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Advanced Functionally Graded Plate-Type Structures Impacted By Blast Loading

Terry Hause, Ph.D.

Research Mechanical Engineer

U.S. Army RDECOM-TARDEC, Warren, MI 48397

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MODELING AND SIMULATION, TESTING AND VALIDATION



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Outline

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2. Basic Assumptions and Preliminaries
3. Types of FGM Models
4. Theoretical Developments
5. Solution Methodology
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1. Motivation

- A longer service life of the structure operating in extreme environments consisting of severe thermal and mechanical loading conditions.
- Combine the properties of two dissimilar Materials
- The load carrying capacity of plate-type structure is enhanced.
- An increased operational life of the structure.

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2. Basic Assumptions and Preliminaries

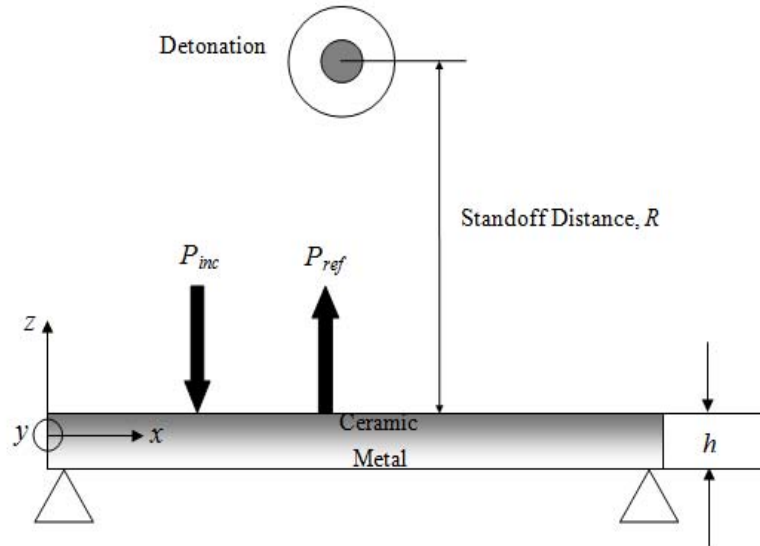


Figure 1. A simply Supported functionally graded (FG) plate shown in 2-D, under an explosive blast

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2. Basic Assumptions and Preliminaries-Continued



Basic Assumptions

- The plate mid surface is referred to a Cartesian Orthogonal System of Coordinates (x, y, z) .
- Z is the thickness coordinate measured positive in the upwards direction from the mid-surface of the plate with h being the uniform plate thickness.
- The plate is assumed to be thin. As a result, the Kirchoff Assumptions, of zero transverse shear stresses, will be assumed leading to the classical plate theory.
- The functionally graded plate is composed of ceramic and metal phases whose properties vary smoothly and continuously across the wall thickness from one surface to another.

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3. Types of FGM Models



Based on a rule of mixtures, the following form of the variation of mechanical properties (Young's Modulus, Density, and Poisson's Ratio) are postulated as

$$P(z) = (P_{\text{ceramic}} - P_{\text{metal}})V_c(z) + P_{\text{metal}}$$

Denotes a Generic Property

Specific Values of the Respective Properties

Volume Fraction

Note : For $V_c(z) = 0$, $P(z) = P_m$ and For $V_c(z) = 1$, $P(z) = P_c$
 $\Rightarrow V_c(z) \in [0,1]$

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3. Types of FGM Models-Continued



Type 1: Symmetric Distribution of the Constituent Phases

The phases vary symmetrically through the wall thickness in the sense of having full ceramic at the outer surfaces of the plate and tending toward full metal at the mid-surface

$$V(z,N) = \left(\frac{z}{h/2}\right)^N \frac{1 + \text{sgn}z}{2} + \left(\frac{z}{-h/2}\right)^N \frac{1 - \text{sgn}z}{2}$$

Where,

$$\text{sgn}(z) = \begin{cases} 0, & \text{if } z = 0 \\ -1, & \text{if } z < 0 \\ +1, & \text{if } z > 0 \end{cases} \implies \text{Signum Function}$$

$N \equiv$ Volume Fraction Index ($0 \leq N \leq \infty$)

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3. Types of FGM Models -Continued

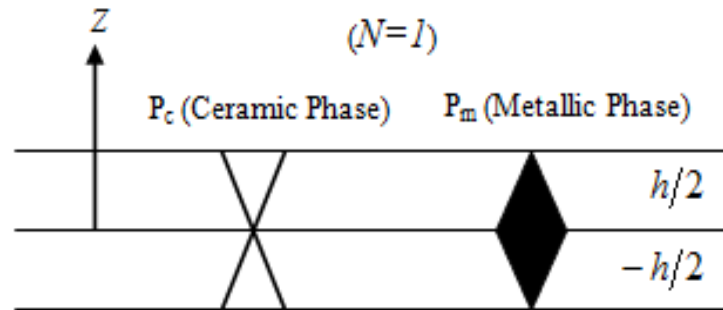


Figure 2. Distribution of the constituent materials through The plate thickness for the symmetric case.

at $z = \pm h/2$, for any N , $V = 1$, and as a result $P(\pm h/2) \Rightarrow P_c$, and for $z = 0$, $V = 0$ and $P(0) \Rightarrow P_m$.

Type 2: Asymmetric Distribution of the Constituent Phases

The phases vary non-symmetrically through the wall thickness and in this case there is Full ceramic at the outer surface of the plate wall and full metal at its inner surface.

3. Types of FGM Models-Continued



$$V(z, N) = \left[\frac{(h - 2z)}{2h} \right]^N$$

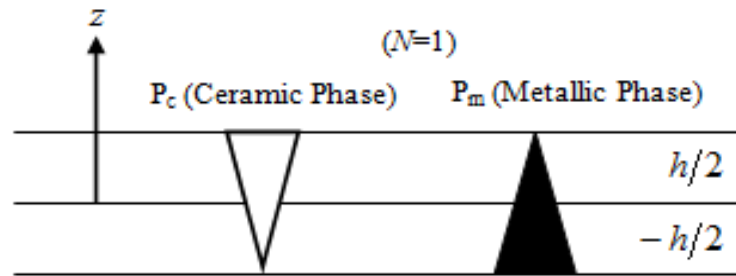


Figure 3. Distribution of the constituent materials through The plate thickness for the asymmetric case.

at $z = +h/2$, $P(h/2) \Rightarrow P_m$, and for $z = -h/2$, $P(h/2) \Rightarrow P_c$. At the Midsurface,

$$z = 0, \text{ and for } k = 1, P(0) = \frac{1}{2}(P_c + P_m)$$

4. Theoretical Developments



3-D Displacement Field

$$u(x, y, z, t) = u_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial x}$$

$$v(x, y, z, t) = v_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial y}$$

$$w(x, y, z, t) = w_0(x, y, t)$$

$u_0, v_0, w_0 \Rightarrow$ 2-D displacement quantities of the mid-surface

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4. Theoretical Developments- Continued



Non-linear Strain-Displacement Relationships

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}$$

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zz} = 0$$

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4. Theoretical Developments- Continued



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Substitution of the displacement field into the strain-displacement relationships
 Gives the 3-D strain measures across the plate thickness as

$$\underbrace{\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}}_{\{\varepsilon\}} = \underbrace{\begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \end{Bmatrix}}_{\{\varepsilon^{(0)}\}} + z \underbrace{\begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}}_{\{\varepsilon^{(1)}\}}$$

3D Strain Measures

2D in-plane strain measures

2D bending Strains

4. Theoretical Developments- Continued



Plane Stress Constitutive Equations

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$$

Where,

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad Q_{12} = \frac{\nu E(z)}{1-\nu^2}, \quad Q_{66} = \frac{E(z)}{2(1+\nu)}, \quad Q_{16}, Q_{26} = 0$$

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4. Theoretical Developments



Governing Equations

$$\int_{t_0}^{t_1} (\delta U + \delta V - \delta K) dt = 0 \Rightarrow \text{Hamilton's Principle}$$

t_0, t_1 are two arbitrary instants in time

U , denotes the strain energy

V , denotes the work done by surface tractions, edge loads, and body forces

K , denotes the kinetic energy of the 3 - D body of the Structure

δ , denotes the variational operator

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4. Theoretical Developments- Continued

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Strain Energy

$$\delta U = \int_{\Omega_0} \int_{-h/2}^{h/2} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{yy} \delta \varepsilon_{yy} + \sigma_{xy} \delta \gamma_{xy}) dz d\Omega_0$$

External Work

$$\delta V = - \int_{\Omega_0} P_t(x, y) \delta w \left(x, y, \frac{h}{2} \right) d\Omega_0 - \int_x \int_{-h/2}^{h/2} \left(\sigma_{yy}^* \delta v + \sigma_{yx}^* \delta u \right) dz dx -$$
$$\int_y \int_{-h/2}^{h/2} \left(\sigma_{xx}^* \delta u + \sigma_{xy}^* \delta v \right) dz dy$$

Kinetic Energy

$$\delta K = \int_{\Omega_0} \int_{-h/2}^{h/2} \rho(z) \dot{W} \delta \dot{W} dz d\Omega_0$$

4. Theoretical Developments- Continued



Equations of motion in terms of Stress resultants

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \frac{\partial}{\partial x} \left(N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y} \right) +$$

$$\frac{\partial}{\partial y} \left(N_{yy} \frac{\partial w_0}{\partial y} + N_{xy} \frac{\partial w_0}{\partial x} \right) + P_t - C\dot{w}_0 = I_0 \ddot{w}_0$$



4. Theoretical Developments- Continued



Where,

$$(N_{ij}, M_{ij}) = \int_{-h/2}^{h/2} \sigma_{ij}(1, z) dz \quad (i, j = x, y, xy) \Rightarrow \text{Stress Resultants and Stress Couples}$$

Simply Supported Boundary Conditions

$$w_0 = M_{xx} = N_{xy} = 0, N_{xx} = N_{xx}^* \text{ on } x = 0, L_1$$

$$w_0 = M_{yy} = N_{yx} = 0, N_{yy} = N_{yy}^* \text{ on } y = 0, L_2$$

For Compressive Edge Loading, $N_{xx}^* = -N_{xx}^0$ and $N_{yy}^* = -N_{yy}^0$



5. Solution Methodology



The first two equations of motion are satisfied by assuming the following

$$N_{xx} = \frac{\partial^2 \phi}{\partial y^2}, N_{yy} = \frac{\partial^2 \phi}{\partial x^2}, N_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

It can be shown, the details of which are not shown here, that the third equation of motion can be expressed in terms of displacements and a stress potential as

$$D\nabla^4 w_0 - \left(\frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} - 2 \frac{\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 w_0}{\partial x^2} \right) + I_0 \ddot{w}_0 + C \dot{w}_0 = P_t$$

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5. Solution Methodology- Continued



Where,

$$D = \frac{E_1 E_3 - E_2^2}{E_1(1 - \nu^2)}$$

One more equation in terms of the transversal deflection and the stress potential is needed. This will come from the strain displacement relationships, by eliminating the in-plane displacements, which is known as the compatibility equation which can be shown to be given by

$$\nabla^4 \phi = E_1 \left[\left(\frac{\partial^2 w_0}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} \right]$$



5. Solution Methodology- Continued



Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \Rightarrow$ Laplacian Operator

Assume a solution form for ϕ, w_0

$$w_0(x, y, t) = w_{mn}(t) \sin \lambda_m x \sin \mu_n y$$

$$\begin{aligned} \phi = & A_{mn}(t) \cos 2\lambda_m x + B_{mn}(t) \cos 2\mu_n y + C_{mn}(t) \cos 2\lambda_m x \cos 2\mu_n y \\ & + D_{mn}(t) \sin 2\lambda_m x \sin 2\mu_n y + \frac{1}{2} N_{xx}^* y^2 + \frac{1}{2} N_{yy}^* x^2 \end{aligned}$$

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5. Solution Methodology- Continued



Where,

$$\lambda_m = m\pi/L_1, \mu_n = n\pi/L_2, \quad m, n = 1, 2, 3, \dots, \text{etc.}$$

Utilizing the compatibility equation and comparing coefficients gives

$$A_{mn}(t) = \frac{E_1 w_{mn}^2(t) \mu_n^2}{32 \lambda_m^2}, B_{mn}(t) = \frac{E_1 w_{mn}^2(t) \lambda_m^2}{32 \mu_n^2}, C_{mn}(t) = D_{mn}(t) = 0$$

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5. Solution Methodology- Continued



Transversal Pressure

Let,

$$P_t = P_{mn}(t) \sin \lambda_m x \sin \mu_n y$$

By integrating over the plate area gives

$$P_{mn}(t) = \frac{4}{L_1 L_2} \int_0^{L_2} \int_0^{L_1} P_t(t) \sin \lambda_m x \sin \mu_n y dx dy$$

or

$$P_{mn}(t) = \frac{16P_t(t)}{\pi^2}, \quad \text{Where, } m, n = 1$$

5. Solution Methodology- Continued



Finally, applying the Galerkin method to the third equation of motion Gives the nonlinear equation differential equation governing the Structural response of FG plates under external excitation.

$$\ddot{w}_{mn}(t) + 2\Delta_{mn}\omega_{mn}\dot{w}_{mn}(t) + \omega_{mn}^2 w_{mn}(t) + \Omega_{mn}w_{mn}^3(t) = \tilde{P}_{mn}(t)$$

$w_{mn}(t) \Rightarrow$ Amplitude of Deflection

$\omega_{mn} = \sqrt{K_{mn}/I_0} \Rightarrow$ The natural frequency of the plate

$\Delta_{mn} = C/2I_0\omega_{mn} \Rightarrow$ Nondimensional damping factor

$\Omega_{mn} = E_1(\lambda_m^4 + \mu_n^4)/16$

$\tilde{P}_{mn}(t) = 16P_t(t)/I_0\pi^2$

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5. Solution Methodology- Continued



$$K_{mn} = \frac{D\pi^4}{L_1^4} (m^4 + 2m^2n^2\psi^2 + n^4\psi^4) + \frac{N_{xx}^*\pi^2}{L_1^2} (m^2 + n^2\Phi\psi^2)$$

Where,

$\psi = L_1/L_2 \Rightarrow$ Aspect Ratio

$\Phi = N_{yy}^*/N_{xx}^* \Rightarrow$ Compressive/Tensile Edge Load Ratio

The governing nonlinear differential equation is solved using the 4th-Order Runge-Kutta method assuming zero initial conditions.

6. Blast Loading



For a free in-air spherical air burst, the pressure profile over time is given in figure 4 as

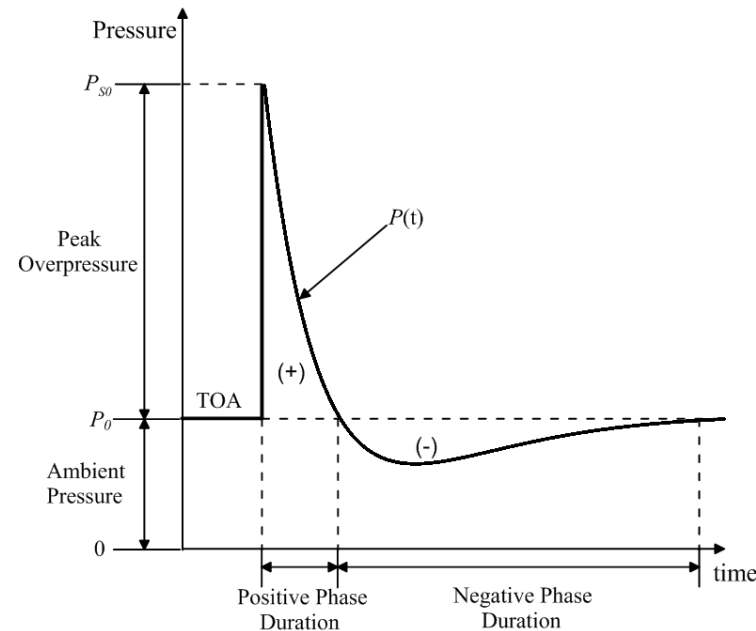


Fig 4. Incident Profile of a blast wave

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6. Blast Loading-Continued

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The wave form shown in figure 4 is given by an expression known as The Friedlander equation and is give as

$$P_t(t) = (P_{so} - P_o) \left(1 - \frac{t - t_a}{t_p} \right) e^{-\alpha \left(\frac{t - t_a}{t_p} \right)}$$

Where,

$$P_{so} = \frac{1772}{Z^3} - \frac{114}{Z^2} + \frac{108}{Z} \Rightarrow \text{Peak Overpressure over ambient}$$

$$Z = R/W^{1/3} \Rightarrow \text{scaled distance} \begin{cases} R \text{ is the Standoff Distance} \\ W \text{ is the equivalent weight of charge} \\ \text{of TNT in terms of kilograms} \end{cases}$$

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6. Blast Loading-Continued

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P_0 is the ambient pressure

t_a is the time of arrival

t_p is the positive phase duration of the blast wave

t is the time

For conditions of STP at sea level, the time of arrival and the positive phase duration can be determined from

Arrival time or positive phase duration

$$\frac{t}{t_1} = \frac{R}{R_1} = \left(\frac{W}{W_1} \right)^{\frac{1}{3}} \Rightarrow \text{Cube root scaling}$$

Arrival time or positive phase duration for a reference explosion of charge weight, W_1

It should be noted that the standoff distances are themselves scaled
According to the cube root law



Validation of the Present Approach

Comparisons are made (Akay, H.U., 1980) who considered explosive blast of Metallic plates under a step loading excitation given by

$$P_t(t) = P[H(t) - H(t - t_0)]$$

$H(t)$ is referred to as the Heaviside Step Function defined as $H(t) = 1$ for $t \geq 0$ and $H(t) = 0$ for $t < 0$

The geometrical and material properties used were

$$L_1 = 2.438m, h = 0.00635m, \psi = 1$$

$$E_m = 70.3GPa, \rho_m = 2547kg/m^3, \nu_{ave} = 0.25$$



7. Results-Continued

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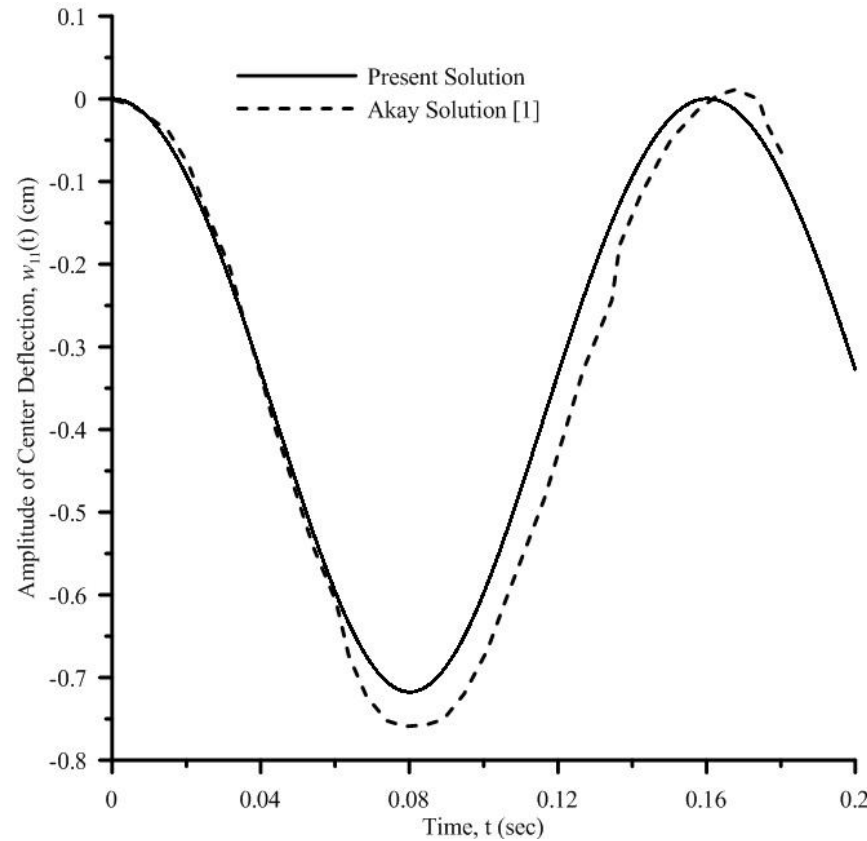


Fig 5. Comparisons of solutions of the time history of the Central deflection Under a Step Load $P = 48.82Pa$, $t_0 = 0.2\text{ sec}$

7. Results-Continued



Present Results

A ceramic-metal functionally graded plate with the following properties were used

Table 1. Material Properties

	Metal (Ti-6Al-4V)	Ceramic (Aluminum Oxide)
E , Modulus (GPa)	105.7	320.24
ν , Poisson's Ratio (Unitless)	0.2981	0.26
ρ , Density (Kg/m ³)	4429	3750

($L_1 = 1$ m, $\psi = 1$, $h = 0.0254$ m)

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7. Results-Continued

Table 2. Air blast parameters versus distance for a one Kilogram TNT spherical air burst (Kingery and Bulmash, 1984).

Standoff Distance, (m)	Arrival Time, (msec)	Positive Phase Duration, (msec)
1.0	0.532	1.79

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7. Results-Continued

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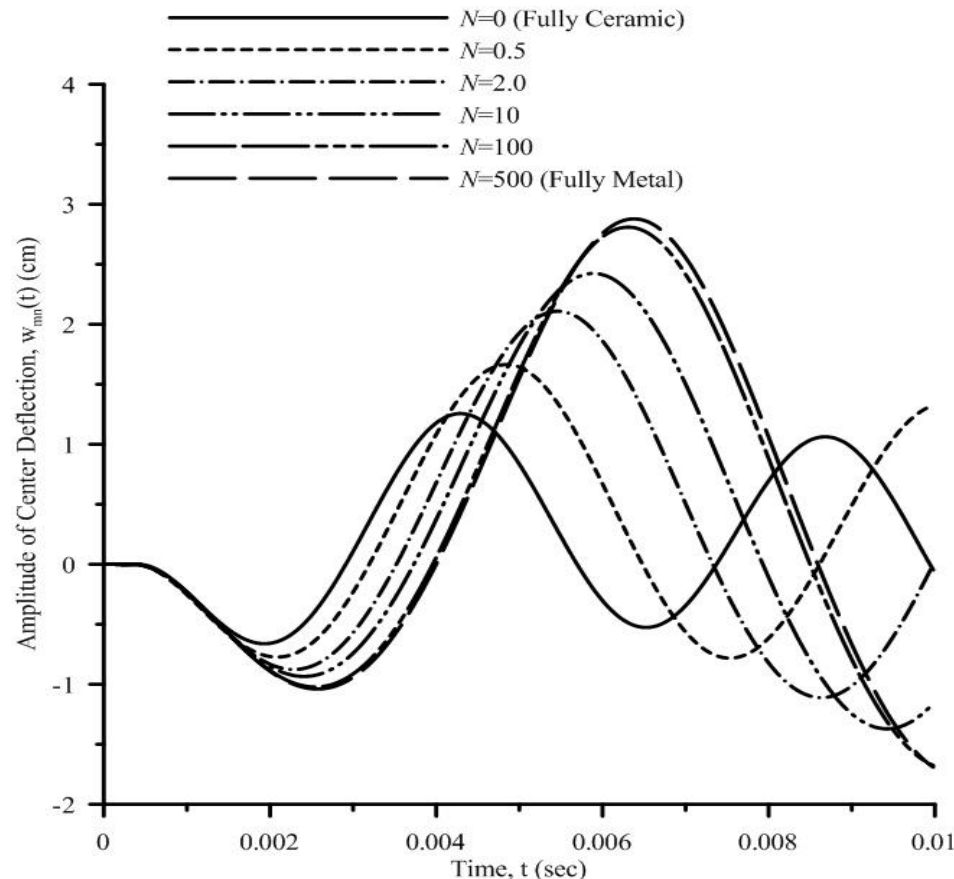


Fig 6. Implications of the volume fraction index on the deflection-time history of an asymmetric FG Plate without damping.

7. Results-Continued

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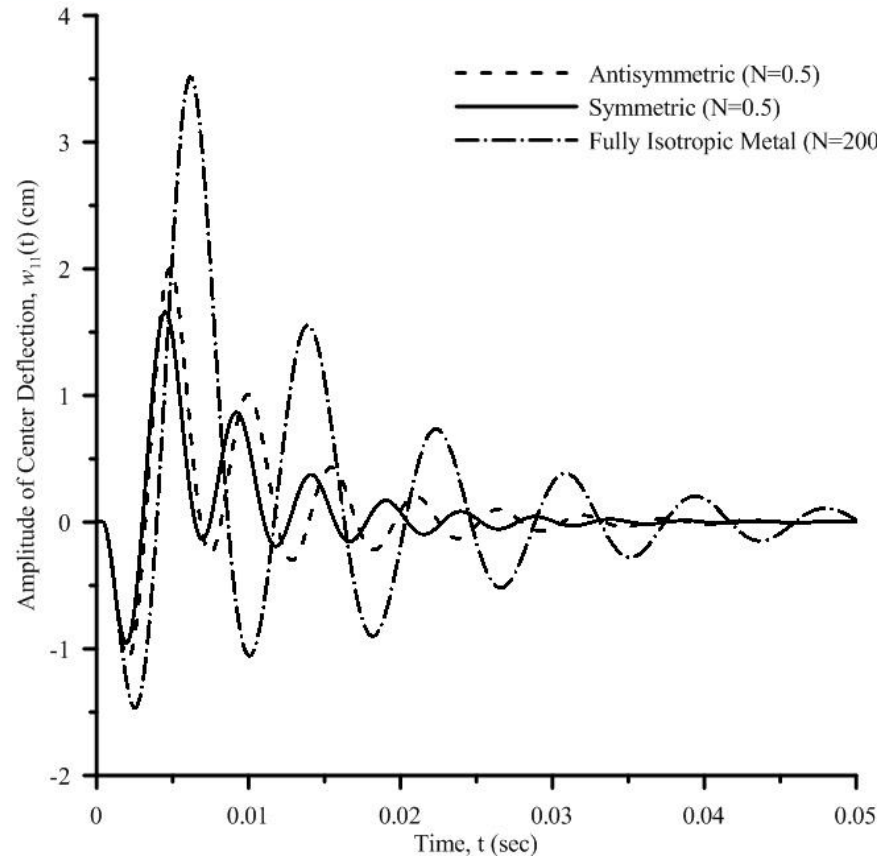


Fig 7. Implications of the symmetry on the deflection-time response of FG plate. $\Delta_{11} = 0.1$

7. Results-Continued

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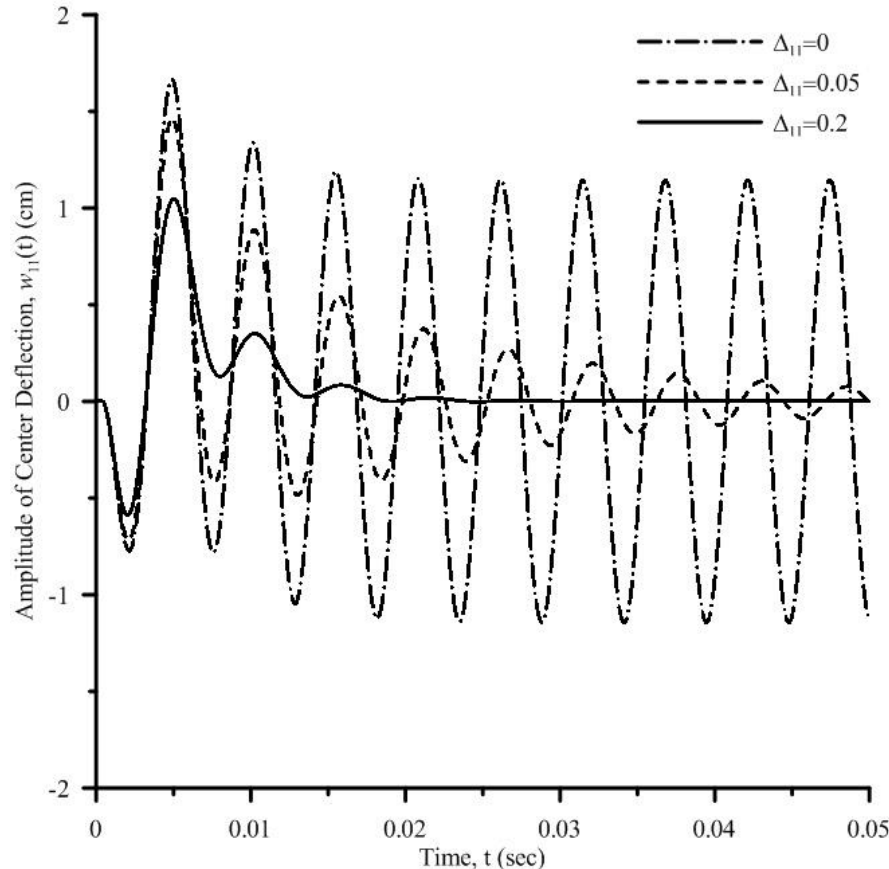


Fig 8. Implications of various amounts of damping on the deflection-time response of an asymmetric FG Plate. (N=0.5)

7. Results-Continued

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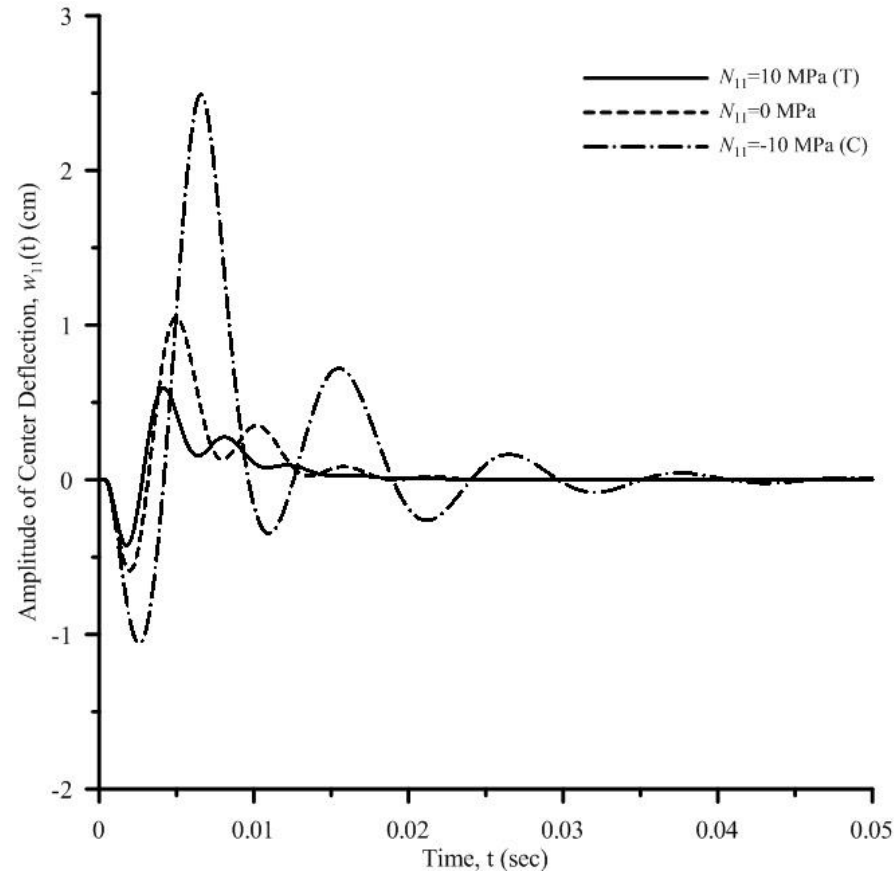


Fig. 9. The implications of the compressive/tensile edge loading on the deflection-time response of an asymmetric FG plate. ($\Delta_{11} = 0.2, N = 0.5$)

7. Results-Continued

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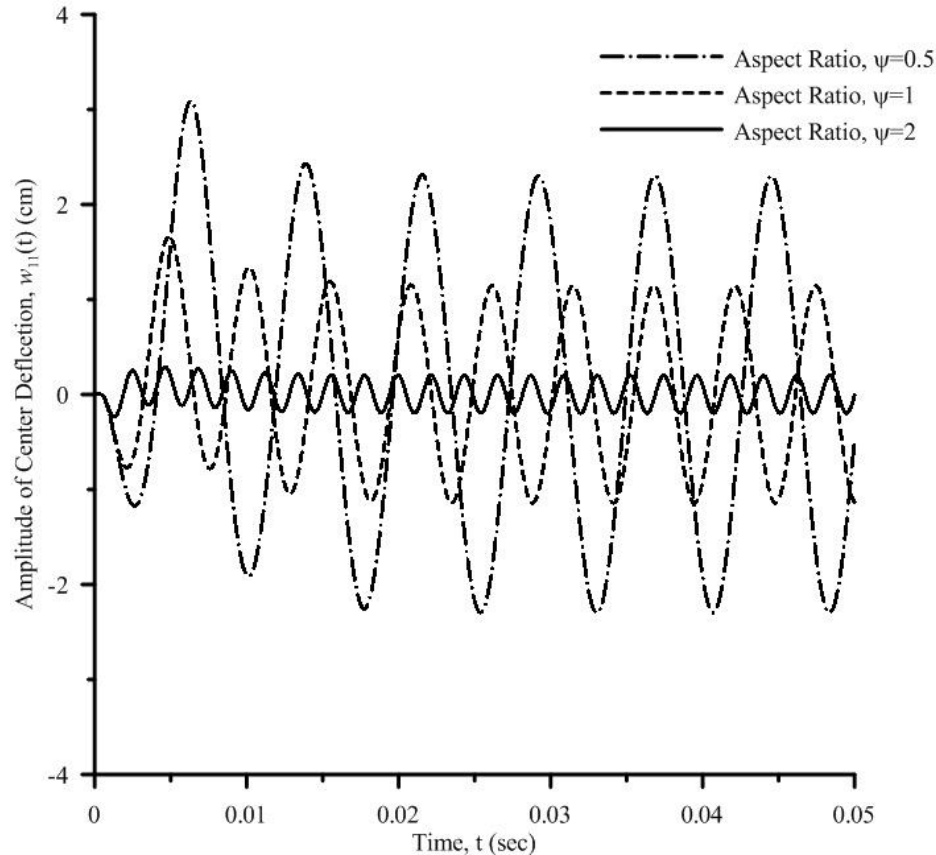


Fig. 10. The effects of various aspect ratios of an asymmetric FG plate on the deflection-time response. ($N=0.5$)

7. Results-Continued

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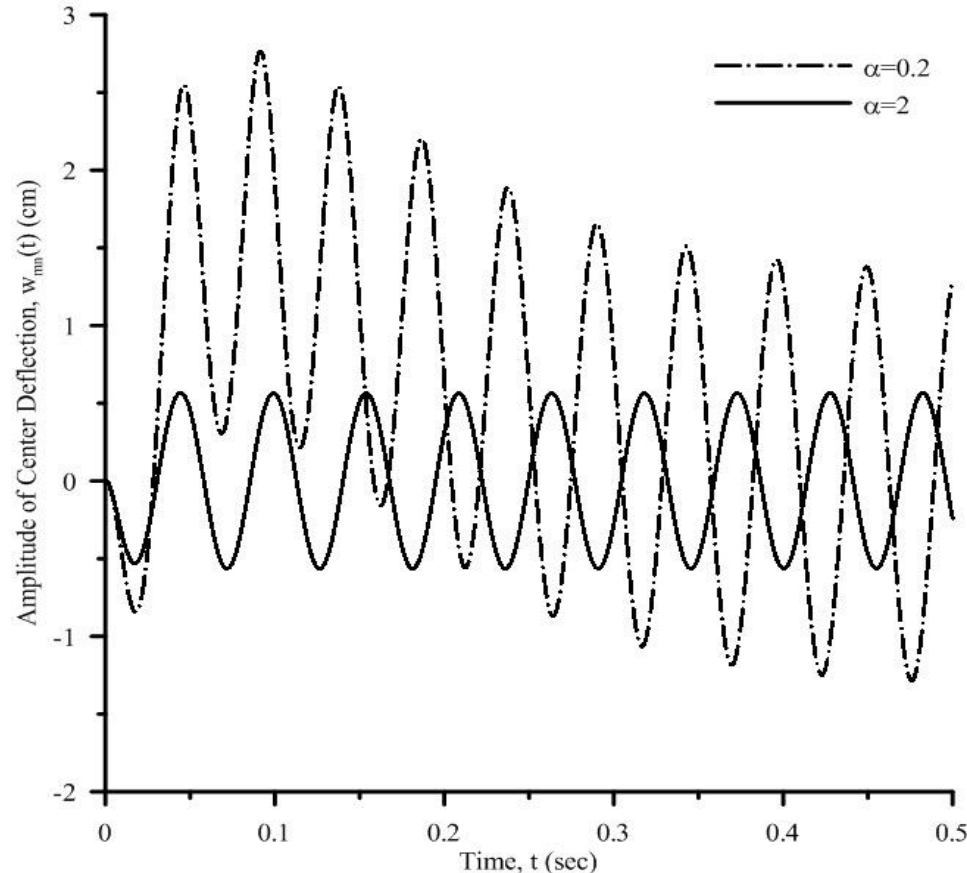


Fig. 11. The implications of the value of the decay parameter on the time-deflection response of a asymmetric FG Plate without damping. (N=0.5)

8. Conclusions

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- A Structural Model of the dynamic response of functionally graded plates exposed to a free in-air blast with simply supported boundary conditions has been presented.
- As the volume fraction of ceramic increases, the deflections become smaller with higher frequencies in contrast to metal phase.
- The symmetric construction due to being stiffer has smaller deflections and higher frequencies as compared to the asymmetric construction
- For a fixed compressive edge load, as the metal concentration increases, the deflections increase with decreasing frequencies.