



**EVALUATION OF COST IMPROVEMENT MODELS  
WHEN PROGRAMS EXPERIENCE UNPLANNED  
PRODUCTION DECREASES**

THESIS

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AFIT/GCA/ENV/10-M04

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**Abstract**

As military and other governmental budgets decline and impacted project deadline changes require instantaneous responses, cost analysts' tasks become more and more formidable. Inaccurate estimates can lead to misappropriation of resources and can thus create delays in goods reaching warfighters. This thesis aims to avail cost estimators of more reliable projection tools and to challenge the status quo of cost estimating, the production rate cost improvement model, when programs face reductions in lot quantities. The findings reveal that the status quo proves efficient under many cost profiles, but clearly does not estimate as well when a program suffers lot quantity reduction coupled with loss of cost efficiency. Prior research recognized the importance of changes in lot quantity to cost estimating, but definitive guidance never surfaced with regards to choosing a model. Monte Carlo simulation allows us to vary cost-affecting variables and isolate conditions where the use of a fixed cost, cost improvement model provides more accurate estimates than does the status quo. While no model for estimation should be discounted without exploration of its usefulness, we argue that the fixed cost model should be considered for use based on its ability to predict increases in average unit cost.

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*Dedicated to my wife and family*

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Tony George

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# EVALUATION OF COST IMPROVEMENT MODELS WHEN PROGRAMS EXPERIENCE UNPLANNED PRODUCTION DECREASES

## I. Introduction

### Background

Cost analysts face formidable obstacles with every new project they encounter. The single most important responsibility of any cost analyst's job is to make sure the calculations and figures they present to project decision-makers are accurate estimates to the best of their abilities. Regardless of exogenous factors weighing on cost analysts, they must sift through all information and develop logical conclusions firmly supported through a mixture of art and science. As military and other governmental budgets decline and project deadline changes require instantaneous responses, cost analysts' tasks become more and more difficult and are subjected to intense scrutiny as well as frequent criticism. Such a climate dictates our assignment: to evaluate the imperfections in the systems, and from our findings, to hone reliable and effective solutions to replace the less accurate protocols in current use. Complacency often stands in the way of providing the best possible estimates; relying on historical processes and estimating procedures can limit exposure to improvements in estimating techniques. We must challenge the status quo whenever feasible. Fortunately, technology aids us in our exploration of new innovative techniques. Developing expertise with estimating software now fits in as part of our job description and is absolutely critical to accurate estimates.

Inaccurate estimates create a large range of problems for program managers and other decision-makers. Current and future budgets base resource allocation schemes on

program cost estimates. Should the estimates be inaccurate, multiple programs face hardships; resources will realistically require reallocation and the overall financial situation will be constrained impacting current projects in addition to future endeavors. Furthermore, as decision-makers face funding issues, the real mission failure comes when the defense acquisition process cannot present the warfighter with products they expect and need. We fear that Air Force cost analysts sometimes use incorrect models and create bad estimates, which generate inefficiency in the process. Within this research work we are attempting to further perfect the overall cost analysis process and current system in use by the Air Force. Our research goal is to provide cost analysts and decision-makers with a more in-depth analysis of cost improvement curve theory to apply to estimates.

### **Purpose of This Study**

Our study involves a very specific situation we cost analysts face. In the current acquisition environment, budget cuts reduce the size of programs and the new cost estimates must adjust for the new fiscal constraints. Breaking our study down to its simplest form yields this focus: given a specific cost improvement curve estimation scenario with two possible models, which model outperforms the other when predicting future production costs? Research, much like real estimates, never turns out as uncomplicated as the above explanation, but that simplistic view, of necessity, frames our research.

More specifically, our research looks at lot production quantities that decrease from the agreed upon quantity after initial production begins. We chose this situation because as budgets become more constrained, many programs lose funding and reduce

order quantities. The reverse situation of increasing order quantities could theoretically happen to a program, but is unlikely in the current environment. Given this scenario we aim to find the best possible adjustments to the original estimate in order to capture the effect on learning and production rate suffered. “Conceptually, production rate should be expected to affect unit cost because of the impact of economies of scale” (Moses, 1990:1). These costs can include but are not limited to: quantity discounts received for ordering larger amounts of material; reduced ordering and processing costs; reduced shipping, receiving, and inspection costs on materials ordered; a greater use of facilities spreading overhead costs over output quantity; and also, the inverse of these costs (Moses, 1991:17-30). These effects reach beyond the range of a simple cost improvement equation and call for the addition of a production rate variable. We refer to these equations as ‘production rate’ and ‘production rate adjustment’ models. Dr. David A. Lee, in his book *The Cost Analyst’s Companion*, presents the two equations we evaluate in our research (Lee, 1997:60-61).

$$C(Q) = T_1 Q^b (R/R_0)^c \quad (1.1)$$

$$C(Q) = (F/R) + T_1 Q^b \quad (1.2)$$

Dr. Lee offers situations where each equation is most useful, but he also presents problems with each of the equations based on program specifics. Dr. Lee explains that Equations 1.1 and 1.2 most aptly respond to the cost drivers given changes in production lot sizes.

Cost analysts must estimate their respective programs with both science and art; we expect the evaluation of these two equations to be no different. Resources available to analysts, including the *Air Force Cost Analysis Handbook*, do not outline exactly how

an estimate should be built. The resources act very similar to a ‘guide’ or template when developing an estimate. We intend to take some of the guesswork out of developing a cost estimate with this research by revealing which equation performs better with decreasing production quantities. While predicting future events cannot be 100% accurate, we can provide indicators showing analysts a better estimating framework.

### **Research Questions**

Given the above scenario, our two equations, and the analysis, we answer the following research questions:

1. What is the current practiced method for rate adjustments when lot order quantities are changed from the manufacturing plant’s designed buy quantity and is that method consistent across the field?
2. What factors/inputs influence each of the specified cost improvement curve equations when lot quantities change?
3. Which of these cost improvement models best estimates the impacts of changes in production buy quantities and what are the driving forces behind each of the estimates?

### **General Approach**

In order to answer the research questions mentioned above, we conducted an extensive literature review to uncover the progression of cost improvement curves, production rate adjustments, and the analysis of these models. Based on the research, we performed a Monte Carlo analysis to evaluate each of the equations and to conclude which inputs to the production process influence the estimate. We also use historic

program data to verify our findings and to better align our research to actual cost estimates. Our cumulative findings allow us to draw conclusions about the application of each of the equations to a cost estimate, and hopefully to aide estimators in making more accurate estimates.

For the purpose of our study, we will be using the term ‘cost improvement curve’ to describe the phenomenon often referred to as learning curve, production cost progress curve, cost-quantity curves, experience curves and cost-progress curves (Department, 2007:8-1). We chose this terminology because the term ‘cost improvement’ encompasses factors beyond the standard concept of ‘learning.’ While we understand the basic structure of cost improvement curves, there have been alterations to the original methods to account for many possible production situations. Academics found the original cost improvement equations needed elaboration because many other factors can affect the amount of learning that takes place in a production. Though in general practice the cumulative quantity is the main cost driver, and as quantity increases the unit cost decreases, other cost drivers are present. In the case of production rate, there is an inverse relationship between production rate and unit cost. As production rate increases, the plant should gain economies of scale and decrease unit cost (and the reverse situation should also hold true).

### *Assumptions*

1. We assume at the point of changing future lot sizes the product requirements remain constant throughout the rest of the programs life. Our simulated data and our equation prediction error will be built upon the assumption of stable product requirements.

Requirements may change during production of an item, which would call for a new cost estimate based on the changing information. We do not model that scenario because providing an encompassing range of possible requirement changes is not feasible.

2. Based on our research, the two equations that Dr. Lee presents best estimate cost improvement when a program suffers changes in production lot sizes. Other research also led us to this conclusion, but there may be a need to explore new equations never presented. We will not be measuring prediction accuracy of every cost improvement equation even though there may be specific conditions with our simulated and actual data where other models outperform our models. We did identify the need to model two other cost improvement models found in the literature review, but the addition of these models does not exhaustively collect all possible cost improvement models and their predictive capabilities. Across the spectrum of varying conditions within our scenario we assume our models will consistently outperform other models.

### *Limitations*

1. Our simulated data and actual data cannot cover every possible situation analysts may face. We generalize our findings to real situations, but we cannot be certain that other factors outside the range of our study are driving cost. Our simulated data represents a ‘normalized’ data set where abnormalities have been removed.

2. Along the same lines as our first assumption, we cannot possibly research all exogenous factors affecting cost. We limit ourselves to the most common cost drivers, but we understand other actual programs can face unique situations. The burden falls on the program analyst to filter the program and decide what factors are truly driving cost, and if our research can be useful.

## **Chapter Summary**

Chapter I provided an overview of our research and states the research questions we hope to answer through our results. In Chapter II we review the history of cost improvement curves and the advancements researchers have made to understand and evaluate the effects of production rates. Based upon the information provided in the literature review, we outline in Chapter III our methods for creating a robust Monte Carlo simulation and evaluation of Equations 1.1 and 1.2. Chapter IV will show the results and analysis of our Monte Carlo simulation and evaluation. Finally, Chapter V will summarize the significant findings of our analysis and highlight potential policy implications to consider.

## **II. Literature Review**

We target two areas in the literature review: first, an overview of cost improvement curves to include production rate variations, and second, previous research performed on production rate variations. Academics have produced extensive research on learning curve theory and application, but we will only focus on research that directly relates to the production rate variations. Our literature review does not serve as a stand-alone document on all variations to learning curve models; for a more in depth explanation of learning curves, readers should reference the many available publications.

### **An Overview of Cost Improvement Curve Theory**

As previously mentioned, for the sake of continuity we will be using the term ‘cost improvement curve’ instead of any of the other acceptable variations. Though we often use the names interchangeably, there are subtle differences in the phrasing and meaning of each variation. For example ‘learning’ describes the efficiencies gained by laborers improving performance at producing an item in a repetitive process, while ‘production cost progress’ explains the process of repetitive production where an increase in the total quantity produced may lower the unit cost of each item. Any recurring (or variable) production costs, including labor, raw materials, and manufacturing costs of an individual item decrease as the total quantity of items produced increases (Department, 2007:8-5). The original forms of the learning curve model do not include fixed costs in the equation, but as we will discuss later, fixed costs can contribute to model accuracy under certain circumstances. ‘Improvement’ refers to the over-arching efficiencies that sometimes cannot be pinpointed but occur in a repetitive process (Department, 2007:8-2).

Each variation of the cost improvement curve involves an important aspect of production and our scenario encompasses many of these variables.

Cost improvement models serve as crucial tools for cost analysts across all fields. The aircraft industry first recognized the usefulness of cost improvement models, uncovering and utilizing the predictive value of modeling learning curves. Cost analysts discovered that these models could be applied almost universally across production and thus, other industries soon followed in practice (Department, 2007:8-4). Generalization of the learning concepts allows for use of the models in calculating expected labor hours, resources, and costs. “In manufacturing, learning curve representations are used to plan manpower needs, set labor standards, establish sales prices, aid make/buy decisions, judge wage incentive payments, evaluate organizational efficiency, develop quantity sales discounts, analyze employee training programs, evaluate capital equipment proposals, predict future production unit costs, and create production delivery schedules” (Smith, 1989:1). The application possibilities range across many different decision points within the acquisition process. The Air Force utilizes cost improvement curve estimates across a range of systems and acquisitions to include airframes, modifications, common avionics acquired for multiple platforms, engines, missiles, and satellite hardware (Department, 2007:8-2). Accurate estimation plays an important role for cost analysts and provides valuable perspective as they evaluate contractor proposals as well. The effects of misestimating can negatively affect the decision-making process; thus, the estimates must be as accurate as possible.

The two most common versions of the cost improvement curve models are the cumulative average model developed by T.P. Wright, and the unit cost theory formulated by James R. Crawford and based on Wright's.

### **The Cumulative Average Model**

In 1936, aircraft researcher T.P. Wright became the father of learning curve theory when he created a model explaining learning-related reductions in airframe construction costs. The basic premise effectively illustrated that when the number of aircraft produced in sequence doubled, the cumulative average direct labor input per aircraft decreased in a regular pattern. His ratio relationship could be modeled exponentially, but also became a linear function when applied to the labor/cost changes that occurred over the sequence of production units (Department, 2007:8-4). The linear function represented Wright's 'learning curve slope'.

Wright's theory can be expressed mathematically as:

$$A(Q) = A_1(Q_c)^b \quad (2.1)$$

where

$A(Q)$  = average cost to produce the first  $Q$  units

$A_1$  = first unit cost (model parameter)

$Q_c$  = total quantity of units produced (whose average cost is to be computed)

$b$  = slope coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

Equation 2.1's framework does not appear complex, but conceptually there are two essential elements that must be recognized and accepted. First, the value of  $A_1$  is described as the 'first unit cost' but is not the actual cost of the first unit of production.  $A_1$  is an estimated model parameter derived from historical values to fit the curve.

Second, the slope refers to the rate of learning, and represents the percentage by which reoccurring labor/cost decreased every time production quantity doubles. “For example, for a slope of 80%, the value of labor/cost for every doubling of the quantity Q is 80% of the value for Q; equivalently, every time the production quantity doubles, the associated hours/cost improves at (is reduced by) a rate of 20%” (Department, 2007:8-6).

Figure 2.1 below depicts the above scenario of 80% slope, starting with a unit 1 cumulative cost of \$1,000.

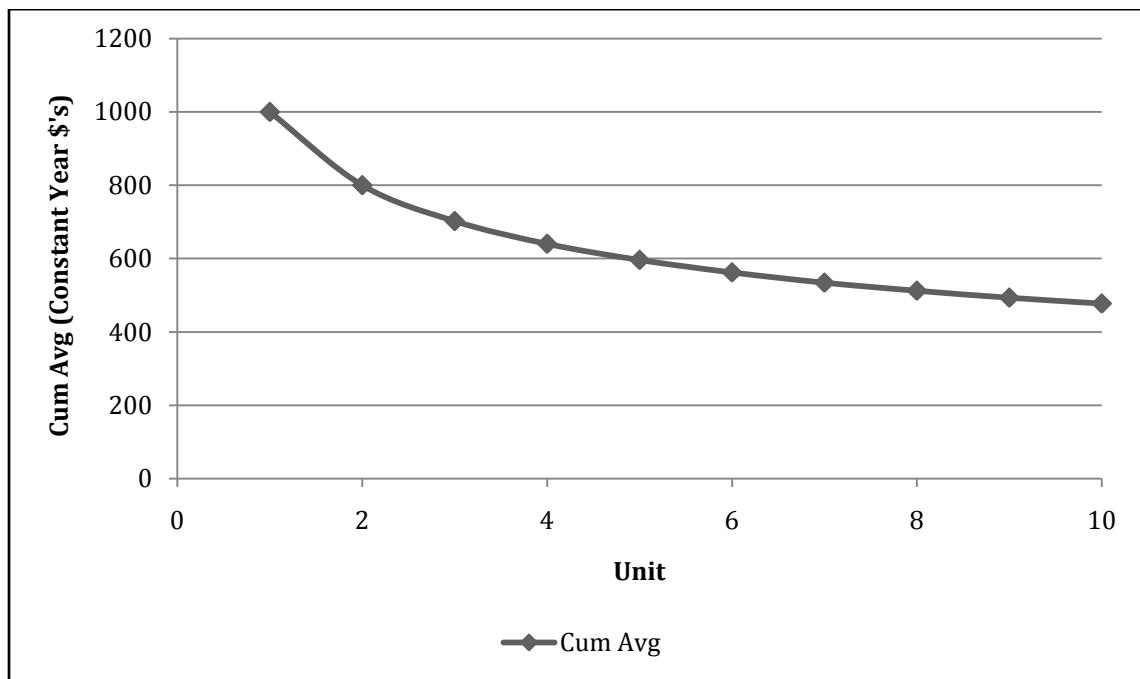


Figure 2.1: Wright Model Plotted on Arithmetic Grids

### The Unit Cost Model

Following WWII, James R. Crawford updated Wright’s model based on information from aircraft production during the war. Very similarly to Wright, Crawford theorized that the cost per unit decreases by some constant percentage (ratio) as the total

number of units produced doubles. Crawford found this model more useful for his applications.

The mathematical expression of Crawford's model is identical to Wright's model though Crawford's definition of terms differs.

$$C(Q) = T_1 Q^b \quad (2.2)$$

where

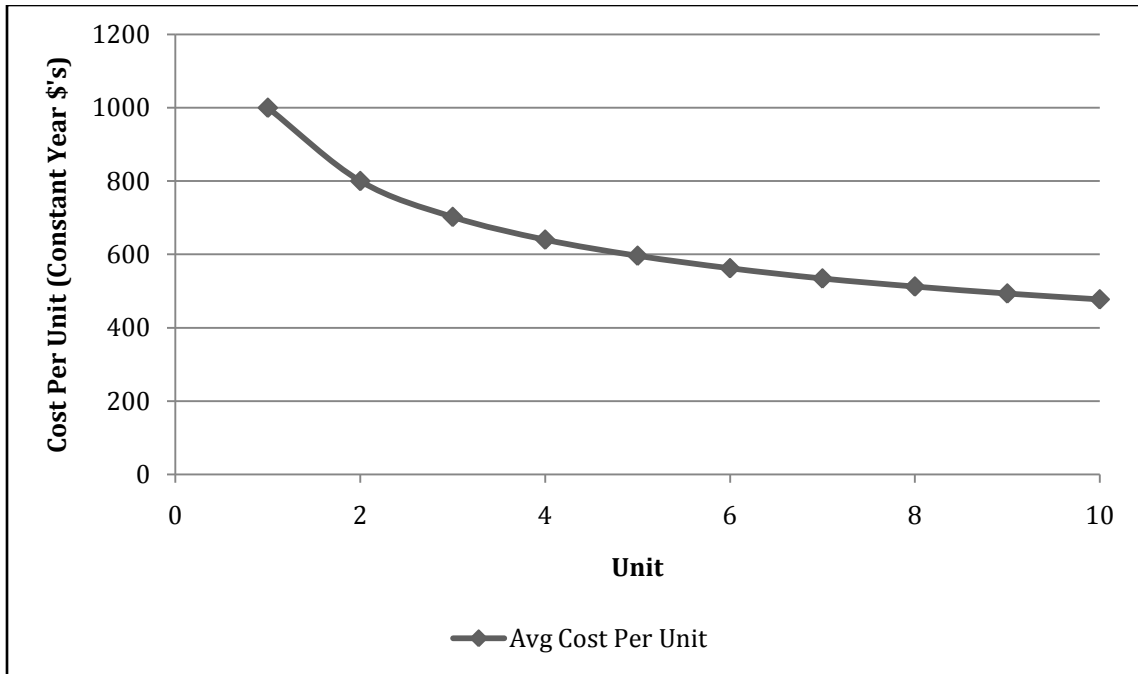
$C(Q)$  = cost to produce the  $Q^{\text{th}}$  unit

$T_1$  = first unit cost (model parameter)

$Q$  = unit number (whose cost is to be computed)

$b$  = slope coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

The subtle difference from Equation 2.1 is notable: in Equation 2.2 the dependent variable  $C(Q)$  is the cost of a specific unit, while in Equation 2.1 the cost is an average across all prior units. These distinctions are reflected in the title description of each model. Figure 2.2 below plots the unit cost curve with a learning slope of 80% and a unit 1 cost of \$1,000. Figure 2.1 above looks identical to Figure 2.2; the only differences are the axes labels and the interpretations of the data.



**Figure 2.2: Crawford Model Plotted on Arithmetic Grids**

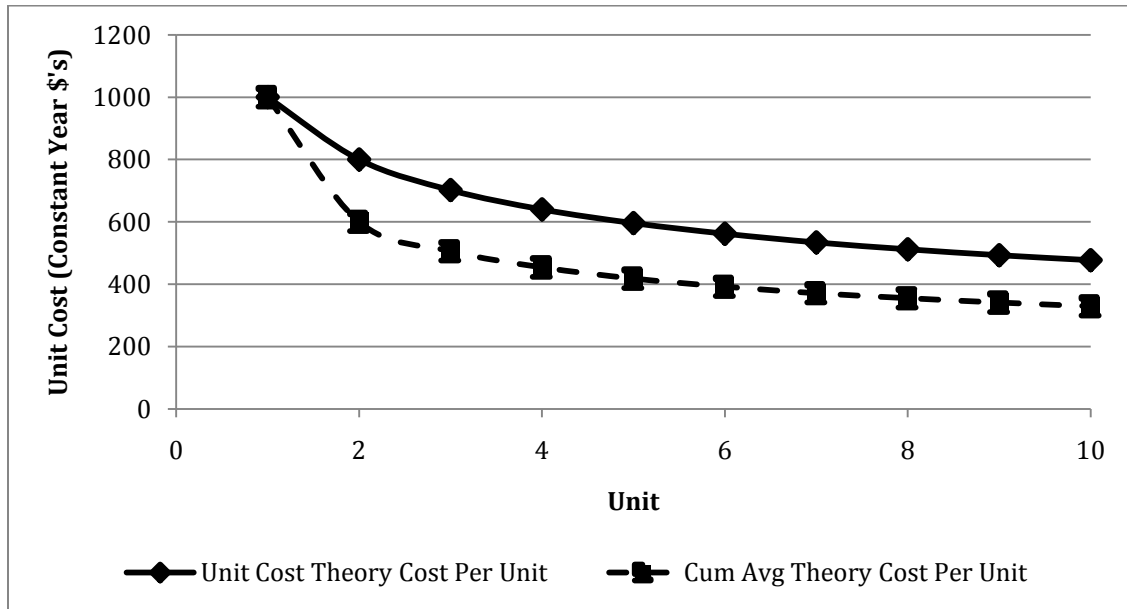
### **Unit Versus Cumulative Average**

Whether one model outperforms the other model depends on the situation, and through mathematical manipulation each model can produce the other's results. While the equations can be applied interchangeably for estimates, the form we choose must remain constant throughout the estimate for accuracy (Anderson, 2003). For example, if the analyst begins the estimate with the unit cost model, he/she must complete all calculations using that chosen model and define the results in unit cost form. If he/she wishes to explain cumulative average results, that application must be employed from the beginning, or else the final numbers must be changed through addition of each unit cost (Anderson, 2003). Analysts must remain true to the use of their chosen cost improvement curves in order to create valid estimates. These two models remain the most commonly applied models because of their simplicity and consistent performance.

When the two models are compared side by side they give the results seen in Table 2.1 and Figure 2.3.

**Table 2.1. Comparison of Cumulative Average and Unit Formulation (Shea, 1994:18)**

		<b>Cum Avg Cost Formulation (Constant Year \$'s) 80% Slope</b>				<b>Unit Cost Formulation (Constant Year \$'s) 80% Slope</b>	
Unit #	Cum Avg Cost	Unit Cost	Unit #	Cum Avg Cost	Unit Cost		
1	1000	1000	1	1000	1000		
2	800	600	2	900	800		
3	702	506	3	834	702		
4	640	454	4	786	640		
5	596	418	5	748	596		
6	562	392	6	717	562		
7	534	371	7	691	534		
8	512	355	8	668	512		
9	493	341	9	649	493		
10	477	329	10	632	477		



**Figure 2.3: Cum Avg. Theory and Unit Cost Theory Plotted By Incremental Unit Cost**  
(Shea, 1994:20)

When looking at Figure 2.3, “the cumulative average cost declines by a constant percentage between double quantities; however, when converted to incremental unit costs, the percent decline is non-constant” (Shea, 1994:18). The converse holds true for the unit cost theory, where the unit cost declines at a constant percentage and the cumulative average cost does not decline at a constant percentage. In reality, creating the two models will show variance because the unit 1 cost will not be the same, but the information highlights the differences of the two equations. Again, the important fact remains that cost analysts must remain consistent in the application of whichever equation form they choose to employ.

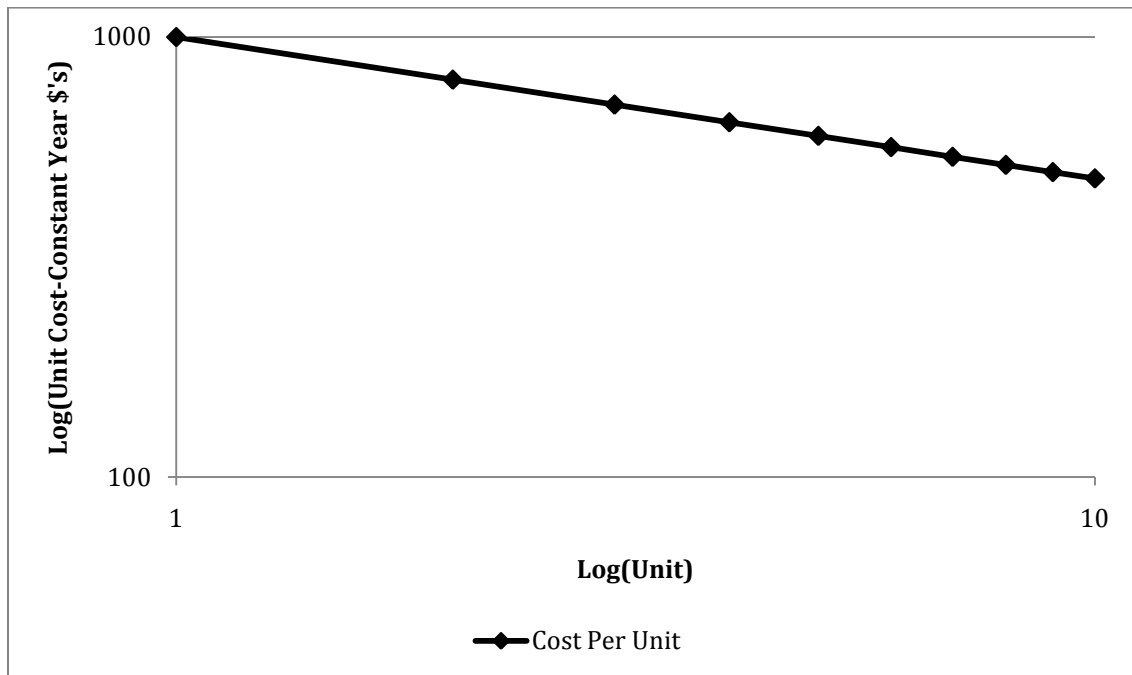
Theoretically, both the cumulative average model and the unit cost model have validity. The basic premise that a laborer’s repetition of task over and over again will cause the laborer to become better at the task can be perceived in performance. For example, an observer could theoretically stand next to the laborer with a stopwatch and could measure the changes in time taken to complete a repetitive process. The efficiency of motion and task completion gained works both in theory and in practice. As we look at variations to the two original models and eventually production rate adjustment models, the cumulative average model and the unit cost model still demonstrate validity, even though there is no feasible way to physically measure all of the inputs and outputs.

### **Linear Transformation**

Both the cumulative average model and the unit cost model are commonly expressed in log-linear form for statistical evaluation purposes. When the exponential curves are transformed to their linear forms, they become more easily understood in presentation and practice. The log-linear form is expressed as:

$$\text{Log } A(Q) = \log A_1 + b \log Q \quad (2.3)$$

When graphically represented, this equation illustrates the linear relationship between cost and learning that the original forms of the equation embody. If we take a unit cost of \$1,000 and a learning slope of 80% just as before, and plot the data on a logarithmic scale, the results are shown in Figure 2.4 below.



**Figure 2.4: Unit Cost Curve Plotted on Logarithmic Grids**

Figure 2.4 not only gives the same information as the arithmetic grid plots, but also can be used to better illustrate the relationship between learning slope and cost necessary for analysis of the equations statistically.

## Variations to Original Models

The wide range of applications for cost improvement curve theory opens the door to variations that more aptly model specific situations. Two equations are no longer sufficient to handle the nature of a product, manufacturing process, business environment, or countless other factors that reach beyond typical learning. Because we do not have clear guidance on which equations to use, we must filter through all possibilities and make determinations based on program specifics and indicators. Dr. Adedeji B. Badiru published an extensive compilation of ‘univariate’ and ‘multivariate’ learning curve models. ‘Univariate’ equations, exemplified by Equations 2.1 and 2.2, calculate projections using single input variables; ‘multivariate’ equations utilize more than one input variable such as processes using Equations 1.1 and 1.2 in Chapter I. While these models are not the focus of our research, the principles driving their conception and application aid the understanding of learning curve theory and how such theory is applied. Simply, Dr. Badiru concludes that there are underlying causes, often unseen and not directly traceable, that affect cost; these models aim to capture the cost changes when the programs exhibit certain symptoms.

The classical developments based on the Wright and Crawford’s original models include:

- The S-Curve
- The Stanford-B Model
- DeJong’s Learning Formula
- Levy’s adaptation formula
- Glover’s learning formula
- Pagel’s exponential function
- Knecht’s upturn model
- Yelle’s product model
- Multiplicative Power Model

Each of these adaptations to the model may outperform the original models given certain program specifics. The cost analysts must investigate the specifics of their respective programs and accordingly apply the appropriate learning curve equations. In the decision-making process, these equations can improve analysts' evaluations of the designs of training programs, the manufacturing economic analyses, the breakeven analyses, the make or buy decisions, the manpower scheduling, the production planning, the labor estimating, the budgeting, and the resource allocation (Badiru, 1991:439-440). Dr. Badiru also explains how analysts must choose between a 'univariate' (a single input usually quantity) and a 'multivariate' model for a cost estimate. The choice rests on many factors related to the actual calculations as well as to the delivery of the information in an intelligible form to decision-makers. Based on the amount of data and the time and statistical software available, the use of multivariate models might not be possible. Univariate models can be applied competently with limited data, but these models may not be capturing all variables affecting cost. Multivariate models require a better knowledge base of statistical data for starters, as well as expertise in the art form of presenting the information. Sometimes a parsimonious solution proves to be more useful in the decision-making process as many non-analysts may prove unable to accurately comprehend complex models. Since both of the models we evaluate in this research are multivariate production rate models, science and art alike must be utilized.

## **Cost Improvement Curve Production Rate**

Expansion of the original cost improvement curve equations called for a term to explain and model effects when manufacturing production rates change throughout the production lifecycle. Hoffmayer's 1974 hypothesis, in a report completed at RAND, proposed that as production rate increased, manufacturers should gain greater efficiency above only 'learning', and that the resulting effect would be a decrease in unit cost (Hoffmayer, 1974:2). Decreases in unit cost can be contributed to "greater specialization of labor, quantity discounts and efficiencies associated with raw materials purchases, and greater use of facilities permitting fixed overhead costs to be spread over a larger output" (Moses, 1991:2). At the time of the Hoffmayer study, cost analysts did not fully understand the concept of production rate and how it affected cost improvement. More recently, the Air Force Cost Analysis Handbook recognizes production rate's link to economies and diseconomies of scale as the production rate increases and decreases (Department, 2007:8-31).

Hoffmayer's study recognized the importance of production rate and the widespread utility an accurate tool to predict cost could provide cost analysts. Karl Hoffmayer and the other RAND authors aimed to create an estimating model to capture the magnitude of costs and/or cost savings realized through changes in production rate (Hoffmayer, 1974:1). To accomplish their goal, the RAND study authors focused on how production rate would affect four major cost elements: manufacturing labor, materials, tooling, and engineering (Hoffmayer, 1974:1). Along with major cost elements, the RAND study focused on manufacturing overhead, which had previously been omitted from cost improvement (learning curve) equations. Beginning with these

five areas, the authors hypothesized that they could find other exogenous factors, outside of the program's control, that altered unit cost with production rate.

Hoffmayer's study determined the causes of changes in production rate to be design problems, cost growth, funding problems, modifications, and other similar factors. These same causes are prevalent throughout the current acquisition process and have directly led to the scenario we are studying, as well as to the two production rate adjustment equations we are evaluating. The unresolved cost-related problems within acquisition programs include: the way in which rate changes were and are achieved; the availability of suppliers; the local labor supply; management policy; the timing of rate changes; plant capacity; plant backlog; and a number of other transitory factors (Hoffmayer, 1974:41). These elements drastically changed the major cost components and overhead allocation. Hoffmayer concluded that while he and the other authors could understand the concepts involved in production rate changes, they were unable to create a useful model fit for all scenarios (Hoffmayer, 1974:41). The most important finding of Hoffmayer's study related to the relationship between production rate changes and overhead.

Hoffmayer and the authors of the RAND study looked at major acquisition programs and concluded the one cost element that is clearly a function of production rate is overhead. The effect presented itself clearly through the RAND study; because overhead costs could reach upwards of 50% of total cost, even minute changes could appear significant. The RAND study finally concluded that, "When the total volume of business is very low, cost can be quite sensitive to [production] rate. When total volume is high, the influence of rate is reduced but still perceptible" (Hoffmayer, 1974:43). Even

following conclusion of their study, the authors found themselves unable to predict the production rate changes with confidence, and thus they could not fully estimate cost with any degree of certainty. The RAND study did frame the idea of production rate changes and led to further investigation and statistical analysis to create a useable estimating model. The Equations 1.1 and 1.2 we are evaluating in our study are based on the same concept found in the RAND study.

### **Production Rate Model**

Production rate models evolved based on the principles described above. Many cost estimating handbooks now include sections describing production rates and the effects they will bear on cost estimates. The Air Force Cost Analysis Handbook presents the most common cost improvement model with a production rate term (Department, 2007:8-31). Equation 2.4 shows the **production rate model**.

$$C(Q) = T_1 Q^b R^c \quad (2.4)$$

where

$C(Q)$  = cost to produce the  $Q^{\text{th}}$  unit

$T_1$  = first unit cost (model parameter)

$Q$  = unit number (whose cost is to be computed)

$b$  = slope coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

$R$  = production rate (number of units produced in a production period)

$c$  = rate coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

Given production rate changes, Equation 2.4 outperforms the basic learning curve model (Moses, 1990:30). The problem rests in determining which independent variables drive the effectiveness of the equation and subsequently allow analysts to predict the equation's usefulness. "Conceptually, production rate should be expected to affect unit cost because of the impact of economies of scale. Higher production rates may lead to several related effects: greater specialization of labor, quantity discounts and efficiencies associated with raw materials purchases, and greater use of facilities permitting fixed overhead costs to be spread over a larger output quantity" (Moses, 1990:1-2). Adding a production rate term to the cost improvement model also creates clear disadvantages. Finding a clear production rate/rate slope to use in the model is the first disadvantage of the production rate model. This problem can be minimized and possibly resolved through statistical analysis. A second disadvantage occurs if the production rate is constantly increasing or decreasing. This situation causes high co-linearity between the unit and rate variables. No clear solution for this problem can be found, so analysts must work the individual problems according to the specifics of their data set and program. The limit on reductions or gains in production is viewed as another disadvantage to the production rate model. Once a plant reaches either minimum or maximum capacity, large expenditures will take place: overtime, expedited material orders, purchase of new capital, hiring more of the labor force, and increased training (Lee, 1997:60). Due to these inadequacies, some cost analysts may find themselves hesitant to utilize the production rate model.

## Evaluation of the Production Rate Model

In a report completed at the Naval Postgraduate School in 1990, O. Douglas Moses evaluated the Production Rate Model. The report, *Learning Curve and Rate Adjustment Models: Comparative Prediction Accuracy Under Varying Conditions*, compared the original unit cost Model (Equation 2.2) developed by Crawford and the production rate model (Equation 2.4) to find out, simply, which equation performs better. Based on available research, Moses hypothesized that a clear relationship existed between cost and production rate, but since the relationship would vary, neither equation outperformed the other outright (Moses, 1990:3). Unable to rely on prior research and practical application because he could not verify the proper application of the production rates used, Moses created simulated data to evaluate the equations.

**Table 2.2. Moses' (1990) Independent Variable Values For Data Simulation (Moses, 1990:13)**

<b>Variable</b>	<b>Levels/Values</b>		
Data History	4	7	10
Variable Cost Learning Rate	75%	85%	95%
Fixed Cost Burden	15%	33%	50%
Production Rate Trend	Level		Growth
Production Rate Instability/Variance	.05	.15	.25
Cost Noise/Variance	.05	.15	.25
Future Production Level	Low	Same	High

Moses derived a generic cost function to create his simulated data, but only solved the cost function for a set number of discrete values as seen in Table 2.2 above. True cost functions do not exist in the manufacturing process. If manufacturers knew their exact cost function, then analysts would not need to develop their own equations. As a result, Moses derived his equation from cost components. Moses' equation allowed him to inject various independent variables into the equation at different levels to create simulated cost data. Using the simulated cost data, Moses assessed each model individually and measured the error in predicting the future lot costs (also created with the cost function). The strength of Moses' generic cost function and independent variables validates the research and his findings. We label his cost simulation function Equation 2.5 and provide the definition of terms below.

$$C(Q) = VC_1(Q^d) + SFC (PR^{-1}) \quad (2.5)$$

where

$C(Q)$  = unit cost

$VC$  = variable cost per unit (learning included)

$SFC$  = standard fixed cost per unit

$Q$  = cumulative quantity

$d$  = parameter, the learning index (same as learning slope)

$PR$  = production rate for any period

The above cost function encompasses the independent variables Moses uses to explore the cost drivers of each equation. His independent variables included data history, variable cost learning rate, fixed cost burden, production rate trend, production

rate instability/variance, cost noise/variance, and future production level (Moses, 1990:13). Moses chose these variables because they most resemble variables analysts could flag in their programs as indicators identifying which cost improvement curve to apply.

Moses did not find overwhelming evidence demonstrating that either tested equation outperforms the other across the entire range of scenarios. Rather, Moses found that under certain conditions each equation estimates production rate superiorly to the other. Moses discovered that the following general tendencies led to reduction in the incidence of prediction errors and to improved accuracy for the production rate equation:

- The number of observations available for the analysis was relatively high
- The amount of fixed cost in total cost was relatively high
- The production rate trend had been growing during the model estimation period
- The period-to-period variability in production rate was relatively large
- Random noise in cost due to unsystematic factors impacting cost was relatively low
- Production volume was expected to be cutback in the future periods for which cost predictions were being made (Moses, 1990:29).

Moses also recognized, “The greatest impact (of changes in the various factors) on relative prediction accuracy (of the learning curve approach and the rate adjustment approach) occurs when cutbacks in future production are anticipated” (Moses, 1990:30). Based on this finding, the most crucial model selection decisions present themselves in a program where future production volume is declining (Moses, 1990:30). Our research deals directly with the above situation. We must acknowledge that Moses was aware of the impact decreases in production rates can have on cost estimates.

By astutely analyzing the interactions of independent variables, Moses’ research also discovered that the factors on the relative prediction tend to be additive. The analysis did not reveal significant scenarios where the basic learning curve function

outperformed the production rate model. The Crawford model did outperform the production rate model, but not consistently nor convincingly. Because Moses created simulated data at discrete values and tested the models against that data, there is no way to know if his findings can be generalized outside of that specific dataset. Our research fills this void by using both Monte Carlo simulation to create an inclusive range of possible scenarios and historical program data to evaluate the production rate adjustment equations.

### **Including Fixed Costs in Cost Improvement Curves**

The longstanding mindset of ‘learning curve’ models states that the models can be applied only to recurring costs. As learning curves evolve into cost improvement curves, this rule reflects that change. Based on the above research done by Hoffmayer and Moses, large fixed costs factored as a proportion of total cost can drastically skew the results of the original cost improvement models (Equations 2.1 & 2.2). Inclusion of a fixed cost variable yields the following **fixed cost model**, which we labeled Equation 1.2:

$$C(Q) = (F/R) + T_1Q^b \quad (1.2)$$

where

$C(Q)$  = cost to produce the  $Q^{\text{th}}$  unit

$T_1$  = first unit cost (model parameter)

$Q$  = unit number (whose cost is to be computed)

$b$  = slope coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

$R$  = rate of production (quantity per time period or lot)

$F$  = fixed cost per lot (model parameter)

Conceptually logical, the fixed cost equation opens the door for experimentation to see if the equation creates valid estimates. Our research plans to highlight the estimating capabilities of Equation 1.2 and lobby that the equation should become more widely accepted.

### **Dr. David Lee's Cost Improvement Models**

Dr. Lee presents Equation 1.1, which we refer to as the **production rate adjustment model**, and which can be seen below with a description of variables.

$$C(Q) = T_1 Q^b (R/R_0)^c \quad (1.1)$$

where

$C(Q)$  = cost to produce the  $Q^{\text{th}}$  unit

$T_1$  = first unit cost (model parameter)

$Q$  = unit number (whose cost is to be computed)

$b$  = slope coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

$R$  = current production rate (number of units produced in a production period)

$R_0$  = planned production rate (production rate prior to production rate decrease)

$c$  = rate coefficient (model parameter) =  $\ln(\text{slope})/\ln(2)$

In *The Cost Analyst's Companion*, Dr. Lee mathematically derives the equations from Chapter I we re-named Equations 1.1 & 2.1. These two equations are the basis for our research. Dr. Lee presents two theoretical situations where his two equations should be applied and will outperform any other variations of the cost improvement curve. Equation 1.1 ( $C(Q) = T_1 Q^b (R/R_0)^c$ ) should be applied when, "Factors of production can change with rate, to keep the facility operating at its designed rate" (Lee, 1997:60). Dr.

Lee states that this might only be possible if production has not yet begun and the manufacturer has not yet constructed the plant (Lee, 1997:60). Equation 1.2 ( $C(Q) = (F/R) + T_1Q^b$ ) should be applied when such things as factory floor space, specialized machinery, and tooling cannot be easily changed without cost (Lee, 1997:61). Dr. Lee concludes that the fixed cost equation (Equation 1.2) is more practical than Equation 1.1 for actual programs facing changes in production rate. Dr. Lee applies Equation 1.1 and Equation 1.2 each to one set of data, but does not fully evaluate them. His book serves more as a compilation of theoretical possibilities rather than an evaluation of methods. Our research will focus on the evaluation of these two equations.

Grouped together, Hoffmayer's report (1974), Moses' report (1990), and Dr. Lee's book (1997) explain the responsiveness of production rate equations to the major cost drivers. Most importantly, the reports explained how the production rate adjustment models more aptly respond to reductions in production lot quantities. The results mentioned above drove the need for our research and helped us to limit and define the number of equations we need to evaluate.

## **Chapter Summary**

This chapter provided an extensive review of cost improvement models. Wright and Crawford developed the cumulative average model and unit cost model, respectively, to explain the concept of learning. Learning describes the efficiencies that are gained as a task is executed repeatedly. The laborers become better at completing the tasks and the unit cost of an item decreases as more units are produced. These two models became widely accepted across many disciplines because of their predictive capabilities and parsimonious constructions. The first models developed by Wright and Crawford

provided the concepts of learning, but did not capture nor incorporate all aspects of production costs.

Later models built upon the concept of learning to thus include improvements in overall production. The improvements explain more than the work done by a single laborer; they capture efficiencies for the entire production plant as a unit. The uniqueness of products and production plants provided impetus for expansion of the original models, and resultantly many different models surfaced. In 1974, Hoffmayer discovered the need to model the effects of production rate (lot quantity) changes on cost improvement. The production rate model attempted to model the findings of Hoffmayer's work and model the economies and diseconomies of scale in production.

Moses evaluated the production rate equation and determined that under certain circumstances the model outperforms the unit cost model (Crawford's model). Further research into the hidden costs of production discovered a conceivable need to include fixed costs in the cost improvement model. Dr. Lee recognized the need for a fixed cost variable, as well as for a slight modification of the production rate model to capture the costs when a program suffers changing lot quantities. In Chapter III, we will build on the work presented in Chapter II to evaluate equations 1.1 and 1.2. Subsequent chapters will explain our results and explore the possible implications of our research.

### **III. Methodology**

Previous chapters discussed the need for accurate cost estimates and the academic progression of cost improvement models. Through our methodology we aim to discover which cost improvement model should be employed in cost estimates with decreasing lot sizes. The works detailed in Chapter II provide much of the structure of our Monte Carlo simulation, our model creation, and our analysis. Moses provides independent variables affecting cost, which can be evaluated in conjunction with the cost estimates to determine the variable's effect, if any, on the estimates. (Moses, 1990:13). O. Douglas Moses identifies the predictive ability of the Crawford unit cost model and production rate model, while Dr. David Lee explains the two cost improvement equations (Equations 1.1 and 1.2) he determines to have the most predictive ability when evaluating decreases in lot quantities. (Lee, 1997:60-61). We are focusing our evaluation on these two equations.

To accomplish our evaluation, we utilize Microsoft Excel with a Visual Basic for Applications (VBA) Macro to create the simulated cost data under varying assumptions, and the Microsoft Excel Premium Solver Platform add-in to optimize each model. While Moses recognized that his cost data simulation limited the usefulness of the research, we intend to demonstrate how the reasonable, yet exhaustive, assumptions of our simulations strengthen our research. (Moses, 1990:30). Microsoft Excel affords us the opportunity to provide a more robust Monte Carlo simulation; thus we have the responsibility to provide an extensive simulation. As more advanced tools become available to analysts, the improved technology furnishes the ability to challenge the status quo of cost estimation, our goal with this research effort.

## Basic Evaluation Structure

The basic structure of our evaluation method can be seen in Figure 3.1.

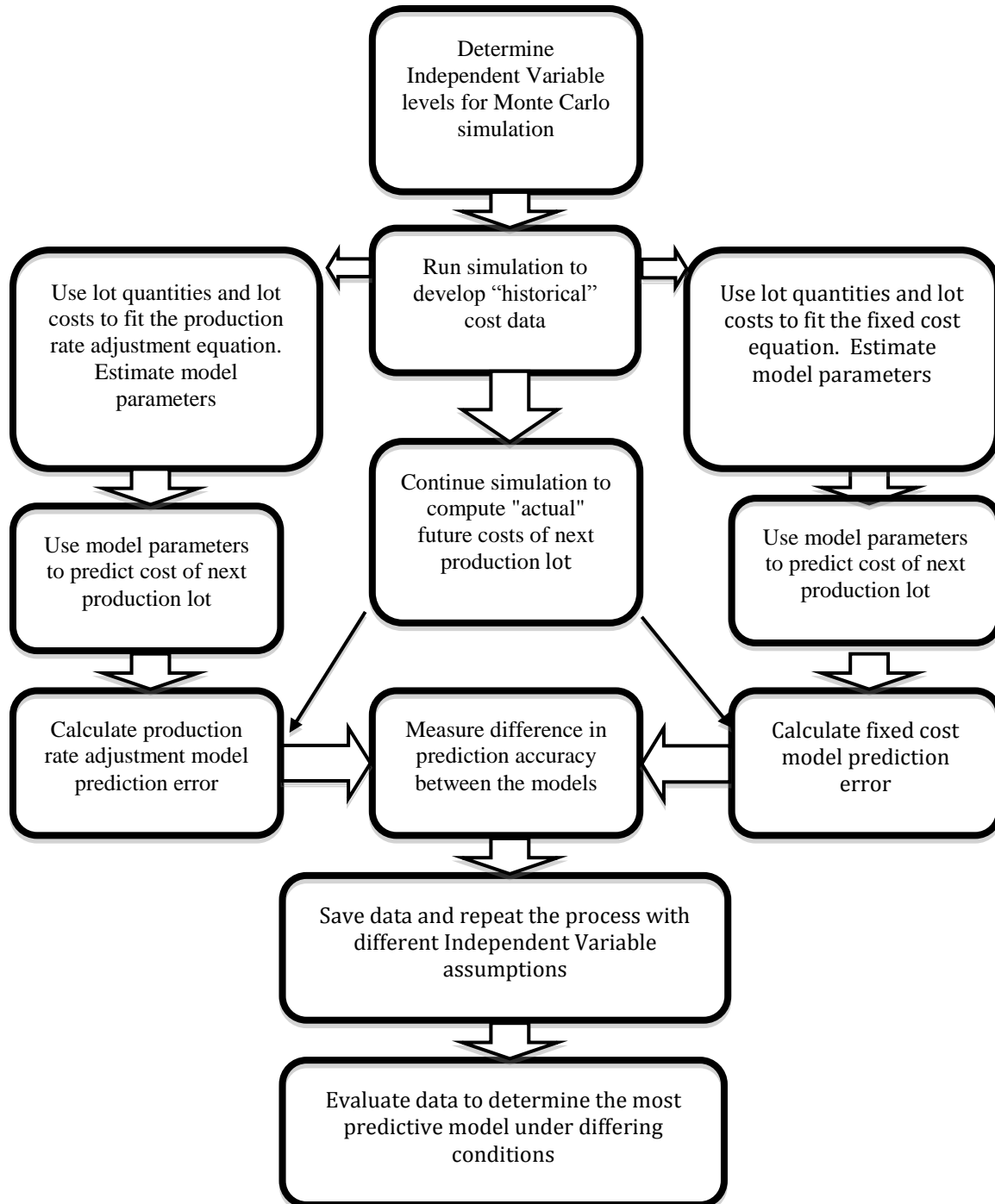


Figure 3.1: Basic Evaluation Structure Flowchart

Our evaluation structure is simple and easy to follow. Following this flow chart allows us to preserve the same structure as we repeat the process under different simulation assumptions. Even though we assembled a simple structure, Microsoft Excel allows us to evaluate the cost improvement equations under complicated assumptions. Our methodology focuses on creating many possible cost profiles and comparing each of the cost improvement models on their predictive capability with the same cost data.

### **The Cost Generating Functions**

Our cost generating functions mimic similar patterns in historical data. We model samples from our simulated production costs against the basic learning curve shown in Chapter II as well as historical costs to ensure similarities. Creating a cost function requires some guesswork because if producers knew their true cost generating functions, then the need for analysts would not exist (Moses, 1990:8). We must do our best to recreate patterns because we do not know true cost functions for each individual situation. The basic production cost structure of any particular item consists of a fixed-cost portion and a variable-cost portion. As mentioned in Chapter II, fixed costs did not originally factor into cost improvement models, but as research grew, analysts acknowledged fixed cost influence on cost improvement models. We used two different cost generating functions. We determined the need for two cost generating functions to avoid favoring either equation thus biasing the analytical process, and the necessity to create different cost scenarios in order to provide a thorough analysis. We sampled construction from the cost improvement research accomplished by Avinger (1987), Moses (1990), and Thomas (1975) to determine our cost functions. The two cost functions are simulated and modeled separately; the simulation results for each cost

function will be discussed in Chapter IV of our research. To account for the inherent fixed cost in every unit, the first cost generating function contains a fixed cost portion:

$$UC = VC_1Q^b + (FC/R) + \varepsilon \quad (3.1)$$

where

UC = unit cost

VC<sub>1</sub> = variable cost of the first unit of production

Q = unit number (cumulative over production life)

b = variable cost learning rate = log(learning slope)/log(2)

FC = fixed cost for the production period, which is the same for every production period

R = production rate/lot quantity

ε = error term

Adding a learning rate to the equations validates the influence of the actual learning that occurs through repetitive production. We assume there are no breaks in production affecting the learning rate, such that each production period maintains the same learning rate in a continuous calculation. We chose to keep the fixed costs for each lot equal to capture the contractual obligations of the manufacturers and to acknowledge their inability to change fixed costs as production rates vary. The producers are tied to manufacturing plant size, labor training costs, administrative costs, raw materials orders, and other fixed costs that cannot be avoided. Normally, as production increases, manufacturers can capitalize on quantity discounts and thus spread fixed costs over more units. When the production rate suffers an unanticipated decrease, the short run fixed

costs cannot be modified. These large fixed costs can only be spread across the smaller number of units, meaning that each unit bears a larger portion of the fixed cost. The fixed cost represents the economies or diseconomies of scale within a production cycle.

Because the fixed cost, cost improvement equation favors Equation 3.1 through the fixed cost variable, we also created a more generic cost function. The second cost-generating function excludes a fixed cost variable.

$$UC = VC_1Q^b + \varepsilon \quad (3.2)$$

where

UC = unit cost

VC<sub>1</sub> = variable cost of the first unit of production

Q = unit number (cumulative over production life)

b = variable cost learning rate = log(learning slope)/log(2)

ε = error term

Equation 3.2 does not include the fixed cost variable; because the learning rate can only reduce the unit cost to a certain level, a stable cost per unit that cannot be eliminated exists. Equation 3.2 is identical to the original learning curve equations developed by Crawford. This equation captures the fundamental aspects of producing a good and allows us to vary our independent variables to create unique cost profiles.

### **Independent Variables**

Multiple independent variables affect cost. Monte Carlo simulation allows us to vary multiple independent variables simultaneously to build cost profiles. The independent variables work within the boundaries of the cost generating functions to

mimic historical costs and to truly test our cost improvement models. We chose the independent variables to be inserted for analysis into our equations: number of lots, lot quantity, cumulative quantity, unit one cost, variable cost learning rate, fixed cost burden, production rate decrease, and noise. In the following subsections we provide explanations for each of these variables and explain how we created our simulations. Through utilization of independent variables, we discover statistical patterns and indications of reliable forecasting. Following the variable descriptions, Table 3.1 shows the distributions and parameters for each variable in the Monte Carlo simulation for each cost-generating function.

#### ***Number of Production Lots (History)***

Number of production lots refers to how many production lots will be used as "historical" data. With lower numbers of production lots, fewer observations are available for modeling possibly affecting how the two cost improvement models perform. It is possible that a given cost improvement model estimates extremely effectively with more data points, but drastically underperforms when a significantly smaller number of data points are available. Models could also be affected by the amount of historical data, thereby overestimating or underestimating consistently based on a certain amount of data history tainting the models' accuracy and validity.

Our Monte Carlo simulation uses uniform discrete distribution ranging from three production lots to ten production lots. The uniform distribution assigns equal probabilities to each value within that range and randomly assigns an integer value. Based on previous academic research, we chose these values because they represent the typical amount of data available for cost improvement modeling. Values below three do

not provide enough data for cost analysts to model, and values above ten do not typically occur because production does not usually span over ten periods. Our range parallels the minimum and maximum values of Moses' research, but he did not account for all values within that range as we do (Moses: 13).

### *Lot Quantity*

The historical lot quantities represent full rate production levels. We do not model ramped up production because the specific situations we address occur during the later life of individual programs. Also, in the early stages of a program, a producer prototypes products and refines the manufacturing process as needed; we do not want to account for the adjustments made before production stabilizes. The simulation adds variations to each production period because production levels are commonly unstable from period to period.

Lot 1 quantities range from a starting value of 15 to an upper ceiling of 60. The integer values within this range are uniformly distributed, which again means that each value holds an equal probability of occurrence through random generation. The range accounts for the need to model programs with high productivity as well as programs with lower product output. Our range is somewhat arbitrary demonstrating potentially extreme maximum and minimum values, but it is arguably representative of possible scenarios, thus offering a truer test of our estimation models.

Each subsequent lot uses the previous lot quantity and a triangular distribution with that value as the mode (center) value. This ensures that each production run builds on information within that scenario. Lot quantities are critical to data simulation but are

not used during analysis. The lot quantity data is summed to produce the cumulative quantity, which can be analyzed against the cost improvement model.

### *Cumulative Quantity*

Measuring cumulative quantity of the production line allows us to observe how the equations behave with different program sizes. We aim to uncover any pattern or statistical significance associated with the models and the models' abilities to predict based on program size. The cumulative quantity builds from the number of lots and the lot quantity data.

### *Unit One Cost ( $UC_1$ )*

The cost of the first unit of production represents how much of the total cost of the first unit of production can be attributed to variable cost and is inherently affected by predictable learning on the production line. The first unit cost determines the starting point for subsequent cost calculations of learning rate and fixed cost burden. Fixed cost is added as a percentage of total cost and is not affected by learning. To capture all relevant cost structures we created an extensive range of simulated values.

We simulated the first unit variable cost through a uniform continuous distribution between 10,000 and 1,000,000 dollars. The value can be interpreted as thousands of dollars to more accurately relate to the flyaway cost of an airframe, but this inclusion does not change the analysis. Every value within that range has an equal probability of becoming the first unit cost as we process individual iterations of our simulation. We wanted ensure with certainty that we tested the predictive ability of the two cost improvement models over a range of large and small values. The smaller values represent low cost items while the large cost items depict major manufacturing items

and/or total production. Though we cannot model every value within that range, we succeed in creating a mixture of values representative of varying cost profiles. The simulation applies a learning rate to each of these cost profiles to create values over a period of time.

### ***Variable Cost Learning Rate (LR)***

Learning Rate describes the actual amount of learning taking place on the production line. As laborers repeat the same processes over a certain time period, the laborers achieve more efficiency and complete the same amount of work in a lesser amount of time than when they initiated the production process. Due to the progressive manufacturing efficiency, the unit variable cost will decrease by a constant percentage as the number of units produced doubles. The variable cost learning rate is the same as the learning rate described in Chapter II, and further examples can be read in Chapter II of this research.

We first chose to use a triangular distribution to model the learning rate with the minimum expected value set as 75 percent learning, the maximum value set as 95 percent learning, and the most likely learning rate as 80 percent. The values are based on similar academic research evaluating cost improvement models (Moses, 1990:8; Avinger, 1987:18). Choosing a triangular distribution allows us to restrict learning rates to values between 75 percent and 95 percent while acknowledging that those minimum and maximum values have a low probability of occurrence. A normal distribution will model similar principles, but will also allow for the occurrence of extremely high and low values not consistent with actual production. For example, if the value of 100 percent learning is randomly selected through a normal distribution, then zero learning will occur

throughout the life-cycle of that production line. Only under the most rare of circumstances will this situation exist and throughout multiple production lots it is unlikely the situation will remain consistent.

Based upon the input from our in-person conversations with Mr. Ken Birkofer and Mr. Doug Mangen, cost analysts with the F-22 Program, we constructed multiple scenarios for the learning rate. In their experience, the learning rate for larger production scenarios stays between 85 percent and 95 percent, while assembly line production is usually 75 percent to 85 percent. Our first distribution spans over the entire range, but we also use distributions to model the two other levels independently. By limiting the learning rates to these values, we can detect if the different levels affect the predictive abilities of Equations 1.1 and 1.2. As mentioned previously, variable costs are the only costs affected by learning, but variable costs are not the only costs composing the cost structure of a unit and an entire production period. Fixed costs play a role in determining the cost of future lots.

#### ***Fixed Cost Burden (FC)***

Fixed cost burden represents the percentage of total cost not affected by learning and held constant throughout all production periods. With higher production rates the production plant gains efficiencies and fixed costs can be distributed across more units to lower the cost per unit. When a plant experiences lower production rates, the fixed costs become a higher percentage of unit costs and efficiencies are lost. In our specific situation, the unanticipated production decreases do not give the manufacturer the opportunity to change the already established fixed cost structure. The manufacturer cannot sell a portion of the manufacturing plant, cut contractual agreements for supply

purchases, reduce storage space, un-train laborers, nor reduce fixed costs in any other way.

We assigned discrete, equal distribution to fixed cost burdens of 10 percent, 20 percent, and 30 percent. The simulation uses the variable costs of the first production lot to create a total cost profile where one of these values is used to represent the fixed cost of the lot. The value calculated from the first lot becomes a consistent fixed cost for each production period, where the amount assigned to each unit depends on the production rate. The proposed cost improvement models claim that there are hidden costs inherent to production that cannot be avoided and that need to be modeled in the cost improvement model. The fixed cost burden provides a value that is consistent throughout production lots, that can be affected by the production rate, and that should be captured by the cost improvement models. For the future production lot that will be estimated, the fixed cost becomes crucial as the production period suffers decreases in production. The production rate decrease determines how many units will bear the burden of the fixed cost.

#### ***Production Rate Decrease for the Estimate Lot (PR)***

The main goal of our research is to demonstrate how the unanticipated cut in production levels will affect the cost improvement model's ability to forecast. Thus, the future production lot that needs to be estimated will display a production decrease affecting actual cost. Production decreases represent any program changes whether need-based or funding-based that can lead to fewer units being required and therefore produced than were previously anticipated and projected. Because our simulated data does not

belong to a program with a history of projected production lot sizes, the final historical lot size serves as the future production level experiencing a decrease.

Our Monte Carlo simulation represents production rate decreases with a discrete, equal distribution of the values 25 percent, 50 percent, and 75 percent. For example, if the final historical production level is 100 units and suffers a 25 percent production rate decrease, then the future production rate will be 75 units. There is no pattern for the amount of decrease programs will face. The amount of decrease depends on the economic and social climate of the time and on the nature of the program. By choosing these values we can simulate low, medium, and high production decreases and thereby measure how the cost improvement models predict future costs. The cost information for future production lots is simulated simultaneously with the historical production lots so the model estimations can be evaluated against 'true' costs.

### *Noise*

Noise represents the unpredictable natures of production situations and estimating costs. Adding the noise variable presents the best option to account for the unknown events of a production run. Noise represents any unforeseen and even unknown events that occur at any time during production.

We create noise based on the following function:

$$\text{Noise} = \text{UC} * e \quad (3.3)$$

where

UC = unit cost for that particular unit

e = randomly generated percentage based on our distribution

Noise affects each unit cost calculation, and in our simulation noise can be any value between -0.05 percent and 15.0 percent of that unit cost. The values are represented by uniform continuous distribution where every value has an equal probability of occurrence through random number generation. Though normal production cycles may not fit this noise distribution where a value of 15.0 percent appears as often as a value closer to zero, we aim to generate a high number of possibilities to observe how each of the models reacts. This range offers a sample with more extreme values to observe, while also providing lower, more conservative, values for evaluation. We did not limit ourselves to one possible distribution of noise; we also simulated a normal distribution with a mean of zero and a standard deviation of 7.5 percent. By using different distributions we can see if and how noise affects the predictive abilities of the equations.

***Table of Simulated Values***

Table 3.1 only shows the first set of assumptions; we modeled assumptions for learning rate and noise at different levels to explore any changes in the results.

**Table 3.1. Independent Variable Descriptions for Monte Carlo Simulation (1<sup>st</sup> Set)**

<b>Independent Variable</b>	<b>Distribution</b>	<b>Range</b>
Number of Production Lots	Uniform Distribution	Min: 3 Lots Max: 10 Lots
Lot Quantity	Lot 1: Uniform Distribution Other Lots: Triangular Distribution	Lot 1 Min: 15 Units Lot 1 Max: 60 Units Other Lots Mode: Previous Lot's Number of Units Other Lots Min: Mode – 5 Other Lots Max: Mode +5
Cumulative Quantity	No Distribution	Summation of Lot Quantities
Unit One Cost	Uniform Distribution	Min: \$10,000 Max: \$1,000,000
Variable Cost Learning Rate	Triangular Distribution	Min: 75.0% Mode: 85.0% Max: 95.0%
Fixed Cost Burden	Discrete Distribution	Possible Outcomes: 10.0%, 20.0%, 30.0%, and 40% of Total Cost
Production Rate Decrease	Discrete Distribution	Possible Outcomes: 25%, 50% and 75% Decreases
Noise	Uniform Distribution	Min: -5.0% Max: 15.0%

### **Monte Carlo Simulation and Model Creation**

Monte Carlo simulation provides us with the ability to model thousands of different cost profiles and then to evaluate the results. Through random number generation within the distribution guidelines for each variable previously mentioned, we create costs that exhibit similar patterns to historical production costs.

We use Microsoft Excel Visual Basic for Applications and Premium Solver Platform from Frontline Systems to create a workbook that allows us to create historical production costs, to model the parameters of each cost improvement equation, to use the model parameters to estimate the next production lot, to simulate the cost of the next

production lot, to measure the prediction errors, and to store simulation and model information.

### ***Visual Basic for Applications (VBA)***

The code creates a program that runs on the command of the user. Based on the structured Microsoft Excel worksheets, the VBA code creates all of our essential data for more accurate evaluation of the cost improvement models. The code runs the simulation, stores snapshots of the simulated conditions (levels for independent variables), runs Solver to find model parameters, stores the model parameters, and stores the estimated values. The process runs in an integrated, fluid motion where the user only needs to input the number of iterations that need to be calculated. Microsoft Excel automatically calculates information while the VBA runs the simulation, and the Solver function finds the optimal model parameters for estimation.

### ***Model Creation***

Model creation follows the guidance offered in The Air Force Cost Analysis Handbook. In our study, models are formulated from historical data points with no discernible knowledge of the inter-workings of the program. Within our research, the only elements of information available are unit cost and lot cost for production. Only one data point is derived from each set of lot data; the points consist of a lot midpoint unit number and an average unit cost per lot. We calculate the lot midpoint using a heuristic for the unit number. “A lot midpoint is the unit number (not necessarily a whole number) that corresponds with the average unit cost for a given lot under the Unit curve formulation” (Handbook: 8-22). Lot midpoints can only be used in the unit cost formula of the cost improvement model, which is consistent with our research because production

rate adjustments must be based on unit cost methods (Handbook: 8-22, 8-32). The lot midpoint heuristic used is:

$$\text{Lot Midpoint} = \frac{(LS+LE+2 + \sqrt{LS*LE})}{4} \quad (3.3)$$

where

LS = cumulative production number of the first unit in the lot

LE = cumulative production number of the last unit in the lot

The lot midpoint and average unit cost create the historical data points to find our model parameters. With those data points we use Premium Solver Platform to find the statistically correct model parameter for estimation.

### ***Premium Solver Platform***

Microsoft Excel offers a Solver add-in called Premium Solver Platform. Solver operates as an optimization tool to find the value of parameters within a model that meets a goal or condition. Frontline System states on their website that Premium Solver Platform “solves every type and size of problem, using built-in and plug-in Solver Engines” (www.solver.com). Premium Solver Platform is “a unique combination of genetic algorithms and classical nonlinear optimization methods” (www.solver.com). As a reminder, our four equations for modeling are:

$$\text{The Production Rate Adjustment Model: } C(Q) = T_1 Q^b (R/R_0)^c \quad (1.1)$$

$$\text{The Fixed Cost Model: } C(Q) = (F/R) + T_1 Q^b \quad (1.2)$$

$$\text{The Crawford Unit Cost Model: } C(Q) = T_1 Q^b \quad (2.2)$$

$$\text{The Production Rate Model: } C(Q) = T_1 Q^b R^c \quad (2.4)$$

Three of our cost improvement models are nonlinear (Equations 1.1, 1.2 and 2.4) and require the more robust solver to calculate the model parameters. More specifically, we need to be certain that Solver can calculate the global minimums while avoiding the local minimum values. To combat this problem, Premium Solver Platform developed a multi-start function to identify global solutions. The multi-start function “can be automatically run many times from judiciously chosen starting points, and the best solution found will be returned as the optimal solution. [...] multi-start methods will converge in probability to the globally optimal solution” (www.solver.com). With the use of Premium Solver Platform we calculate the model parameters for each model on each set of production costs.

Solver requires that a condition be optimized to find the model parameters. We chose to calculate the Sum of Squared Error (SSE) for each model fitting the data, and to minimize SSE to define the optimum model parameters. SSE identifies the error for each data point, squares the error, and then adds the resulting product to the values of other data points. By choosing different values for the model parameters, the SSE can be minimized thus reducing the fitting error. For the production rate adjustment equation (Equation 1.1) cost improvement model we optimize the values of  $b$ ,  $c$ , and  $T_1$ . For the fixed cost equation, (Equation 1.2) cost improvement model we optimize  $b$ ,  $F$ , and  $T_1$ . The values of  $T_1$  and  $F$  are not the true Unit one values or fixed costs amount, but rather  $T_1$  and  $F$  are theoretical parameters established to create the best estimating model. Solver converged, based on probability, to the optimum value of these model parameters in order to minimize SSE. Premium Solver Platform generates outputs based upon the Solver iterations’ simulated data, delineating the steps of the program.

### ***Limitations***

Modeling every possible production scenario is impossible and an obvious limitation. We attempt to include all variables affecting costs along with making reasonable selections of values for the variables, while heavily relying on the power and accuracy of Premium Solver Platform to calculate the correct model parameters. All indicators lead us to believe that the program converges on the best possible solution, but we do not have the means to personally test and analyze every model. Any anomalies within the data will be researched and explained in the analysis. The assumptions for accuracy of fit, equal variance, and independence will not be evaluated with our results. We focus on the predictive ability of each of the models; testing the assumptions of thousands of model iterations is simply not feasible. We have done our best to create sound methodology for our Monte Carlo simulation and model creation, but acknowledge that unknown or unrecognized factors can impact the research.

### **Model Evaluation with Historical Data**

The Air Force Cost Analysis Agency provided unit recurring flyaway data for a number of aircraft platforms. Summary information represents data that was collected, normalized and analyzed by RAND Corporation. Any anomalies have been explained and removed from the data to identify a cost structure that can be modeled. The summary data provides lot quantity data and average unit costs for each lot. We will model these observations using the same methods mentioned above for our simulated data. We will use lot midpoint and average unit cost data points to run Solver Premium Platform and to solve for model parameters. Future costs to measure the predictive

accuracy are unknown, thus we will withhold the data point where a production decrease takes place and will use that as the ‘future’ data point.

## **Measures of Error**

### ***Mean Percentage Error (MPE)***

MPE is calculated through the following equation:

$$\text{MPE} = \frac{\text{Actual Cost} - \text{Estimated Cost}}{\text{Actual Cost}} * 100 \quad (3.4)$$

We used MPE to measure the fitting error as well as the forecast error. Each data point’s MPE is calculated separately; the MPE values are then averaged to reach a final value. The value illustrates the bias of the model by the positivity or negativity of the final MPE value. A positive value indicates that the model underestimates the data, while a negative value signals overestimation. The fitting error illustrates how well the model fits the historical data. While MPE can be used as an indication of accurate forecasting by the model, the MPE does not guarantee that the model will forecast precisely.

We also calculate the MPE of the forecast to compare that value to the fitted MPE. Large differences in the value show that even though the model fits the historical data well, the model is no indication of future costs.

### ***Mean Absolute Percentage Error (MAPE)***

Mean Absolute Percentage Error takes the absolute value of each MPE and sums those absolute values. MAPE reveals the extent that the fit and forecast vary from the actual value without taking into account whether the equations overestimate or underestimate. Again, the MAPE fitting the historical data does not guarantee the model will provide an accurate forecast, but the MAPE can be used as an indication of the

predictive ability of the model. Measuring the difference between the fit error and the forecast error provides insight into the validity of the model and the model's ability to predict future values.

We use SSE, MPE, and MAPE to reveal information about each of the models. Evaluating these values with the independent variable values demonstrates how well the models are forecasting and which variables affect the individual model's ability to perform.

### **Chapter Summary**

In this chapter we built upon the literature reviewed in Chapter II to develop our methods for evaluating Equations 1.1 and 1.2. We will be using Monte Carlo simulation to create production costs that can then be modeled. In our Monte Carlo simulation we will vary eight different independent variables to create an all-inclusive set of possible cost profiles. Number of lots, lot quantity, cumulative quantity, unit one cost, variable learning rate, fixed cost burden, production rate decrease, and noise will all be varied in our simulation. The simulated data will mimic normalized historical data used for cost estimating. For each set of production costs we will model the production rate adjustment equation (Equation 1.1) and the fixed cost model (Equation 1.2) using the Microsoft Excel add-in Premium Solver Platform. The model parameters discerned using Solver will be implemented to predict a future lot, and the error of that prediction will be measured. Using historical data provided by AFCAA, we intend to verify our findings and to determine which equation better predicts future costs.

Chapter IV shows the results of our Monte Carlo simulation and the model estimation for the simulated and historical data. We introduce the measures of error

results for each simulation to determine which variables affect each model's forecast. Chapter IV provides a summary of our analysis, while Chapter V highlights the most significant aspects of our research and the potential cost analysis policy implications.

## IV. Results and Discussion

Previous chapters included the purpose of our research, an extensive literature review on cost improvement curves, and our methodology for our cost improvement models. In this chapter, we focus on the results of our Monte Carlo simulation, our cost improvement model creation, and our model evaluation. We display a random example from each of our simulated production functions and compare individual production profiles to historical cost profiles for validity. Our analysis of the Monte Carlo simulation datasets includes measures of error, accuracy plots, statistical analysis, and patterns revealed within the models. Our results illustrate that the production rate cost improvement model outperforms the other cost improvement models overall. Originally we hypothesized that Equations 1.1 and 1.2 would furnish better results than would the other models, but this did not prove to be true. For this reason we include the results of the Crawford unit cost model and the production rate model when applicable in this chapter. The analysis also demonstrates that fixed cost model produces statistically equal forecasts when production cycles possess high fixed costs and suffers losses of efficiency when production decreases occur. We validate the performance of the fixed cost model with historical data where the fixed cost model consistently provides more accurate estimates than do the other models. Our evaluation of the cost improvement models includes estimations of normalized historical data; the results remain consistent with our findings.

## Monte Carlo Simulation Results

Our cost generation functions produce reasonable data for our analysis. Patterns found in historical data provided by the Air Force Cost Analysis Agency fuel our simulations as mentioned in Chapter III, substantiating the validity of our results. The following subsections elucidate random examples from each of our cost functions.

### *Production Cost Simulation: Fixed Cost Function*

The below dataset displayed in Figure 4.1 and Table 4.1 represents a random production run from our simulation. We cannot individually plot each of the thousands of dataset simulations from our research, but our example illustrates the basic structure evident throughout our research. The thin black line represents each unit cost throughout production, while the thick black line depicts the lot midpoints and the average unit cost per lot utilized for model creation.

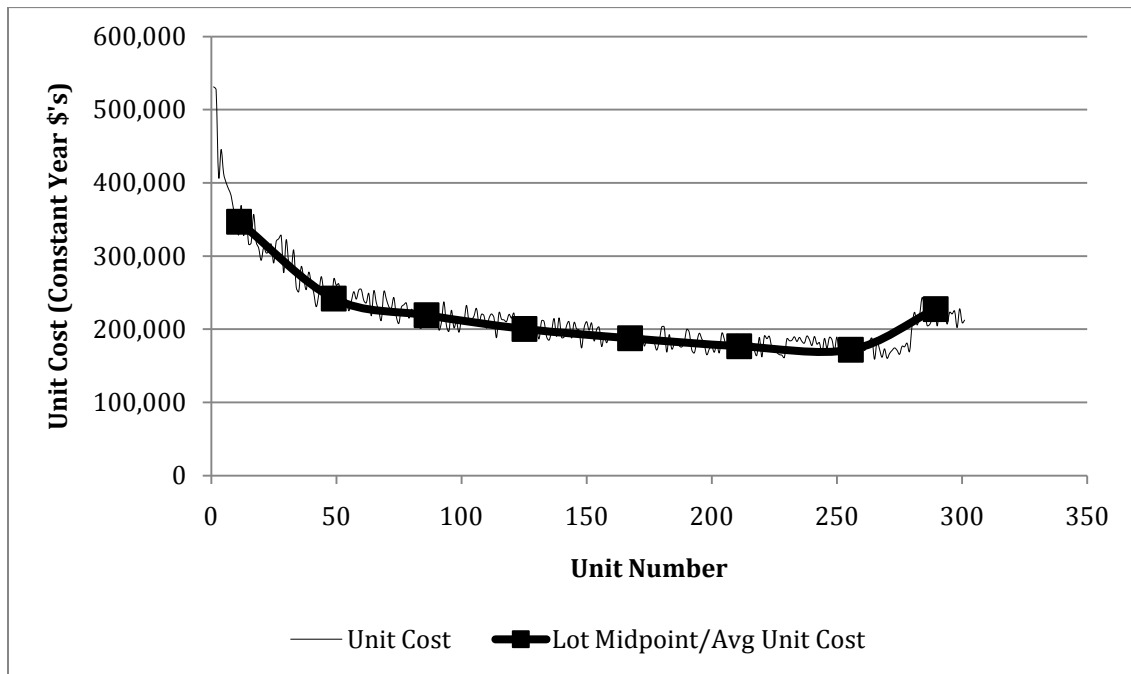


Figure 4.1: Fixed Cost Production Simulation Example

Figure 4.1 tracks the path of a traditional learning curve production, but breaks that pattern in the final lot where the production rate decreases. The production rate decrease triggers the average unit cost for that production lot to increase because each unit must shoulder a larger portion of the fixed cost. The change in the cost represents our hypothesized loss of efficiency associated with production decreases. Appendix A manifests this dataset in its entirety. Table 4.1 presents the value of each input variable used in the construction of the model. The input variable noise is not included in Table 4.1 because we appended noise to each unit cost at different levels.

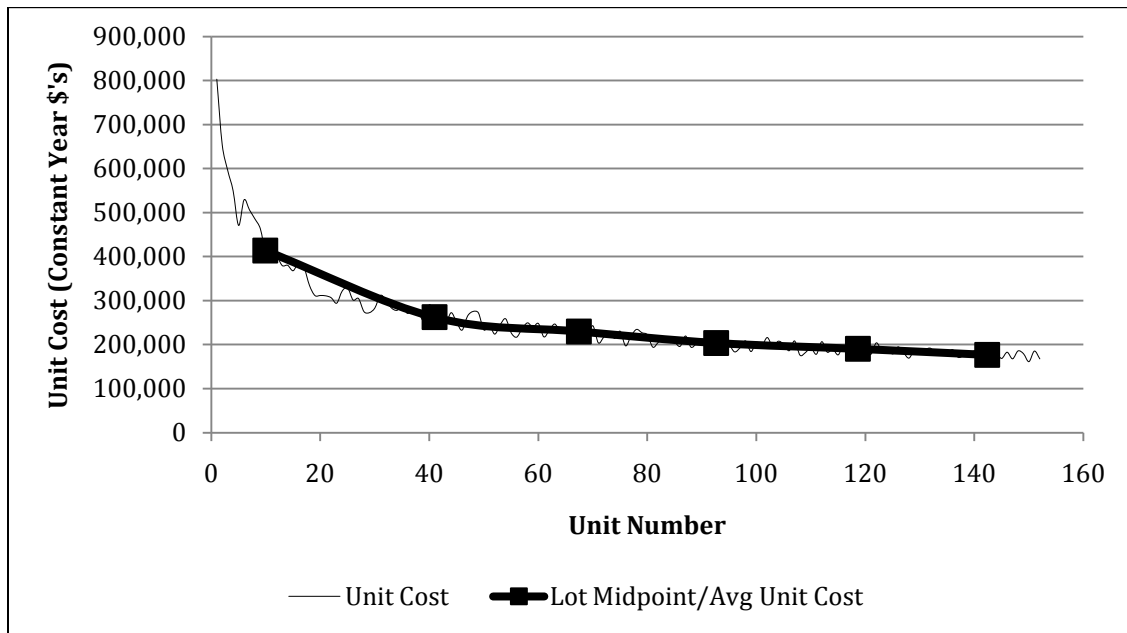
**Table 4.1. Fixed Cost Production Simulation Example Input Variables**

<b>Variable</b>	<b>Value</b>
Number of Lots To Be Modeled (History)	7
Variable Cost Starting Point	\$445,579
Cumulative Units	301 units
Learning Slope	84%
Fixed Cost Burden	20%
Percent Production Decrease	50%

***Production Cost Simulation: Cost Function with No Fixed Cost***

Our research focuses on modeling a production decrease and the inherent loss of efficiency when production lines must adjust to unexpected volume variations. The plot of our second cost generating function, Figure 4.2, does not include a fixed cost variable. Although Figure 4.2's plot does not expose a loss of efficiency, the simulated datasets maintain the validity in the evaluation of the cost improvement models. When we

compare the production accuracy from this production model to the fixed cost production results, we can illustrate the behavior of each model under varying conditions.



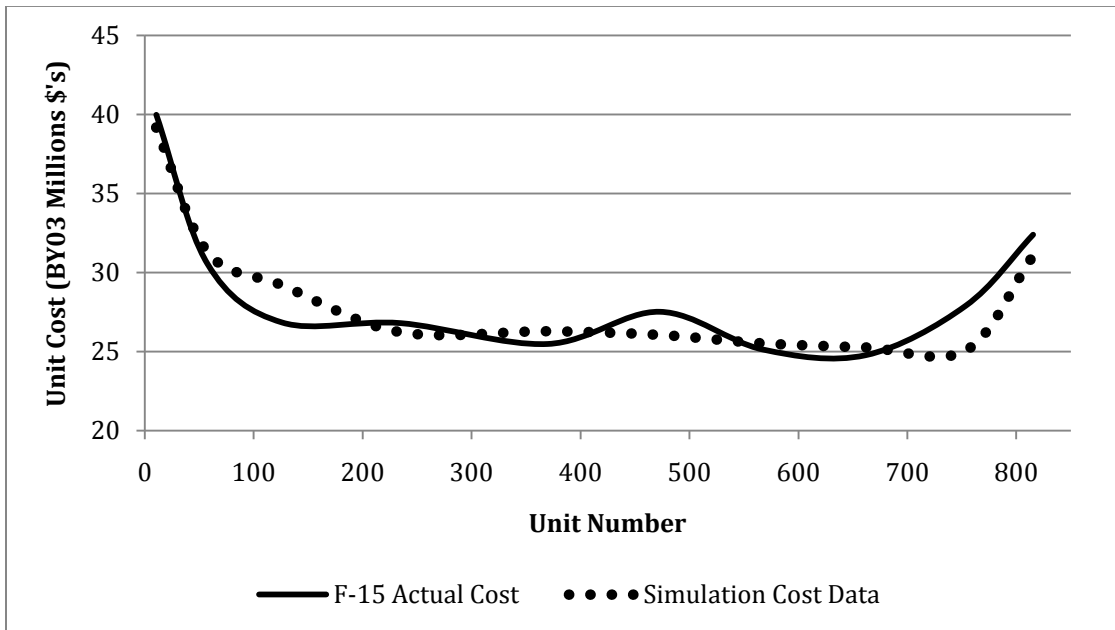
**Figure 4.2: Production Simulation Example with No Fixed Cost**

### *Historical Production Cost*

Our fixed cost production closely aligns with the hypothesized patterns described in Chapter III, but it is essential that we demonstrate with certainty that our production clearly mimics historical patterns. Utilizing the normalized dataset provided by the Air Force Cost Analysis Agency (AFCAA), we can successfully model a program's production decrease. The F-15 program suffered production decreases in 1981 and 1982; Figure 4.3 shows a plot of the lot midpoints and average unit costs for the program from 1973 through these production decreases. The pattern in Figure 4.3 similarly conforms to production costs from our simulation where the production decrease induces the average unit cost for those respective lots to correspondingly increase. We cannot be certain of

the true extent to which a fixed cost burden existed in the F-15 program at that time, nor ascertain the exact cause of the rise in average unit cost, but Figure 4.3 does reveal a significant loss of efficiency. By ensuring that our production datasets mimic patterns found in historical programs, we feel confident that the findings can be generalized beyond simulated data.

The lot quantities associated with the F-15 program fall outside of the range of our simulated conditions, but by matching the lot quantities we can create cost data that almost exactly matches the F-15 scenario. While Figure 4.3 reveals actual historical data, we have also added a simulated cost profile to demonstrate the validity of our Monte Carlo simulation.



**Figure 4.3: F-15 (1973-1982) Flyaway Production Cost (BY03 Million \$'s) Compared to Simulated Cost Data**

Though the profiles do not exactly mimic each other, they demonstrate similar cost patterns and the cost data is represented by a 4% mean absolute percentage

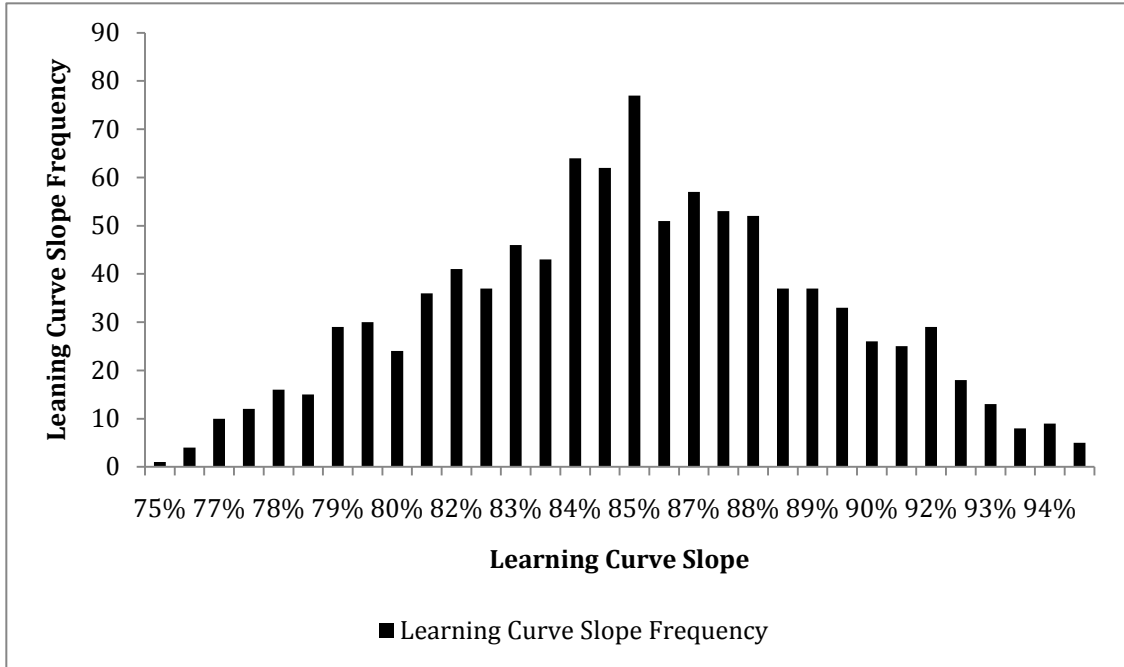
difference. Table 4.2 below shows the simulated conditions that created the cost profile portrayed above. Each of the independent variables falls within the range used for our simulations shown in Chapter III, thus our simulated data accurately represents historic cost data.

**Table 4.2. Simulated conditions Creating Historically Accurate Cost Profile**

<b>Simulated Variable</b>	<b>Variable Level</b>
Number of Lots	10
Unit 1 Cost (Million \$'s)	\$39
Variable Learning Rate	94%
Fixed Cost Burden	15%
Production Decrease	44%

***Independent Variable Simulation Results***

When implementing Monte Carlo simulation, each of the independent variables produces expected values. One of our simulations for learning curve slope represents a triangular distribution with a minimum value of 75 percent, a mode of 85 percent and a maximum value of 95 percent. The resulting simulation of 1,000 iterations produces the following results.



**Figure 4.4: Learning Curve Distribution from Simulation (1,000 Iterations)**

Distributions for the remaining generated independent variables are displayed in Appendix B.

### **Model Comparison**

#### ***Production with a Fixed Cost Variable***

We model the fixed cost production simulation example shown above (Figure 4.1) with the results from the Crawford unit cost model, production rate model, production rate adjustment model, and the fixed cost model. Table 4.3 and Table 4.4 present the simulated production costs and the model results. Though Table 4.3 and Table 4.4 are only one example of the many different production runs we completed and modeled, this particular data articulates patterns consistent throughout our simulations and modeling. Table 4.3 displays the dollar values from each of the models and Table 4.4 presents the percentage error so that the differences among the models can be clearly established.

**Table 4.3. Model Comparison Example (Constant Year \$'s)**

<b>Lot</b>	<b>Lot Midpoint</b>	<b>Actual Avg. Unit Cost</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
1	11	\$346,746	\$345,194	\$344,792	\$344,910	\$348,211
2	49	\$241,971	\$243,422	\$247,087	\$247,206	\$235,854
3	86	\$219,049	\$213,162	\$217,931	\$218,102	\$220,927
4	125	\$200,380	\$195,175	\$199,462	\$199,417	\$196,864
5	167	\$187,576	\$182,304	\$186,615	\$186,512	\$186,173
6	211	\$176,854	\$172,642	\$177,118	\$177,012	\$180,962
7	256	\$172,069	\$165,033	\$169,588	\$169,472	\$176,315
8 (Estimated Lot)	289	\$227,575	\$160,271	\$172,406	\$232,081	\$357,259

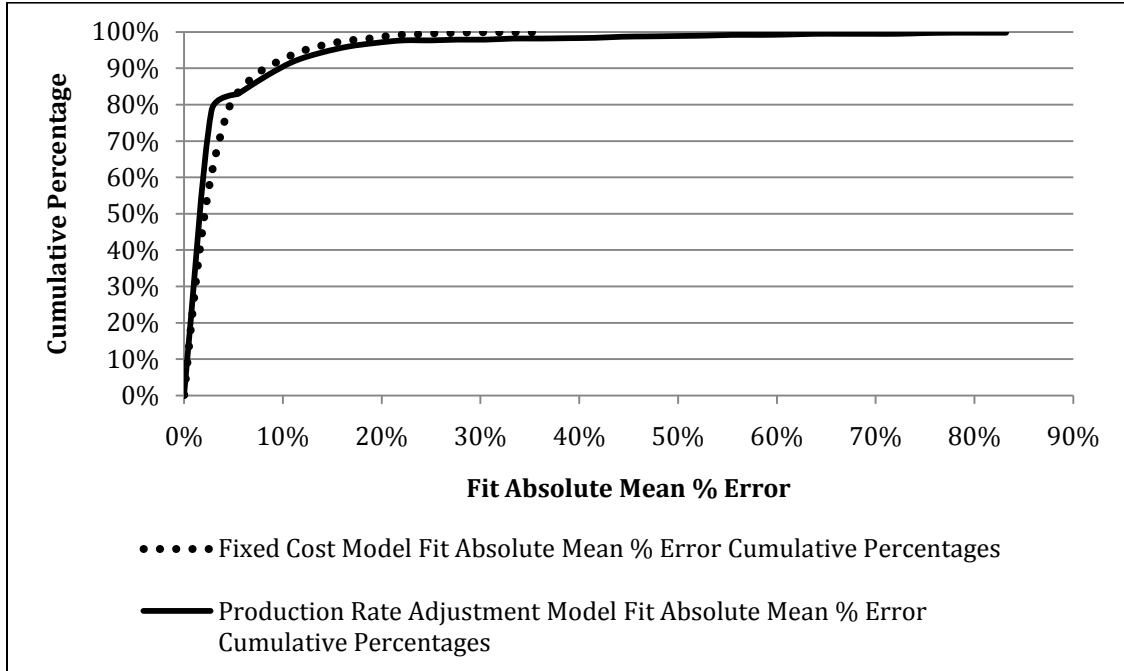
**Table 4.4. Model Comparison Example: Fit and Forecast % Error**

<b>Lot</b>	<b>Lot Midpoint</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
1	11	0.45%	0.56%	0.53%	-0.42%
2	49	-0.60%	-2.11%	-2.16%	2.53%
3	86	2.69%	0.51%	0.43%	-0.86%
4	125	2.60%	0.46%	0.48%	1.75%
5	167	2.81%	0.51%	0.57%	0.75%
6	211	2.38%	-0.15%	-0.09%	-2.32%
7	256	4.09%	1.44%	1.51%	-2.47%
8 (Estimated Lot)	289	29.57%	24.24%	-1.98%	-56.99%

In this example the production rate adjustment model predicts future cost more accurately than does any of the other models. Throughout our analysis each equation proves to possess forecasting capabilities; each equation predicts better than other models on specific generated iterations, but some models prove to produce more consistent forecasts.

### ***Model Fit Accuracy***

Fit accuracy does not appear to significantly vary between the models. Table 4.4 establishes that each of the models fits the data for lots 1 through 7 extremely well, and that each of the simulations' iterations under the different assumptions evidences the same results. Plots depicting the cumulative percentages of model accuracy for the production rate adjustment equation and fixed cost equation for the first set of simulation assumptions (triangular distribution learning slope 75.0 percent to 95.0 percent and noise continuous, uniform distribution -5.0 percent to 15.0 percent) verify that each of the equations fits the data well. Plots for the Crawford unit cost model and the production rate model can be viewed in Appendix C. The Crawford unit cost model and the production rate model produce similar results. Figure 4.5 and other plots of cumulative percentages illustrate that at any chosen point along the y-axis, we can be a certain percentage confident (y-axis value) that the model error will be less than or equal to the value on the x-axis. When comparing the models, the model with a curve appearing further to the left at the higher cumulative percentage adduces a lower error from actual values. Thus, in Figure 4.5 the models fit the data relatively closely, but the fixed cost model offers us a higher confidence that the model will provide a lower fit error.



**Figure 4.5: Fixed Cost and Production Rate Adjustment Model Fit Absolute Mean % Errors for Fixed Cost Production**

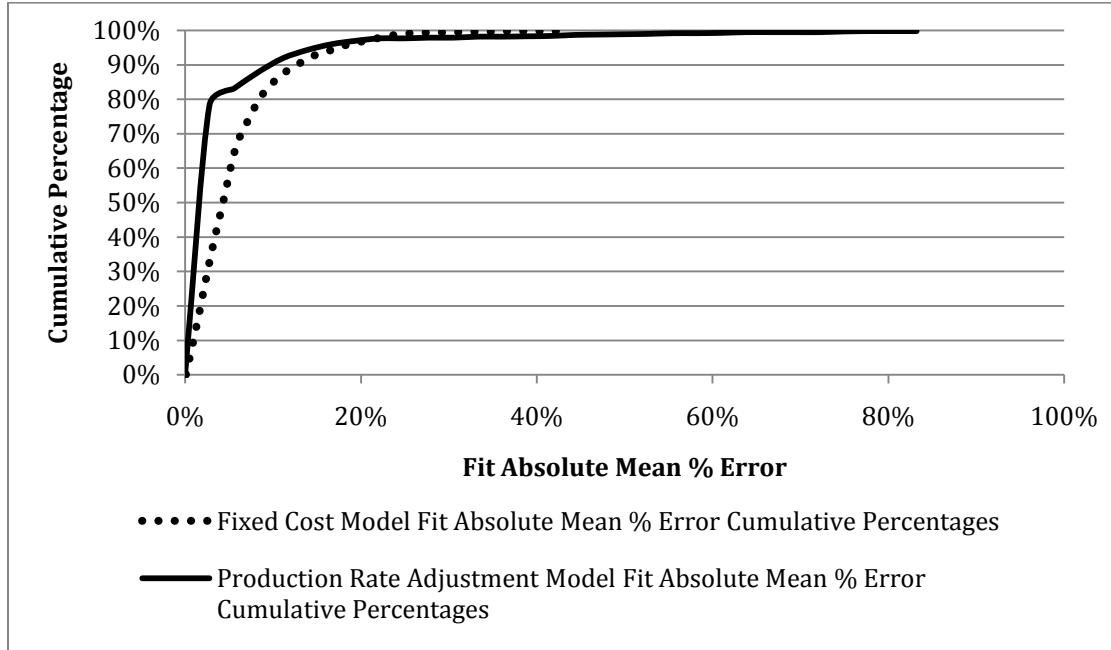
The cumulative percentages for fit error confirm that each of the models fit the data extremely well. Statistically, a test of equal means for the absolute mean percentage fit error establishes that the means are not equal, but the cumulative chart verifies that the models all fit the data well. Table 4.4 discloses the cumulative percentages of the four models. When translated, the percentages mean that we can be confident by a certain defined percentage that the model will return a fit error equal to or less than that identified fit error. For example, for the Crawford unit cost model, we can be 75 percent confident that the fit error for any model will be less than or equal to 2.0 percent under these simulated conditions.

**Table 4.5. Fit Absolute Mean % Error Cumulative Percentages for Production with Fixed Costs**

<b>Cumulative Percentage</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>25%</b>	1%	1%	1%	1%
<b>50%</b>	1%	1%	1%	2%
<b>75%</b>	2%	2%	2%	4%
<b>95%</b>	26%	14%	14%	12%

The production rate model and the production rate adjustment model have the same fit errors because they fit data applying the same model parameters. The slight deviation shown in Table 4.4 stems from very small differences in the model parameters ascertained by Solver (rounding). Running each of the equations separately with Solver to ensure the program identifies the same parameters, the equations perform properly. We find that even though there are slight differences, the means of each of the fit errors are statistically equal. The differences between the two models can be verified during estimation where the production rate adjustment model adds the variable to account for changes in lot quantities.

Under different assumptions, the fit errors yield similar results. We do not uncover any significant differences in fit error when the independent variables vary; the fit errors remain consistently low. Figure 4.8 evinces the fixed cost model fit absolute mean percentage errors for our production simulation without a fixed cost variable; Figure 4.9 illustrates the fit error for the production rate adjustment model. This production simulation differs only by the exclusion of the fixed cost variable; all other independent variables are simulated with the same ranges.



**Figure 4.6: Fixed Cost and Production Rate Adjustment Model Fit Absolute Mean % Errors For Production without Fixed Costs**

The plots appear almost identical to the fit error for the production simulation with a fixed cost variable because the major cost changes do not surface until the production decrease occurs in the estimated lot. The cumulative percentages for the fit absolute mean percentage errors for each of the four models are the focus subject of Table 4.6.

**Table 4.6. Fit Absolute Mean % Error Cumulative Percentages for Production without Fixed Costs**

Cumulative Percentage	Crawford Unit Cost Model	Production Rate Model	Production Rate Adjustment Model	Fixed Cost Model
25%	1%	1%	1%	2%
50%	1%	1%	1%	4%
75%	1%	2%	2%	7%
95%	57%	16%	16%	17%

The fixed cost model yields a slightly higher fit error than do the other three models until the 95% cumulative percentage where it demonstrates a significantly better fit than do the other models. Crawford’s unit cost model strictly mimics the cost production function we simulate except for the error term. As a result, we expect a low fit error.

The fit errors for each of the models prove to be very low, and each of the models performs equally well. The major differences among the models are evidenced during forecasting where we discover truly significant differences among the models.

***Model Forecasting Error***

We measure the mean percentage error for each of the model forecasts to determine if any of the models demonstrates a consistent bias to over or underestimate future production cost. Table 4.7 catalogs the results for each of the models from the production simulation with a fixed cost component.

**Table 4.7. Model Mean Percentage Error for the Production with Fixed Cost**

	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>Forecast MPE</b>	36%	19%	-209%	-45%

Table 4.7 provides a snapshot of each of the models to determine whether or not the individual model tends to over or underestimate production cost. The value shown in the table is an average, and we discover that the value can be influenced by extremely large values. Obviously, as a number based on 1,000 observations, an outlier will prove to exhibit less influence. Table 4.7 reveals that the Crawford unit cost model and the

production rate model tend to underestimate, while the production rate adjustment model and the fixed cost model tend to overestimate.

Our next step is to measure the forecast mean absolute percentage error. Just as with the model fit errors, we look at the cumulative percentages to determine our confidence levels in individual models. Table 4.8 presents the absolute percentage errors for all four models when forecasting the production cycle including a fixed cost component. The corresponding cumulative percentage histogram plots can be seen in Appendix D.

**Table 4.8. Forecast Absolute Mean % Error Cumulative Percentages for Production With Fixed Costs**

<b>Cumulative Percentage</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>25%</b>	18%	9%	20%	17%
<b>50%</b>	33%	18%	54%	33%
<b>75%</b>	55%	36%	161%	65%
<b>95%</b>	69%	57%	861%	154%

Each model presents data points that appear to be outliers. For example, with the production rate model, we find a model with a fit absolute mean percentage error of 28 percent and a corresponding forecast absolute mean percentage error of 152 percent. This data point proves to be the largest for the production rate model by approximately 50 percent. To test the effects on the cumulative percentages, we remove this data point and recalculate the percentages. The resulting cumulative percentages do not change from 9 percent, 18 percent, and 36 percent. So, even though each of the models produces some estimates that appear to be outliers, those values provide minimal, if any, impact on

our overall evaluations of the models. We do not remove any apparent outliers because the points do not affect the overall results, and Solver finds the apparent outlier models to best fit the generated data.

Based on the forecast errors for the production simulation with a fixed cost component, the production rate model outperforms the other three models. Using our second cost production function, we discover similar results. Table 4.9 provides the mean percentage error for each of the models to measure any bias across all estimates. Table 4.10 shows the forecast error cumulative percentages for each of the models when the production simulation does not include a fixed cost variable. Because the production rate model so closely mirrors the Crawford unit cost model, except for the Noise variable, we expect the Crawford model to outperform the other models. We use the Crawford model as a check on our simulation and as a comparison tool for our other models. Table 4.8 shows that the Crawford model slightly underestimates cost while each of the other models drastically overestimates cost.

**Table 4.9. Model Mean Percentage Error For Production without Fixed Cost**

	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>Estimate MPE</b>	7%	-20%	-292%	-152%

**Table 4.10. Forecast Mean Absolute % Error Cumulative Percentages for Production  
without Fixed Costs**

<b>Cumulative Percentage</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>25%</b>	1%	5%	34%	49%
<b>50%</b>	1%	13%	93%	109%
<b>75%</b>	2%	29%	212%	260%
<b>95%</b>	74%	60%	814%	392%

As expected, the Crawford unit cost model produces accurate cost estimates. The production rate model produces very consistent results demonstrating similar accuracy with both cost production functions. The production rate adjustment model and fixed cost model do not predict cost accurately with this function. Each of these models hypothetically accounts for changes in the normal production pattern, and this cost function does not create changes. These two models should not be used if production remains consistent because they will model aspects of production that are not present and will thus create inaccurate forecasts.

***Model Performance under Differing Simulated Conditions***

Monte Carlo simulation allows us to vary the assumptions of our production cost profiles to evaluate each of the model’s reaction, if any, to varying conditions. We vary the learning curve slope as well as the noise distribution to simulate different production costs, neither of which produces results different from those previously presented.

Overall, the production rate model outperforms the other models. This does not hold true when we isolate the variables and the fixed cost burden, nor when production decreases. We notice that even though the production rate model appears to forecast

better than each of the other models, production with a high fixed cost variable causes the fixed cost model to forecast equally well and even considerably better under some circumstances. As a reminder, our fixed cost variable captures the inefficiency that can be gained when production faces an unplanned decrease in the number of units produced in a lot. The inefficiencies can include fewer units to absorb high fixed costs, penalties from suppliers for reducing order quantities, overhead associated with a production plant, costs associated with re-tooling the production line, labor costs associated with re-tooling the production line, and loss of learning because the labor force must adjust to the new conditions. We cannot simulate each of these possible scenarios individually, so we capture the related costs in our fixed cost variable. This variable affects both the production costs and the model's performance.

Table 4.11 displays the forecast absolute mean percentage error cumulative percentages for production with a fixed cost burden of 40 percent, the highest fixed cost burden we simulate.

**Table 4.11. Forecast Absolute Mean % Error Cumulative Percentages for Production with a 40% Fixed Cost Burden**

<b>Cumulative Percentage</b>	<b>Crawford Unit Cost Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>25%</b>	21%	11%	24%	9%
<b>50%</b>	37%	23%	59%	20%
<b>75%</b>	62%	45%	169%	35%
<b>95%</b>	69%	60%	832%	58%

The data display in Table 4.11 substantiates that at 25 percent, 50 percent, and 75 percent cumulative percentages, we can be confident that the fixed cost model will

generate a more accurate forecast than does any other model. In fact, further simulation reveals that a fixed cost burden of 33 percent is the inflection point between the fixed cost model and the production rate model. The production rate model will produce more accurate estimates when the fixed cost burden falls below 33 percent, and the fixed cost model creates more accurate estimates when fixed costs exceed 33 percent. Analysis shows that the MAPE of the fixed model is negatively correlated with fixed cost burden at a level of -.5. This value means that as fixed cost rises, the MAPE will become smaller (the model becomes more accurate). The Crawford unit cost model and the production rate model show a slight positive correlation to fixed cost, .3 and .2 respectively; each of these models demonstrates less accuracy as fixed costs rise. The production rate adjustment model did not show any correlation to the fixed cost variable.

When we look even further into the simulated conditions, we notice that the amount of production decrease also affects which model establishes the most accurate estimations.

**Table 4.12. Forecast Absolute Mean % Error Cumulative Percentages for Production with a 40% Fixed Cost Burden and A 50% Production Decrease**

Cumulative Percentage	Model Error			
	Crawford Unit Cost Model	Production Rate Model	Production Rate Adjustment Model	Fixed Cost Model
25%	35%	14%	19%	12%
50%	38%	24 %	68%	25%
75%	41%	33%	178%	34%
95%	46%	43%	845%	42%

## **Modeling Historical Data**

We model four sets of data provided by the AFCAA. As previously discussed, the data has been normalized by RAND to return the dollar figures to base year and to remove any anomalies that should not be modeled as pattern. In each of these four sets of data, the respective program suffers a production decrease resulting in an increase to the average unit cost for the next production lot. The programs where we observed this pattern are the F-15 program, the F-16 program, and the F-18 program. None of the production decreases come at the end of the production life cycle; the decreases occur within the first 10 years of the program. We model the F-15 twice having observed that the program suffers a minor production decrease, which is then followed the next year by a significantly larger decrease in production. Using the small decrease in production during the model fit provides us with information about how the models react to the changes and then forecast the changes to follow in the next year. Each of the historical datasets is displayed in Appendix G.

Modeling historical data verifies our findings from the simulated data. Most importantly, this modeling certifies that the fixed cost model outperforms any of the other models overall. We anticipate this result; when these military aircraft production programs suffer sudden yield decreases, there are major losses of efficiency that cause average unit costs to increase. The production rate model also performs well with estimation results quite similar to the fixed cost model. Table 4.13 exhibits the percentage error for each of the models and reveals that the production rate adjustment model is the only model manifesting a significant bias. The resultant negative values mean that the model tends to overestimate consistently across each of the datasets. No

other model produces an overwhelming bias in any direction as can be seen by the averages at the bottom of each column.

**Table 4.13. Model Percentage Error for Each Historical Dataset**

	<b>Crawford Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>F-15</b>	-1%	11%	-36%	11%
<b>F-15(2)</b>	25%	9%	19%	-1%
<b>F-16</b>	15%	9%	-80%	-11%
<b>F-18</b>	-16%	-21%	-113%	-17%
<b>Average</b>	6%	2%	-53%	-4%

When looking at the absolute percentage error for each of the models, the fixed cost model does not outperform the other models in every instance, but it does outperform the other models overall. While the Crawford unit cost model and the production rate model do provide lower errors at times, the fixed cost model establishes a lower average across all four datasets. The averages shown at the bottom of Table 4.14 indicate how each of the models estimates the historical data.

**Table 4.14. Model Absolute Percentage Error for Each Historical Dataset**

	<b>Crawford Model</b>	<b>Production Rate Model</b>	<b>Production Rate Adjustment Model</b>	<b>Fixed Cost Model</b>
<b>F-15</b>	1%	11%	36%	11%
<b>F-15(2)</b>	25%	9%	19%	1%
<b>F-16</b>	15%	9%	80%	11%
<b>F-18</b>	16%	21%	113%	17%
<b>Average</b>	14%	12%	62%	10%

Although these results are only snapshots of a couple of programs, the historical patterns and the model results match the findings from our simulated data. When a program suffers a production decrease that causes average unit cost to increase, the fixed

cost model provides more accurate estimates than do any of the other four models in our research. We are aware that every program contains specific attributes and cost drivers that might only be found through a grassroots cost analysis, but our research conceivably can provide some insight into the behaviors of cost with a production decrease.

### **Chapter Summary**

Our Monte Carlo simulation for production costs closely resembles patterns found within historical data. While each of the cost improvement models fits the data extremely well, the forecast errors greatly differ among the models. Throughout all of the simulations' conditions, the production rate model forecasts with the least error, and thus outperforms other cost improvement models. When programs suffer from high fixed costs, the fixed cost model captures the inefficiencies with production decreases and forecasts equally as well as the production rate model. Though the fixed cost model appears to forecast slightly better for a program with high losses of efficiency, statistically the absolute mean percentage forecast errors are equal for both models. The historical datasets validate our findings and actually reveal that the fixed cost model outperforms each of the other models. In Chapter V we discuss the strengths, limitations, policy implications, and possible future research based upon our research.

## **V. Conclusions**

Chapter V first focuses on the strengths of our research; we cover the highlights from Chapter IV and the most essential takeaways. Second, we look at the limitations of our research. Though we identified our limitations in Chapter I, we feel that we need to revisit these limitations due to their significance to the results of our research. Third, we discuss the policy implications of our research followed by possible future research based upon our findings.

### **Strengths**

Our main focus in our research is to evaluate cost improvement models and challenge the status quo of models used in estimating. In an environment of restricted budgets, short deadlines, and production decreases, coupled with increased pressure to produce accurate estimates, the introduction of a fixed cost model offers opportunities for more accurate estimates under certain program conditions. When a program suffers a production decrease and a subsequent loss of efficiency, the fixed cost model provides more accurate estimates than do other models. The loss of efficiency can be due to high fixed costs or changes in the production line, and that impediment results in higher average unit cost for the reduced production lot. Based on our analysis of historical data, loss of efficiency took place within past programs and estimates could have benefited from utilization of the fixed cost model. We also clearly illustrate that over the entire range of possible programs and cost profiles, the production rate model provides the most accurate estimates. We reach these conclusions through thousands of iterations of possible cost profiles and by altering variables affecting cost for these iterations.

We built our research on a solid foundation of academics that scrutinized similar topics. Our literature review discusses in depth the works of other authors who explored cost improvement curves, and we use that research as a foundation for our methodology. The confidence in our results also stems from the thoroughness of our Monte Carlo simulation and our ability to create a wide range of possible production cost patterns. Varying independent variables through thousands of possible combinations creates an accurate depiction of how each of the models estimates. Isolating changes in the independent variables allows us to detect situations where the fixed cost model furnishes more accurate estimates than do the other models. We hypothesized that during a production decrease with a large loss of efficiency, the fixed cost model would outperform other models; our analysis proves this to be true. By offering more insight into cost improvement modeling, we hope that cost analysts are able to dispense more timely and accurate forecasts to aid the defense acquisition system in delivering assets to the warfighter.

### **Limitations**

Though we are confident that our Monte Carlo simulation covers a wide range of potential cost profiles, we cannot possibly simulate every practicable condition. We feel that we have generalized the simulation to focus on the more common cost drivers, but other exogenous factors can engender changes in cost and our simulation does not deal with these inconsistencies. We assume that our dataset has been normalized so that all inconsistencies have been eliminated, but in reality each program exhibits unforeseen uniqueness.

We are also limited by the actuality that we cannot evaluate every potential cost improvement model available to cost analysts. There could be circumstances where another model outperforms our models, but we chose to restrict our research due to the infeasibility of evaluating every model available.

### **Policy Implications**

We hope to enhance materials such as *The Air Force Cost Analysis Handbook* such that they furnish more specific direction to cost estimators. The flexibility within resources forces analysts to explore available methods and discover the most appropriate models for their respective programs, but sometimes the expansive options impede discovery. Searching all possible avenues for cost estimation takes time not often realistic in these pressurized, time-conscious situations. Our research acts as a more accurate ‘guide’ for costs analysts. While other cost improvement models should not be discounted early in the process, our research identifies the validity of the fixed cost model and the need for the model’s inclusion in common practice. Such is the case especially when programs face reductions as the F-22 program currently faces. Instructional materials presenting the Crawford unit cost model, the production rate model, and the production rate adjustment model need to also include the fixed cost model. We have constructed a foundation elucidating circumstances where the fixed cost model proves most useful; dissemination of that information is essential.

### **Further Research**

More intensive examination should be done regarding application of the fixed cost model. First, the model needs to be explored with specific datasets to check the statistical characteristics of the model. Our research did not include statistical validation

of the model due to the voluminous number of different datasets we modeled. In order to further the validation of the model, statistical characteristics should be studied. Our research explores ‘at point’ estimates with the fixed cost model, but a more in-depth analysis of the model should target the behavior of confidence intervals.

Further Research could direct efforts toward discovering a more accurate cost improvement model. As conditions change and programs face new challenges, the current cost improvement models may not be sufficient. None of the models we evaluate present an overwhelming accuracy with either the simulated or the historical data. Thus, perhaps a newly formulated model would replicate and project cost more accurately. Our research and further research should re-examine cost analysis methodologies and strive to improve current methods of cost estimating and to provide more complete and accurate estimates.

## Appendix A: Fixed Cost Production Function Simulation Example Dataset

Table A.1. Fixed Cost Production Function Simulation Example

Lot Number	Lot Quantity	Cum Quantity	Lot Midpoint	Actual Lot Cost (\$)	Lot Average Unit Cost (\$)
1	32	32	11.08	11,095,882	346,746
2	36	68	48.94	8,710,967	241,971
3	37	105	86.06	8,104,799	219,049
4	41	146	125.20	8,215,572	200,380
5	43	189	167.34	8,065,774	187,576
6	44	233	210.95	7,781,591	176,854
7	45	278	255.53	7,743,111	172,069
8 (Estimated Lot)	22	300	289.40	5,006,658	227,575

## Appendix B: Independent Variable Distributions from Monte Carlo Simulation

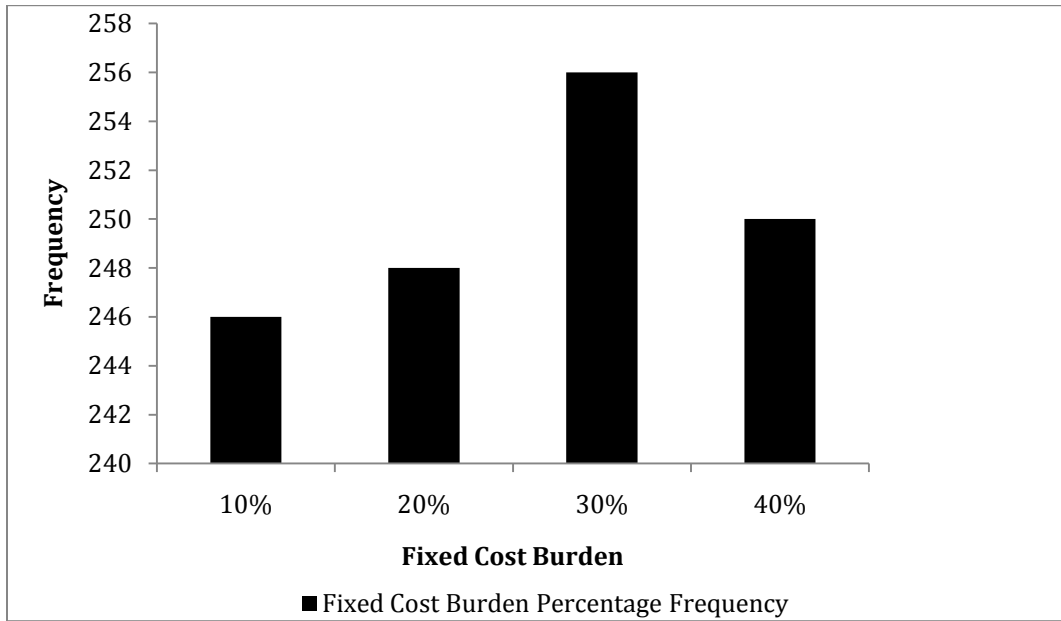


Figure B.1. Fixed Cost Burden Distribution From Simulation (1,000 Iterations)

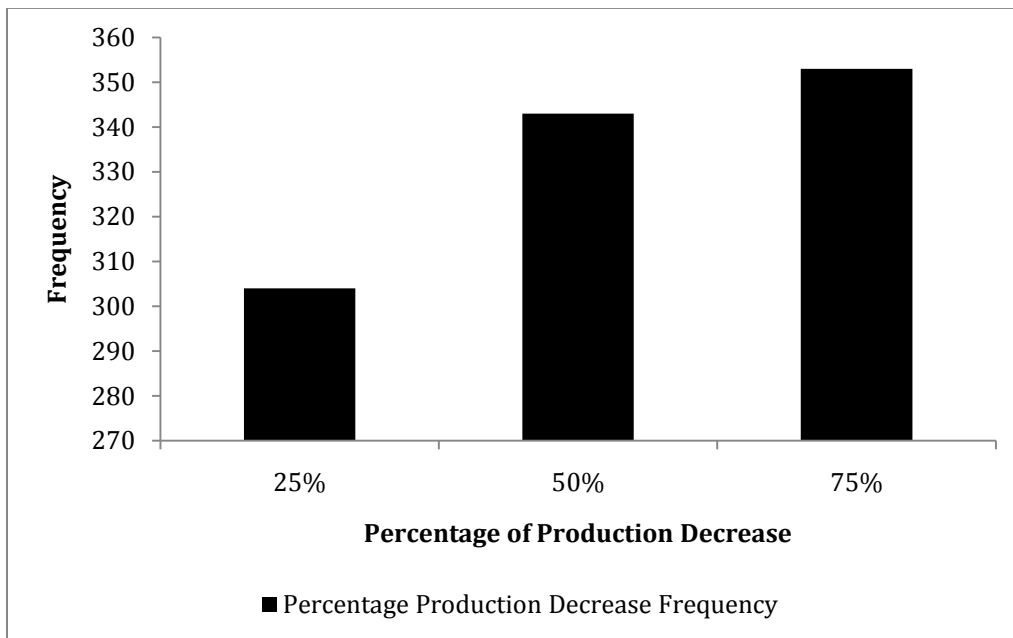
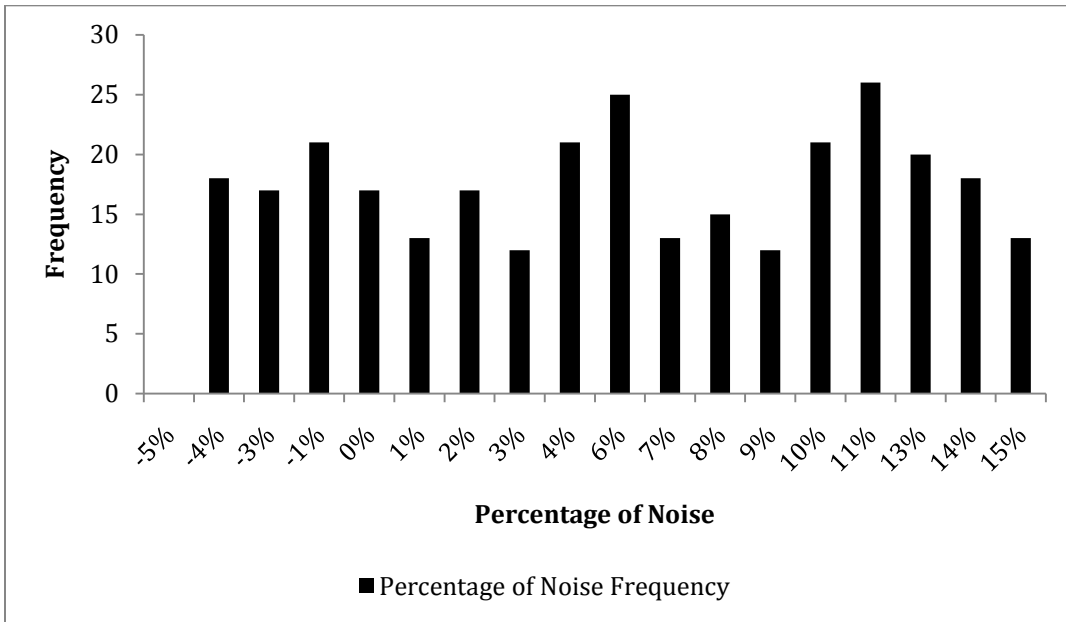
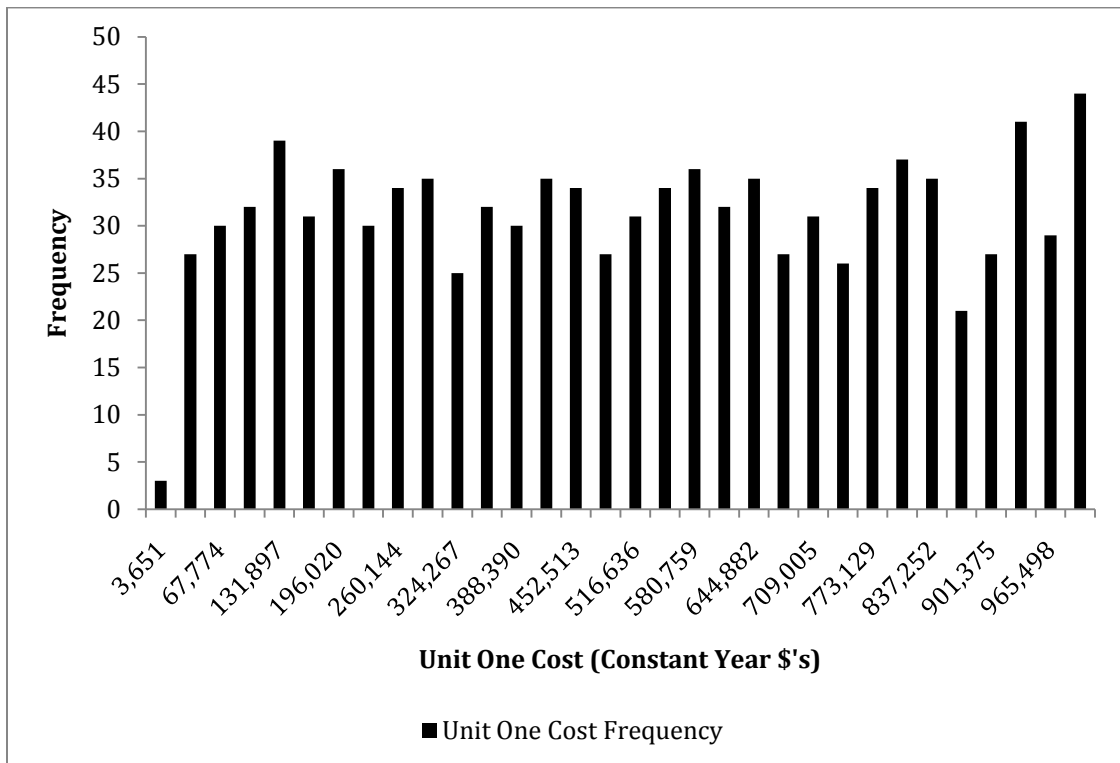


Figure B.2: Production Decrease Distribution From Simulation (1,000 iterations)



**Figure B.3: Noise Distribution from A Single Production Simulation**



**Figure B.4: Unit One Cost Distribution from Simulation (1,000 Iterations)**

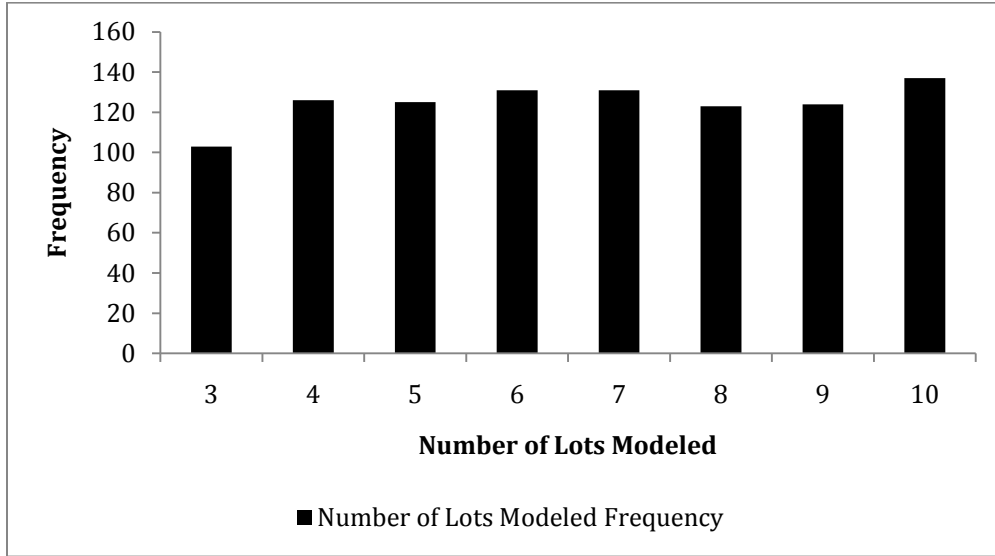


Figure B.5: Number of Lots Modeled From Simulation (1,000 Iterations)

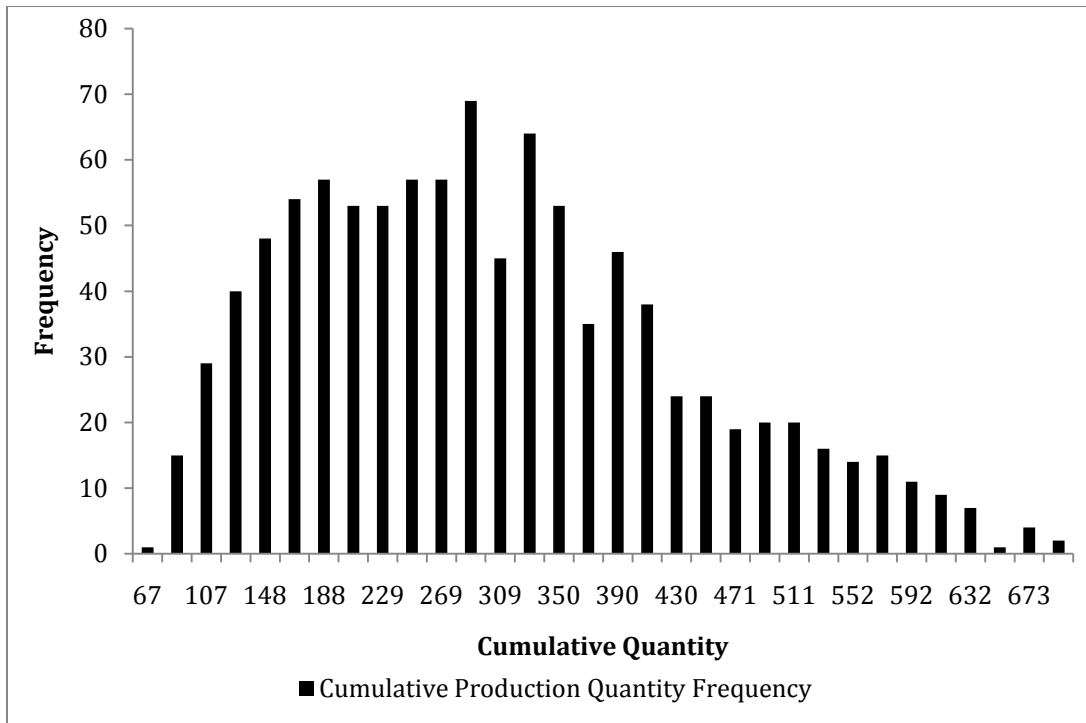
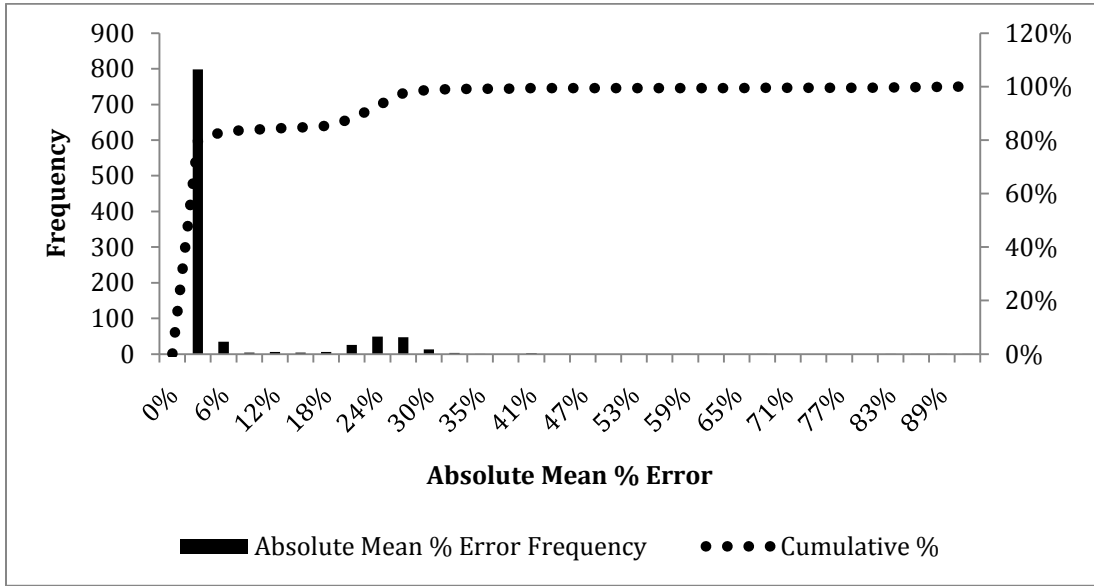
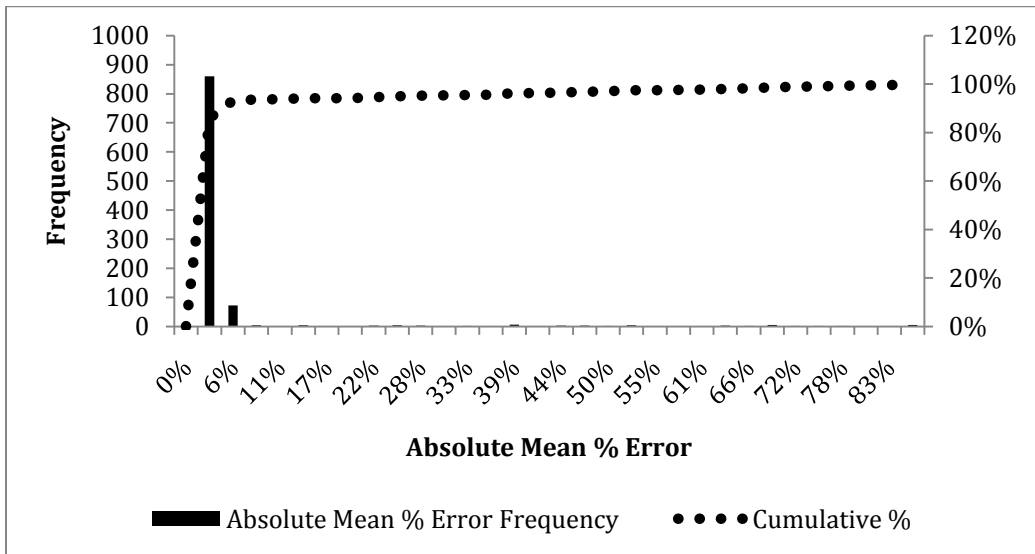


Figure B.6: Cumulative Quantities From Simulation (1,000 Iterations)

**Appendix C: Model Fit Absolute Mean % Error Plots From Production with Fixed  
Cost Variable**

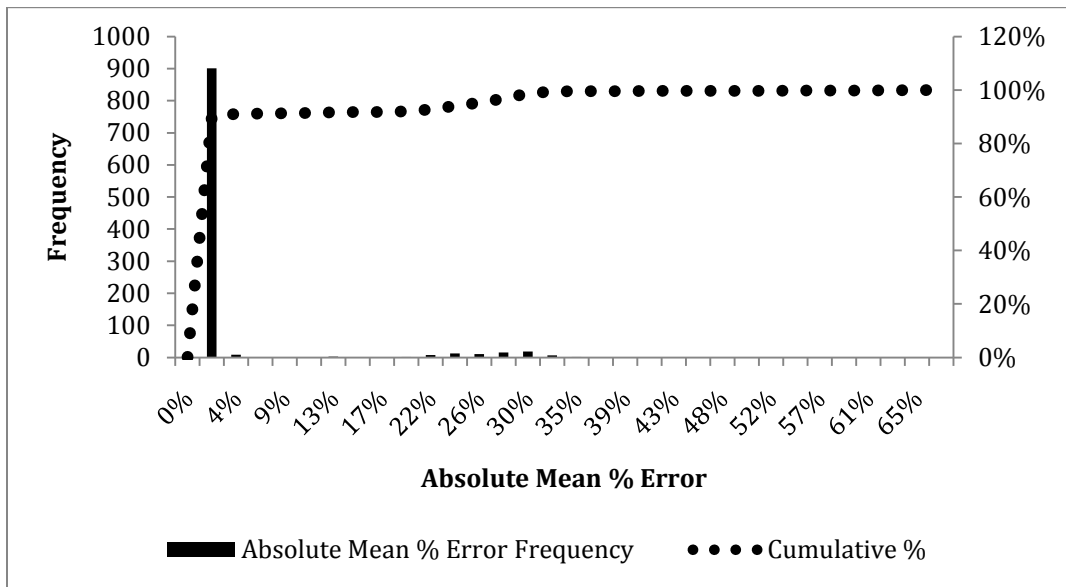


**Figure C.1: Production Rate Model Fit Absolute Mean % Error For Production With Fixed  
Cost Variable**

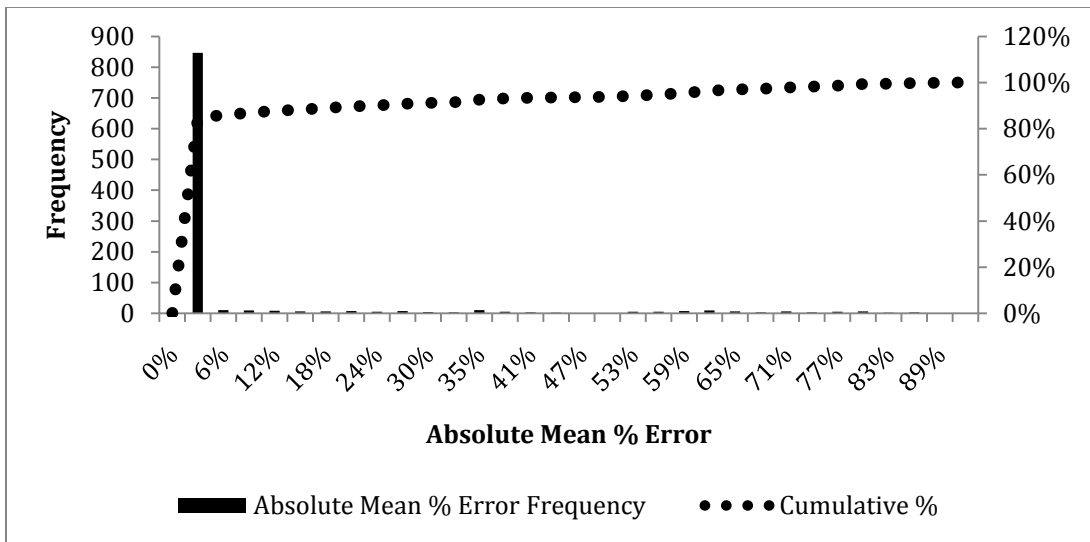


**Figure C.2: Crawford Unit Cost Model Fit Absolute Mean % Error For Production With  
Fixed Cost Variable**

**Appendix D: Model Fit Absolute Mean % Error Plots From Production without  
Fixed Cost Variable**

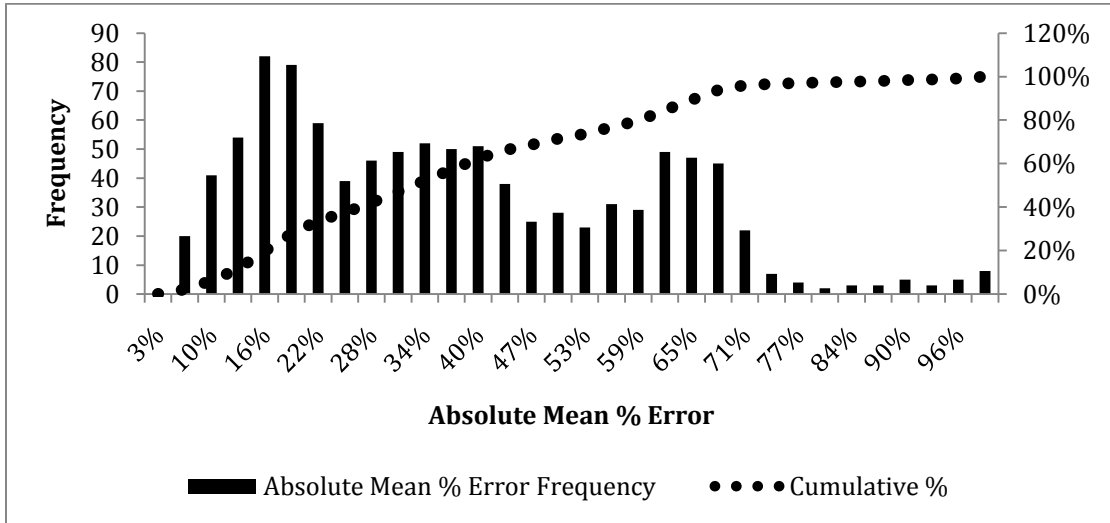


**Figure D.1: Production Rate Model Fit Absolute Mean % Error From Production Without  
Fixed Cost Variable**

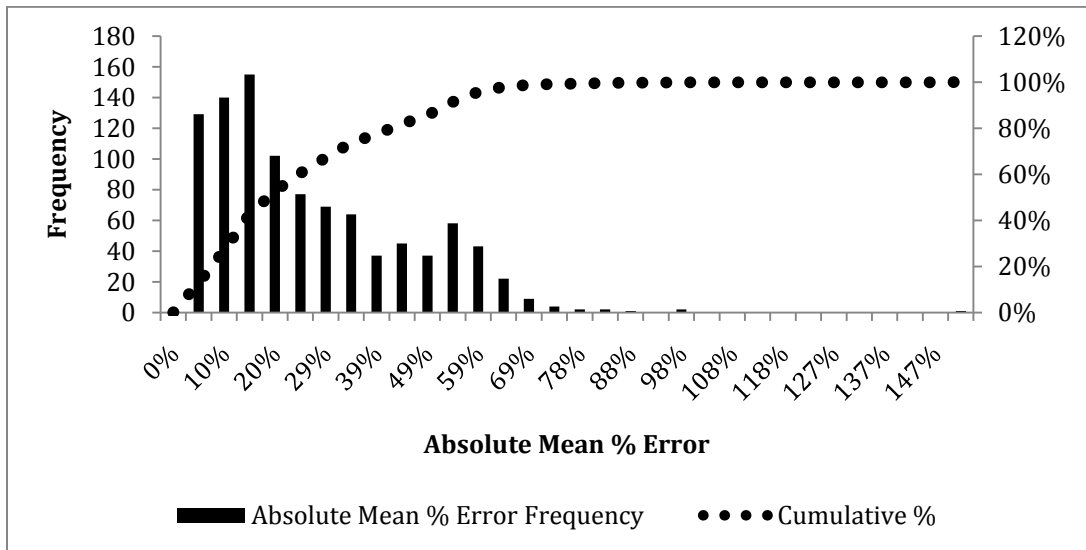


**Figure D.2: Crawford Unit Cost Model Fit Absolute Mean % Error From Production  
Without Fixed Cost Variable**

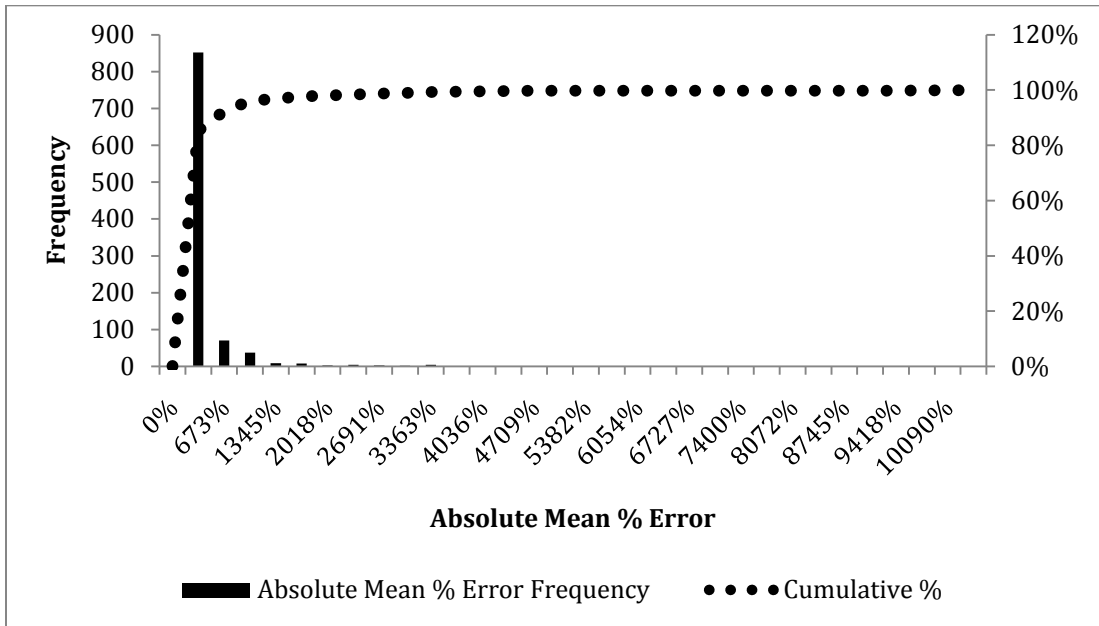
**Appendix E: Model Forecast Absolute Mean % Error Plots From Production with  
Fixed Cost Variable**



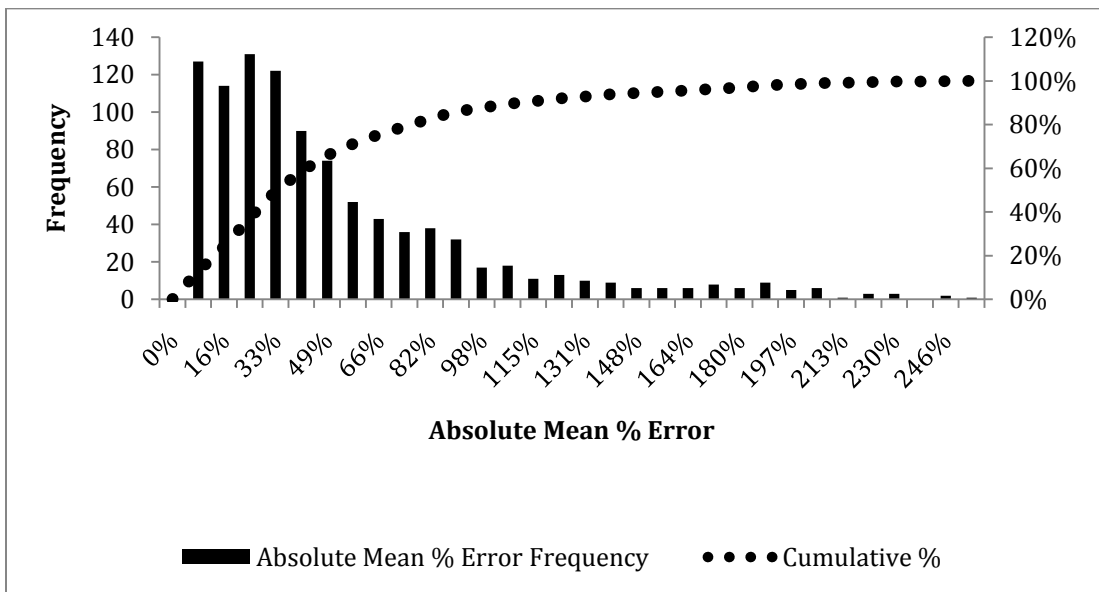
**Figure E.1: Crawford Unit Cost Model Forecast Absolute Mean % Error From Production  
With Fixed Cost Variable**



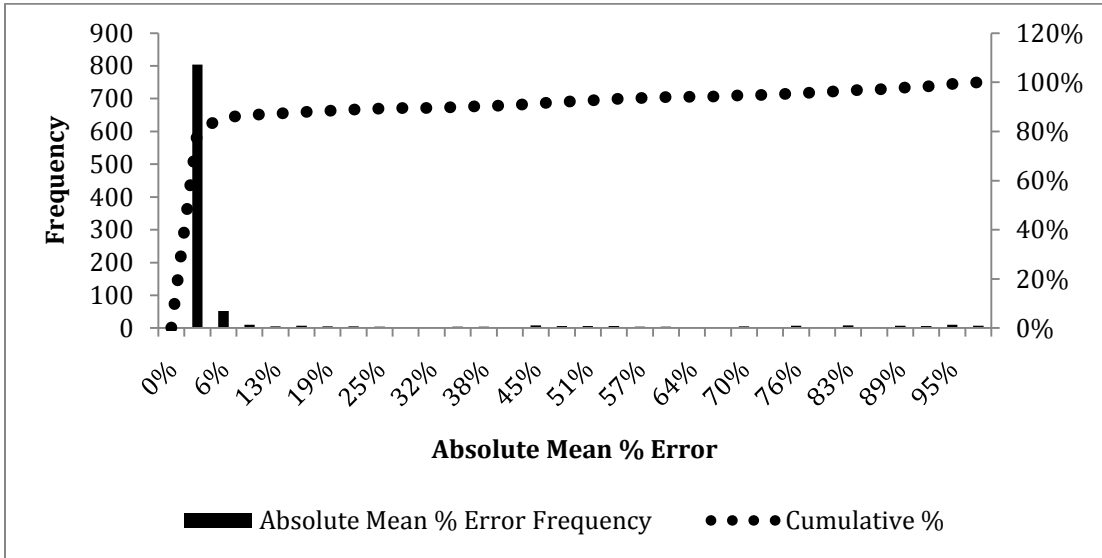
**Figure E.2: Production Rate Model Forecast Absolute Mean % Error From Production  
With Fixed Cost Variable**



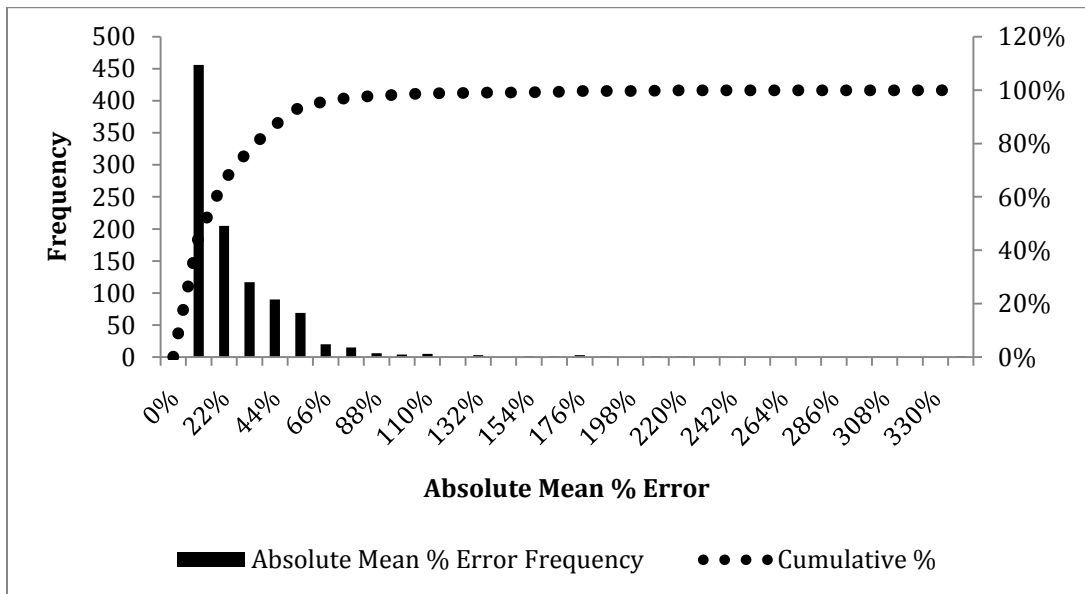
**Figure E.3: Production Rate Adjustment Model Forecast Absolute Mean % Error From Production With Fixed Cost Variable**



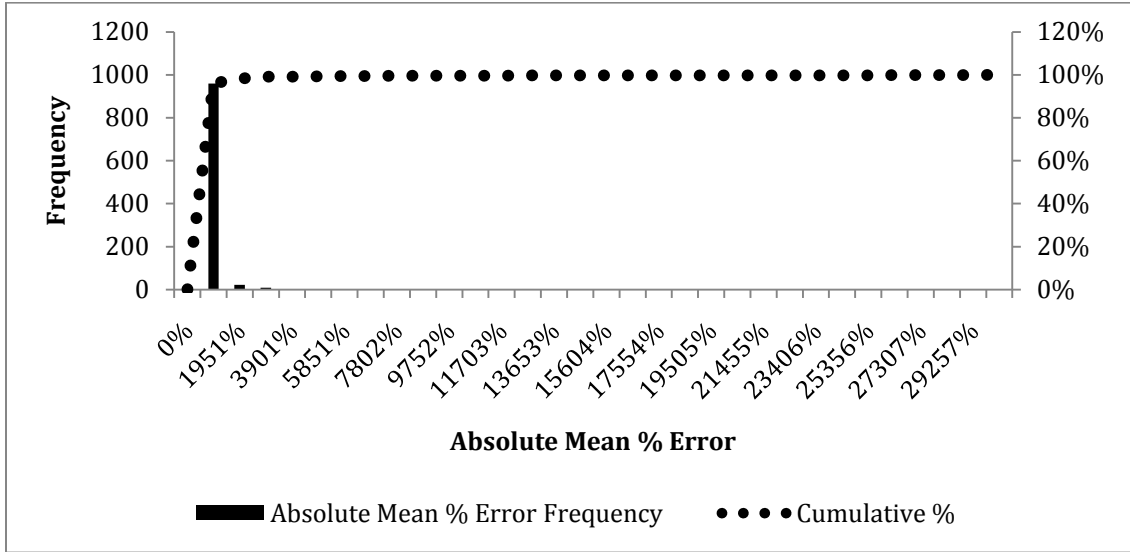
**Appendix F: Model Forecast Absolute Mean % Error Plots From Production  
without Fixed Cost Variable**



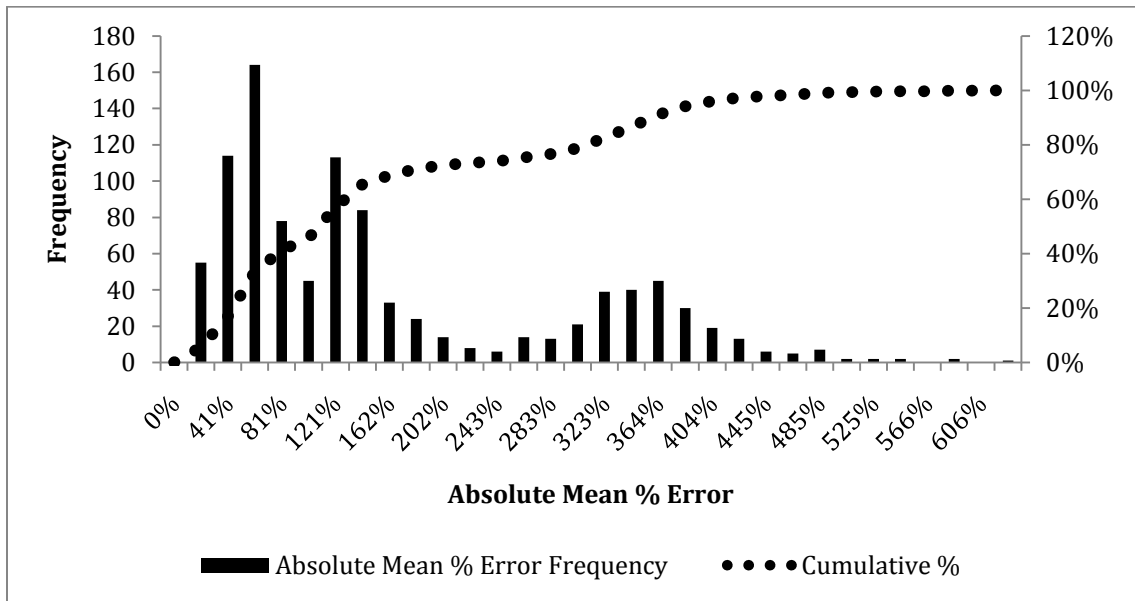
**Figure F.1: Crawford Unit Cost Model Forecast Absolute Mean % Error From Production  
Without Fixed Cost Variable**



**Figure F.2: Production Rate Model Forecast Absolute Mean % Error From Production  
Without Fixed Cost Variable**



**Figure F.3: Production Rate Adjustment Model Forecast Absolute Mean % Error From Production Without Fixed Cost Variable**



**Figure F.4: Fixed Cost Model Forecast Absolute Mean % Error From Production Without Fixed Cost Variable**

## Appendix G: Historical Datasets

Table G.1. F-15 Dataset

Year	Lot Quantity	First Unit	Last Unit	Lot Midpoint	Lot Average Unit Cost (BY03 Million \$'s)
1973	30	1	30	11.9	40.0
1974	62	31	92	58.7	30.6
1975	72	93	164	126.8	26.8
1976	153	165	317	236.8	26.8
1977	108	318	425	370.2	25.5
1978	97	426	522	473.2	27.5
1979	91	523	613	567.4	25.1
1980	98	614	711	662.0	24.8
1981 Estimated Lot	85	712	796	753.7	27.9

**Table G.2. F-15 Dataset (2)**

<b>Year</b>	<b>Lot Quantity</b>	<b>First Unit</b>	<b>Last Unit</b>	<b>Lot Midpoint</b>	<b>Lot Average Unit Cost (BY03 Million \$'s)</b>
1973	30	1	30	11.9	40.0
1974	62	31	92	58.7	30.6
1975	72	93	164	126.8	26.8
1976	153	165	317	236.8	26.8
1977	108	318	425	370.2	25.5
1978	97	426	522	473.2	27.5
1979	91	523	613	567.4	25.1
1980	98	614	711	662.0	24.8
1981	85	712	796	753.7	27.9
1982 Estimated Lot	38	797	834	815.5	32.4

**Table G.3. F-16 Dataset**

<b>Year</b>	<b>Lot Quantity</b>	<b>First Unit</b>	<b>Last Unit</b>	<b>Lot Midpoint</b>	<b>Lot Average Unit Cost (BY03 Million \$'s)</b>
1978	202	1	202	76	16.2
1979	250	203	452	319	12.3
1980	287	453	739	590	12.6
1981	276	740	1015	874	12.7
1982	242	1016	1257	1134	13.2
1983	168	1258	1425	1341	14.3

**Table G.4. F-18 Dataset**

<b>Year</b>	<b>Lot Quantity</b>	<b>First Unit</b>	<b>Last Unit</b>	<b>Lot Midpoint</b>	<b>Lot Average Unit Cost (BY03 Million \$'s)</b>
1979	9	1	9	4	89.2
1980	25	10	34	20	60.9
1981	79	35	113	69	38.5
1982	87	114	200	155	33.5
1983	125	201	325	260	28.8
1984	134	326	459	390	26.2
1985	145	460	604	530	23.4
1986	138	605	742	672	22.6
1987	109	743	851	796	22.4

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14. ABSTRACT  As military and other governmental budgets decline and impacted project deadline changes require instantaneous responses, cost analysts' tasks become more and more formidable. Inaccurate estimates can lead to misappropriation of resources and can thus create delays in goods reaching warfighters. This thesis aims to avail cost estimators of more reliable projection tools and to challenge the status quo of cost estimating, the production rate cost improvement model, when programs face reductions in lot quantities. The findings reveal that the status quo proves efficient under many cost profiles, but clearly does not estimate as well when a program suffers lot quantity reduction coupled with loss of cost efficiency. Prior research recognized the importance of changes in lot quantity to cost estimating, but definitive guidance never surfaced with regards to choosing a model. Monte Carlo simulation allows us to vary cost-affecting variables and isolate conditions where the use of a fixed cost, cost improvement model provides more accurate estimates than does the status quo. While no model for estimation should be discounted without exploration of its usefulness, we argue that the fixed cost model should be considered for use based on its ability to predict increases in average unit cost.					
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