

# THREE DIMENSIONAL PSEUDOVORTICITY FIELD IN THE WEST SPITSBERGEN CURRENT

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## 1. INTRODUCTION

Dynamical features of polar oceans, captured by traditional treatment of hydrographic data sets, are only two fields: dynamical height of sea surface relative to certain depth, and geostrophic currents by assuming a certain level of no motion. Much information about the polar water is lost by such a treatment.

In fact, the ocean flow is not purely geostrophically balanced. It contains two parts: geostrophic currents and ageostrophic circulations. The geostrophic currents are obtained from the hydrographic data sets (traditional physical oceanographic treatment), and the ageostrophic circulation is forced by the geostrophic flow (called the geostrophic forcing) and surface wind field. Therefore, the three-dimensional circulation (both geostrophic and ageostrophic) can also be calculated by the hydrographic and surface wind data sets.

There are potentially significant errors with the traditional treatment of the hydrographic data (e.g., only computing geostrophic currents), particularly in regions having strong temperature and salinity gradient, such as in the west Spitsbergen current. Neither geostrophic current nor dynamical height can provide detail information about the three-dimensional flow field near the west Spitsbergen current. In order to diagnose the three dimensional flow field, a new theory should be adapted.

## 2. C-VECTOR CONCEPT

### 2.1 General Ideas

In meteorology, the Q-vector concept proposed by Hoskins et al. (1978) and related analysis methods have been very useful in understanding and diagnosing the synoptic and frontal vertical circulation. However, the barotropic part of the rotational ageostrophic flow is excluded in the Q-vector equation. In order to overcome this deficiency, a C-vector concept was proposed by Xu (1992) for the atmospheric mesoscale moist frontogenesis.

In physical oceanography, the C-vector concept has a great potential in diagnosing three-dimensional ageostrophic flow fields. With the Boussinesq approximation, the geostrophic currents ( $U_g, V_g$ ) are computed from the CTD data by

$$f \frac{\partial U_g}{\partial z} = \frac{g}{\rho_0} \frac{\partial \rho}{\partial y} = \frac{g}{\rho_0} \frac{\partial \sigma_t}{\partial y} = -\frac{\partial b}{\partial y}, \quad (1a)$$

$$f \frac{\partial V_g}{\partial z} = -\frac{g}{\rho_0} \frac{\partial \sigma_t}{\partial x} = -\frac{g}{\rho_0} \frac{\partial \sigma_t}{\partial x} = \frac{\partial b}{\partial x} \quad (1b)$$

where  $\rho_0 = \rho_0(z)$ , is the mean density field,  $\rho$  is the density deviation from  $\rho_0$  (i.e.,  $\rho_0 + \rho$  is the observed density

field),  $\sigma_t = \rho_0 + \rho - 1000$ , and  $b = -g\rho/\rho_0$ , is the buoyancy force. The geostrophic currents are not the real currents. The basic equations, describing the coastal water flow without adiabatic source of buoyancy and induced by the wind stress, are

$$\left( \frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) u - f(v - V_g) = \frac{\partial Y^x}{\partial z} \quad (2a)$$

$$\left( \frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) v + f(u - U_g) = \frac{\partial Y^y}{\partial z} \quad (2b)$$

$$\left( \frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) b + N^2 w = 0 \quad (2c)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2d)$$

where  $\vec{V} = (u, v, w)$ ,  $f$  is the Coriolis parameter,  $N$  is the Brunt-Vaisala frequency, and  $(Y^x, Y^y)$  is turbulent momentum flux caused by the surface wind stress ( $\tau^x, \tau^y$ ),

$$Y^x|_{z=0} = \frac{\tau^x}{\rho_0}, \quad Y^y|_{z=0} = \frac{\tau^y}{\rho_0} \quad (3)$$

The three-dimensional flow in coastal regions can be decomposed into geostrophic currents ( $\vec{V}_g$ ) and ageostrophic circulation ( $\vec{V}_a$ )

$$\vec{V} = \vec{V}_g + \vec{V}_a \quad (4)$$

After the decomposition, the basic equations (2a)-(2d) should have the following form:

$$\mathcal{H}(\vec{V}_a) = \mathcal{R}(\vec{V}_g, \rho, Y^x, Y^y) \quad (5)$$

where  $\mathcal{H}, \mathcal{R}$  are certain differential operators. If the real forms of these operators ( $\mathcal{H}, \mathcal{R}$ ) are obtained, we can use (5) to compute the three-dimensional ageostrophic circulation  $\vec{V}_a$  from the hydrographic data ( $\vec{V}_g, \rho$ ) and surface wind data.

### 2.2 Diagnostic System for a Weak Ageostrophic Flow

If the ageostrophic flow is assumed weak compared to the geostrophic currents:

$$|\vec{V}_a| \ll |\vec{V}_g| \quad (6a)$$

which leads to

$$\frac{\partial}{\partial t} + \vec{V} \cdot \nabla \approx \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla, \quad (6b)$$

the basic equations for the coastal water (2a)-(2d) on f-plane become

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$$-fv_a = \frac{\partial Y^x}{\partial z} - \left( \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla \right) U_g \quad (7a)$$

$$fu_a = \frac{\partial Y^y}{\partial z} - \left( \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla \right) V_g \quad (7b)$$

$$N^2 w_a = - \left( \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla \right) b \quad (7c)$$

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial w_a}{\partial z} = 0 \quad (7d)$$

The ageostrophic pseudo-vorticity equations can be derived by the cross-derivatives:

$$-\frac{\partial}{\partial z} (f^2 v_a) + \frac{\partial}{\partial y} (N^2 w_a) = 2C_1 \quad (8a)$$

$$\frac{\partial}{\partial z} (f^2 u_a) - \frac{\partial}{\partial x} (N^2 w_a) = 2C_2 \quad (8b)$$

$$\frac{\partial}{\partial x} (f^2 v_a) - \frac{\partial}{\partial y} (f^2 u_a) = 2C_3 \quad (8c)$$

where

$$C_1 \equiv -f \left( \frac{\partial U_g}{\partial y} \frac{\partial V_g}{\partial z} - \frac{\partial V_g}{\partial y} \frac{\partial U_g}{\partial z} \right) + \frac{f}{2} \frac{\partial^2 Y^x}{\partial z^2} \quad (9a)$$

$$C_2 \equiv -f \left( \frac{\partial U_g}{\partial z} \frac{\partial V_g}{\partial x} - \frac{\partial V_g}{\partial z} \frac{\partial U_g}{\partial x} \right) + \frac{f}{2} \frac{\partial^2 Y^y}{\partial z^2} \quad (9b)$$

$$C_3 \equiv -f \left( \frac{\partial U_g}{\partial x} \frac{\partial V_g}{\partial y} - \frac{\partial V_g}{\partial x} \frac{\partial U_g}{\partial y} \right) - \frac{f}{2} \frac{\partial}{\partial z} \left( \frac{\partial Y^x}{\partial x} + \frac{\partial Y^y}{\partial y} \right) \quad (9c)$$

The C-Vector,  $\vec{C} \equiv (C_1, C_2, C_3)$ , computed from the geostrophic currents

$(U_g, V_g)$  which is obtained from the hydrographic data set, and the wind data  $(Y^x, Y^y)$ . This diagnostic model for the three-dimensional ageostrophic circulation can be shown in Figure 1.

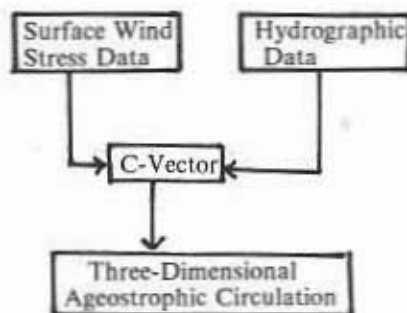


Fig. 1 Schematic diagram for the computation of ageostrophic circulation.

### 3. PHYSICAL SIGNIFICANCE OF C-VECTOR

#### 3.1 C-Vector as an Ageostrophic Vortex Line

The mathematical connection between the C-vector

$$\nabla \cdot \vec{V}_a = 0 \quad (10b)$$

where  $Ro \equiv U/(fL)$  is the Rossby number,  $H/L = W/U = f/N$  is chosen for the scaling, and  $\vec{C}$  is nondimensional here. The ageostrophic vorticity is proportional to the C-vector, therefore, a C-vector streamline can be viewed as an ageostrophic vortex line (Figure 2).

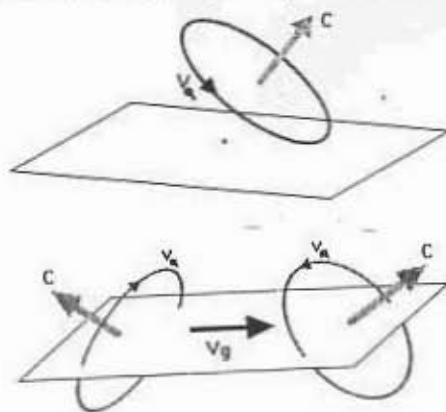


Fig. 2. Ageostrophic circulation induced by  $\vec{C}$  in three-dimensional space.

#### 3.2 Vertical Velocity Induced by the Vertical Vorticity of C-Vector

The vertical velocity equation is obtained from  $-\vec{k} \cdot \nabla \times (10a)$ :

$$\nabla^2 w_a = -2Ro \vec{k} \cdot \nabla \times \vec{C} \quad (11)$$

which indicates that the vertical velocity is induced by the vertical vorticity of  $\vec{C}$  (Figure 3).

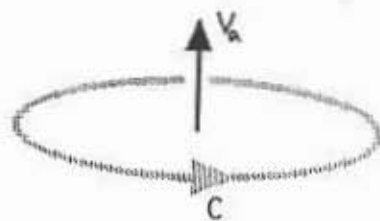


Fig. 3. Vertical velocity  $w_a$  induced by horizontal rotation (vorticity) of  $\vec{C}$ .

### 4. DATA SET FOR C-VECTOR COMPUTATION

The CTD data set collected during RV/VALDIVIA cruise 54 in March-April 1987 (Quadfasel and Ungewiß, 1988) is used for computing the C-vector in the west Spitsbergen current. The following data set description is from Quadfasel and Ungewiß (1988). From 16 March to 5 April 1987 a large-scale hydrographic survey of the eastern Greenland Sea and Fram Strait was carried out from RV/VALDIVIA. The major aim of VALDIVIA cruise 54 was to map the vertical distrib-

ution of temperature, salinity, and dissolved oxygen in the Greenland Sea as a measure of the large-scale circulation and transport. A secondary objective was to search active convection events. Along seven sections a total of 73 CTD profiles were taken (Fig.4). Four of these sections crossed the Arctic front that separates the Greenland Sea gyres from the warm and salty northward flowing Norwegian Atlantic and west Spitsbergen Currents. The sections were designed to form three closed boxes to allow calculation of transport budgets. Usual station spacing was 30 nautical miles except along the Fram Strait section at  $78^{\circ} 25' N$  and across the Hovgaard Fracture Zone, where the spacing was decreased to less than 15 miles. All CTD profiles were run to within 5 m of the bottom. The ME-Kiel Multisonde CTD (No 73) used for the hydrographic measurements provided data with a resolution of 0.2 dbar for pressure,  $0.0015^{\circ}C$  for temperature, and  $0.002ms/cm$  for conductivity. It was supplemented by a Hydrobios Rosette water sampler equipped with 20 bottles, 10 of which carried protected and unprotected reversing thermometers. Salinity samples were analyzed by use of a Guildline Autosol Salinometer. Altogether 572 water samples and 279 thermometer readings were taken, providing in-situ calibration values for the CTD. The CTD-data were processed at the Institute für Meereskunde, Hamburg with the established procedure, including conversion of raw data into physical units using the polynoms based on precruise laboratory calibration, elimination of all "upcast" data in the downcast profiles that were introduced through ship heaving, application of the second level calibration polynoms based on the comparison of bottle and in-situ CTD values, and elimination of spikes in the profiles by running a 7 point median filter (Sy, 1985). The resulting accuracies of the calibrated CTD data are then 4 dbar for pressure,  $0.006^{\circ}C$  for temperature, and  $0.004ms/cm$  for conductivity, corresponding to  $0.005g/kg$  for salinity.

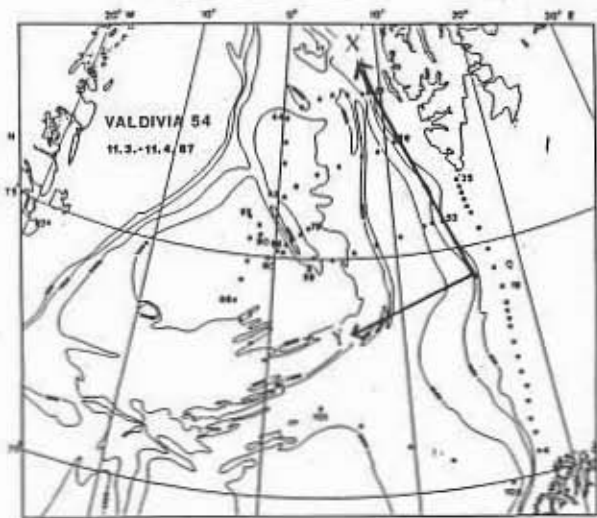


Fig.4. Location of the CTD stations occupied during RV/VALDIVIA cruise 54 in the eastern Greenland Sea and Fram Strait from 16 March to 5 April 1987.

## 5. PSEUDOVORTICITY FIELD IN THE WEST SPITSBERGEN CURRENT

The data set contains three dimensional  $\sigma_t$  field in the area shown in Fig.4. Two cross-sections of  $\sigma_t$  along the Stations 44-51, and the Stations 26-41 (Figs.5-6) show quite strong density gradients in the west Spitsbergen current at  $p < 2500$  dbar.

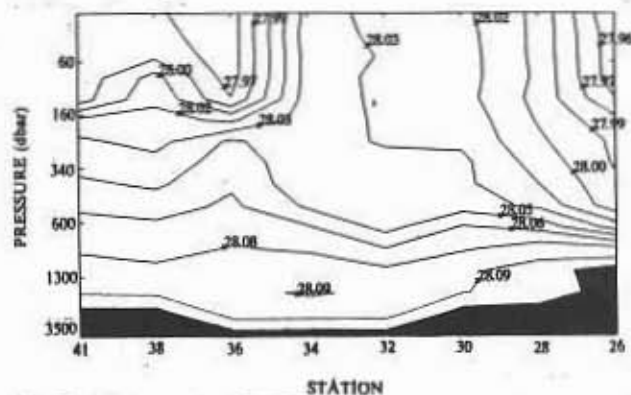


Fig.5  $\sigma_t$  along stations 26-41 during RV/VALDIVIA cruise 54 from 16 March to 5 April 1987.

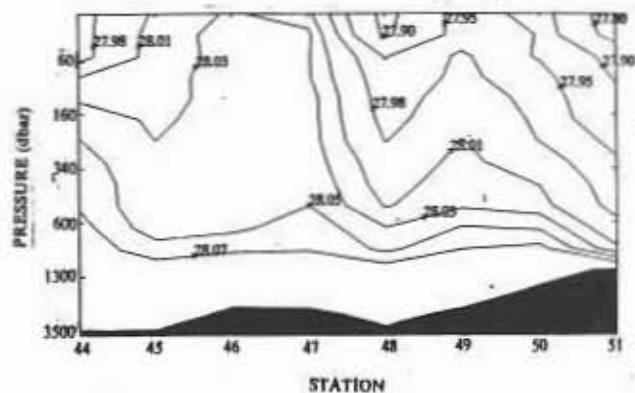


Fig.6  $\sigma_t$  along stations 44-51 during RV/VALDIVIA cruise 54 from 16 March to 5 April 1987.

Taking 2500 dbar as a reference level, geostrophic current ( $U_g, V_g$ ) is obtained from integrating (1a,b). Here, the coordinate system is indicated in Fig.4. As an example, Figs.7 and 8 show ( $U_g, V_g$ ) in the vertical cross-section along stations 44-51. Due to lack of surface wind observation during RV/VALDIVIA cruise 54 in 1987, the wind stress contribution is neglected in the C-vector computation. Therefore, the C-vector can be computed from the three dimensional  $U_g$  and  $V_g$  fields (9a,b,c). Fig.9 shows the  $C_x$  values in the vertical cross-section along stations 44-51. From the C-vector definition (8),  $C_x$  is the pseudo-vorticity in x-direction (i.e., in alongshore direction), which approximately indicates the cross coastal vertical circulation pattern (Fig.9). Four cells can be clearly seen from Fig.10. Furthermore, a relatively weak vertical circulation appears from 340-1300 dbar near station 47.

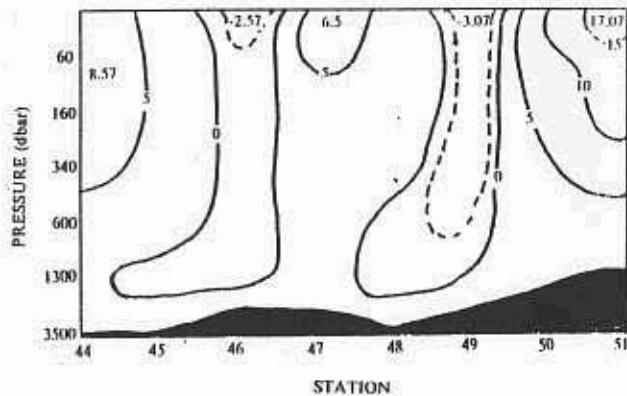


Fig.7. Geostrophic current  $U_g$  (cm/s) along the stations 44-51 (2500 dbar as reference level).

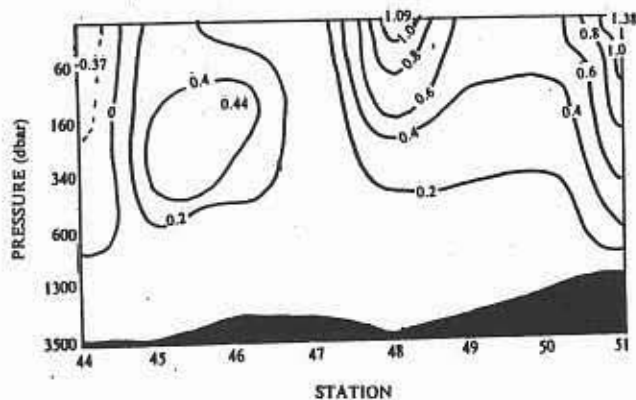


Fig.8. Geostrophic current  $V_g$  (cm/s) along the stations 44-51 (2500 dbar as reference level).

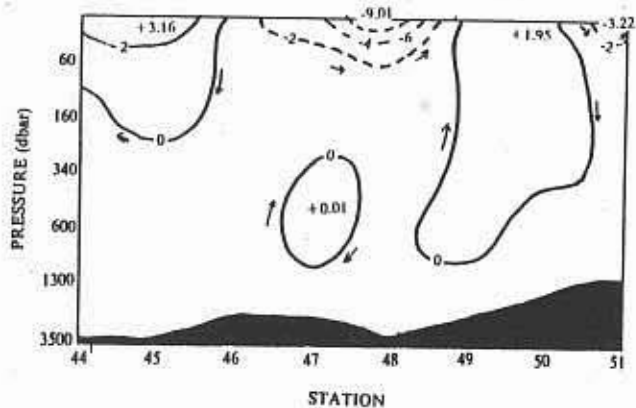


Fig.9. X-component of pseudo-vorticity field  $C_x/f^2$  (in  $10^{-7}s^{-1}$ ) along the stations 44-51. The arrows only help us to understand +/- vorticity, and are not the velocity in the cross section.

## 6. DISCUSSION

(a) In this study, a new concept (C-vector) has been introduced into polar oceanography. A three dimensional pseudo-vorticity field can be constructed from CTD measurement. We can either directly visualize three dimensional circulations from the pseudo-vorticity field, or solve the equations (8a,b,c) to get three dimensional flow field.

(b) The requirements for this technique are  $f = const$ , and  $|\vec{V}_a| \ll |\vec{V}_g|$  (weak ageostrophic component). Therefore, the C-vector method can be applied to mid-latitude oceans.

(c) In the future, we can use satellite altimetry data to determine the geostrophic current at the ocean surface  $(U_g, V_g)_{z=0}$ , and then to integrate the "thermal wind" relation (1) from the ocean surface to certain depth  $z$ . Such a treatment can eliminate an ambiguous concept of "level of no motion" in physical oceanography.

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