Categories of Situations for Level 2 Fusion

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Abstract - ***In this paper we use category theory to define a situation. We seek a mathematical formalism to discuss a situation which will enable tightly integrated sensor management and planning systems.***

Keywords: Category Theory, Situation, Action

1 Introduction

The Air Force Research Laboratory (AFRL) is refining its basic research portfolios in response to the emerging requirements of Network-Centric Warfare. As guided by Air Combat Command, Air Force Special Operations, and counter-Weapons of Mass Destruction efforts, AFRL is seeking to stimulate research in Information Forensics and Process Integration with the goal of tightening the integration of data collection services and the military's hierarchical planning systems.

Active Intelligence, Surveillance, and Reconnaissance (ISR) techniques are of particular interest—specifically, intrusive techniques that are reliably informative though subtle and do not exceed the degree of confrontation required by the circumstances. Targets that use evasive tactics must be flushed out and engaged rapidly, often in the presence of innocents. Targets hit with non-kinetic weapons must be probed to assess and even elicit interpretable behaviors to ensure that desired effects have been achieved.

Networked Operations require skill sets beyond message passing and subscription services; AFRL is exploring the mathematics and semiotics that enable confirmatory sensing and interrogation, information forensics, authoritative presentation, learning for prediction, and formalisms addressing the composition of information to discern intent and innovation. Interrogation techniques address distributed sensing strategies that subtly pulse sources for actionable information—strategies that draw out the enemy and reveal their intent and weaknesses in order to exploit them.

In data-rich environments like the Global Information Grid, the tactical operator (or analyst) is working with short time lines and limited resources with which to access "heads up" information. Tactical operators cannot tolerate distractions so the challenge is to resolve large amounts of data from a diverse set of sensors and compose that data into readily assessable, usable forms. The point of these systems is to provide an expanded sense of presence and to present new information so that it can be fluidly assimilated into the operator's world model. Friendly sources of confusion and contradiction will be facts of life within network-centric warfare and must be cleverly addressed to prevent fratricide. In networked operations, the Air Force will take on the primary responsibility of mining local data collections (performed by the Army, Navy, Marines, Coast Guard, etc.) to resolve the bird's eye view of the operational battlefield. The Air Force's success in meeting this responsibility will determine the power of our operational networks and their utility for many and varied users.

1.1 Motivation

Rigorous studies and applications of "situational awareness", "situational analysis", and "situational assessment"are being researched. To improved on these we believe that one needs a definition of situation that the entire community can embrace. It is this quest that motivates us. In addition, we believe that category theory will help in the description. We wish to create a definition of situation that is mathematical in nature, yet captures the essence of the definition.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **JUL 2006**	2. REPORT TYPE	3. DATES COVERED **00-00-2006 to 00-00-2006**

4. TITLE AND SUBTITLE **Categories of Situations for Level 2 Fusion**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Institute of Technology,Department of Mathematics and Statistics,2950 Hobson Way,Wright-Patterson AFB,OH,45443-7765**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
9th International Conference on Information Fusion, 10-13 July 2006, Florence, Italy. Sponsored by the International Society of Information Fusion (ISIF), Aerospace & Electronic Systems Society (AES), IEEE, ONR, ONR Global, Selex - Sistemi Integrati, Finmeccanica, BAE Systems, TNO, AFOSR's European Office of Aerospace Research and Development, and the NATO Undersea Research Centre.

14. ABSTRACT
see report

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

1.2 Problem Statement

Create a definition of situation that is general enough that one can study the properties of a situation without having to study the specific situation. Demonstrate how category theory aids in this generalization.

2 Background

In this section we give a short literature review of definitions of situation, then some background on category theory.

2.1 Definitions of a Situation

What is a situation? We give some definitions from the literature.

Dictionary

From the Merriam-Webster's Medical Dictionary, © 2002 Merriam-Webster, Inc.

situation n.

1. the general state of things; the combination of circumstances at a given time; "the present international situation is dangerous"; "wondered how such a state of affairs had come about"; "eternal truths will be neither true nor eternal unless they have fresh meaning for every new social situation"- Franklin D. Roosevelt [syn: state of affairs]
2. a condition or position in which you find yourself; "the unpleasant situation (or position) of having to choose between two evils"; "found herself in a very fortunate situation" [syn: position]
3. a complex or critical or unusual difficulty; 'the dangerous situation developed suddenly"; "that's quite a situation"; "no human situation is simple"
4. physical position in relation to the surroundings; "the sites are determined by highly specific sequences of nucleotides" [syn: site]
5. a job in an organization; "he occupied a post in the treasury" [syn: position, post, berth, office, spot, billet, place]

Computer Science

In situational calculus, situation is defined as structured part of the reality that an agent manages to pick out and/or to individuate. (See [6])

Nourani's [11] definition is: a situation consists of a nonempty set D, the domain of the situation, and two mappings: g, h, where g is a mapping of function letters into functions over the domain as in standard model theory. The mapping h maps each predicate letter, pn, to a function from Dn to a subset of $\{t, f\}$, to determine the truth value of atomic formulas as defined below. The logic has four truth values: the set of subsets of $\{t, f\}$ specifically, $\{\{t\}, \{f\}, \{t, f\}, \varnothing\}$. The latter two corresponding to inconsistency, and lack of knowledge of whether it is true or false.

FUSION Conferences

Roy [14] does not define situation, but considers five basic situation elements:

Environment - which is not defined;

Entity - an existing thing (as contrasted with its attributes), something that has independent, separate, self-contained, and/or distinct existence and objective or conceptual reality;

Event - something that happens (especially a noteworthy happening);

Group - a number of individuals (entities and/or events) assembled together or having some unifying relationship, i.e. an assemblage of objects/events regarded as a unit;

Activity - embedding the ideas of action, movement and motion. The term activity is appropriate when something has the quality or state of being active, i.e., when something is characterized by action or expressing action as distinct from mere existence or state.

Maupin and Jousselme [7] point out the Roy's model omits to mention agents and processes that are central notions around which situation awareness can be articulated.

Other Literature

Pew [13] defines situation as the following: "A situation is a set of environmental conditions and system states with which the participant is interacting that can be characterized uniquely by a set of information, knowledge, and response options."

McMichael and Jarrad [9] define a situation to be the estimates states. They construct a situation tree.

Joint Directors Laboratory

The JDL definition of Situation Assessment (SA) is the "estimation and prediction of entity states on the basis of inferred relations among entities." Situations associate observation with consequence and enable us to make connections between what we want to do and what we might know. When a situation is well understood, it is possible to identify actionable information—i.e., observable precursors to events of consequence—and to convey this information with authority and effectiveness. From the JDL's definition of SA, we infer their definition of situation to be a collection of entities with relations between the individual entities.

2.2 Category Theory

The branch of mathematics known as Category Theory quite naturally takes advantage of relationships and objects. In fact, the basic definition of a category includes a definition of a directed graph as well. Other useful elements will become apparent later, but exploring the full power of category theory in order to produce a theory of fusion is part of the research. In this section, we have drawn upon various authors' presentations to explain the basics of category theory [1, 2, 3, 4].

Definition 1 (Category) *A category $\mathcal{C}$ consists of the following:*

A1. A collection of objects denoted $\mathbf{Ob}(\mathcal{C})$.

A2. A collection of arrows denoted $\mathbf{Ar}(\mathcal{C})$.

A3. Two mappings, called Domain (*dom*) and Codomain (*cod*), which assign to an arrow $f \in \mathbf{Ar}(\mathcal{C})$ a domain and codomain from the elements of $\mathbf{Ob}(\mathcal{C})$. Thus, for arrow f, given by $O_1 \xrightarrow{f} O_2$, $dom(f) = O_1$ and $cod(f) = O_2$.

A4. A mapping assigning each object $O \in \mathbf{Ob}(\mathcal{C})$ an unique arrow 1_O called the identity arrow, such that

$$O \xrightarrow{1_O} O$$

and such that for any existing element x of O, we have that

$$x \xmapsto{1_O} x.$$

A5. A map, $\circ$, called composition, $\mathbf{A} \times \mathbf{A} \xrightarrow{\circ} \mathbf{A}$. Thus, given $f, g \in \mathbf{Ar}(\mathcal{C})$ with $cod(f) = dom(g)$ there exists an unique $h \in \mathbf{Ar}(\mathcal{C})$ such that $h = g \circ f$.

Notice that Axioms A1 - A3 define a directed graph, where the objects are the nodes and the arrows are the directed edges of the graph. Axioms A3-A5 lead to the associative and identity rules:

- **Associative Rule**. Given appropriately defined arrows f, g and $h \in \mathbf{Ar}(\mathcal{C})$ we have that

$$(f \circ g) \circ h = f \circ (g \circ h). \tag{1}$$

- **Identity Rule**. Given arrows $A \xrightarrow{f} B$ and $B \xrightarrow{g} A$, then there exists 1_A such that $1_A \circ g = g$ and $f \circ 1_A = f$.

Definition 2 (Subcategory) *A subcategory $\mathcal{B}$ of the category $\mathcal{A}$ is a category whose objects are some of the objects of $\mathcal{A}$ and whose arrows are some of the arrows of $\mathcal{A}$, such that for each arrow f in $\mathcal{B}$, $dom(f)$ and $cod(f)$ are in $\mathbf{Ob}(\mathcal{B})$, along with each composition of arrows, and an identity arrow for each element of $\mathbf{Ob}(\mathcal{B})$.*

A category of interest is the category **Set**. The objects of **Set** are sets, its arrows are all total functions, and the composition is usual composition of functions. Clearly this construct has identity arrows and the associative rule applies, so it is, indeed, a category.

Another useful categorical construct is a **functor**.

Definition 3 (Functor) *A **functor** $\mathfrak{F}$ between two categories $\mathcal{A}$ and $\mathcal{B}$ is a pair of mappings $\mathfrak{F} = (\mathfrak{F}_{\mathbf{Ob}}, \mathfrak{F}_{\mathbf{Ar}})$ such that*

$$\mathbf{Ob}(\mathcal{A}) \xrightarrow{\mathfrak{F}_{\mathbf{Ob}}} \mathbf{Ob}(\mathcal{B}) \tag{2}$$

$$\mathbf{Ar}(\mathcal{A}) \xrightarrow{\mathfrak{F}_{\mathbf{Ar}}} \mathbf{Ar}(\mathcal{B})$$

while preserving the associative property of the composition map and preserving identity maps.

Thus, given categories $\mathcal{A}, \mathcal{B}$ and functor $\mathfrak{F} : \mathcal{A} \to \mathcal{B}$, if $A \in \mathbf{Ob}(\mathcal{A})$ then there exists a $B \in \mathbf{Ob}(\mathcal{B})$ such that

(i) $\mathfrak{F}_{\mathbf{Ob}}(\mathrm{A}) = \mathrm{B}$.

Given arrows $f, g, h, 1_A \in \mathbf{Ar}(\mathcal{A})$ such that $f \circ g = h$ is defined, then there exists arrows $f', g', h', 1_{\mathrm{B}} \in \mathbf{Ar}(\mathcal{B})$ such that

(ii) $\mathfrak{F}_{\mathbf{Ar}}(f) = f'$, $\mathfrak{F}_{\mathbf{Ar}}(g) = g'$.

(iii) $h' = \mathfrak{F}_{\mathbf{Ar}}(h) = \mathfrak{F}_{\mathbf{Ar}}(f \circ g) = \mathfrak{F}_{\mathbf{Ar}}(f) \circ \mathfrak{F}_{\mathbf{Ar}}(g) = f' \circ g'$.

(iv) $\mathfrak{F}_{\mathbf{Ar}}(1_{\mathrm{A}}) = 1_{\mathfrak{F}_{\mathbf{Ob}}(\mathrm{A})} = 1_{\mathrm{B}}$.

In general, if a functor between two categories of fusion can be developed or discovered, it could possibly demonstrate an isomorphism between the two.

Finally, we need the definition of a natural transformation between functors.

Definition 4 (Natural Transformation) *Given categories $\mathcal{A}$ and $\mathcal{B}$ and functors $\mathfrak{F}$ and $\mathfrak{G}$ with $\mathcal{A} \xrightarrow{\mathfrak{F}} \mathcal{B}$ and $\mathcal{A} \xrightarrow{\mathfrak{G}} \mathcal{B}$, then a **Natural Transformation** is a family of arrows $\nu = \{\nu_{\mathrm{A}} | \mathrm{A} \in \mathcal{A}\}$ such that for each $f \in \mathbf{Ar}(\mathcal{A})$, $\mathrm{A} \xrightarrow{f} \mathrm{A}'$, $\mathrm{A}' \in \mathcal{A}$, the square*

$$\begin{tikzcd} \mathfrak{F}(\mathrm{A}) \arrow[r, "\nu_A"] \arrow[d, "\mathfrak{F}(f)"'] & \mathfrak{G}(\mathrm{A}) \arrow[d, "\mathfrak{G}(f)"] \\ \mathfrak{F}(\mathrm{A}') \arrow[r, "\nu_{\mathrm{A}'}"] & \mathfrak{G}(\mathrm{A}') \end{tikzcd} \tag{3}$$

commutes. We then say the arrows $\nu_{\mathrm{A}}, \nu_{\mathrm{A}'}$ are the components of $\nu : \mathfrak{F} \longrightarrow \mathfrak{G}$, and call ν the natural transformation of $\mathfrak{F}$ to $\mathfrak{G}$.

Definition 5 (Functor Category $\mathcal{A}^{\mathcal{B}}$) *Given categories $\mathcal{A}$ and $\mathcal{B}$, the notation $\mathcal{A}^{\mathcal{B}}$ denotes the category of all functors $\mathfrak{F}$, $\mathcal{B} \xrightarrow{\mathfrak{F}} \mathcal{A}$. This category has all such functors as objects and the natural transformations between them as arrows.*

Definition 6 (Product Category) *Let $\{\mathcal{C}_i\}_{i=1}^n$ be a finite collection of categories, then*

$$\prod_{i=1}^{n} \mathcal{C}_i = \mathcal{C}_1 \times \mathcal{C}_2 \times \cdots \times \mathcal{C}_n$$

is the corresponding product category. The objects are Cartesian products of objects (i.e., ordered n-tuples of object)

$$\mathbf{Ob}(\prod_{i=1}^{n} \mathcal{C}_i) = \{(O_1, O_2, \ldots, O_n) : O_k \in \mathbf{Ob}(\mathcal{C}_k) \forall k\}$$

the arrows are Cartesian products of arrows (i.e., ordered n-tuples of arrows)

$$\mathbf{Ar}(\prod_{i=1}^{n} \mathcal{C}_i) = \{(f_1, f_2, \ldots, f_n) : f_k \in \mathbf{Ar}(\mathcal{C}_k) \forall k\}$$

and the composition $\overset{\pi}{\circ}$ is

$$\begin{aligned} & (f_1, f_2, \ldots, f_n) \overset{\pi}{\circ} (g_1, g_2, \ldots, g_n) \\ = \ & \left(f_1 \overset{1}{\circ} g_1, f_2 \overset{2}{\circ} g_2, \ldots, f_n \overset{n}{\circ} g_n \right). \end{aligned}$$

3 Main Results

We build our definition of situation upon the JDL's definition which we infer to be a collection of entities with a collection of relations between the entities.

Let $\mathcal{E}$ be a collection of entities so that $e \in \mathcal{E}$ is an individual entity. Let $\mathscr{E}$ be an algebra of subsets of $\mathcal{E}$, possibly a σ-algebra of subsets of $\mathcal{E}$, then $(\mathcal{E}, \mathscr{E})$ is a measurable space. An entity can be a physical object or non-physical. The set $E \in \mathscr{E}$ will be a set of entities. Some literature will call this a group (of entities) but *group* means a very special algebraic construction, thus, we will not use the word group to refer to a set of entities. The algebra of subsets of $\mathcal{E}$ will capture the granularity (or aggregation) that is discussed in [5]. The specific application will dictate what an entity will be. For two sets of entities, $E_1, E_2 \in \mathscr{E}$ there may be a relation f between them. We use an arrow to denote the relation f and represent it with the diagram

$$E_1 \xrightarrow{f} E_2 \ .$$

It is possible that $E_1 = \{e_1\}$, a singleton set. The direction of the arrow is important since it implies a certain relation. There are relations that are unary that "point" back to itself

$$E \xrightarrow{f} E \ .$$

The relations between entities that tie the entities together is the key idea here. We believe that a question(s) (or queries) will determine the relationships between two entities. For example, given two entities e_1 and e_2, a question q could be, "does $e_1$has information about e_2"? If the answer is yes, then a relationship has been established. We will represents this with an arrow as in the diagram

$$e_1 \xrightarrow{q} e_2$$

More precisely, we will use this diagram to represent this situation. Of course, there are other questions that one can ask of the entity sets. We will consider only the subsets from the algebra of entities $\mathscr{E}$. For this paper we will consider only questions that have affirmative answers or negative answer. Given a question q and entities e_1 and e_2 we need some operation that will determine if the question is answered affirmative or not by entities e_1, e_2 or, better said, the query q concerning entities e_1, e_2 is true (or false). Therefore, we define the label set $\mathcal{L} = \{\text{yes, no}\}$. We define the Truth *answer* mapping (lower case)

$$\begin{aligned} & \operatorname{ans}(q, e_1, e_2) \\ = \ & \begin{cases} \text{yes} & \text{if} \quad \text{query } q \text{ concerning } e_1, e_2 \text{ is true} \\ \text{no} & \text{if} \quad \text{query } q \text{ concerning } e_1, e_2 \text{ is false} \end{cases} . \end{aligned}$$

The domain of definition of the Truth answer mapping has $\mathcal{E}$ (entities) and $\mathcal{Q}$ (queries), so $\operatorname{ans} : \mathcal{Q} \times \mathcal{E} \times \mathcal{E} \to \mathcal{L}$. At this point, we do not specify how $\mathcal{Q}$ is determined and will discuss this later.

We can extend this idea to a collection of entities, E. We say the set $E \in \mathscr{E}$ answers, or concerns the query q if

$$\operatorname{Ans}(q, E) = yes$$

and similarly, say the set $E \in \mathscr{E}$ does not concern the query q if

$$\operatorname{Ans}(q, E) = no.$$

We extend the notation as well and define the Truth answer mapping (upper case)

$$\operatorname{Ans}(q, E) = \begin{cases} \text{yes} & \text{if} \quad E \text{ answers } q \\ \text{no} & \text{if} \quad E \text{ does not answers } q \end{cases}$$

The mapping Ans will be used since the singleton set $E = \{e\}$ is equivalent to the entity e. Thus,

$$\operatorname{Ans} : \mathcal{Q} \times \mathscr{E} \to \mathcal{L}.$$

Definition 7 *Let $q \in \mathcal{Q}$ be a question. The two entities $e_1, e_2 \in \mathcal{E}$ are said to be related if they answer the question q affirmatively. That is,* $\operatorname{ans}(q, e_1) = yes$ *and* $\operatorname{ans}(q, e_2) = yes$.

Two entity sets $E_1, E_2 \in \mathscr{E}$ are said to be related if they answer the question q affirmatively. That is, $\operatorname{Ans}(q, E_1) = yes$ *and* $\operatorname{Ans}(q, E_2) = yes$.

Remarks 8 *Given a question q the collection of entities relevant to q yield a fully connected graph where the nodes are the entities and the edges are the relationships generated by the single question.*

It follows that a collection of questions and a collection of entities should generate relations between entities. Furthermore, not every pair of entities may have a relations. Thus, the corresponding diagram would not be fully connected.

We believe that the use of queries will "build" or "discover" the situation (see [12] for queries that assist is situation assessment.)

Definition 9 (Situation) *Let $(\mathcal{E}, \mathscr{E})$ be a measurable space of entities. A situation $\mathcal{S}$ over $(\mathcal{E}, \mathscr{E})$ is a category $\mathcal{S} = (\mathbf{Ob}(\mathcal{S}), \mathbf{Ar}(\mathcal{S}), \mathbf{Id}(\mathcal{S}), \circ)$ where the:*

- *objects in* $\mathbf{Ob}(\mathcal{S})$ *are entity sets in* $\mathscr{E}$.

- *arrows in* $\mathbf{Ar}(\mathcal{S})$ *are collections of relations between the pairs of entity sets.*

- *identity arrows* $\mathbf{Id}(\mathcal{S})$ *are collections of relations on individual entity sets.*

- *composition* $\circ$ *is a mapping on* $\mathbf{Ar}(\mathcal{S})\times\mathbf{Ar}(\mathcal{S})$ *into* $\mathbf{Ar}(\mathcal{S})$ *such that for* $\mathbf{f},\mathbf{g} \in \mathbf{Ar}(\mathcal{S})$ *with* $cod(\mathbf{f}) = dom(\mathbf{g})$ *there exists an unique* $\mathbf{h} \in \mathbf{Ar}(\mathcal{C})$ *such that* $\mathbf{h} = \mathbf{g} \circ \mathbf{f}$.

We use the bold symbols to denote a collection of relations, e.g., $\mathbf{f} = \{f_1, f_2, \ldots, f_L\}$ and $\mathbf{g} = \{g_1, g_2, \ldots, g_M\}$ then $\mathbf{h} = \{h_1, h_2, \ldots, h_N\}$ where h_n is the relation formed from two or more relations. For example, maybe $h_1 = \{f_1, f_3\} \circ \{g_2, g_3, g_7\}$. Thus, the single relation h_1 is the result of combining the relations f_1, f_3 that have domain, say, A (a set of entities) and codomain B (another set of entities), with the relations g_2, g_3, g_7 that have domain B and codomain C, then the (single) relation h_1 has domain A and codomain C. Notice that the cardinality of the collections are not the same.

An identity arrow for the object $E \in \mathscr{E}$, denoted $\mathbf{1}_E$, is a collection of unary relations on E, that is, $\mathbf{1}_E = \{1_E^{(1)}, 1_E^{(2)}, \ldots, 1_E^{(L))}\}$ (the positive integer L depends on the specific set E.) such that for any arrow $\mathbf{f} \in \mathbf{Ar}(\mathcal{S})$ with domain E and codomain E' then

$$\begin{aligned}
\mathbf{1}_E \circ \mathbf{f} &= \{1_E^{(1)}, 1_E^{(2)}, \ldots, 1_E^{(L)}\} \circ \{f_1, f_2, \ldots, f_L\} \\
&= \{1_E^{(1)} \overset{1}{\circ} f_1, 1_E^{(2)} \overset{2}{\circ} f_2, \ldots, 1_E^{(L)} \overset{L}{\circ} f_L\} \\
&= \{f_1, f_2, \ldots, f_L\} \\
&= \mathbf{f}
\end{aligned}$$

and for $\mathbf{1}_{E'} = \{1_{E'}^{(1)}, 1_{E'}^{(2)}, \ldots, 1_{E'}^{(L)}\}$

$$\begin{aligned}
\mathbf{f} \circ \mathbf{1}_{E'} &= \{f_1, f_2, \ldots, f_L\} \circ \{1_{E'}\} \\
&= \{f_1 \circ 1_{E'}^{(1)}, f_2 \circ 1_{E'}^{(2)}, \ldots, f_L \circ 1_{E'}^{(L)}\} \\
&= \{f_1, f_2, \ldots, f_L\} \\
&= \mathbf{f}.
\end{aligned}$$

The converse of the statement above is the following. Specifically, "a collection of entities that have relationships suggest a collections of questions".

Next we consider actions. Let a denote a mapping that takes an entity $e \in \mathcal{E}$ as an input and yields another event $e' \in \mathcal{E}$ and its output. We write this as $a : \mathcal{E} \to \mathcal{E}$. How does an action change a collection of entities, and its relationships? In particular, how does an action change (or effect) a situation? The action may (or may not) change the entities, and may (or may not) change the relationships between the entities.

Definition 10 (Effect) *Let* a *be an action and* $\mathcal{S}$ *be a situation. Define the effect mapping by*

$$\mathrm{Eff}(a, \mathcal{S}) = \mathcal{S}'$$

that yields a new situation $\mathcal{S}'$.

The idea of the effect mapping is to produce a new situation. The new situation $\mathcal{S}'$ could have new relations between the previous entities, or maybe relations are removed. The new situation could have new entities not in situation $\mathcal{S}$, or maybe entities are removed. To determine if the action really caused a change one needs a mapping that determines the difference between two situations. Let Diff denote a difference mapping between two situations, so that

$$\mathrm{Diff}(\mathcal{S}, \mathcal{S}')$$

is a nonnegative real number that quantifies the difference. We desire that this mapping to be symmetric and positive definite.

Definition 11 (Difference) *Let* $\mathcal{S}, \mathcal{S}'$ *be situations and* Diff *satisfy the following properties:*

1. $\mathrm{Diff}(\mathcal{S}, \mathcal{S}') \geq 0$, *(nonnegativity);*
2. $\mathrm{Diff}(\mathcal{S}, \mathcal{S}') = \mathrm{Diff}(\mathcal{S}', \mathcal{S})$, *(symmetry).*
3. $\mathrm{Diff}(\mathcal{S}, \mathcal{S}') > 0$ *if and only if* $\mathcal{S} \neq \mathcal{S}'$, *(positive definite).*

Therefore, if

$$\mathrm{Diff}(\mathcal{S}, \mathrm{Eff}(a, \mathcal{S})) = 0$$

then action a did not cause the situation to change. If

$$\mathrm{Diff}(\mathcal{S}, \mathrm{Eff}(a, \mathcal{S})) > 0$$

then action a did cause the situation to change by an amount d. It appears that the difference mapping is almost a metric.

Remarks 12 *An action* a *may not effect all entities and relations in the situation. Thus, some actions have no effect.*

An example of a Diff mapping is to minimize over all functors $\mathfrak{F}$ that act on situation $\mathcal{S}$,

$$\begin{aligned}
&\min_{\mathfrak{F}} |card(\mathbf{Ob}(\mathfrak{F}(\mathcal{S}))) - card(\mathbf{Ob}(\mathcal{S}'))| \\
&+ |card(\mathbf{Ar}(\mathfrak{F}(\mathcal{S}))) - card(\mathbf{Ar}(\mathcal{S}'))|.
\end{aligned}$$

Suppose one chooses the sequence of actions $\{a_1, a_2, ..., a_N\}$ to act on the original situation $\mathcal{S}_0$ then the result would be a sequence of situations

$$\{\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_N\}.$$

4 Conclusions

Category theory can be used to define a situation and represent it. Thereby, one can manipulate operations on situations and represent these operations as well.

Acknowledgements

We wish to acknowledge the financial support of the U.S. Air Force Research Laboratory, Sensor Directorate, Automatic Target Recognition Branch (AFRL/SNAT) at Wright-Patterson AFB, Ohio; the Air Force Office of Scientific Research (AFOSR) at Arlington, Virginia; and the Air Force Air Combat Command (ACC) at Langley AFB, Virginia.

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