

Computationally Efficient Resampling of Nonuniform Oversampled SAR Data

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Abstract— A computationally efficient technique is developed to resample SAR data oversampled in the slow-time domain for a platform accelerating along the track. The algorithm segments the data between uniformly spaced points on the synthetic aperture, then performs a weighted average on the demodulated SAR data in each segment to adjust the effective sample position on the aperture. The weighting is derived using a first-order Taylor series approximation and used parameters from the velocity and acceleration of the platform and the radar waveform.

I. INTRODUCTION

Traditional synthetic aperture radar (SAR) processing algorithms assume that radar data are collected on a platform that is traveling in a straight line at a constant velocity [1,2,3]. For large aircrafts, this is often a good assumption. However, for small slow-moving platforms, such as robotic vehicles or unmanned aerial vehicles (UAV), this assumption may not be true. Factors such as user input, the path of the road, or environmental factors such as wind can change the velocity of the platform. To form well-focused SAR images using traditional processing algorithms, techniques need to be developed to compensate for these effects. In addition, these factors can have a significant effect on coherent change detection (CCD) algorithms and ground moving target indicator (GMTI) algorithms based upon dual-aperture SAR processing algorithms [4]. For these algorithms, clutter is removed by subtracting the SAR images formed in each aperture. If the sampling rates are not uniform across both apertures, then the performance of clutter cancellation algorithms is degraded [5].

SAR processing algorithms based upon techniques such as polar formatting or wavefront reconstruction need to resample the processed data so that it is distributed uniformly in k -space [6,7]. The resampling can be performed using algorithms based upon sinc reconstruction, which is computational expensive for large data sets. The algorithm developed in this paper does not eliminate this step, but it reduces the amount of data needed to be resampled with a minimal impact on the signal-to-noise ratio (SNR).

To generate SAR data, a radar is typically flown along a straight path at a constant velocity as shown in Fig. 1. A radar waveform is transmitted at a constant pulse repetition frequency (PRF) as the platform moves along the y -axis, or equivalently, along the track. The reflected radar waveform is sampled in time and in space as the platform moves along the synthetic aperture. The sampling in time of a radar pulse is referred to as fast-time and the sampling across the aperture at different locations is referred to as slow-time. The position of the radar on the y -axis at the start of each PRI is denoted by u . For a stripmap SAR, the antenna beam is pointed at a constant angle relative to the direction of motion, so the patch of ground illuminated changes as the platform moves.

SAR reconstruction algorithms will generate an image centered around the point (X_c, Y_c, Z_c) with the n th target located at (X_n, Y_n, Z_n) . The size of the SAR image is typically constrained between $2Y_0$ in cross range, and $2X_0$ in down range relative to the center of the image [8].

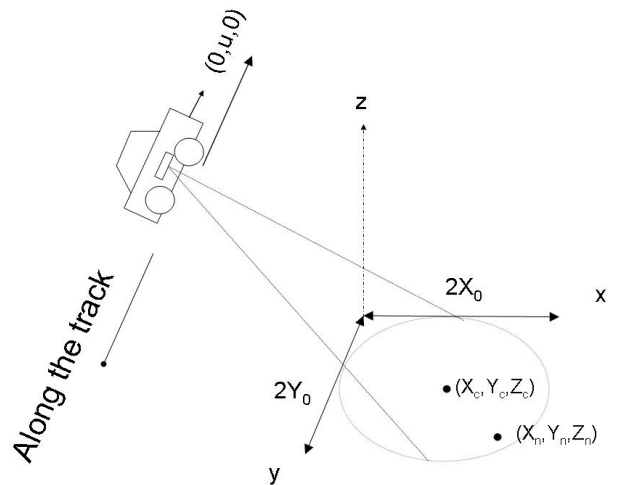


Figure 1. SAR collection geometry.

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14. ABSTRACT A computationally efficient technique is developed to resample SAR data oversampled in the slow-time domain for a platform accelerating along the track. The algorithm segments the data between uniformly spaced points on the synthetic aperture, then performs a weighted average on the demodulated SAR data in each segment to adjust the effective sample position on the aperture. The weighting is derived using a first-order Taylor series approximation and used parameters from the velocity and acceleration of the platform and the radar waveform.					
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This paper develops a computationally efficient technique to resample SAR data collected on an accelerating platform. The algorithm assumes that the radar data are significantly oversampled in slow-time. For robotic vehicles and small UAVs, the speed of the platform and available power are likely to be small. The slow platform speed will increase the oversample rate in slow-time and allow flexibility in designing waveforms with acceptable ambiguity functions. Coherent integration of the returned signals will improve the SNR.

The signal processing for the resampling algorithm is performed in two steps. First, slow-time data that has been demodulated is grouped together in uniformly spaced segments that can contain a variable number of samples. Next, a weighting is applied to the demodulated data in each segment, and the results are summed. Now, standard SAR image reconstruction algorithms can be applied to the resampled data.

II. SIGNAL PROCESSING

SAR imagery is generated by coherently processing the sampled radar data. After down conversion, the SAR data is modeled as

$$s(\omega, u) = P(\omega) \sum_N \sigma_n \exp(-j2k\sqrt{(X_n^2 + (Y_n - u)^2 + Z_n^2)}) \quad (1)$$

where ω is the frequency, $k = \omega/c$ is the wave number; c is the speed of light, $P(\omega)$ is the Fourier transform of the transmitted signal $p(t)$; σ_n is the amplitude and X_n , Y_n and Z_n are the position of the n^{th} scattering center; and u is the position of the platform along the track [8]. The first step for processing this signal is to multiply it by the complex conjugate of $P(\omega)$. For waveforms such as linear chirp, $P(\omega)$ is flat over the frequencies of interest, and the term can be removed from (1). Next, the bandwidth of the received signal is reduced by multiplying (1) by the complex conjugate of the reference signal given by

$$S_0(\omega, u) = \exp(-j2k\sqrt{(X_c^2 + (Y_c - u)^2 + Z_c^2)}) \quad (2)$$

where X_c , Y_c , and Z_c are the coordinates of the center of the region of the image. This results in a demodulated signal in the slow-time domain that can be approximated using the first-order Taylor series by

$$s_c(\omega, u) = S(\omega, u) S_0^*(\omega, u) \approx \sum_N \sigma_n \exp(-j2k \frac{((X_n - X_c)X_c + (Y_n - Y_c - u)Y_c + (Z_n - Z_c)Z_c)}{R_c}) \quad (3)$$

where $s_c(\omega, u)$ is the demodulated radar signal and $*$ denotes complex conjugate. The Nyquist sample rate for the demodulated radar signal is given by

$$\Delta u \leq \frac{R_c \pi}{k(2Y_0)} \quad (4)$$

where R_c is the range to the scene center and $2Y_0$ is the maximum cross range extent of the target relative to the scene center.

The motion of the platform is modeled and simulated using

$$u(m) = L + VmT + \frac{1}{2}A(mT)^2, \quad m=0 \dots M-1 \quad (5)$$

where L , V , and A are the position, velocity, and acceleration of the platform on the y -axis; and T is the PRI of the transmitted waveform or equivalently, the slow-time sample interval. Since A is generally nonzero, the sample positions along the track are not uniformly spaced, which is undesirable.

The radar data associated with M slow-time samples can be grouped into N equally spaced segments from 0 to $u(M-1)$, where $M > N$. The desired location of the slow-time sample positions are given by

$$u'(n) = u(0) + (\Delta u' - \Delta \bar{u})/2 + \Delta u'n, \quad n=0, \dots, N-1 \quad (6)$$

where

$$\Delta u' = \left[(u(M-1) - u(0)) / \left(N - \frac{N}{M} \right) \right] \quad (7)$$

and

$$\Delta \bar{u} = (u(M-1) - u(0)) / (M-1). \quad (8)$$

The boundaries between each segment are given by

$$u(0) + (\Delta u' - \Delta \bar{u})/2 + \Delta u'(n-1/2), \quad n=1, \dots, N-1. \quad (9)$$

The demodulated data can be multiplied by a window function to adjust the effective slow-time sample location in each segment, as shown below:

$$u'(n) = \sum_{s=0}^{s=S_n-1} w_n(s) u(s + M'(n)) \quad (10)$$

with the constraint that

$$\sum_{s_n} w_n(s) = 1, \quad (11)$$

where w_n is the window function for the n^{th} data segment with S_n elements, and $M'(n)$ is the index associated with the first pulse in the n^{th} data segment.

Fig. 2 shows an example of the slow-time data being segmented for $M=17$ and $N=3$. The boundaries for each segment are shown in red, the desired location of the sample locations are shown with light blue circles, and the window functions are shown with three black lines. Intuitively, the window functions would more heavily weight the data that was more sparsely sampled and less heavily weight the data that was more closely sampled.

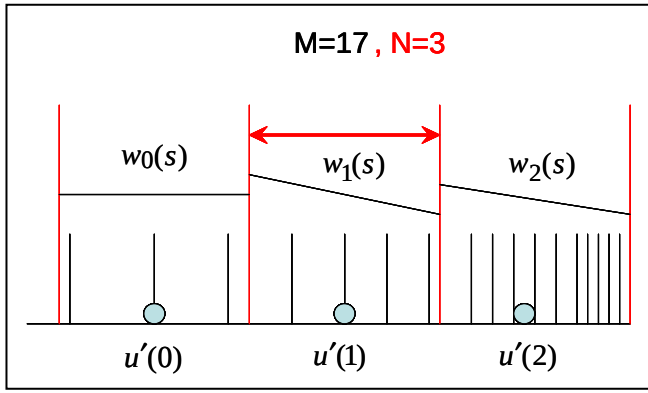


Figure 2. Illustration of segmenting and windowing of slow-time demodulated radar data.

A simple window function, for which an analytical expression can be derived for its parameter, is given by

$$w_n(s) = \frac{1}{S_n} + q_n \left(s - \frac{(S_n - 1)}{2} \right), \quad s=0, \dots, S_n - 1 \quad (12)$$

where S_n is the number of samples in the n^{th} interval, s is the index of the sample, and q_n corresponds to the slope of the line in the n^{th} window function. Multiplying the data by this window function and summing is equivalent to performing a weighted average. If the window function is equal to $1/S_n$ for all values of s , this is equivalent to a simple average.

The unknown parameter in the window function is determined by observing that for small changes in u , the average phase of the demodulated radar signal equals the phase of the average of the demodulated radar signal. This approximation is based upon the first-order Taylor series expansion given by

$$\exp(j(\phi + \varepsilon)) \approx \exp(j\phi)(1 + j\varepsilon) \quad (13)$$

$$\text{when} \quad \frac{\varepsilon^2}{2} \ll 1. \quad (14)$$

The slope in (12) can be adjusted so that the effective slow-time sample position of the data in each segment is its center. This result is obtained by equating the desired signal for a scatterer located at an arbitrary crossrange location calculated from (2) and (6) to the weighted average location:

$$\exp(-j2k \left(\frac{Y_n u'(n)}{\sqrt{(X_c^2 + (Y_c - u'(n))^2 + Z_c^2)}} \right)) = \sum_{s=0}^{S_n-1} w_n(s) \exp(-j2k \left(\frac{Y_n u(s + M'(n))}{\sqrt{(X_c^2 + (Y_c - u(s + M'(n)))^2 + Z_c^2)}} \right)) \quad (15)$$

where Y_n is the y-coordinate of an arbitrary scattering center.

The slope of the window function is solved for analytically by making a small angle approximation, expanding the

expressions in the numerator of the exponents in (15) while assuming the denominators in the exponents are equal, and then summing the terms on the right hand side using the formulas for the finite sequences as shown [9]:

$$\sum_{n=1}^N n = \frac{1}{2} N(N+1) \quad (16)$$

$$\sum_{n=1}^N n^2 = \frac{1}{6} N(N+1)(2N+1) \quad (17)$$

$$\sum_{n=1}^N n^3 = \left(\frac{1}{2} N(N+1) \right)^2 \quad (18)$$

Using these formulas, the slope for the n^{th} weighting function described in (12) is equal to

$$q_n = \frac{u'(n) - L'(n) - V'(n)T(M-1)/2 - AT^2(M-1)(2M-1)/12}{\left(\frac{V'(n)T(M(M-1)(2M-1)/6 - M(M-1)^2/4 + AT^2(M^2(M-1)^2/8 - M(M-1)^2(2M-1)/24))}{1} \right)} \quad (19)$$

where $L'(n)$ and $V'(n)$ are the position and velocity of the platform along the track at the beginning of the n^{th} segment. Nonuniformly sampled demodulated radar data is resampled with effective uniform sampling using the results from (19).

III. SIMULATIONS

Simulations were run to demonstrate the effect of the resampling algorithm on SAR image quality. Imagery was processed using the wavefront reconstruction algorithm with the data simulated on a platform with constant velocity and compared to results for data simulated on an accelerating platform. The data simulated on the accelerating platform was processed with no resampling and resampling based upon an unweighted and a weighted average of the demodulated data. As expected, data simulated on an accelerating platform that was processed with no resampling did not image properly and is not analyzed further.

The simulated SAR imagery contained 9 point scatter targets that were centered at a range of 100 m and at a squint angle of 30°. One target is located at the center of the image at (86.6, 50) m and 8 targets are located at the corners of two squares centered in the image with sides of length 34 m and 20 m. All the targets had an amplitude of 1.0. The platform motion was simulated with a velocity of 3 m/s and an acceleration of approximately 1 m/s² along the y-axis. The acceleration was randomly varied with a uniform distribution from 0.9 to 1.1 m/s². The radar center frequency was 3 GHz and the waveform had a bandwidth of 300 MHz, and a PRF of 1.15 KHz. For an acceleration of 1 m/s², 4176 samples in slow-time were simulated over an aperture of ~17.5 m. The targets were sampled in fast-time at the Nyquist rate for the region of spatial support. The data were oversampled in slow-time by a factor of 20, and then reduced to a rate that was oversampled by a factor of 1-9. The oversample rate was

based on the Nyquist criteria for the least favorable position of a scatter within the region of spatial support (the largest square). No noise, vibration, antenna patterns, or other affects were included in the simulated data.

SAR images are reconstructed using a wavefront reconstruction algorithm [8]. Fig. 3 shows a SAR image generated with uniform sampling in slow-time and oversampled by a factor of 2. Fig. 4 shows a SAR image generated with data that is sampled nonuniformly in slow-time with an average oversampled rate that is a factor of 20 over Nyquist and resampled to a rate is a factor of 2 over Nyquist using a weighted average. The scattering centers should have sinc-like sidelobes. The center of the black circles show the actual locations of the scattering centers. The color scales are in dB. The SAR image shown in Fig. 4 is very similar to the ideal SAR image shown in Fig. 3. Small difference in the sidelobes of can be seen, particularly in the right-hand corner of the images.

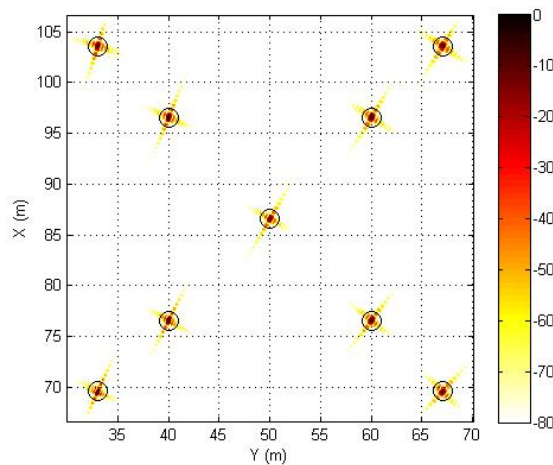


Figure 3. SAR image with uniform sampling along the track.

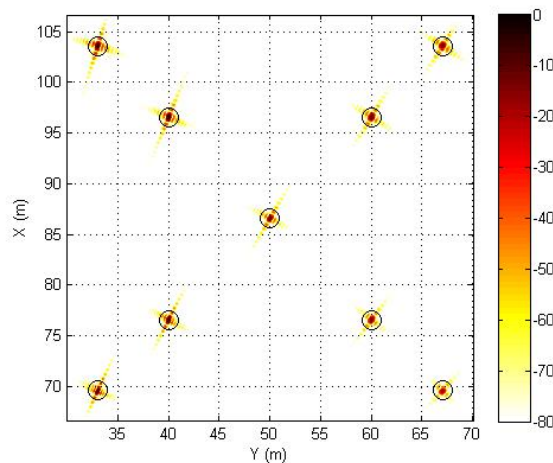


Figure 4. SAR image generated with resampled data calculated using weighted averaging.

To quantify the performance of the resampling algorithm, the ratio of the power difference between SAR images calculated using uniformly sampled data and nonuniformly

sampled data to the power of the SAR image calculated using uniformly sampled data is calculated for different slow-time resampling rates. The power differences between the SAR images are calculated both coherently and noncoherently. The resample data is calculated using both a simple average and a weighted average of the demodulated data. The average nonuniform oversample rate is a factor of 20 over Nyquist for all the runs. The effective sample positions along the aperture for the resampled data are matched to the sample positions of the uniformly sampled data. The results are calculated for resample rates between 1 – 9 and are averaged over 100 trials with randomly varying accelerations. The results are shown in Fig. 5 for the noncoherent power difference and Fig. 6 for and coherent power difference. The blue line shows the results for resampling performed with a weighted average and the red line shows the results for resampling with a simple average.

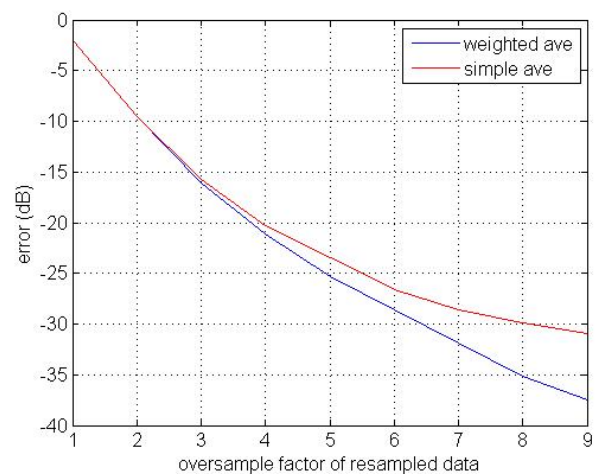


Figure 5. Noncoherent difference between SAR imagery generated with uniform sampling and nonuniform sampling that was resampled.

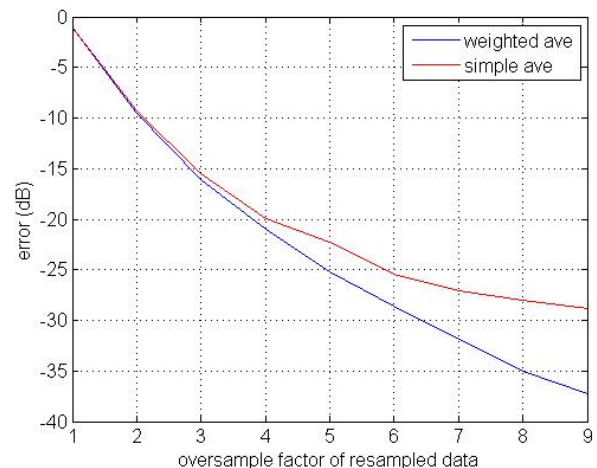


Figure 6. Coherent difference between SAR imagery generated with uniform sampling and nonuniform sampling that was resampled.

These results show that at low resample rates, the small angle approximation made in (14) is not valid and the errors in both techniques are large. For example, an oversample rate of 5 requires $0.2 \ll 1$, which is marginally satisfied. As the

oversample rate is increased, the results for both averaging methods improve for the coherent and noncoherent calculations. The improvement is better for resampling with a weighted average compared to a simple average.

IV. CONCLUSION

A computationally efficient technique is developed to resample SAR data oversampled in the synthetic aperture or slow-time domain for a platform accelerating along the track. The algorithm segments the data between uniformly spaced points on the synthetic aperture, then performs a weighted average on the demodulated data in each segment to adjust the effective sample position on the aperture to the desired location. The weighting is derived using a first-order Taylor series approximation and it uses parameters from the velocity and acceleration of the platform and the radar waveform.

The calculations based upon a weighted average performed better than a simple average as the oversample rate of the resampled data is increased. The weighted averaging will slightly degrade SNR, but this effect was not evaluated in the simulation.

The computational complexity of the algorithm is determined by the demodulation of the data performed in (3) and the weighted averaging of demodulated data performed in (10). The demodulation of the data requires that the down converter radar signal modeled in (1), whose length is determined by the spatial support of the SAR image in range and the bandwidth of the transmitted signal, is multiplied by a complex reference signal of the same length. For the simulations, this was 132 elements. The weighted average calculation multiplies the demodulated data by a scalar, then sums the results. For a slow-time oversample rate of 20 and a

resample rate of 5, typically, 4 samples were averaged for each frequency and segment.

This technique could be adapted to other scenarios and improved by using different window functions and different segmentation schemes. For example, to achieve greater decimation of the data, a second-order Taylor series expansion in could be performed on the demodulated data, and an arbitrary offset could be added to the linear weighting function.

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