

## A MESOSCALE AIR-ICE-OCEAN FEEDBACK MECHANISM FOR THE ICE DRIFT IN THE MARGINAL ICE ZONE

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### ABSTRACT

Ice drift in the marginal ice zone (MIZ) is a very important feature of air-ice-ocean interaction at high latitude. Thermally generated surface winds, blowing from ice to water (ice breeze) with some deflection due to the earth rotation, force the ice drift and ocean currents near the MIZ. By changing the surface temperature gradient, the ice motion and the ocean currents feed back on the surface winds. A coupled air-ice-ocean theoretical model for the MIZ is employed to discuss the ice drift pattern with such a feedback mechanism. The steady-state solutions show that an off-ice and divergent wind field not only produces a dilation of the MIZ (as people generally think), but also generates a compaction of MIZ for some circumstances. An ice divergence/convergence criterion is found. The time-dependent solutions show that the ice motion exhibits two bifurcations. First, it bifurcates into decaying and growing modes. Second, the growing mode bifurcates into non-oscillatory and oscillatory states. Finally, the model predicts the ice edge upwelling.

### NOMENCLATURE

A	ice concentration
$a_{kj}$	integral constant
$b_k$	integral constant
$C_{ai}$	air drag coefficient on ice, 0.0036
$C_{aw}$	air drag coefficient on water, 0.0012
$C_{wi}$	water drag coefficient on ice, 0.00086 m/s
$DT_0$	mean surface temperature difference across the MIZ
g	gravitational acceleration, 9.81m/s <sup>2</sup>
$g^*$	reduced gravitational acceleration, 0.005m/s <sup>2</sup>
$H_i$	mean ice thickness
$H_w$	equilibrium depth of the pycnocline
$h_w$	thickness of the upper ocean
k	wave number

L	length scale twice the MIZ width, 200 km
R	rate for Rayleigh friction and Newtonian cooling, $2.08 \times 10^{-5}/s$
$Ri$	atmospheric Richardson number
U	scale of ice breeze wind
$u_a, v_a$	ice breeze wind
$u_i, v_i$	ice drift velocity
$u_w, v_w$	ocean current
$\alpha$	angle between surface wind and surface temperature gradient
$\beta$	angle between ice velocity and surface wind
$\delta$	thickness of atmospheric Ekman layer
$\zeta$	nondimensional ice edge displacement
$\theta^a$	surface temperature
$\theta_{a0}^a$	characteristic surface temperature, 270°K
$\lambda_{kj}$	eigenvalue for vertical structure
$\mu_k$	eigenvalue for time evolution
$\nu_k$	air vertical eddy viscosity, 5 m <sup>2</sup> /s
$\rho_a$	air density, 1.29 kg/m <sup>3</sup>
$\rho_i$	ice density, 920 kg/m <sup>3</sup>
$\rho_w$	water density, 1000 kg/m <sup>3</sup>
$\Omega$	angular velocity of the Earth rotation, $0.7292 \times 10^{-4}/s$ .

### INTRODUCTION

The importance of ice breeze in the MIZ is discussed as follows. Schmidt (1947) used a simple linear model to depict seabreeze/landbreeze phenomena and concluded that for a maximum land-water temperature gradient of 4.3°K/100km, a maximum landbreeze intensity of about 2 m/s would be reached. From observations in the Southern Bering Sea, Reynolds et al. (1985) estimated that ice floes drift to the right of the wind by approximately 30° at about 4% of the wind speed at 3m. The ice breeze is analogous to the landbreeze. Therefore, 1°K/100km surface temperature gradient roughly produces 0.5 m/s ice breeze, which in

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turn generates about 2 cm/s ice flow. Paquette and Bourke (1979), during a summer cruise in the Chukchi Sea, found the sea surface temperature to change by over 6°K across a distance of about 25 km as the ice concentration fell from seven-eighths cover to zero. This is an enormous gradient by most oceanic standards (McPhee, 1983). Under such a condition it is found that the ice breeze is around 12 m/s (Schmidt's estimation), and the associated ice drift is about 48 cm/s (Reynold et al's estimation). The coupled air-ice-ocean model contains three parts: a thermally forced boundary layer air flow, a mechanically driven ice drift, and a reduced gravity ocean. The three components are linked through the surface temperature gradient and various interfacial stresses.

#### THERMALLY FORCED BOUNDARY LAYER AIR FLOW

A K-theory approach planetary boundary layer air model with modified Boussinesq approximation and with constant stress sublayer treated by Kuo (1973) and Chu (1986, 1987a,b,c) is employed to compute surface wind (ice breeze) thermally driven by surface temperature gradient across the ice edge. The x-axis is in the cross ice-edge direction (positive iceward), and the y-axis is parallel to the ice edge, as shown in Fig.1. It is considered that spatial variations in the MIZ are much large perpendicular to the ice edge than parallel to it, and hence derivatives with respect to y are assumed to be zero. As discussed by Chu (1986) the waterward/iceward migration of the MIZ increases/decreases the surface temperature gradient. The effects of ice flow on the surface temperature gradient can be depicted by (Chu, 1987b)

$$\partial^2 \theta_a / \partial t \partial x = (\pi D T_0 / L^2) u_i(x, t) \quad (1)$$

where  $\theta_a$  is a surface temperature,  $u_i$  is an ice drift velocity in the cross ice-edge direction,  $D T_0$  is a characteristic surface temperature difference across the MIZ, and  $L$  is a length-scale twice the MIZ width. Subscripts 'a' and 'i' denote atmosphere and ice, respectively. The surface temperature is separated into a steady state and a time-dependent part. Both parts are decomposed into Fourier sine series. The fact that the waterward monotonically increasing surface temperature often observed near the MIZ (Muench, 1983) implies that the steady state only has the first mode, i.e.,

$$\theta_a(x, t) = \sum_K \theta_{ak}(t) \sin k\pi x/L + \theta_a \sin \pi x/L \quad (2)$$

where  $\theta_{ak}$  and  $\theta_a$  are Fourier coefficients. Integrating Kuo's (1973) planetary boundary layer model with slip and kinematic boundary conditions at the surface and finite condition at the top, the thermally driven surface wind is given by (Chu, 1986, 1987a,b,c)

$$u_a(x, t) = \sum_K u_{ak} \zeta_{ik}(t) \cos k\pi x/L + u_{a1} \cos \pi x/L \quad (3)$$

$$v_a(x, t) = \sum_K v_{ak} \zeta_{ik}(t) \cos k\pi x/L + v_{a1} \cos \pi x/L \quad (4)$$

where  $(u_a, v_a)$  is a surface wind vector, and  $u_{ak}$

and  $v_{ak}$  are constants defined by

$$u_{ak} = U \sum_{j=1}^3 a_{kj} / \lambda_{kj}, \\ v_{ak} = U (b_k - 2f_0 \sum_{j=1}^3 a_{kj} / \lambda_{kj}), \quad k=1, 2, \dots \quad (5)$$

where  $f_0 = \sin \varphi$ ,  $\varphi$  is the latitude, and

$$U = g \delta D T_0 / (2L \Omega \theta_{a0}) \quad (6)$$

is the scale of ice breeze wind. Here  $\delta = (v/\Omega)^{1/2}$  is the Ekman depth,  $v$  is the air vertical eddy viscosity, and  $\theta_{a0}$  is a characteristic air temperature at surface. According to Kuo (1973),  $v$  is taken as 5 m<sup>2</sup>/s. The eigenvalues  $\lambda_{kj}$  are the roots with negative real parts of following sixth order algebraic equations:

$$\lambda^6 + 4f_0^2 \lambda^2 - 4k^2 \pi^2 Ri = 0, \quad k=1, 2, \dots \quad (7)$$

where

$$Ri = (\delta N / 2L \Omega)^2 \quad (8)$$

is the air Richardson number.  $N$  is the air Brunt-Vaisaila frequency. The integral constants  $a_{kj}$  and  $b_k$  in (5) are the roots of the following nonhomogeneous linear algebraic equations

$$\sum_{j=1}^3 a_{kj} = 0, \\ \sum_{j=1}^3 \lambda_{kj} (1 - M \lambda_{kj}) a_{kj} = 0, \\ -2f_0 \sum_{j=1}^3 (a_{kj} / \lambda_{kj}) (1 - M \lambda_{kj}) + b_k = 0, \\ 2k\pi Ri \sum_{j=1}^3 a_{kj} / \lambda_{kj}^2 = -1/k. \quad (9)$$

where  $M$  is a measure of the effective depth of the constant stress-sublayer (Kuo, 1973):  $M = v / (C_{ai} U \delta)$ , here  $C_{ai}$  is an air drag coefficient on ice. The ice-ocean system is mechanically driven by the surface wind, therefore, ice velocity should have the same Fourier components as the wind, i.e.,

$$u_i(x, t) = \sum_K u_{ik}(t) \cos k\pi x/L + u_i \cos \pi x/L \quad (10)$$

$$v_i(x, t) = \sum_K v_{ik}(t) \cos k\pi x/L + v_i \cos \pi x/L \quad (11)$$

where  $(u_i, v_i)$  is the ice velocity vector,  $u_{ik}$ ,  $v_{ik}$ ,  $u_i$ , and  $v_i$  are Fourier coefficients.  $\zeta_{ik}(t)$  in (3)-(4) is defined as nondimensional ice edge displacement (Chu, 1986, 1987b), i.e.,

$$L d \zeta_{ik} / dt = u_{ik}(t) \quad (12)$$

If  $f_0$  (i.e., Latitude),  $Ri$ , and  $M$  are known, the eigenvalues  $\lambda_{kj}$  and the integral constants  $a_{kj}$  and  $b_k$  are easily obtained by solving the algebraic equations (7) and (9).

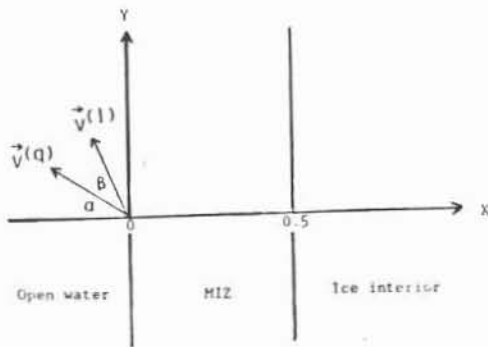


Fig. 1. The coordinate system and air deflection angle  $\alpha$ , ice turning angle  $\beta$

### FREE ICE DRIFT MODEL

Roed and O'Brien (1983) showed that the internal ice stress and the nonlinear terms aren't crucial for the MIZ dynamics, therefore, the ice is nearly free drift. The momentum equations of the ice drift are given by

$$\partial u_i / \partial t - f v_i = \gamma_{ai} u_a + \gamma_{wi} (u_w - u_i) \quad (13)$$

$$\partial v_i / \partial t + f u_i = \gamma_{ai} v_a + \gamma_{wi} (v_w - v_i) \quad (14)$$

where the terms on the right-hand side represent the water and air stresses on ice, respectively. Here

$$\gamma_{ai} \equiv \rho_a C_{ai} U / (\rho_i H_i), \quad \gamma_{wi} \equiv \rho_w C_{wi} / (\rho_i H_i) \quad (15)$$

$C_{wi}$  is a dimensional (m/s) water drag coefficient on ice (Roed and O'Brien, 1983), and  $\rho_a$ ,  $\rho_i$ , and  $\rho_w$  are the densities of air, ice, and water, respectively.  $H_i$  is a mean ice thickness.

### REDUCED GRAVITY OCEAN MODEL

Suppose that the ocean is composed of two layers, in which the lower layer is deep enough for motion in the lower layer to be vanishingly small. Such a model is referred to as a reduced gravity model. Furthermore, the model has the simplest form of dissipation, namely, Rayleigh friction with decay rate  $R$  and Newtonian cooling, also with decay rate  $R$ . The momentum and continuity equations for the ocean are written by (Gill, 1982; Roed and O'Brien, 1983)

$$(\partial / \partial t + R) u_w - f v_w = -g^* \partial h_w / \partial x + (1-A) \gamma_{aw} u_a + A \gamma_{iw} (u_i - u_w) \quad (16)$$

$$(\partial / \partial t + R) v_w + f u_w = (1-A) \gamma_{aw} v_a + A \gamma_{iw} (v_i - v_w) \quad (17)$$

$$(\partial / \partial t + R) h_w + H_w \partial u_w / \partial x = 0 \quad (18)$$

where the second and third terms on the right-hand side of (16) and (17) represent the air and ice stresses on water, and  $g^*$  is a reduced gravitational acceleration.  $H_w$  the equilibrium depth of the pycnocline,  $h_w$  the thickness of the upper layer,  $\gamma_{iw} \equiv C_{wi} / H_w$ , and  $\gamma_{aw} \equiv \rho_a C_{aw} U / \rho_w H_w$ ,  $C_{aw}$  is an air drag coefficient on water.  $A$  is the ice compactness or ice concentration

(i.e., the fraction of area covered by ice). Hibler and Ackley (1983) took the 50% limit of  $A$  as the ice edge. Inside the MIZ, the ice concentration,  $A$ , decreases waterward very slowly from the boundary between pack ice and the MIZ (where  $A \sim 1$ ) to some place near the ice edge, and then diminishes very rapidly to the ice edge. If the area we focus on is not very close to the ice edge, we may set  $A \sim 1$ . The portion of air stress on the water,  $(1-A) \gamma_{aw} v_a$ , is then neglected compared to ice stress on the water,  $A \gamma_{iw} (v_i - v_w)$ . Setting  $A=1$  and eliminating  $h_w$  from (16) and (18), the momentum equations (16) and (17) become

$$(\partial / \partial t + R) u_w - f v_w - (g^* H_w / RL^2) \partial^2 u_w / \partial x^2 = \gamma_{iw} (u_i - u_w) \quad (19)$$

$$(\partial / \partial t + R) v_w + f u_w = \gamma_{iw} (v_i - v_w) \quad (20)$$

### SOLUTIONS

If the ice-ocean is considered as one system, the air stress is the only forcing term in (13), (14), (19), and (20). The ice velocity and the ocean current should have the same Fourier components as the surface wind since our system is linear. According to (3) and (4) the solutions have the following forms:

$$u_i(x, t) = \sum_k u_{ik} \exp(\mu_k t) \cos k\pi x / L + \bar{u}_i \cos \pi x / L \quad (21a)$$

$$v_i(x, t) = \sum_k v_{ik} \exp(\mu_k t) \cos k\pi x / L + \bar{v}_i \cos \pi x / L \quad (21b)$$

$$u_w(x, t) = \sum_k u_{wk} \exp(\mu_k t) \cos k\pi x / L + \bar{u}_w \cos \pi x / L \quad (21c)$$

$$v_w(x, t) = \sum_k v_{wk} \exp(\mu_k t) \cos k\pi x / L + \bar{v}_w \cos \pi x / L \quad (21d)$$

where  $u_{ik}$ ,  $v_{ik}$ ,  $u_{wk}$ ,  $v_{wk}$  ( $k=1, 2, \dots$ ),  $\bar{u}_i$ ,  $\bar{v}_i$ ,  $\bar{u}_w$ , and  $\bar{v}_w$  are the Fourier coefficients, and  $\mu_k$  ( $k=1, 2, \dots$ ) are the eigenvalues of the system. Substituting (21a)-(21d) into (13), (14), (19), and (20) we have a set of nonhomogeneous linear algebraic equations for the steady-state Fourier coefficients ( $\bar{u}_w$ ,  $\bar{v}_w$ ,  $\bar{u}_i$ , and  $\bar{v}_i$ ):

$$[R + \pi^2 g^* / (RL^2) + C_{iw}] \bar{u}_w - f \bar{v}_w - \gamma_{iw} \bar{u}_i = 0, \quad (22a)$$

$$f \bar{u}_w + (R + C_{iw}) \bar{v}_w - \gamma_{iw} \bar{v}_i = 0, \quad (22b)$$

$$-\gamma_{wi} \bar{u}_w + \gamma_{wi} \bar{u}_i - f \bar{v}_i = \gamma_{ai} \bar{u}_a \quad (22c)$$

$$-\gamma_{wi} \bar{v}_w + \gamma_{wi} \bar{v}_i + f \bar{u}_i = \gamma_{ai} \bar{v}_a \quad (22d)$$

and the dispersion relations for the eigenvalues  $\mu_k$ :

$$\begin{vmatrix} \mu_k + \gamma_{wi} & -f & \gamma_{ai} u_{ak} / L & \gamma_{wi} & 0 \\ f & \mu_k + \gamma_{wi} & \gamma_{ai} v_{ak} / L & 0 & \gamma_{wi} \\ 1 & 0 & \mu_k & 0 & 0 \\ \gamma_{iw} & 0 & 0 & \mu_k + R_1 & -f \\ 0 & \gamma_{iw} & 0 & f & \mu_k + R_0 \end{vmatrix} = 0, \quad (23)$$

where  $R_0 = R + C_{iw}$ ,  $R_1 = R_0 + g^* H_w \pi^2 k^2 / RL^2$ .

## MEAN ICE DIVERGENCE/CONVERGENCE CRITERION

In this section we discuss the steady state solutions (22a)-(22d). Chu (1987c) shows that the absolute value of ice breeze deflection angle (angle between surface temperature gradient and surface wind),  $|a|$ , increases with increasing latitude and decreases with increasing  $Ri$ . At  $65^\circ$  (N and S) latitude it varies from  $41.8^\circ$  when  $Ri = 0.001$  to  $a = 27.5^\circ$  when  $Ri = 0.04$ . The angle between surface wind and ice velocity,  $\beta$  (as shown in Fig. 2), is defined by

$$\beta = \tan^{-1}(\bar{v}_i/\bar{u}_i) - \alpha \quad (24)$$

where  $u_i$  and  $v_i$  are the roots of linear algebraic equations (22a)-(22d). An ice divergence/convergence criterion is simply

$$|\alpha + \beta| \begin{cases} > \pi/2 & \text{ice convergence} \\ < \pi/2 & \text{ice divergence} \end{cases} \quad (25)$$

Fig. 2 indicates the mean ice divergence/convergence criterion,

$$\tan^{-1}(\bar{v}_i/\bar{u}_i) = \pi/2 \quad (26)$$

for different  $Ri$  in the  $(H_i, H_w)$  plane. The curves separate the parameter plane  $(H_i, H_w)$  into ice convergence and ice divergence parts. We can differentiate between ice divergence and ice convergence due to the ice breeze in the MIZ by the parameters  $Ri$ ,  $H_i$ , and  $H_w$ . Ice divergence appears in the small  $H_i$  and large  $H_w$  area, however on the contrary, ice convergence appears in the large  $H_i$  and small  $H_w$  region. The critical curve ( $|\alpha + \beta| = \pi/2$ ) moves from lower left to upper right in the  $(H_i, H_w)$  plane from less to more stable atmosphere. If the drag coefficient  $C_{wi}$  is doubled, the turning angle  $\beta$  only has minor changes (less than 10%).

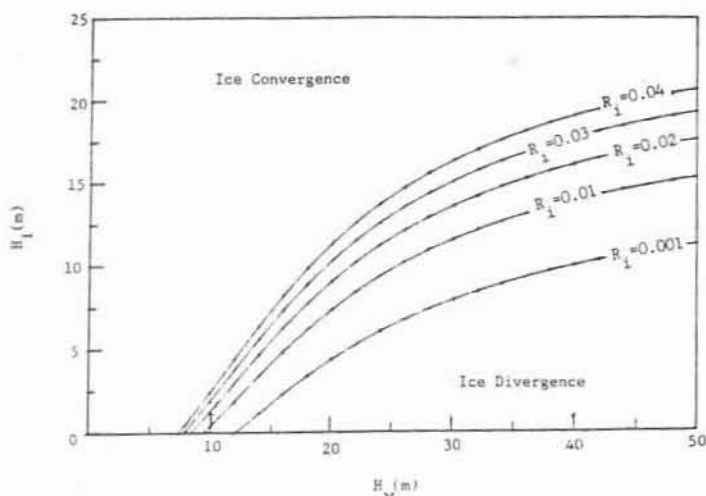


Fig. 2.  $H_i$ - $H_w$  curves for zero ice divergence/convergence\* for five different values of  $Ri$  (after Chu, 1987a).

## TWO BIFURCATIONS OF TIME-DEPENDENT ICE DRIFT

Ice drift observations in the Greenland Sea from 1978 April 28 to September 3 (Vinje, 1982) show two different types of ice motion: oscillatory and nonoscillatory. The mesoscale air-ice-ocean feedback mechanism may explain this phenomena. If the ocean is deep enough such that  $C_{iw}$  can be neglected against  $R$ , the dispersion relation (23) is simplified as

$$\begin{vmatrix} \mu_k + R + \pi^2 k^2 g^* H_w / RL^2 & -f \\ f & \mu_k + R \end{vmatrix} = 0, \quad (27)$$

representing inertial-gravity water waves, and

$$\begin{vmatrix} \mu_k + C_{wi} & -f & C_{ai} u_{ka} / L \\ f & \mu_k + C_{wi} & C_{ai} v_{ka} / L \\ 1 & 0 & \mu_k \end{vmatrix} = 0 \quad (28)$$

showing the ice drift patterns. The roots  $\mu_k$  ( $k=1, 2, \dots$ ) of the cubic equations (28) give the e-folding time dependence of the  $k$ th component of ice velocity. The instability criterion for the  $k$ th mode of ice motion in the MIZ can be written

$$\text{Re}(\mu_k) < 0 \quad (29a)$$

the  $k$ th mode of ice velocity decreases with time,

$$\text{Re}(\mu_k) = 0 \quad (29b)$$

neutral, and

$$\text{Re}(\mu_k) > 0 \quad (29c)$$

the  $k$ th mode of ice velocity increases with time. We define the time-increasing mode of ice velocity as an unstable mode.

The oscillation criterion for the  $k$ th mode of ice motion is given by

$$\text{Im}(\mu_k) = 0, \text{ (nonoscillatory)} \quad (30a)$$

$$\text{Im}(\mu_k) \neq 0, \text{ (oscillatory)} \quad (30b)$$

We compute all the roots of (28) for different values of the parameters  $H_i$ ,  $N$ ,  $DT_0$ , and obtain three roots  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  at each point of the parameter space  $(H_i, N^2, DT_0)$ . Here  $\mu_1$  is real throughout that space, and  $\mu_2$  and  $\mu_3$  are real for some values of  $(H_i, N^2, DT_0)$  and are complex else where.

Fig. 3 shows the surface  $\mu_1 = 0$  for  $k=1$  in the three dimensional space  $(H_i, N^2, DT_0)$ . This surface divides the space into two parts corresponding to growing and decaying modes. The growing mode generally appears when  $DT_0$  exceeds some critical value around  $10^\circ\text{C}$ . This critical value decreases with  $H_i$ , and becomes very small (around  $1^\circ\text{C}$ ) in the region of small mean ice-thickness  $H_i$  ( $H_i < 1.5$  m). Ice motion corresponding to the eigenvalue  $\mu_1$  is nonoscillatory.

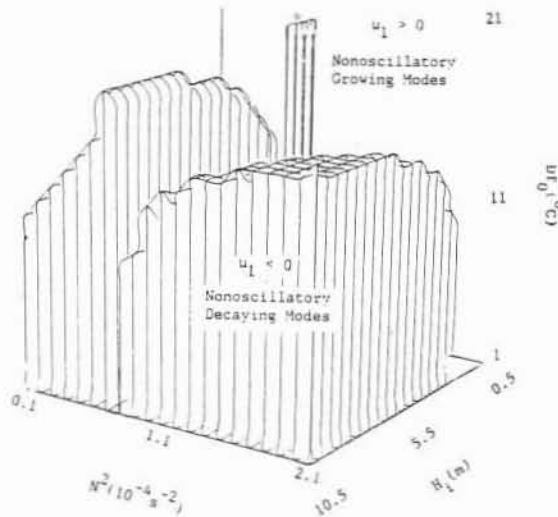


Fig. 3. Separation of nonoscillatory decaying and growing modes referring to real eigenvalues  $\mu_1$  for  $k=1$  (after Chu, 1986).

Fig. 4 indicates the surface of  $\text{Re}(\mu_2)=0$  (or  $\text{Re}(\mu_3)=0$ ) for  $k=1$  in the parameter space ( $H_1, N^2, DT_0$ ). This surface separates the whole space into growing and decaying parts. The growing mode is present when  $DT_0$  exceeds some critical value which is a function of  $H_1$  and  $N^2$  (when  $H_1=4.5$  m and  $N^2=0.00011/\text{s}^2$ , the critical value is  $3^\circ\text{C}$ ).

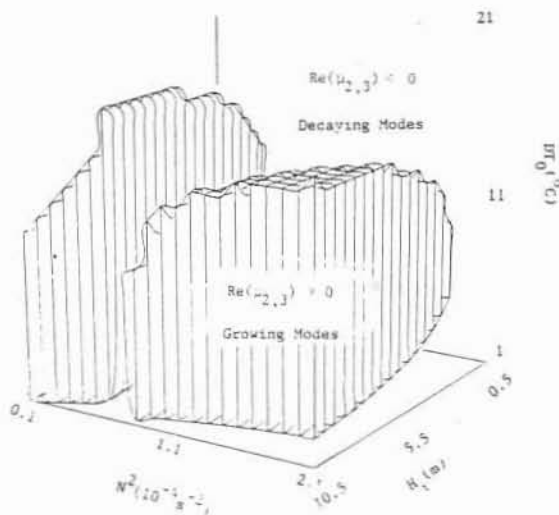


Fig. 4. Separation of decaying and growing modes referring to eigenvalues  $\mu_2$  and  $\mu_3$ .

Fig. 5 reveals the segregation of nonoscillatory and oscillatory modes relating to  $\mu_2$  and  $\mu_3$ . Comparing Fig. 5 with 4, we find that the decaying mode of  $\mu_2$  and  $\mu_3$  is generally nonoscillatory whereas the growing mode of  $\mu_2$  and  $\mu_3$  is generally oscillatory.

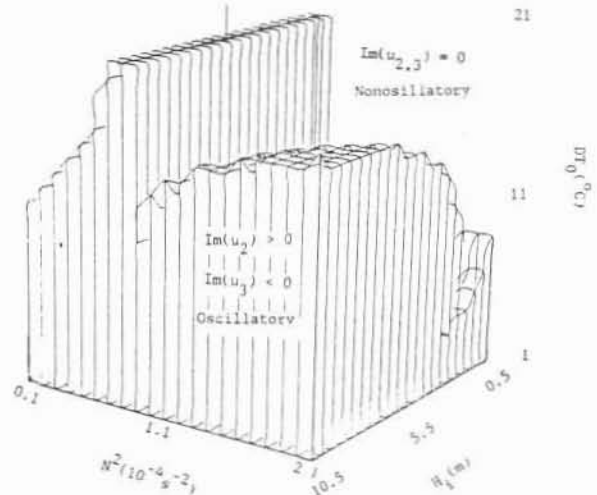


Fig. 5. Separation of nonoscillatory and oscillatory states referring to eigenvalues  $\mu_2$  and  $\mu_3$  for  $k=1$  (after Chu, 1986).

Fig. 3-5 show the following results: (i) both nonoscillatory and oscillatory modes share a common area (restricted to the region of small  $DT_0$ ) in the parameter space; however, (ii) nonoscillatory and oscillatory growing modes occupy different regions in the parameter space. The segregation of the two modes depends largely on  $DT_0$  and  $H_1$ .

Whether ice motion grows or decays substantially depends on a first critical value of the parameter  $DT_0$  (when  $N=1.45 \times 10^{-2} \text{s}^{-2}$  and  $H_1=2.5$  m, this critical value is  $5^\circ\text{C}$ ). When  $DT_0$  is smaller than the first critical value, the motive force is so small that it cannot overcome dissipative effects and does not make ice motion unstable. If  $DT_0$  becomes large enough to overcome the dissipative effects of friction, ice motion becomes larger.

Whether ice motion is oscillatory or nonoscillatory largely depends on  $DT_0$  and the properties of ice. If  $DT_0$  exceeds the first critical value but does not reach a second critical value which mostly depends on  $N$  (i.e., when  $N=0.0145/\text{s}$ , the second critical value is  $14^\circ\text{C}$ ), and when ice is thin (generally during summer) the ice motion is nonoscillatory, however, when the ice is thick (generally during winter) the ice motion is oscillatory. If  $DT_0$  exceeds the second critical value, only the nonoscillatory growing mode appears.

For the nonoscillatory growing mode the time during which the ice doubles its speed is

$$T_1 = \ln(2/\mu_1), \quad (\mu_1 > 0) \quad (31)$$

The doubling time treated as a function of  $DT_0$  (for  $H_1=1$  m,  $N=0.01/\text{s}$ ) is shown in Fig. 6, which displays a decrease of  $T_1$  with an increase of  $DT_0$ .  $T_1$  changes from 2.2 to 0.18 hr as  $DT_0$  varies from  $5^\circ$  to  $22^\circ\text{C}$ .

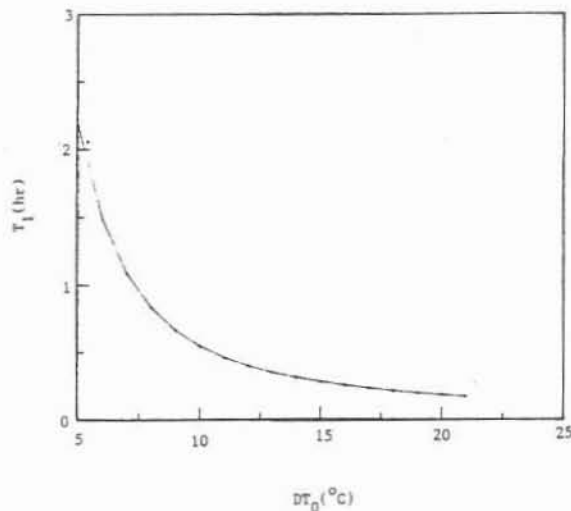


Fig. 5. Dependence of doubling time on  $DT_0$  for the nonoscillatory growing mode for  $H_i=1m$  and  $N=10^{-2}s^{-1}$  (after Chu, 1986).

The oscillatory growing mode is ice motion due to  $\mu_2$  (or  $\mu_3$ ), whose real part is positive and the imaginary part is different from zero. The growing oscillatory mode is located in the area where  $DT_0$  is larger than the first critical value and smaller than the second critical value. As in the nonoscillatory case the growth rate  $Re(\mu_{2,3})$  increase with an increase of  $DT_0$  (increase of forcing). The period of the oscillatory mode is defined by

$$T_p = 2\pi / |\text{Im}(\mu_{2,3})| \quad (32)$$

Fig. 7 shows the doubling time and the period as a function of  $DT_0$  for  $H_i=5m$  and  $N=0.01/s$ . The doubling time decreases monotonically with an increase of  $DT_0$ . However, the period increases slightly when  $DT_0$  varies from  $5^\circ C$  to  $10^\circ C$ , and then increases rapidly with  $DT_0$ . When  $DT_0 = 12.5^\circ C$  the period is nearly one day.

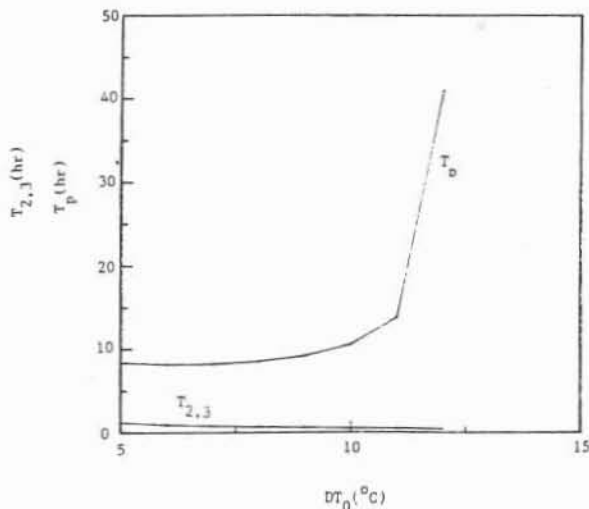


Fig. 7. Dependence of doubling time and period on  $DT_0$  for oscillatory growing mode for  $H_i=5m$  and  $N=10^{-2}s^{-1}$  (after Chu, 1986).

## ICE EDGE UPWELLING

Buckley et al. (1979) reported a pronounced wind-driven upwelling along the edge of the ice pack in an expedition into the Arctic Ocean north of Spitsbergen. The aim of this section is to investigate the effect of the air-ice-ocean feedback mechanism on ice edge upwelling. If the sinusoidal type surface temperature perturbation across the MIZ doesn't change as the ice edge moves in a speed  $u_i$ , the parameterization of surface thermal condition (1) becomes

$$0_a(x,t) = 0_{a0} - DT_0 \sin \pi [(x-u_i t)/L] \quad (33)$$

The negative sign in (34) comes from the x-axis pointing iceward (Fig. 1). Let  $\xi = (x-u_i t)/L$ , the surface wind driven by the horizontal temperature gradient (33) is calculated by

$$u_a|_{z=0} = u_{a1} \cos \pi \xi, \quad (34)$$

$$v_a|_{z=0} = v_{a1} \cos \pi \xi, \quad (35)$$

where  $u_{a1}$  and  $v_{a1}$  are computed by (5). Neglecting inertial oscillation and assuming the ocean velocities to be small (Roed and O'Brien, 1983), the momentum equations for the reduced gravity ocean model (19) and (20) are written by

$$Ru_w - fv_w = -g \delta h_w / \partial x + [1 - H(\xi)] C_{aw} u_{a1} \cos \pi \xi + C_{iw} (u_i - u_w) H(\xi) \quad (36)$$

$$(\partial / \partial t + R)v_w + fu_w = [1 - H(\xi)] C_{aw} v_{a1} \cos \pi \xi + C_{iw} (v_i - v_w) H(\xi) \quad (37)$$

where  $H(\xi)$  is the Heaviside function defined as

$$H(\xi) = \begin{cases} 1, & \text{if } \xi > 0, \text{ (MIZ)} \\ 0, & \text{if } \xi < 0, \text{ (open water)} \end{cases} \quad (38)$$

The equations for the free drift ice (13), (14) and for the reduced gravity ocean (36), (37), and (18) are the basic equations for this section. The solution is

$$\delta h_w / \partial t = \begin{cases} -(H_w / \lambda) [C_1 \exp(-L|\xi|/\lambda) + (\lambda\pi/L) (\beta_{11} \cos \pi \xi - \beta_{12} \sin \pi \xi)], & (\xi < 0) \\ -(H_w / \lambda) [C_2 \exp(-L|\xi|/\lambda) + (\lambda\pi/L) (\beta_{21} \cos \pi \xi - \beta_{22} \sin \pi \xi)], & (\xi > 0) \end{cases} \quad (39)$$

where the parameters are defined by

$$\lambda = (gH_w)^{1/2} / f,$$

$$\beta_{11} = \pi C_{aw} u_{a1} / f^2 \lambda [1 + (\lambda\pi/L)^2],$$

$$\beta_{12} = C_{aw} v_{a1} / f [1 + (\lambda\pi/L)^2],$$

$$\beta_{21} = \pi C_{iw} u_i^2 / f^2 \lambda [1 + (\lambda\pi/L)^2],$$

$$\beta_{22} = C_{iw} v_i / f [1 + (\lambda\pi/L)^2],$$

$$C_1 = (\beta_{22} - \beta_{12}) / 2 + \lambda\pi (\beta_{21} - \beta_{11}) / (2L)$$

$$C_2 = -(\beta_{22}-\beta_{12})/2 + \lambda\pi(\beta_{21}-\beta_{11})/(2L) \quad (40)$$

where  $u_i$ ,  $v_i$  are computed from nonhomogeneous linear algebraic equations (22a)-(22d). Eq. (39) shows that the ice breeze disturbance, which is local to the MIZ, will cause the reduced gravity ocean model to adjust to differential Ekman transport (both from changes in stress at the ice/ocean boundary and from horizontal variation in wind) by increasing or decreasing thickness. The rate of shallowing (thickening) is equivalent to an upwelling (downwelling). This effect of ice breeze on ice-edge upwelling/downwelling can be parameterized in terms of a differential surface heating condition, i.e.,

$$\partial h_w/\partial t = A_1 \exp(-L|\xi|/\lambda) + A_2[\partial\theta_a/\partial\xi - B(\theta_a - \theta_{a0})] \quad (41)$$

where

$$\begin{aligned} A_1 &= -(H_w C_1/\lambda)[1-H(\xi)] + (H_w C_2/\lambda)H(\xi), \\ A_2 &= (H_w/LDT_0)\beta_{11}[1-H(\xi)] + \beta_{21}H(\xi), \\ B &= \pi(\beta_{12}/\beta_{11})[1-H(\xi)] + (\beta_{22}/\beta_{21})H(\xi). \end{aligned} \quad (42)$$

Eq. (41) shows the relationship between ice-edge upwelling/downwelling and the surface temperature distribution across the ice edge.

We compute  $\partial h_w/\partial t$  as a function of  $x$  and  $t$  for  $H_i=1$ , and  $6$  m, respectively, as shown in Fig. 8. It is seen that upwelling ( $\partial h_w/\partial t < 0$ ) appears near the ice edge. The minimum value of  $\partial h_w/\partial t$  (maximum upwelling) reaches  $18.3 \text{ m day}^{-1}$  for  $H_i = 1$  m, and  $19.8 \text{ m day}^{-1}$  for  $H_i=6$  m. If the distance between two lines of  $-5 \text{ m day}^{-1}$  is taken as the width of ice-edge upwelling, it is found that this width decreases with the increase of mean ice-thickness  $H_i$ . The upwelling width is around  $40 \text{ km}$  for  $H_i = 1$  m, and  $25 \text{ km}$  for  $H_i = 6$  m. These values are quite comparable to the baroclinic radius of deformation  $\lambda$  ( $23 \text{ km}$ ).

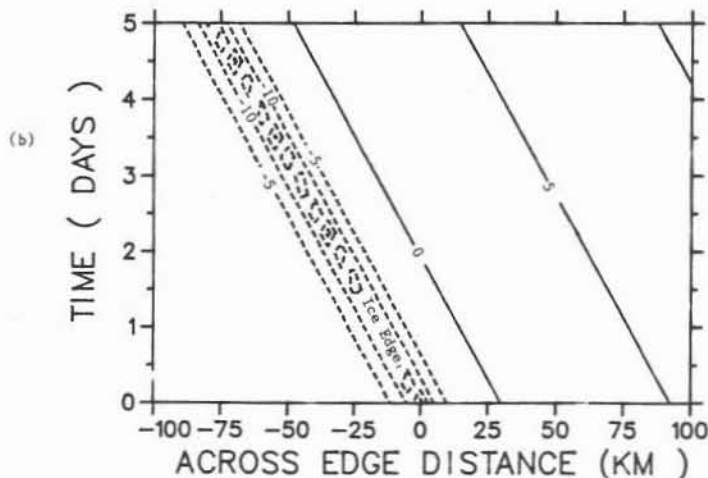
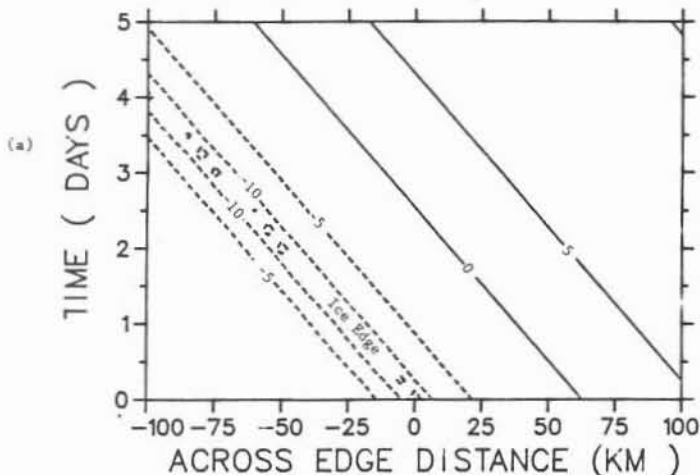


Fig. 8.  $\partial h_w/\partial t$  ( $\text{m day}^{-1}$ ) as a function of  $x$  and  $t$  for (a)  $H_i = 1$  m, and (b)  $H_i = 6$  m.

## CONCLUSIONS

(a) This air-ice-ocean coupled model is intended to depict only the mesoscale processes of air-ice-ocean interactions in the MIZ. The synoptic scale pressure gradient may additionally produce surface winds in the MIZ, and large-scale ocean current may also drive ice drift. These processes are, however, beyond the scope of this paper. Nevertheless, where the ice-to-open-water temperature gradient is strong, the mesoscale feedback mechanism discussed here may become as strong as, or stronger than, the synoptic scale and oceanic forcings.

(b) The model shows the different effects of ice breeze on ice flow, and gives out an ice divergence/convergence criterion in the MIZ.

(c) The ice motion has two bifurcations. First, it bifurcates into a decaying or growing mode, which depends in most cases on the mean surface temperature difference  $DT_0$  representing the strength of the forcing. When  $DT_0$  is small, the decaying mode exists. However, when  $DT_0$  exceeds the first critical value, the growing mode appears. Secondly, the growing mode bifurcates into nonoscillatory and oscillatory states depending on  $DT_0$  and the properties of ice.

(d) Surface winds, thermally generated by the temperature gradient across the ice edge, cause the reduced gravity ocean model to adjust to differential Ekman transport both from changes in stress at the ice/ocean boundary and from horizontal variation in winds. Such differential Ekman transport generates the ice edge upwelling.

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