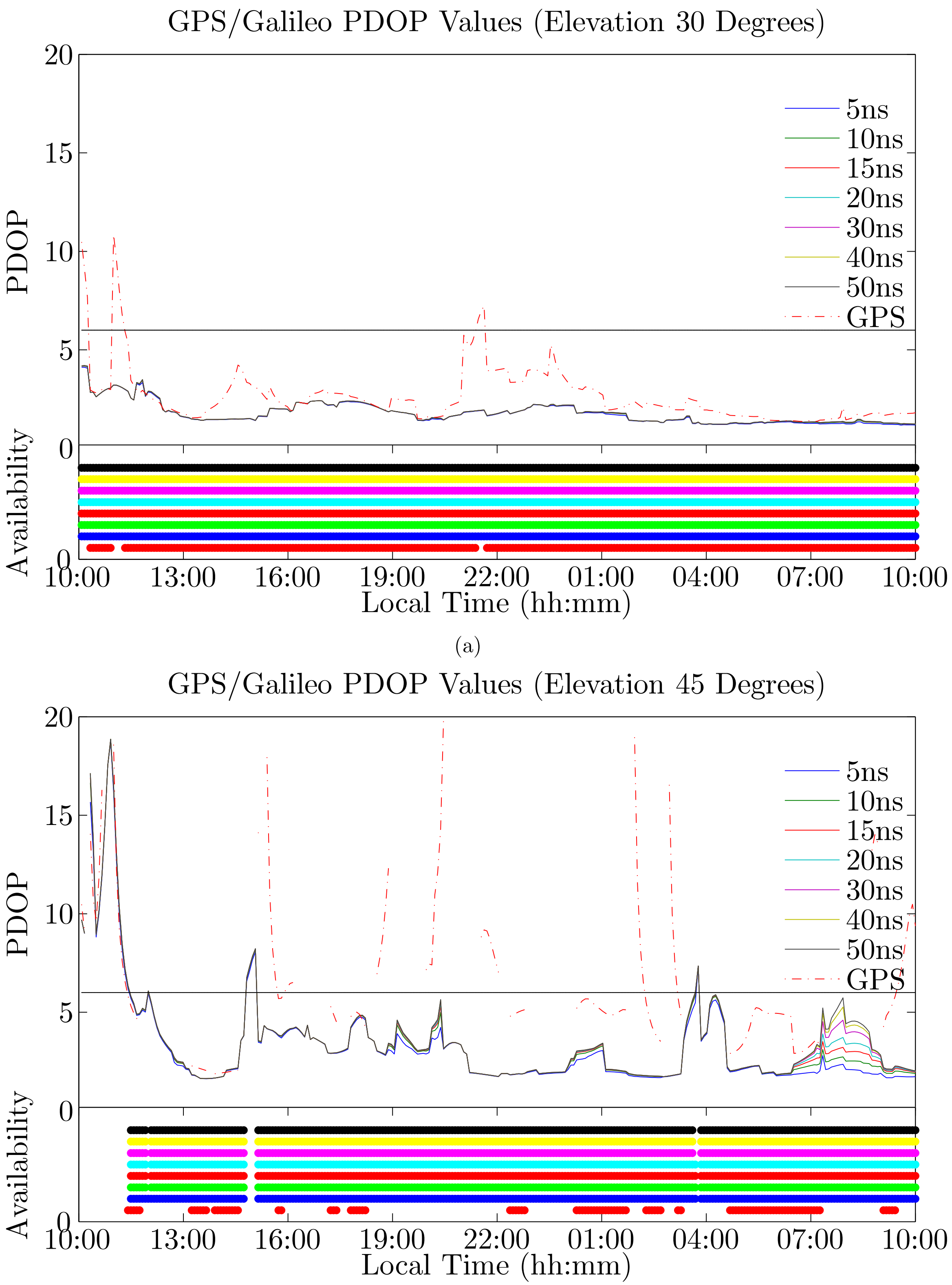


Figure 4.10: GPS/GLONASS Availability



(a)

(b)

Figure 4.11: GPS/Galileo PDOP Values for Elevation Masks 30° and 45°

Table 4.2: Elevation Masks and Combined GPS/GLONASS Availability with Time Bias

| Elevation | GPS | 5ns | 10ns | 15ns | 20ns | 30ns | 40ns | 50ns |
|---|---|---|---|---|---|---|---|---|
| 20° | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 25° | 99.3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 30° | 96.4 | 96.89 | 95.16 | 94.46 | 94.46 | 94.46 | 94.46 | 94.46 |
| 35° | 82 | 93.77 | 93.08 | 92.39 | 92.04 | 94.04 | 92.04 | 92.04 |
| 40° | 56.74 | 86.16 | 82.35 | 81.31 | 80.97 | 80.97 | 80.97 | 80.97 |
| 45° | 34.6 | 78.89 | 78.2 | 77.85 | 77.16 | 76.47 | 76.47 | 76.47 |
| 50° | 21.79 | 49.83 | 45.33 | 41.87 | 41.87 | 41.52 | 41.18 | 41.18 |
| 55° | 17.3 | 41.87 | 39.45 | 36.68 | 35.64 | 35.29 | 34.95 | 34.95 |
| 60° | 12.45 | 26.99 | 25.95 | 24.57 | 24.57 | 23.88 | 23.53 | 23.53 |
| 65° | 0 | 4.15 | 2.42 | 2.42 | 2.42 | 2.08 | 2.08 | 2.08 |
| 70° | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 75° | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

GPS/Galileo for 45° elevation mask. GPS has bigger values and sometimes no PDOP values for this elevation mask, but GPS/Galileo has good overall availability as 92.39% and 91.7% for 5ns and 50ns time biases, respectively. For 50° elevation angle it can provide 73.01% availability for 5ns time bias and 70.24% availability for 50ns time bias. Higher elevation masks decreases availability of GPS/Galileo and finally for 65° PDOPs are bigger than 6 or not available most of the time and it means that it is not useful for navigation over this elevation angle. In Figure 4.12 it can be seen that there are so many gaps and navigation solution can not be obtained after this point. Table 4.3 and Figure 4.13 summarize the availability as a function of mask angle and offset uncertainty.

## *4.6 Combined GPS/Compass Results*

GPS/Compass results are very similar to GPS/GLONASS results. For low elevation angles GPS PDOP values are better than GPS/Compass PDOP values unless there is 5ns time bias. Figure 4.14(a) shows GPS and GPS/Compass PDOP values for 20° elevation mask.

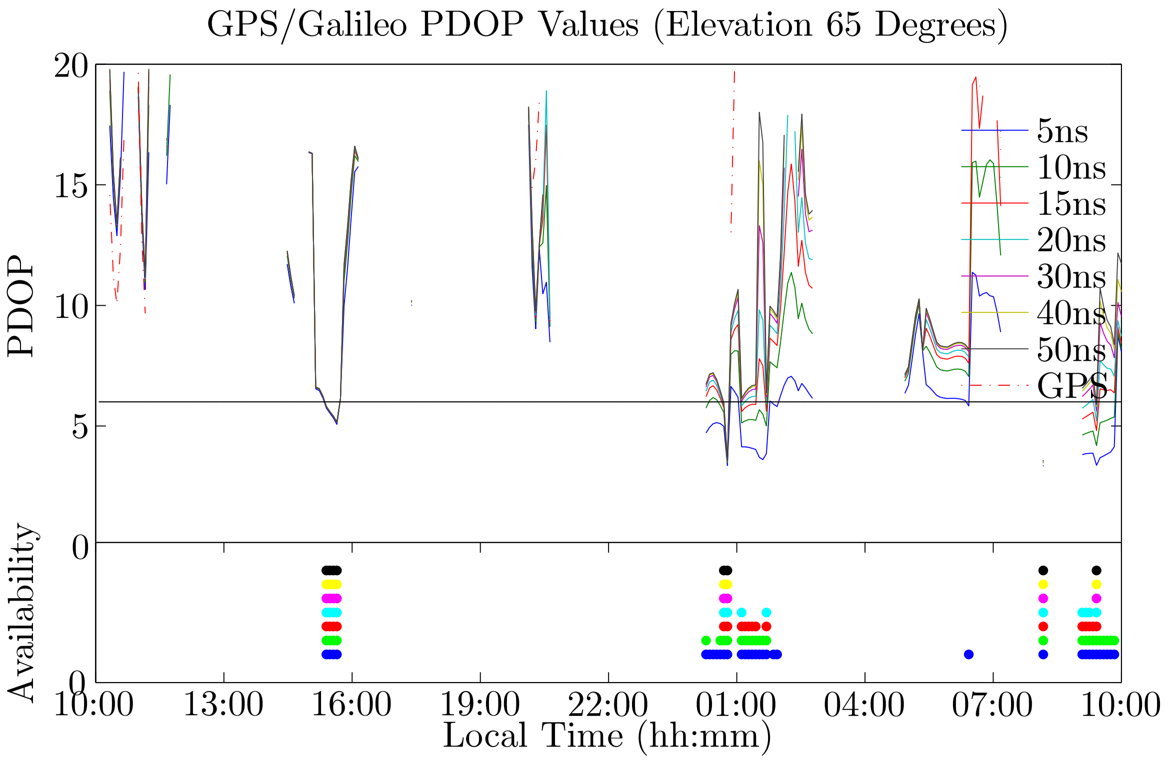


Figure 4.12: GPS/Galileo PDOP Values for 65° Elevation Mask

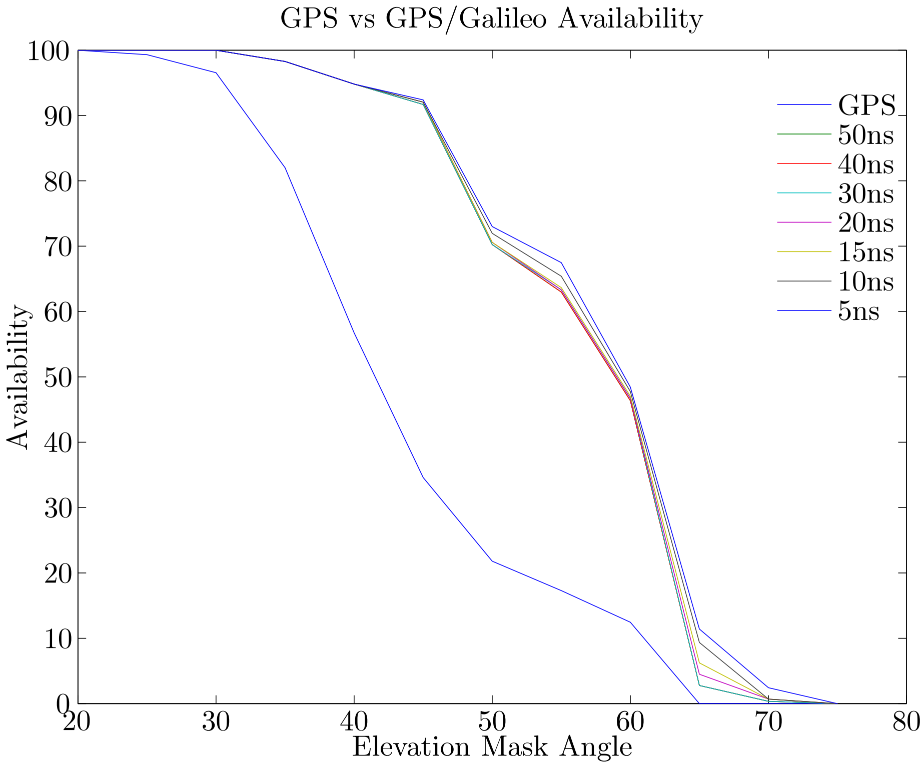
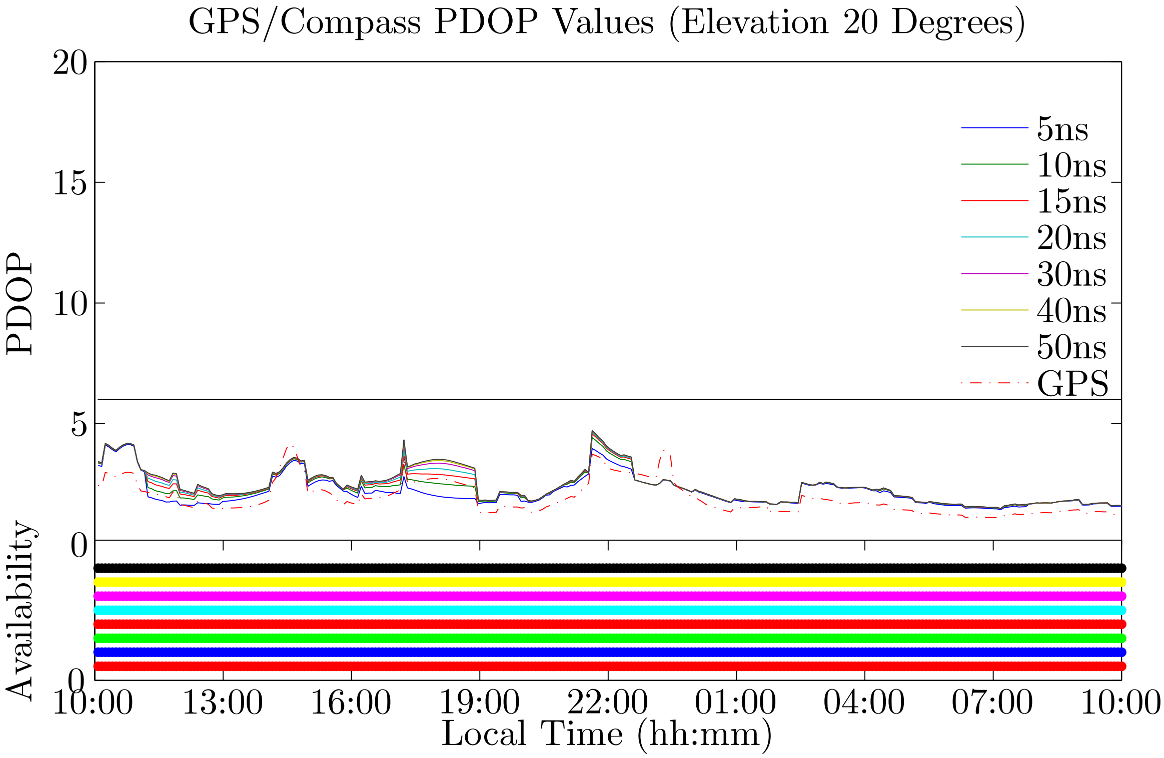
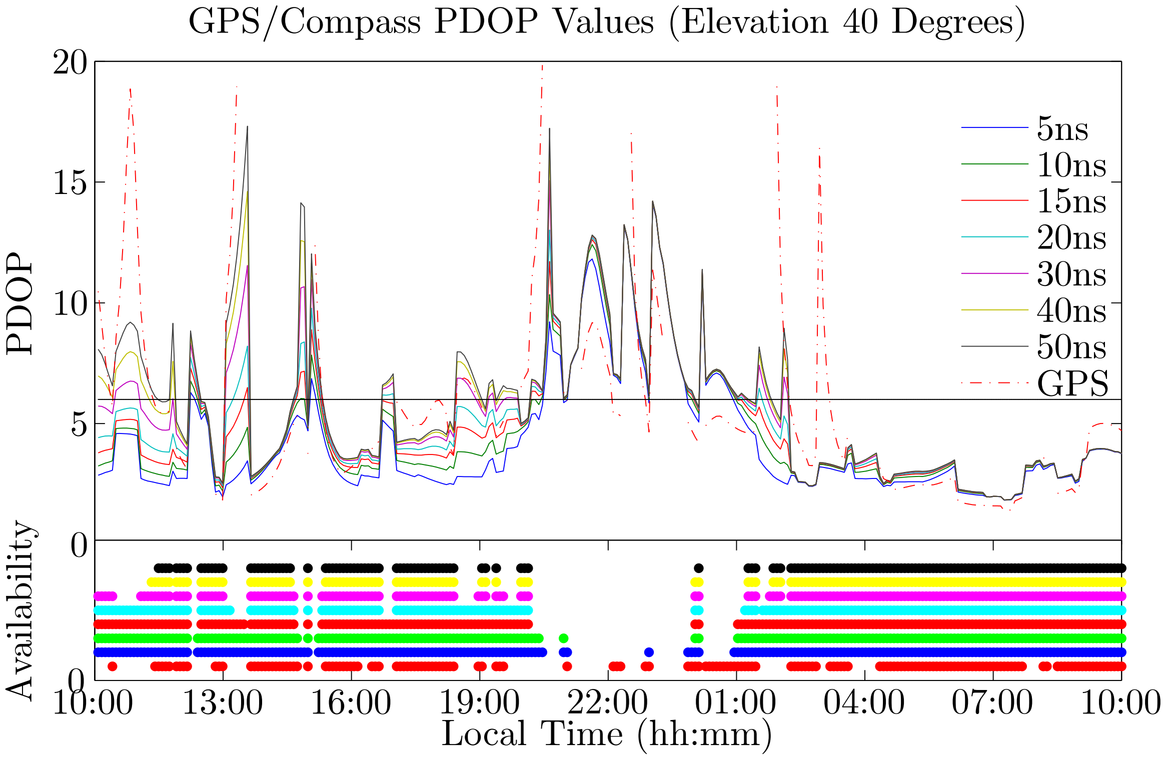


Figure 4.13: GPS/Galileo Availability



(a)



(b)

Figure 4.14: GPS/Compass PDOP Values for Elevation Masks 20° and 40°

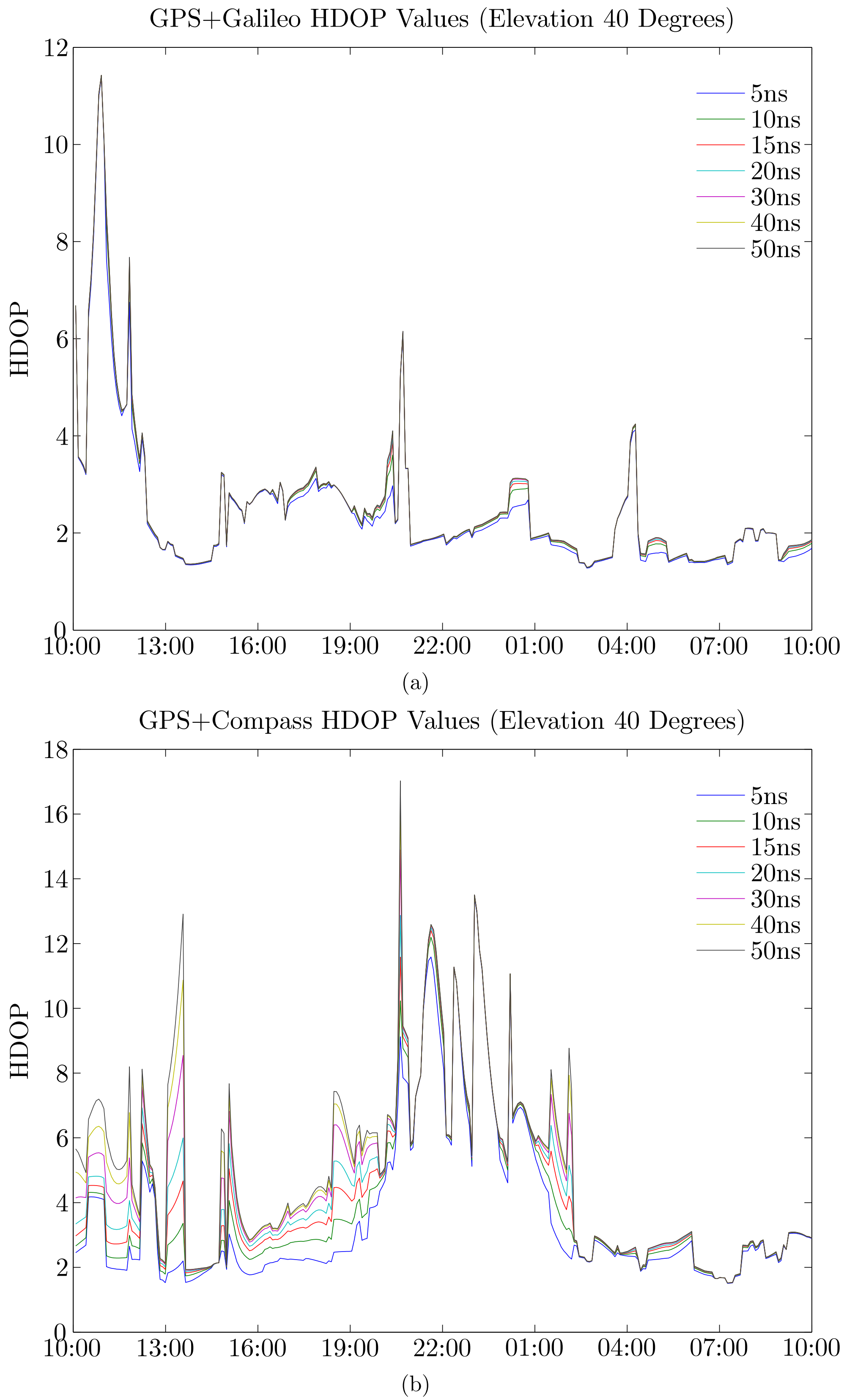
Table 4.3: Elevation Masks and Combined GPS/Galileo Availability with Time Bias

| Elevation | GPS | 5ns | 10ns | 15ns | 20ns | 30ns | 40ns | 50ns |
|---|---|---|---|---|---|---|---|---|
| 20° | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 25° | 99.3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 30° | 96.4 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 35° | 82 | 98.27 | 98.27 | 98.27 | 98.27 | 98.27 | 98.27 | 98.27 |
| 40° | 56.74 | 94.81 | 94.81 | 94.81 | 94.81 | 94.81 | 94.81 | 94.81 |
| 45° | 34.6 | 92.39 | 92.04 | 92.04 | 92.04 | 91.7 | 91.7 | 91.7 |
| 50° | 21.79 | 73.01 | 71.97 | 70.59 | 70.59 | 70.24 | 70.24 | 70.24 |
| 55° | 17.3 | 67.47 | 65.4 | 63.67 | 63.32 | 63.32 | 62.98 | 62.98 |
| 60° | 12.45 | 48.44 | 47.75 | 47.06 | 46.71 | 46.71 | 46.37 | 46.37 |
| 65° | 0 | 11.42 | 9.34 | 6.23 | 4.5 | 2.77 | 2.77 | 2.77 |
| 70° | 0 | 2.42 | 0.69 | 0.69 | 0.69 | 0.35 | 0.35 | 0.35 |
| 75° | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

For 40° elevation mask when GPS PDOP and availability begins to degrade, GPS/Compass provides 79.93% availability if the time bias is not exceed 10ns. Figure 4.14(b) shows this situation. After 60° PDOPs are bigger than 6 or not available most of the time. In Figure 4.16 it can be seen that there are many gaps and a navigation solution can not be obtained after this point. Table 4.4 and 4.17 summarize the results as a function of elevation mask and time offset uncertainty. It is worth to note that time offset uncertainty plays a more significant role for GPS/Compass as shown in Figure 4.17 due to the distribution of Compass satellites in the sky according to GPS at the moment of signal reception. This conclusion is based on HDOP analysis of combined GPS/Compass. Figure 4.15 shows the HDOP values for GPS/Galileo and GPS/Compass. It is obvious that GPS/Compass HDOP values varying significantly depending on time bias compared to GPS/Galileo values.

## *4.7 Combined GPS/GLONASS/Galileo Results*

Combined GPS/GLONASS/Galileo is advantageous for availability and PDOP values compared to GPS. As the number of available satellites increases the availability increases and PDOP values decrease. Although combined GPS/GLONASS/Galileo



(a)

(b)

Figure 4.15: GPS/Galileo and GPS/Compass HDOP Values for Elevation Mask 40°

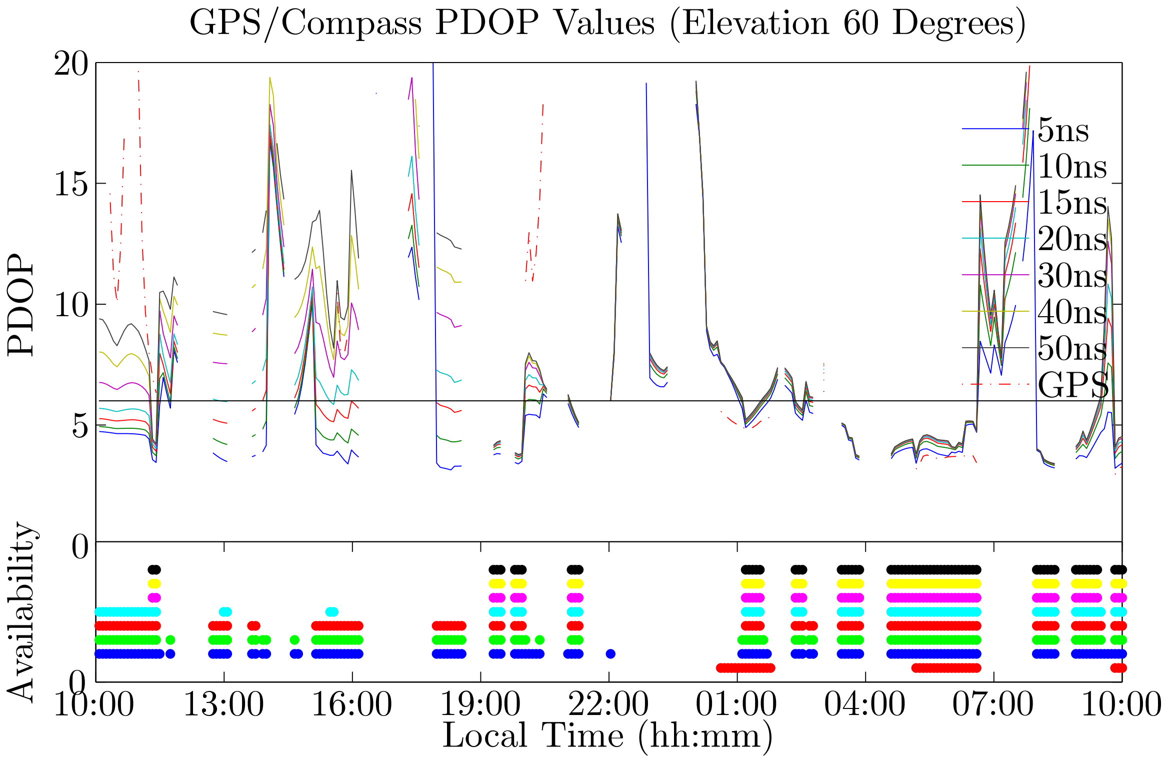


Figure 4.16: GPS/Compass PDOP Values for 60° Elevation Mask

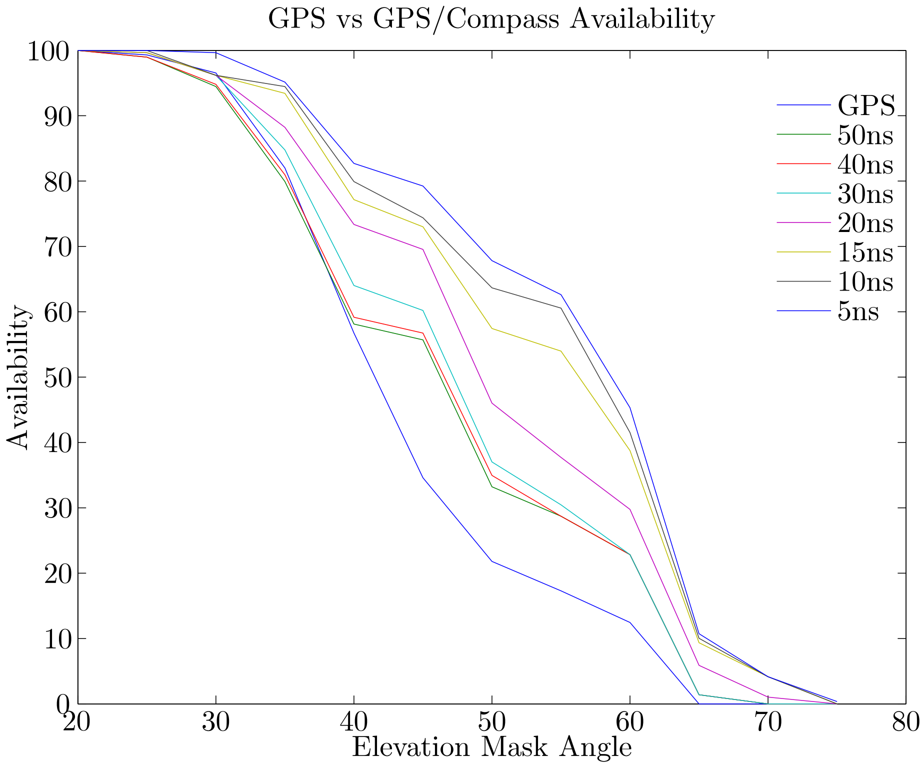


Figure 4.17: GPS/Compass Availability

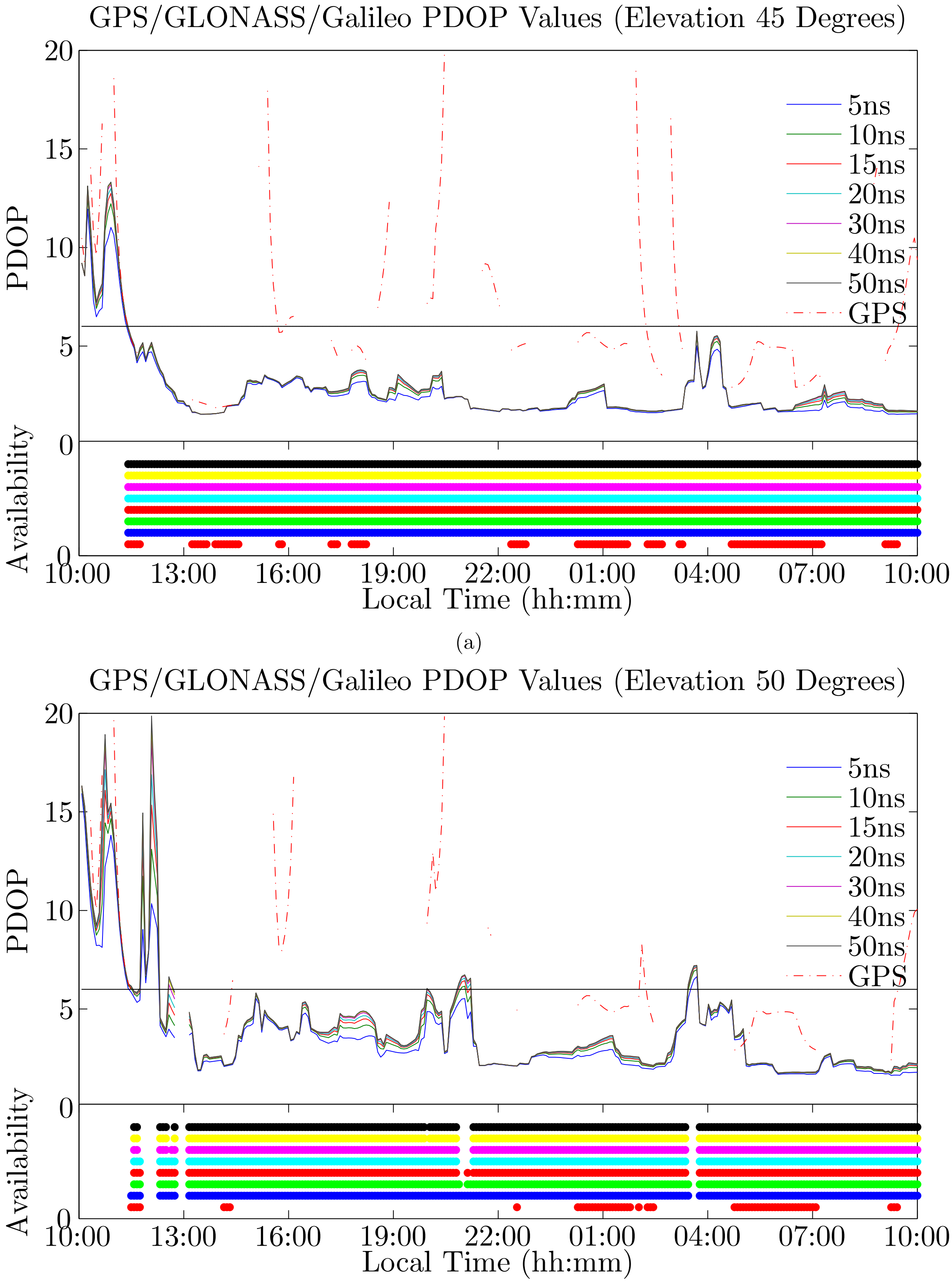
Table 4.4: Elevation Masks and Combined GPS/Compass Availability with Time Bias

| Elevation | GPS | 5ns | 10ns | 15ns | 20ns | 30ns | 40ns | 50ns |
|---|---|---|---|---|---|---|---|---|
| 20° | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 25° | 99.3 | 100 | 100 | 99.65 | 99.65 | 99.65 | 98.96 | 98.96 |
| 30° | 96.4 | 99.65 | 96.19 | 96.19 | 96.19 | 96.19 | 94.81 | 94.46 |
| 35° | 82 | 95.16 | 94.46 | 93.43 | 88.24 | 84.78 | 80.97 | 79.93 |
| 40° | 56.74 | 82.7 | 79.93 | 77.16 | 73.36 | 64.01 | 59.17 | 58.13 |
| 45° | 34.6 | 79.24 | 74.39 | 73.01 | 69.55 | 60.21 | 56.75 | 55.71 |
| 50° | 21.79 | 67.82 | 63.67 | 57.44 | 46.02 | 37.02 | 34.95 | 33.22 |
| 55° | 17.3 | 62.63 | 60.55 | 53.98 | 37.72 | 30.45 | 28.72 | 28.72 |
| 60° | 12.45 | 45.33 | 41.52 | 38.75 | 29.76 | 22.84 | 22.84 | 22.84 |
| 65° | 0 | 10.73 | 10.03 | 9.34 | 5.88 | 1.38 | 1.38 | 1.38 |
| 70° | 0 | 4.15 | 4.15 | 4.15 | 1.04 | 0 | 0 | 0 |
| 75° | 0 | 0.35 | 0 | 0 | 0 | 0 | 0 | 0 |

PDOP and availability is good beginning from low elevation masks, the real contribution of combined GPS/GLONASS/Galileo can be seen for higher elevation masks where GPS availability is very low. Figure 4.18(a) shows GPS/GLONASS/Galileo PDOP values and GPS PDOP values for 45° elevation angle. It can be seen that when GPS has 34.6% availability, GPS/GLONASS/Galileo has 94.46% availability up to 50ns time bias. For elevation mask 50° GPS has 21.79% availability and GPS/GLONASS/Galileo has 89.62% for time bias equal to 5ns, 85.81% availability for time bias equal to 50ns. Figure 4.18(b) shows this situation. For elevation mask 65° GPS has no availability, but GPS/GLONASS/Galileo has 44.29% for time bias equal to 5ns and 27.34% availability for time bias equal to 50ns. Figure 4.19 shows this situation. Table 4.5 and 4.20 summarize the results as a function of elevation mask and time offset uncertainty.

## *4.8 Combined GPS/GLONASS/Galileo/Compass Results*

Combined GPS/GLONASS/Galileo/Compass provides very good availability and PDOP values for especially extreme elevation mask cases. Figures 4.21 and 4.22 represents the GPS and other combined systems' availability for 50°, 65°, 70° and



(a)

(b)

Figure 4.18: GPS/GLONASS/Galileo PDOP Values for Elevation Masks 45° and 50°

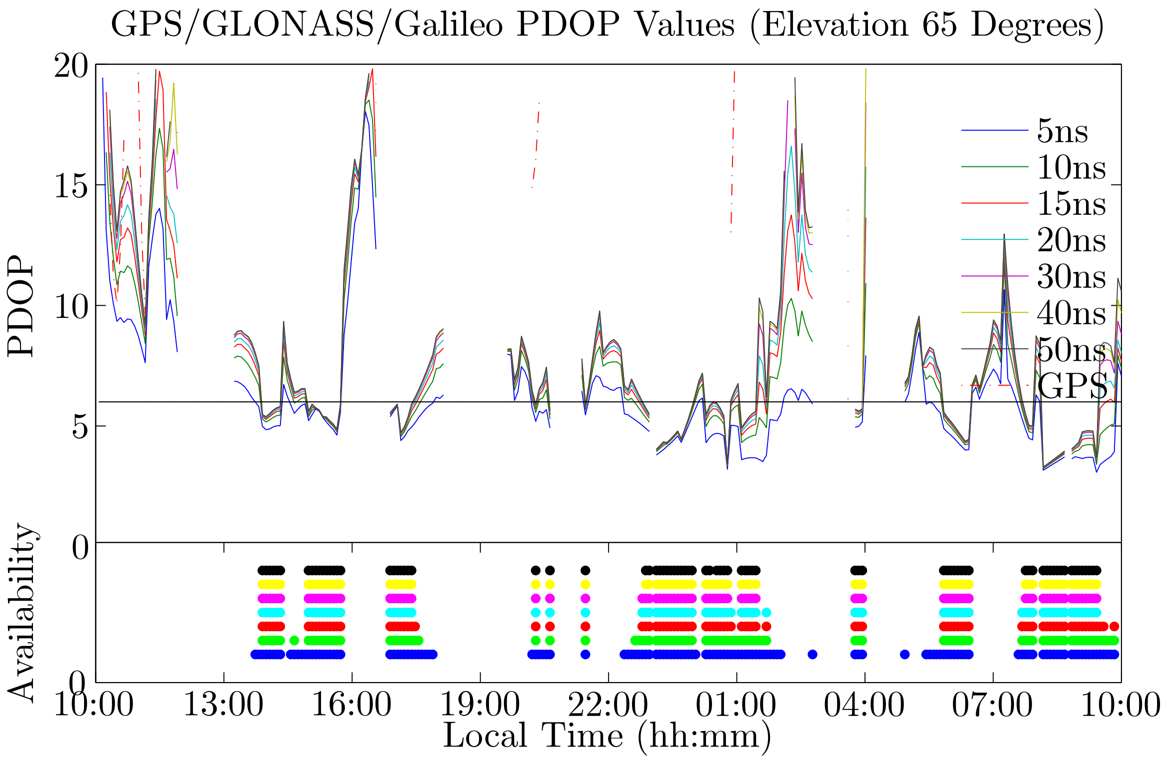


Figure 4.19: GPS/GLONASS/Galileo PDOP Values for 65° Elevation Mask

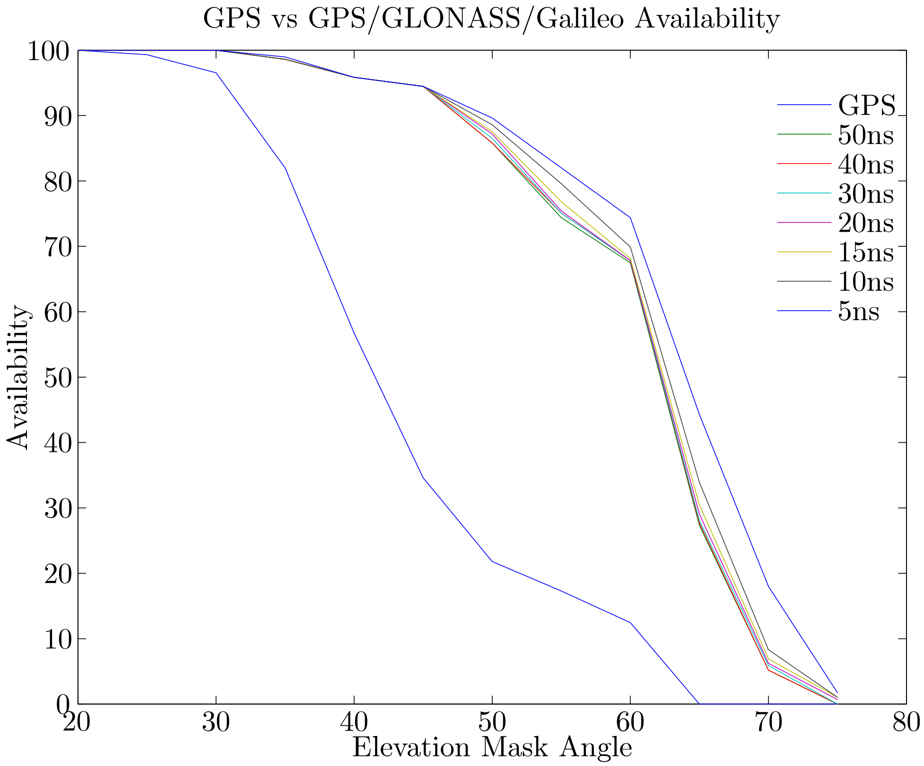


Figure 4.20: GPS/GLONASS/Galileo Availability

Table 4.5: Elevation Masks and Combined GPS/GLONASS/Galileo Availability with Time Bias

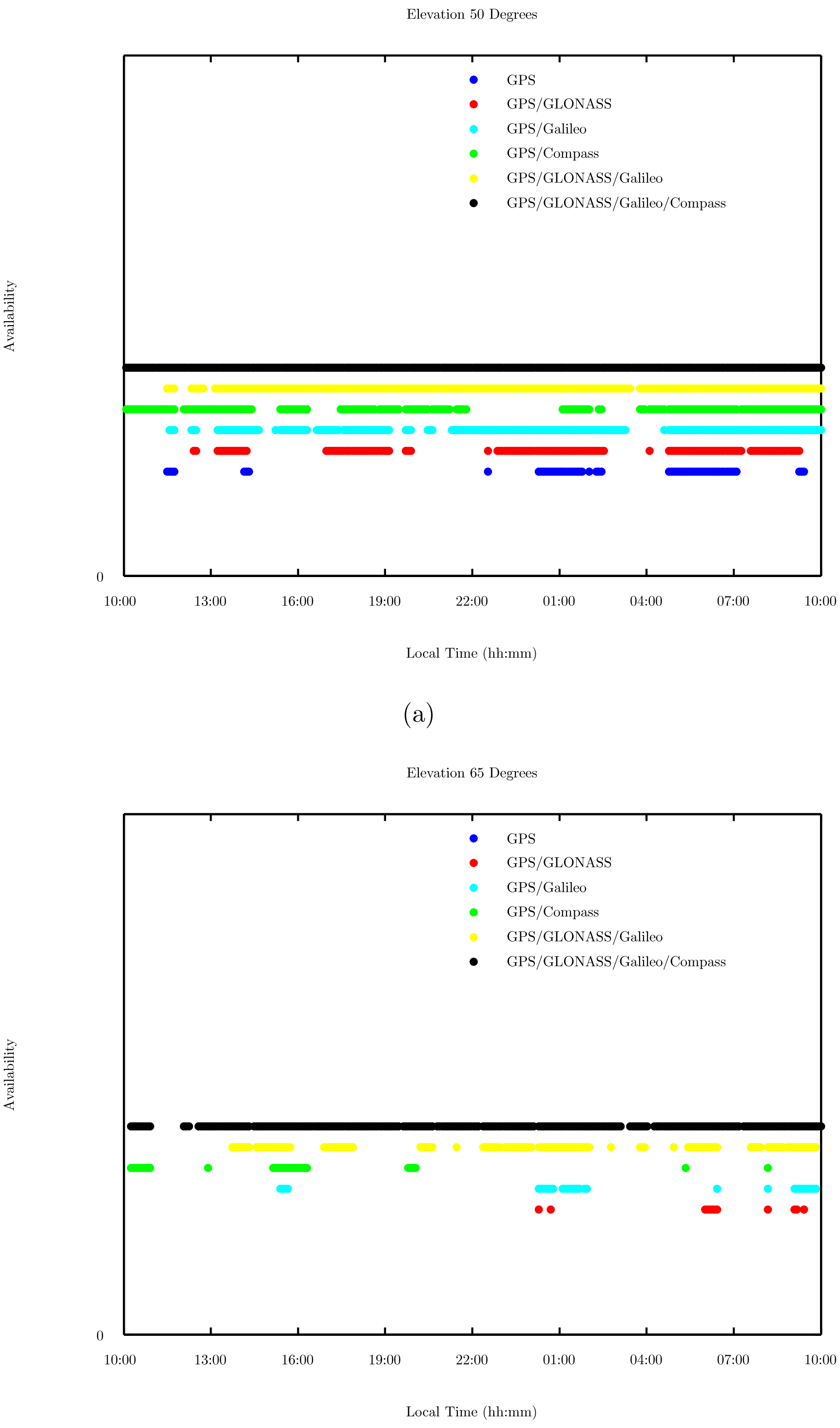
| Elevation | GPS | 5ns | 10ns | 15ns | 20ns | 30ns | 40ns | 50ns |
|---|---|---|---|---|---|---|---|---|
| 20° | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 25° | 99.3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 30° | 96.4 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 35° | 82 | 98.96 | 98.62 | 98.62 | 98.62 | 98.62 | 98.62 | 98.62 |
| 40° | 56.74 | 95.85 | 95.85 | 95.85 | 95.85 | 95.85 | 95.85 | 95.85 |
| 45° | 34.6 | 94.46 | 94.46 | 94.46 | 94.46 | 94.46 | 94.46 | 94.46 |
| 50° | 21.79 | 89.62 | 88.58 | 87.54 | 87.2 | 86.51 | 85.81 | 85.81 |
| 55° | 17.3 | 82.01 | 79.58 | 76.82 | 75.43 | 75.09 | 75.09 | 74.39 |
| 60° | 12.45 | 74.39 | 69.9 | 68.17 | 67.82 | 67.82 | 67.82 | 67.47 |
| 65° | 0 | 44.29 | 33.91 | 30.45 | 29.07 | 28.03 | 27.68 | 27.34 |
| 70° | 0 | 17.99 | 8.3 | 6.92 | 6.23 | 5.88 | 5.19 | 5.19 |
| 75° | 0 | 1.73 | 1.04 | 0.69 | 0 | 0 | 0 | 0 |

75°. It can be seen that GPS/GLONASS/Galileo/Compass has the best availability compared to others. For 65° elevation mask when GPS has no availability, GPS/GLONASS/Galileo/Compass has 89.97% availability for 5ns time bias and 47.06% availability for 50ns time bias. Figure 4.23(a) shows this situation.

For 70° elevation mask, when GPS has no availability, GPS/GLONASS/Galileo/Compass has 57.44% availability for 5ns time bias and 17.99% availability for 50ns time bias. Figure 4.23(b) shows this situation. For 75° elevation mask when GPS has no availability GPS/GLONASS/Galileo/Compass has 28.37% availability for 5ns time bias and 5.19% availability for 50ns time bias. Figure 4.24 shows this situation.

## *4.9 Combining GPS and Galileo with Minimum Satellites*

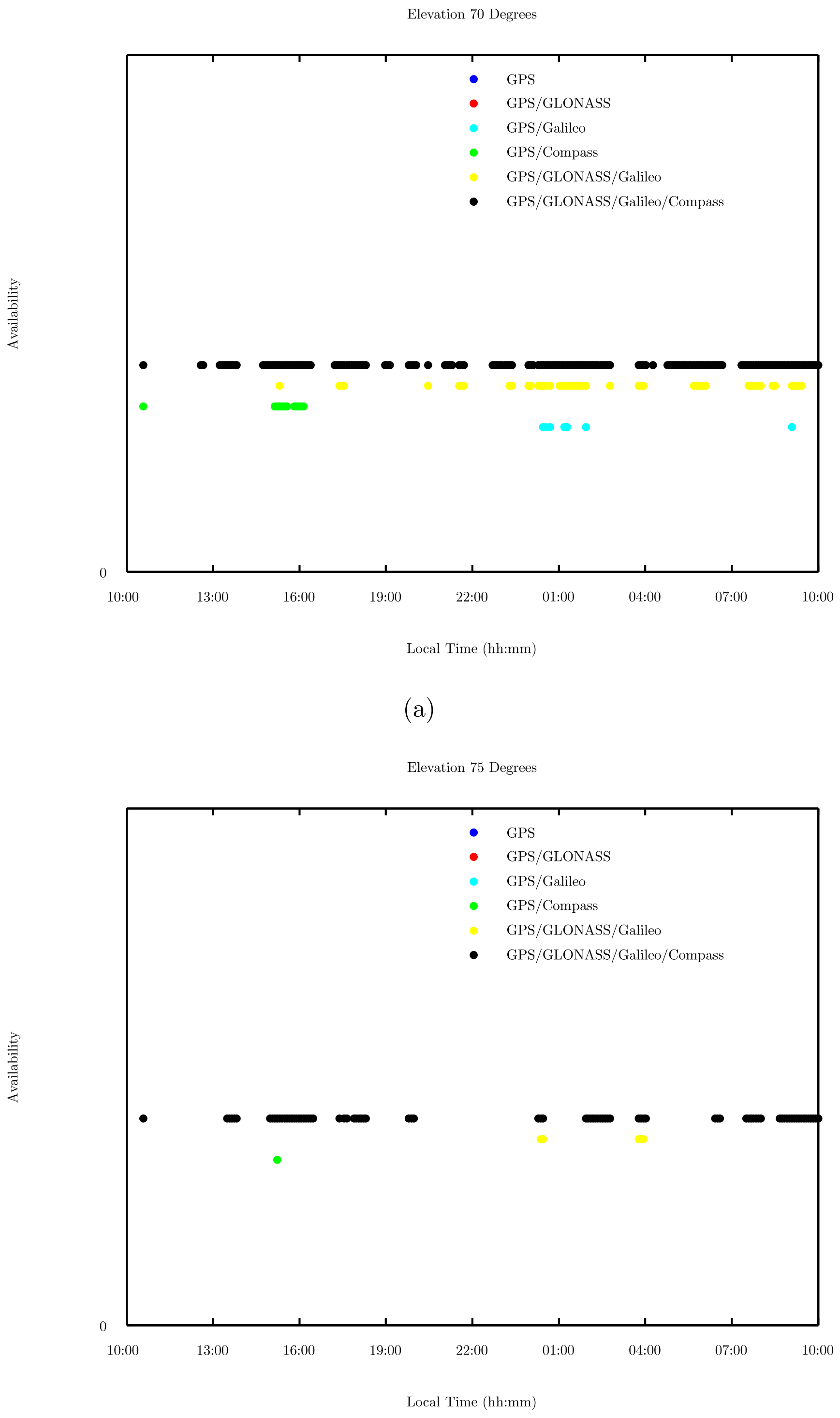
GPS and Galileo systems are very compatible to each other. The results show that average PDOP values for GPS/Galileo is better than other combinations like GPS/GLONASS and GPS/Compass. However dependency on another system is a drawback. A limited simulation is created to provide information what is gained in terms of availability, if only 1, 2 or 3 satellites from Galileo is used, so that dependency



(a)

(b)

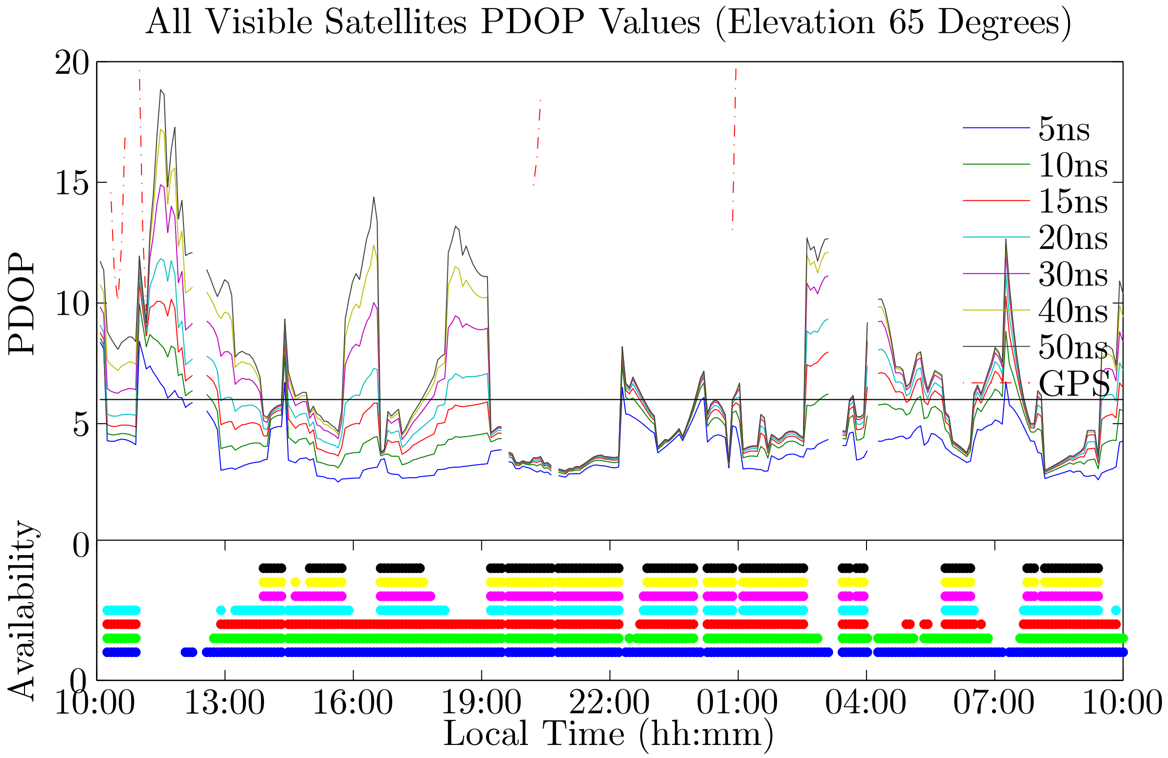
Figure 4.21: GPS and GPS/GLONASS/Galileo/Compass Availability for Elevation Masks 50° and 65°



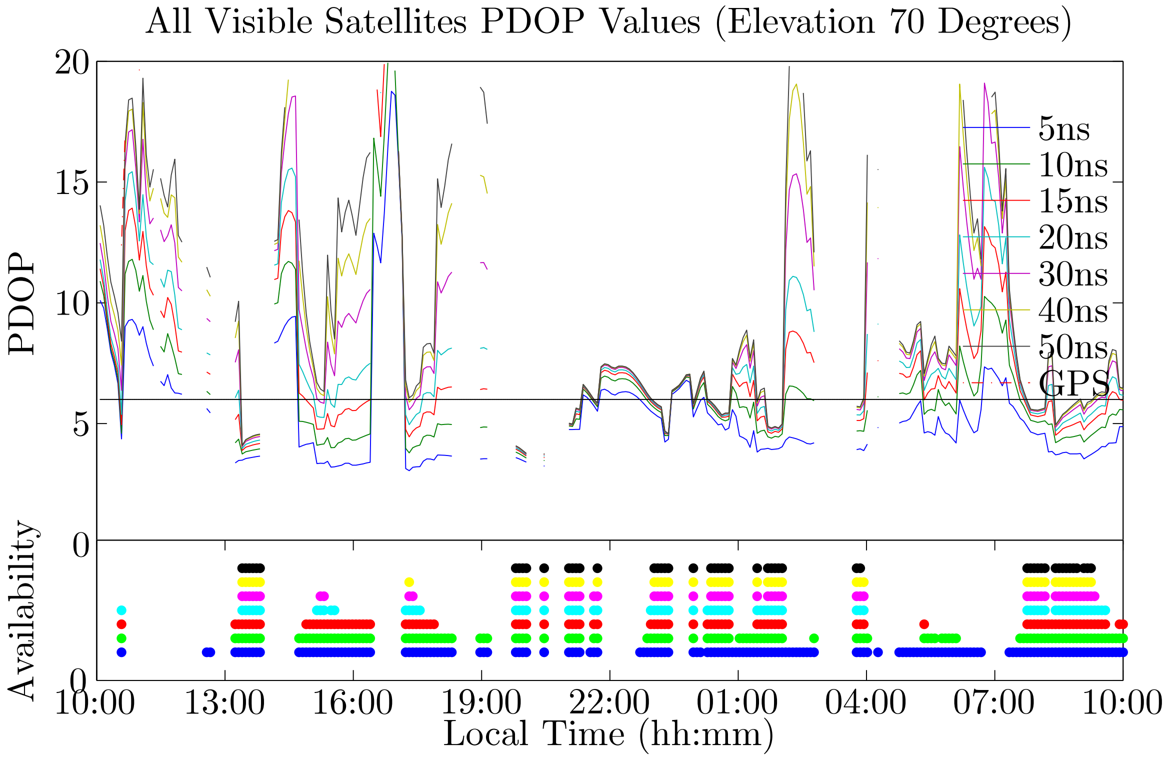
(a)

(b)

Figure 4.22: GPS and GPS/GLONASS/Galileo/Compass Availability for Elevation Masks 70° and 75°



(a)



(b)

Figure 4.23: GPS/GLONASS/Galileo/Compass PDOP Values for Elevation Masks 65° and 70°

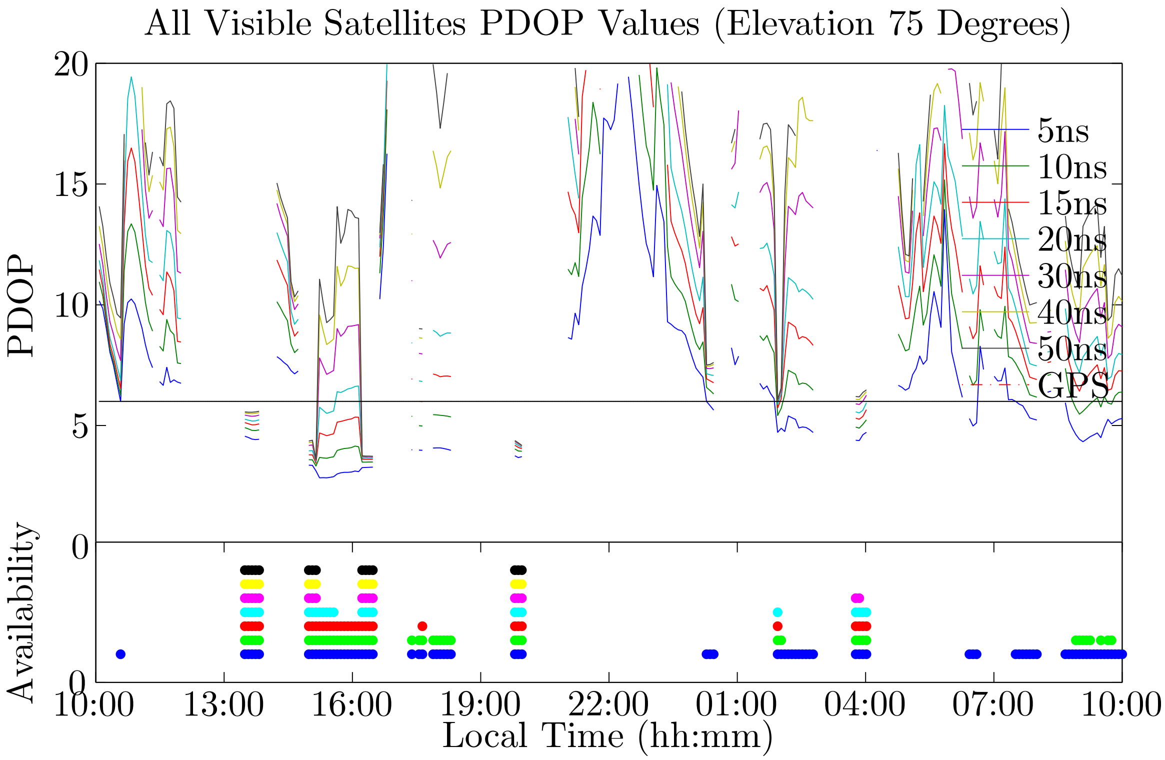


Figure 4.24: GPS/GLONASS/Galileo/Compass PDOP Values for 75° Elevation Mask

Table 4.6: Elevation Masks and GPS/GLONASS/Galileo/Compass Availability with Time Bias

| Elevation | GPS | 5ns | 10ns | 15ns | 20ns | 30ns | 40ns | 50ns |
|---|---|---|---|---|---|---|---|---|
| 20° | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 25° | 99.3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 30° | 96.4 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 35° | 82 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 40° | 56.74 | 100 | 100 | 100 | 100 | 100 | 98.62 | 97.23 |
| 45° | 34.6 | 100 | 100 | 100 | 100 | 100 | 97.23 | 95.5 |
| 50° | 21.79 | 100 | 100 | 100 | 100 | 98.62 | 93.43 | 93.08 |
| 55° | 17.3 | 99.31 | 99.31 | 98.62 | 96.19 | 89.62 | 83.74 | 82.01 |
| 60° | 12.45 | 98.96 | 98.96 | 97.92 | 95.16 | 88.24 | 83.04 | 82.01 |
| 65° | 0 | 89.97 | 82.7 | 73.01 | 60.9 | 50.17 | 47.75 | 47.06 |
| 70° | 0 | 57.44 | 44.98 | 34.26 | 25.61 | 20.76 | 19.03 | 17.09 |
| 75° | 0 | 28.37 | 17.3 | 11.42 | 8.65 | 5.88 | 5.19 | 5.19 |

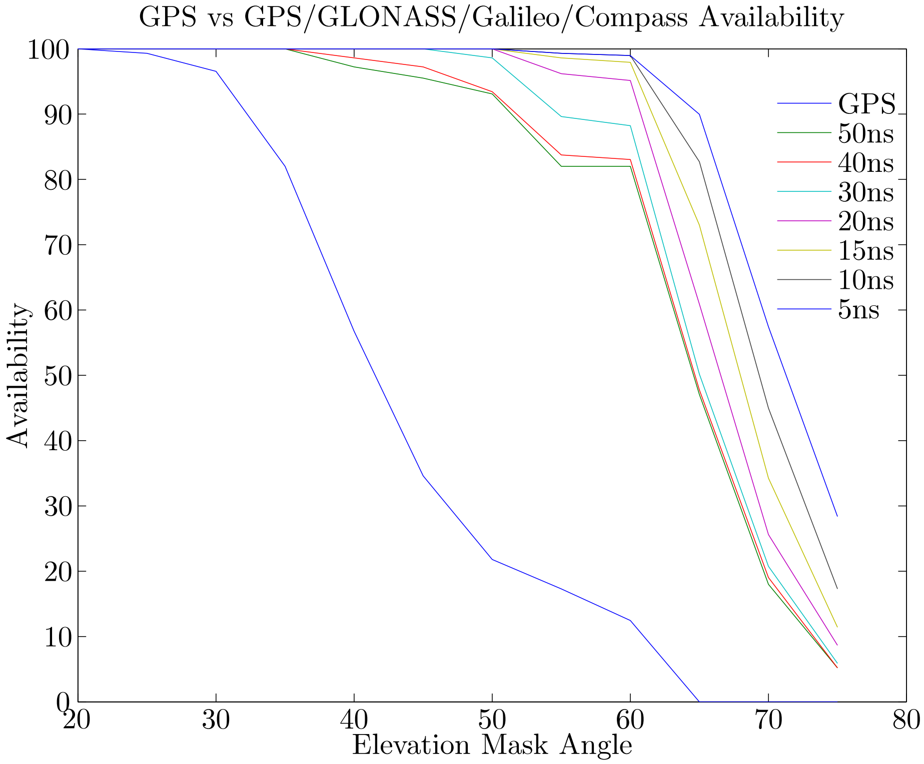


Figure 4.25: GPS/GLONASS/Galileo/Compass Availability

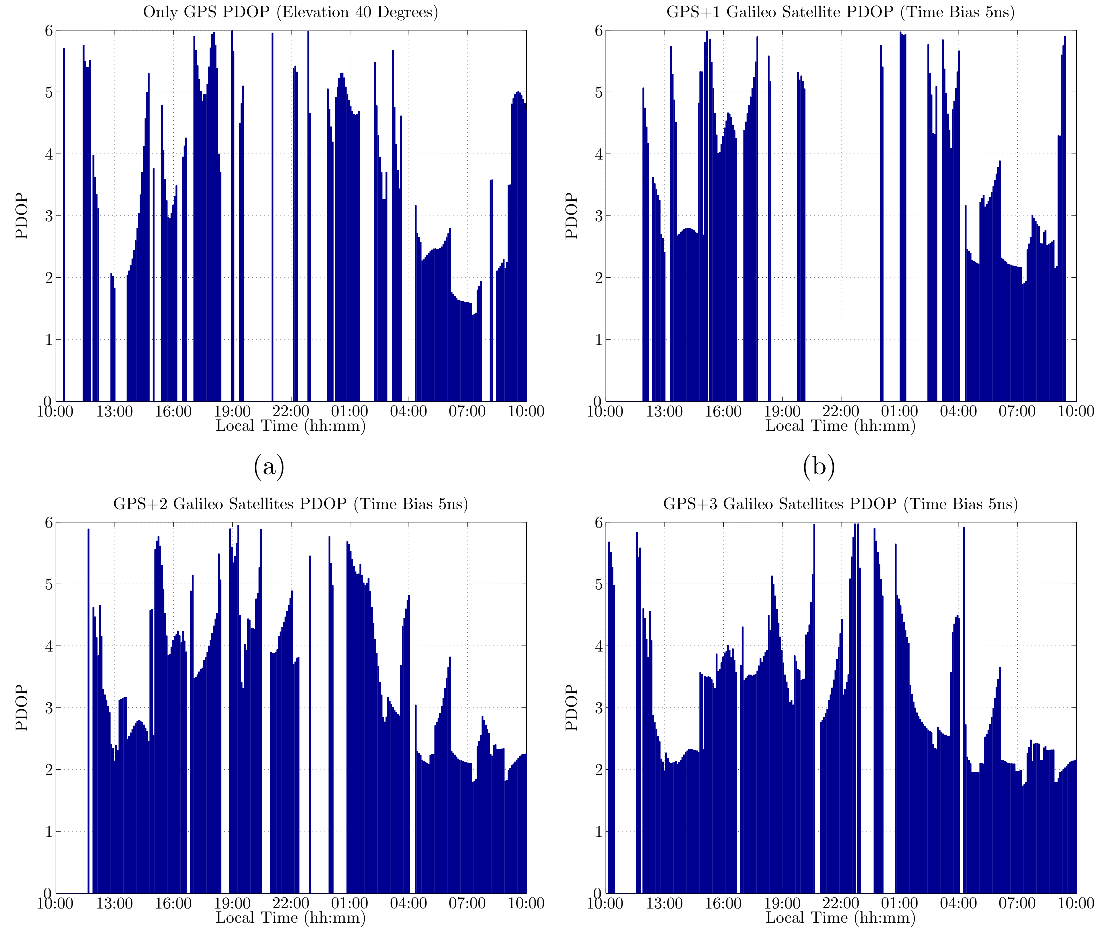
Table 4.7: GPS and Additional Galileo Satellites' Availability Values for Elevation Angles 40° and 50°

| Elevation | GPS | +1 Galileo | +2 Galileo | +3 Galileo |
|---|---|---|---|---|
| 40° | 56.75 | 52.94 | 78.89 | 86.85 |
| 50° | 21.79 | 22.14 | 34.6 | 59.16 |

on other satellites will be minimized. The results are shown in Table 4.7 and Figures 4.26 and 4.27. It can be clearly seen that adding 1 more satellite does not affect availability so much, however adding more than one satellite, even limited to 2 or 3 can, increase the availability of GPS very much, especially for high elevation angles.

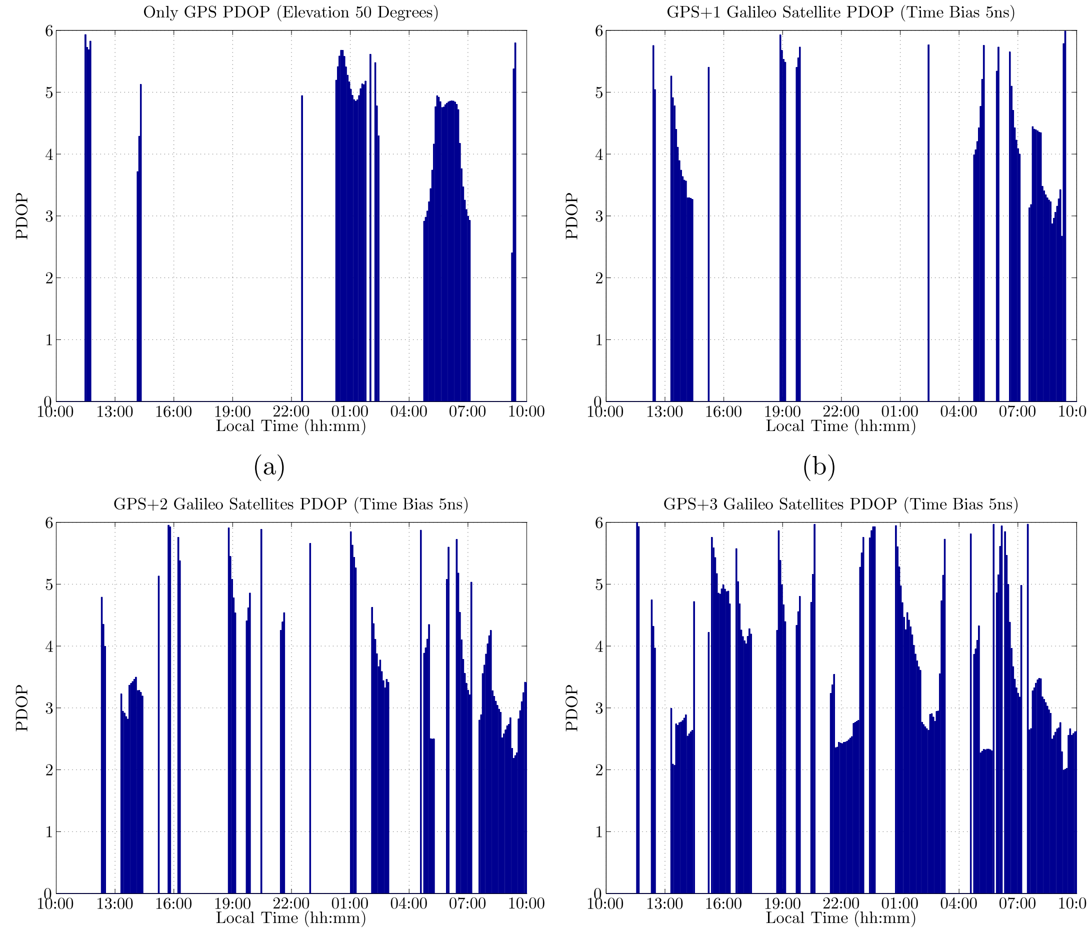
## *4.10 Summary*

In this chapter, the obtained data and analysis of the given. GPS only, GPS/-GLONASS, GPS/Galileo, GPS/Compass, GPS/GLONASS/Galileo and GPS/GLONASS/-



(a) (b) (c) (d)

Figure 4.26: GPS and Additional 1,2, and 3 Galileo Satellites PDOP Values with 5ns Time Bias for 40°



(a) (b) (c) (d)

Figure 4.27: GPS and Additional 1,2, and 3 Galileo Satellites PDOP Values with 5ns Time Bias for 50°

Galileo/Compass PDOP values for different elevation mask angles are showed and results discussed. In Chapter 5, the conclusions and the recommendations for future work is given.

# V. Conclusions and Recommendations

## *5.1 Conclusions*

Based on the research results presented in this thesis, GPS performance degrades for moderate elevation mask angles and at extreme elevation mask angles the navigation information from GPS is not useful due to low availability or high PDOP values. Urban and mountainous can create these kind of high elevation mask environment. GPS can be used alone until elevation mask angle 35° with 82% availability. This corresponds to operations near a three story building. For elevation mask angle 40° availability drops down to 56.74%. To solve this problem some additional satellites from other GNSS constellations can be included in the calculations to improve accuracy and availability. Combined GPS/GLONASS and GPS/Compass is showed little advantage for low elevation mask angles, but for high elevation masks up to 60° there are some benefits of these systems.

Combined GPS/Galileo was expected to be better than compared to combined GPS/GLONASS and GPS/Compass due to its designated interoperability with GPS. Results showed that GPS/Galileo has better performance beginning from low elevation mask angles up to 60°. If it is considered that the uncertainty in the GPS-Galileo time bias will be kept in 5ns then availability will be 48.44% for elevation mask 60° when GPS has only 12.45% availability. If all satellites are included in calculations, then availability gets very good. Even if elevation mask angle increases to 70°, availability will be 57.44% for 5ns time bias and 17.99% for 50ns time bias. Also adding two or three satellites from Galileo constellation can improve the availability of GPS while minimizing the dependence on very much to Galileo system. Uncertainty in time offset was shown to have little impact on performance except for the GPS/Compass combination.

Although there are some significant benefits in adding satellites in calculations, it must be realized that adding satellites from other constellations forces dependence on these systems. There is a trade off between getting good accuracy, availability and dependency on other systems. For low elevation mask angles GPS can be used alone and for high elevation angles some additional satellites can be used in calculations to get better accuracy for period of time. Because in high elevation mask angles GPS is largely denied, some enhancement may be necessary to provide continued operations.

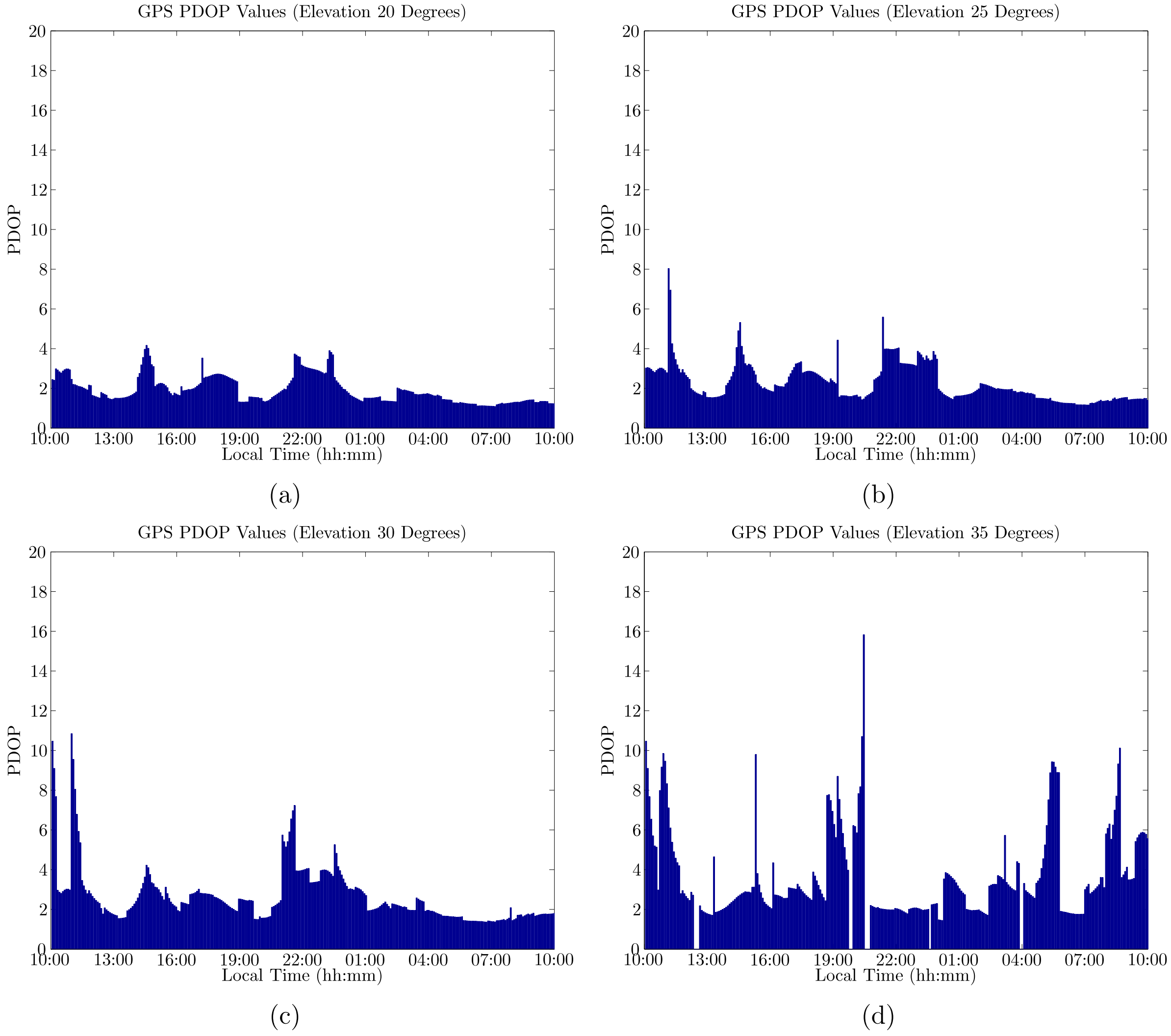
## *5.2 Recommendations*

In thesis calcutaions, ephemeris data is taken from STK. For the future studies when all constellations become operational, results can be validated by real ephemeris data and receivers.

For combining GPS with 1, 2 or 3 Galileo satellites, the satellites were selected randomly. A logical flow chart can be done to select the best satellites for a certain time frame.

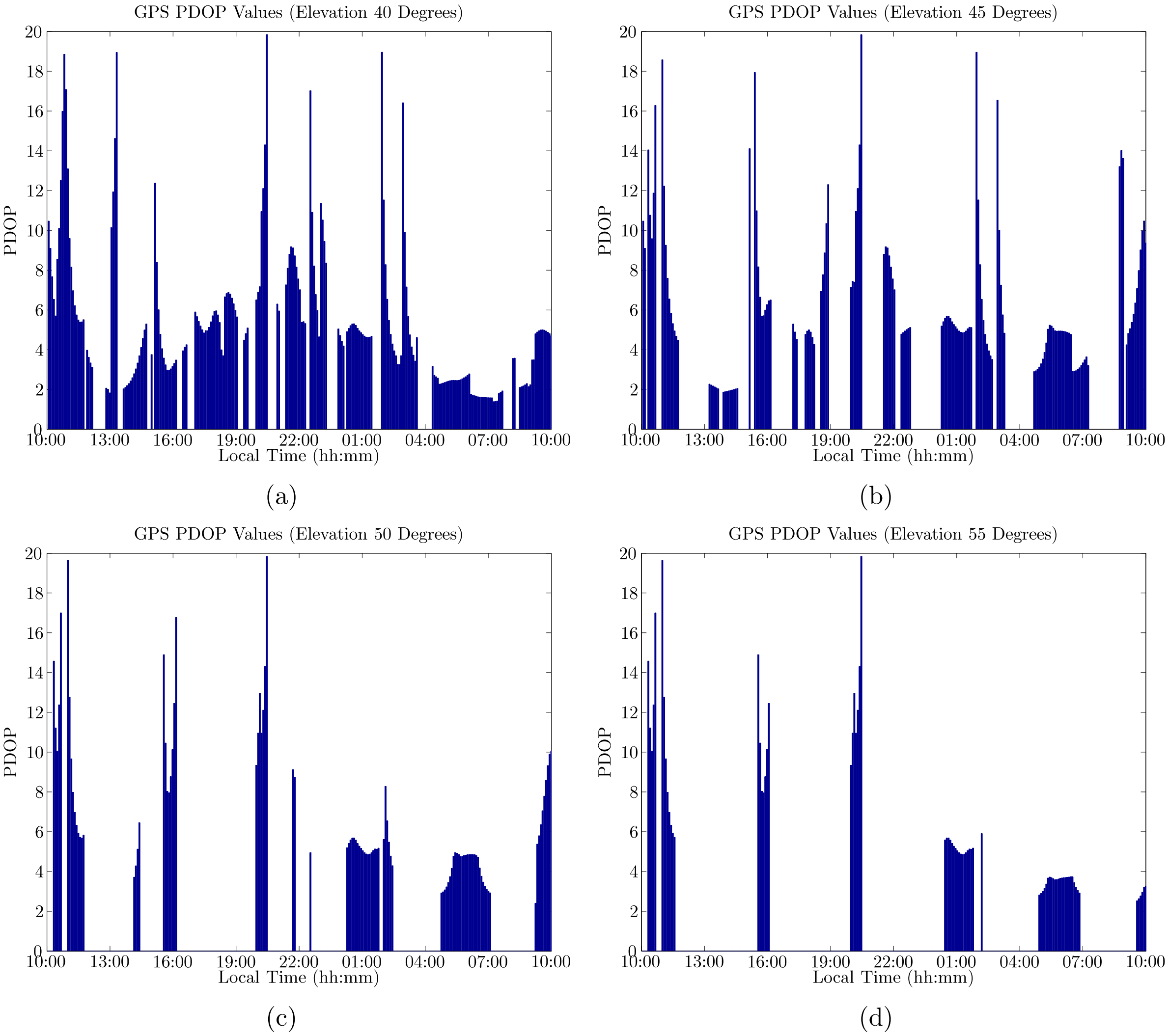
# *Appendix A. GPS PDOP Plots*

This appendix includes plots of GPS position dilution of precision (PDOP) for different elevation angles.



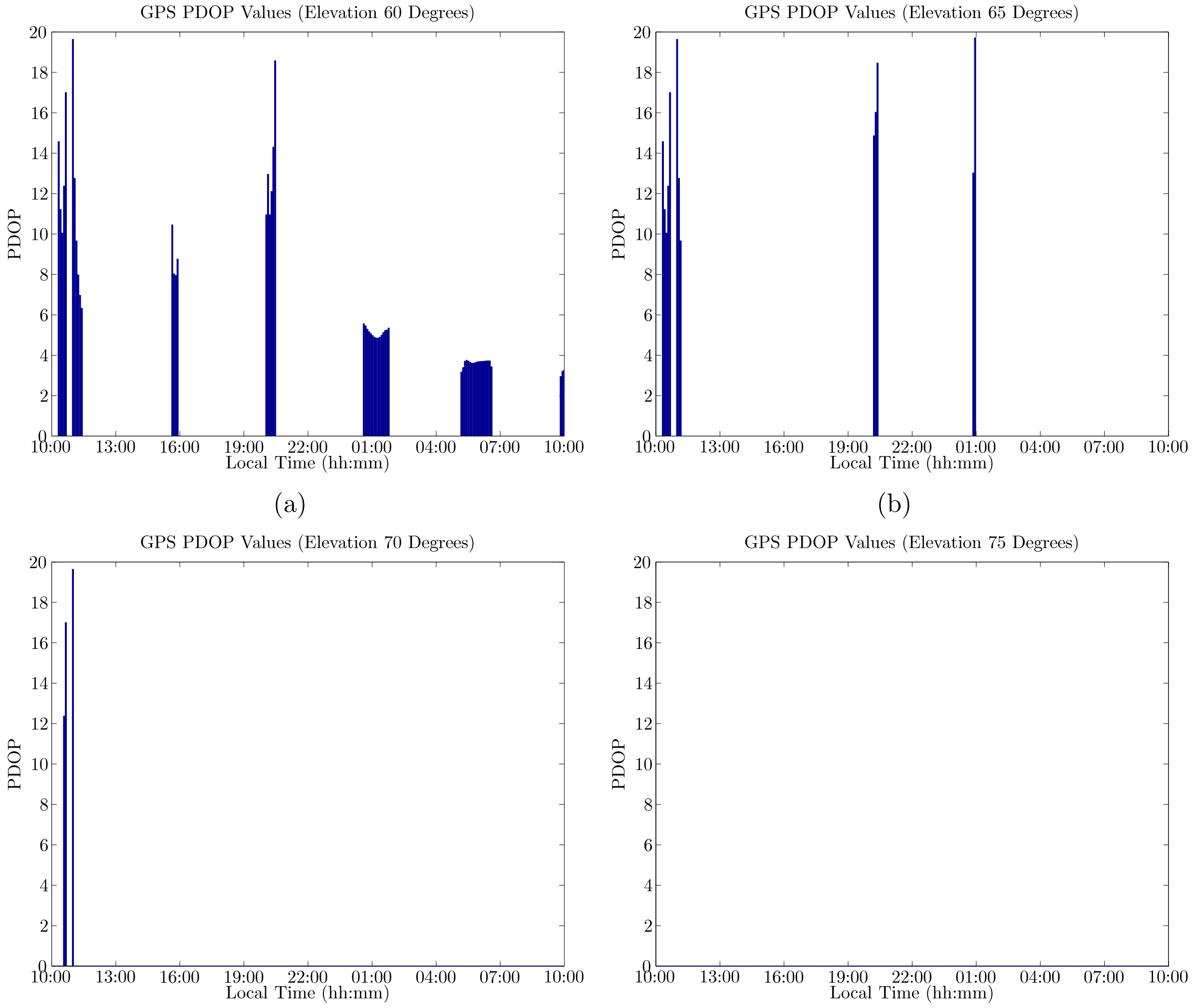
(a) (b) (c) (d)

Figure A.1: GPS PDOP Values for Elevation Masks 20°, 25°, 30° and 35°



(a) (b) (c) (d)

Figure A.2: GPS PDOP Values for Elevation Masks 40°, 45°, 50° and 55°



(a) (b)

(c) (d)

Figure A.3: GPS PDOP Values for Elevation Masks 60°, 65°, 70° and 75°

# *Vita*

Lieutenant Yasin Ali MUTLU graduated from Maltepe Military High School in Izmir, Turkey. He entered undergraduate studies at the Turkish Air Force Academy, Istanbul and graduated with a Bachelor of Science degree in Electronics Engineering in August 2004. After the Academy he attended Interceptor Controller Officer Training at the Turkish Air Force Air Defense School in Izmir, Turkey. Following Interceptor Controller Officer, he was assigned to Çanakkale Radar Site in May 2005. After serving 3 years, he attended the Graduate School of Engineering and Management, Air Force Institute of Technology in August 2008.

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In modern warfare navigation services are very important. GPS is currently providing service for accurate navigation, except in some areas, especially urban areas, where GPS signals cannot always be tracked by users. In these cases some additional navigation support could be provided by other global navigation satellite systems. If GPS is combined with other navigation systems than the navigation gap will be minor.

In this thesis, the effect of combining GPS with other satellite navigation systems, specifically GLONASS, Galileo and Compass, is evaluated in terms of availability and position dilution of precision (PDOP) values. First, satellite constellations are simulated in Satellite Tool Kit (STK) to generate ephemeris data. A street scenario is then established for simulating different elevation mask angles to represent urban and mountainous areas. The performance of the combined system is also evaluated as a function of the uncertainty in the time offset between systems.

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