

# Probabilistic Multi-Hypothesis Tracking for Distributed Multi-Static Active Sonar

Christian G. Hempel, Ph.D

(401) 832-8648

[hempelcg@npt.nuwc.navy.mil](mailto:hempelcg@npt.nuwc.navy.mil)

Naval Undersea Warfare Center Division Newport

1176 Howell Street

Newport, Rhode Island 02841-1708

*Abstract – The PMHT algorithm is sufficiently mature to be applicable to mono-static active sonar in [1] and other researchers have investigated PMHT for multi-sensor systems. An important modification of PMHT to utilize echo amplitude information in addition to range and bearing measurements was developed in [2]. In multi-static active systems different source/receiver combinations will observe the same target at different aspect angles and different signal to noise ratios. Any multi-static tracking algorithm attempting to use amplitude information must model the effects of target aspect on the amplitudes of the observed echoes. This paper presents a scheme to use an aspect dependent model of target strength to adapt the parameters in the distribution for target echo amplitude. Results on simulated data demonstrating the improvement that can be achieved with the new method are also presented.*

## 1. Introduction

### 1.1 Background

The Probabilistic Multi-Hypothesis Tracking (PMHT) algorithm first developed by Striet and Luginbuhl [3] has been extended in [4] and [5] to better cope with the practical issues of track initialization, clutter, contact maneuvers, and track management. Using these methods the author has successfully applied PMHT to monostatic active sonar applications, see [1]. In addition to these issues multi-static active tracking systems must also select an appropriate processing architecture, model data registration errors, exploit knowledge of the sensor coverage regions, and account for the variability of target SNR observed by different source/receiver combinations.

Multi-static tracking systems can be built on three different processing architectures: merged, blocked, and

sequential. In the merged architecture all of the registered measurements from each receiver (i.e., detections or clusters referenced to a common point) are collected into one data structure and used to update the existing tracks and initialize new tracks. The PMHT algorithm can employ a merged framework, see [6], because it allows for the possibility that more than one measurement is from each target. The merged architecture is the easiest to implement but it ignores any possible correlation between measurements. The blocked measurement architecture separates the measurements according to source, receiver, and waveform and stacked measurement vectors are used to update the tracks. The blocked architecture can accommodate correlated measurements but it requires that the individual measurements be well registered to ensure measurements from the same object are stacked together. The sequential architecture uses sorted measurements to perform multiple updates to the tracks. Each full update cycle consists of one individual update for each type of measurement in the batch. The sequential architecture can also accommodate measurements with correlated errors, see [7], and it may be less sensitive to data registration errors. The PMHT algorithm presented here is based on the sequential architecture.

Modeling the errors arising from the spatial data registration process is an important issue for any practical multi-static system. The ability to bring all of the measurements (i.e., individual waveform detections) arising from all possible combinations of source and receiver to a common frame of reference is essential to successful multi-static target tracking. Modern sonar buoys incorporate GPS and improved compasses that should significantly reduce the errors in the bi-static range and bearing measurements. However, when two or more receiver coverage areas overlap the errors in the

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE <b>JUL 2007</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2007 to 00-00-2007</b>	
4. TITLE AND SUBTITLE <b>Probabilistic Multi-Hypothesis Tracking for Distributed Active Sonar</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Naval Undersea Warfare Center Division Newport, 1176 Howell Street, Newport, RI, 02841-1708</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>9th International Conference on Information Fusion, 10-13 July 2006, Florence, Italy. Sponsored by the International Society of Information Fusion (ISIF), Aerospace &amp; Electronic Systems Society (AES), IEEE, ONR, ONR Global, Selex - Sistemi Integrati, Finmeccanica, BAE Systems, TNO, AFOSR's European Office of Aerospace Research and Development, and the NATO Undersea Research Centre.</b>					
14. ABSTRACT <b>see report</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>5</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

measurements from that area will be correlated due to common source location uncertainty, see [8]. Error in the bi-static range due to inaccurate speed of sound will be modeled as part of the measurement error covariance. For the simulated data considered here the positions of the sources and receivers are all assumed to be known accurately and hence the contribution of the error in source position to the bi-static range is small relative to the other sources of error.

In a multi-static system each combination of source and receiver will be able to detect targets of interest in a different region. In general some of the regions may overlap and there may be sizeable gaps between other regions. Overlapping regions offer the possibility of detecting a target of interest faster and tracking it more accurately by combining the echoes from both regions in one track. Combining measurements in this way also requires a model of target SNR as function of aspect because echo amplitude is used in PMHT as an input to the association weights and the track detection scheme. Different source/receiver combinations will typically observe different values of SNR on the same target and inaccurate modeling of target SNR will degrade the performance of the tracking and track detection methods. Knowledge of the gaps between detection regions is also important because it is desirable to propagate tracks of legitimate targets from one detection region to the next. Without accurate knowledge of the detection regions most track detection schemes would kill any track that exits a detection region.

## 1.2 Purpose

The study reported here investigates the effect of target aspect on target SNR and tracking performance in distributed multi-static active sonar systems where the data is centrally collected and registered to a common frame of reference. The case where a significant amount of clutter is present is of particular interest. A method to use estimates of target aspect derived from track state estimates and a model of bi-static target strength to adapt the parameters in the distribution for target echo amplitude is presented. The tracking performance improvement afforded by the proposed method is quantified by a Monte Carlo study using simulated multi-static data with injected clutter.

## 2. PMHT for Distributed Active Systems

### 2.1 Mono-static PMHT

In this section the elements of PMHT that are relevant to subsequent sections of this paper will be presented. The derivation of the original PMHT algorithm is well described in [3] and is based on the so called independent assignment model; each measurement has some non-zero prior probability of being from any one of the targets present independent of the origin of all the other measurements. Under this assignment model it is entirely possible for all of the measurements to

originate from any one of the targets but that hypothesis is almost always far less likely than more sensible assignments. The advantage of the independent assignment model is that when it is used in conjunction with the Expectation Maximization method it avoids having to enumerate a large number of candidate measurement assignment hypotheses and instead only requires the calculation of the posterior probabilities that the  $r$ 'th measurement at time  $t$  originated from target  $s$  as

$$(1) \quad w_{str} = \frac{\pi_s N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\sum_{m=1}^M \pi_m N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})}$$

In [2] and [4] the above formula is modified to employ amplitude information and account for uniformly distributed clutter

$$(2) \quad w_{str} = \frac{\pi_s f_1(\mathbf{a}_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\frac{\pi_0}{V} f_0(\mathbf{a}_{rt}) + \sum_{m=1}^M \pi_m f_1(\mathbf{a}_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})}$$

where  $V$  is the volume of the association gate and  $f_0(\mathbf{a}_{rt})$  and  $f_1(\mathbf{a}_{rt})$  are the distributions for the echo amplitudes for clutter and target respectively. In this study the target echo amplitudes are Rayleigh distributed;

$$(3) \quad f_1(\mathbf{a}) = \frac{\pi \mathbf{a}}{2(1+\theta)} e^{-\pi \mathbf{a}^2 / 2(1+\theta)}$$

and the clutter distribution is a mixture of a unit mean Rayleigh and a non-unit mean Rayleigh;

$$(4) \quad f_0(\mathbf{a}) = \frac{\alpha \pi \mathbf{a}}{2} e^{-\pi \mathbf{a}^2 / 4} + \frac{(1-\alpha) \pi \mathbf{a}}{2(1+\theta_c)} e^{-\pi \mathbf{a}^2 / 2(1+\theta_c)}$$

The heavier tailed Rayleigh component is used to model the presence of high amplitude clutter in the normalized data. As described in [9] the basic PMHT algorithm amounts to iterating the following three steps:

1. Compute the association weights,  $w_{str}$ , for each measurement and target at each time step in batch.
2. Using the weights compute a measurement centroid and associated error covariance matrix (a.k.a. the synthetic measurement and covariance) for each target at each time step in the batch.
3. Update the track (i.e., the batch sequence of state estimates) for each target with a Kalman smoother on the synthetic measurements and error covariance matrices.

### 2.2 Multi-static PMHT

In distributed multi-static active sonar systems the variability of target SNR observed by different source/receiver combinations is determined by a number

of factors including source and receiver capabilities, sensor and target geometry and environmental acoustic conditions. All of these factors play a role in determining whether or not a target is in a location where it can be detected by multiple sensors. When a target can be detected by more than one sensor it is natural to want to combine the measurements from the different sensors to achieve the best possible tracking performance. If the measurements are collected at some central location and registered to a common frame of reference then it is possible to use all of them in one tracking algorithm. This approach, however, is more complicated in multi-static systems because each sensor will typically observe the target at a different SNR due to differing propagation paths, interference levels, and target aspects. Figure 1 is a plot of mono-static target strength as a function of aspect from [10]. It shows that there is an 18dB difference in target strength between broadside and forward end fire aspects.

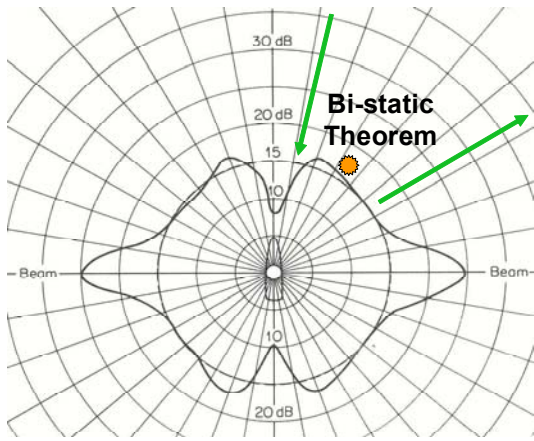


Figure 1. Mono-static target strength vs. aspect.

In the presence of clutter these effects can cause tracking algorithms, including PMHT, that use amplitude information in the data association stage to perform less than optimally. For example, suppose that two identical but separated sensors can detect a target's echo with comparable interference levels and propagation path lengths but significantly different target strengths due to different target aspects. If the same SNR value is used in equation (3) for both sensors then the association weights for the measurements from at least one sensor will be incorrect. Moreover, if a significant amount of clutter is present then it is quite likely that a false measurement will receive a greater association weight than it should. This in turn, can cause loss of track.

Mono-static systems that attempt to use measurements from multiple band separated active waveforms in the same tracking algorithm must contend with the same issue when there are different levels of interference in each frequency band. In [11] the author presents a method to adapt the SNR parameters in equation (3) to differing levels of interference for mono-static systems. This paper applies that method to multi-static systems to adapt to differing target aspects. For simplicity it is assumed that there is one source and two

identical receivers each employing the same waveform detector (e.g., matched filter) and that the differences in interference level and propagation path length observed by each receiver are negligible. It is also assumed that an estimate of course (and hence target aspect) is available from the current track state estimates. The method can be extended to more general multi-static situations but that is the subject of future work. Under these circumstances the difference in observed SNR by each receiver will be due to differing target aspect. The method also assumes that the maximum possible target SNR,  $\theta_{\max}$ , is known. Although this assumption may seem unreasonable the mono-static version of this method is based on a similar assumption and in [11] it is shown that the track detection performance gain is robust to errors in  $\theta_{\max}$ .

Let  $D_1$  and  $D_2$  be the reduction in target strength (in linear scale) from the maximum possible (e.g., broadside aspect) for each receiver as determined by a bi-static target strength model (e.g., Figure 1) using the estimated target aspects. The values for SNR to be used in equation (3) for each receiver are then given by

$$(5) \quad \theta_1 = D_1 \theta_{\max} \text{ and } \theta_2 = D_2 \theta_{\max}$$

When the value for  $\theta_{\max}$  is correct equation (5) will provide the correct SNR values for equation (3) and ensure that the association weights computed using equation (2) are also correct. When the value for  $\theta_{\max}$  is incorrect the SNR values given by equation (5) will also be incorrect but will at least have the correct ratio. In practice this property alone has proven sufficient to provide significant tracking and detection performance gain.

### 3. Tracking Performance Analysis

#### 3.1 Simulated Data

The simulated contact level multi-static data used in this analysis was developed by the NATO Undersea Research Center (NURC), see [12], and consists of two sources, two towed receivers and one target as shown in Figure 2.

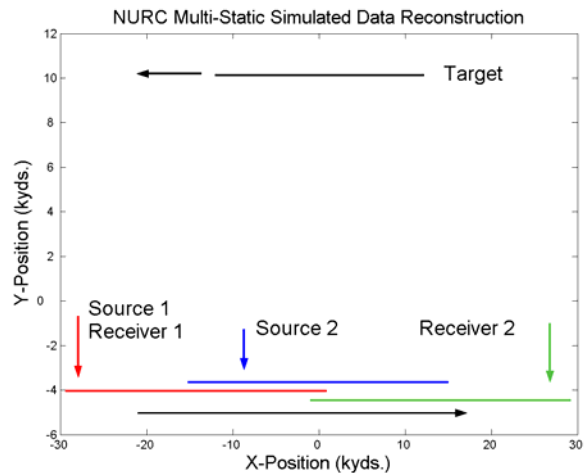


Figure 2. Ground truth information for NURC simulated multi-static data set.

Both sources transmit once per minute and each source and receiving platform maintains a known constant velocity throughout the entire 180 ping scenario. This multi-static system generates three distinct bi-static combinations of source and receiver and one mono-static combination. The contact level measurements are sorted according to source/receiver combination and can be registered to a common frame of reference. However, a relatively small bias of  $1^\circ$  was added to some of the bearing values in addition to random bearing and range errors having known variances. The target also exhibits a perfectly constant velocity trajectory but the implementations of PMHT applied here do not exploit that fact and incorporate a piecewise linear white noise acceleration model with  $\sigma = 0.01\text{yd}/\text{sec}^2$ . The amplitudes of the target are Rayleigh distributed where the SNR parameter for source receiver combination is varied to model the effect of target aspect. The data also contains relatively modest amount of unit mean Rayleigh clutter with spatial density  $\lambda=4.0\text{e-}08$  detections/yard<sup>2</sup>.

In this study the performance of PMHT with and without the adaptive scheme defined by equation (5) is analyzed adding varying amounts of heavy tailed clutter to the NURC simulated multi-static data set. For the adaptive scheme  $\theta_{\max} = 64$  (i.e., 18dB) and bi-static target strength values are obtained by applying the Bi-static Theorem [10] to the mono-static target strength values in figure 1. The amount of heavy tailed clutter is Poisson distributed for a range of spatial densities and the amplitude is Rayleigh distributed with  $\theta_c = 64$ . The spatial density of the heavy tailed clutter was varied from zero to  $2.0\text{e-}07$  detections/yard<sup>2</sup>.

The maximum value for the clutter density corresponds to an average of one clutter detection for every five million square yards. For each value of clutter density 100 Monte Carlo runs were conducted and tracking performance tabulated.

### 3.2 Results

Table 1 shows the results of 100 Monte Carlo runs for both versions of PMHT and each value of clutter density.

Spatial density of heavy tailed clutter (detections/square yard.)	Adaptive PMHT		Non-adaptive PMHT	
	% Tracks dropped	RMS Position error (yds.)	% Tracks dropped	RMS Position error (yds.)
0.0	0	394.2	0	407.0
0.5e-07	1	436.5	6	526.8
1.0e-07	2	475.5	16	564.4
1.5e-07	6	493.0	21	628.0
2.0e-07	11	548.2	28	715.5

Table 1. Monte Carlo tracking performance results for adaptive and non-adaptive PMHT

A track was declared lost if at any update the current state deviated from the ground truth by more than 3000 yards. The RMS position error is computed from the Monte Carlo runs that hold track. The adaptive PMHT demonstrates a clear performance improvement over the non-adaptive version for all nonzero values of heavy tailed clutter density; less than half as many dropped tracks and a substantial reduction in RMS position error over the life of the track. When no heavy tailed clutter is added to the data the difference in tracking performance is negligible. This particular result is not really surprising because throughout the NURC data set at least one combination of source and receiver was providing high SNR target detections.

### 3.3 Conclusions

The results in section 3.2 clearly show that adapting the distribution for target echo amplitude to the observed target aspect improves tracking performance in multi-static active sonar systems. In the near future the author plans to incorporate more sophisticated target strength models and to generalize this method to adapt to differing levels of interference, propagation path length, and environmental conditions. Moreover, analysis of track detection performance is also planned.

### Acknowledgement

This research was supported by the Office of Naval Research under grant Number N0001406WR20122.

### 4. References

- [1] C. Hempel, S. Doran, "A PMHT Algorithm for Active Sonar". Proceedings of the Conference on Acquisition, Tracking, and Pointing XVIII, held at the SPIE Defense & Security Symposium 2004, Orlando, FL, 12-16 April 2004, SPIE vol. 5430.
- [2] D. Dunham, R. Hutchins, "Tracking Multiple Targets in Cluttered Environments with a Probabilistic Multi-Hypothesis Tracker," Proceedings of the Conference on Acquisition, Tracking and Pointing XI, held at the SPIE Annual International Symposium on Aerosense, Orlando, FL, 21-25 April 1997, SPIE vol. 3086, pp. 284-295.
- [3] R. Streit, T. Luginbuhl, "Probabilistic Multi-Hypothesis Tracking," NUWC-NPT Technical Report 10,428, Naval Undersea Warfare Center Division, Newport, RI, 15 February 1995.
- [4] P. Willett, Y. Ruan, R. Streit, "The PMHT for Maneuvering Targets," Proceedings of the Conference on Signal and Data Processing 1998, held at the SPIE Annual International Symposium on Aerosense, Orlando, FL, 14-16 April 1998, SPIE vol. 3373.

- [5] T. Luginbuhl, Y. Sun, P. Willett, "A Track Management System for the PMHT Algorithm," in Proceedings of the 4<sup>th</sup> International Conference on Information Fusion, Montreal, Canada, August 2001.
- [6] C. Rago, P. Willett, R. Streit, "Direct Data Fusion Using the PMHT," in Proceedings of the 1995 American Control Conference, Seattle, WA, 21-23 June 1995, pp.1678-1682.
- [7] Y. Bar-Shalom, X. Rong Li, T. Kirubasrajan, "Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software," John Wiley & Sons, Inc., 2001.
- [8] S. Coraluppi, "Multistatic Sonar Localization Analysis," NURC Technical Report SR-377, June 2003.
- [9] M. Efe, Y. Ruan, P. Willett, "The Pedestrian PMHT," Proceeding of the Fifth International Conference on Information Fusion, Annapolis, MD, July 8-11, 2002.
- [10] R. J. Urick, "Principles of Underwater Sound, 3<sup>rd</sup> Edition," McGraw-Hill Book Company, 1983.
- [11] C. Hempel, "Multi-waveform Track Detection: Theory and Performance Prediction," IEEE Transactions AES, to appear.
- [12] S. Coraluppi, D. Grimmett, R. Laterveer, "NURC Simulated Dataset for MSTWG," NURC Technical Report, 23 August 2005.