

Bayesian Parametric Approach for Multichannel Adaptive Signal Detection

Pu Wang, Hongbin Li, and Braham Himed

Abstract—This paper considers the problem of space-time adaptive processing (STAP) in non-homogeneous environments, where the disturbance covariance matrices of the training and test signals are assumed random and different with each other. A Bayesian detection statistic is proposed by incorporating the randomness of the disturbance covariance matrices, utilizing *a priori* knowledge, and exploring the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix. Specifically, the Block-Toeplitz structure of the covariance matrix allows us to model the training signals as a multichannel auto-regressive (AR) process and hence, develop the Bayesian parametric adaptive matched filter (B-PAMF) to mitigate the training requirement and alleviate the computational complexity. Simulation using both simulated multichannel AR data and the challenging KASSPER data validates the effectiveness of the B-PAMF in non-homogeneous environments.

Index Terms—Parametric adaptive matched filter, Bayesian detection, space-time adaptive signal processing, non-homogeneous environments.

I. INTRODUCTION

Traditional space-time adaptive processing (STAP) usually deals with homogeneous environments, where the test signal is assumed to share the same covariance matrix with the training signals [1]–[4].

To account non-homogeneous environments, a number of models have been proposed. One is the partially homogeneous environment, which assumes the training signals share the covariance matrix with the test signal up to an unknown scaling factor [5], [6]. This model can be considered as a special case of the generalized eigenrelation (GER) [7]. Another one is the compound-Gaussian model, which assumes the training signals are a product of a texture (scalar) and a Gaussian vector. The texture is used to simulate power differences among the signals from range bins [8], [9]. More recently, a new class of non-homogeneous environments for adaptive signal detection emerges. This non-homogeneous environment is characterized by treating disturbance covariance matrices of both the test signal and training signals as random matrices and ensuring that they are different in probability one [10]–[14]. Following the Bayesian non-homogeneous model in [10], an adaptive matched filter has been derived by replacing the exact covariance matrix of the test signal by its maximum a

posteriori (MAP) estimate in the matched filter [10]. It has been shown that, by accounting the heterogeneity knowledge, the Bayesian adaptive matched filter (B-AMF) outperforms the standard AMF in the non-homogeneous environment. For applications of this Bayesian non-homogeneous model that employ space-time adaptive processing, the training requirement of the sample covariance matrix (SCM)-based B-AMF cannot be met due to the scarcity of the training signal. For example, with $J = 11$ spatial channels and $N = 32$ coherent pulses of the KASSPER dataset and assuming an instantaneous RF bandwidth of 500 KHz, $K = JN = 352$ training signals calls for the training range over a 200-km range, which is not practical [15], [16].

In this paper, while preserving the Bayesian non-homogeneous environment, we further explore the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix which allows the block LDU decomposition [17], and hence enables the disturbances to be modeled as a multichannel auto-regressive (AR) process [17]–[23]. The resulting Bayesian parametric adaptive matched filter (B-PAMF) reduces the joint spatial-temporal whitening of the SCM-based B-AMF to successive spatial and temporal whitening. As a result, it facilitates the STAP in the non-homogeneous environments, and reduces the excessive training requirement of the B-AMF. Moreover, the B-PAMF is able to incorporate heterogeneities of the signals and utilize available *a priori* knowledge to the decision statistic. The effectiveness of the B-PAMF is verified by using the simulated multichannel AR data and the high fidelity KASSPER data [15].

II. SIGNAL MODEL

Assume J spatial channels, N temporal pulses, and K training range cells. The problem of interest is to detect a $JN \times 1$ multichannel signal $\mathbf{s}$ with *unknown* amplitude α in the presence of spatially and temporally correlated disturbance $\mathbf{d}_0$:

$$\begin{aligned} H_0 : \quad & \mathbf{x}_0(n) = \mathbf{d}_0(n), n = 0, 1, \dots, N-1, \\ H_1 : \quad & \mathbf{x}_0(n) = \alpha \mathbf{s}(n) + \mathbf{d}_0(n), n = 0, 1, \dots, N-1, \end{aligned} \quad (1)$$

where $\mathbf{d}_0 = [\mathbf{d}_0^T(0), \mathbf{d}_0^T(1), \dots, \mathbf{d}_0^T(N-1)]^T$, and $\mathbf{s}$ and $\mathbf{x}_0$ are similarly defined. In this paper, the signal model makes the following assumptions:

- **AS1 (Multichannel AR Process):** The disturbances in both test and training signals are modeled as a multichannel AR process [17]–[19]:

$$\mathbf{d}_k(n) = -\sum_{i=1}^P \mathbf{A}^H(i) \mathbf{d}_k(n-i) + \boldsymbol{\varepsilon}_k(n), k = 0, \dots, K, \quad (2)$$

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This work was supported by the Air Force Research Laboratory (AFRL) under Contract FA8750-05-2-0001 and the Air Force Office of Scientific Research (AFOSR) under Grant FA9550-09-1-0310.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE MAY 2010		2. REPORT TYPE		3. DATES COVERED 00-00-2010 to 00-00-2010	
4. TITLE AND SUBTITLE Bayesian Parametric Approach for Multichannel Adaptive Signal Detection				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Research Laboratory, AFRL/RVRT, 2241 Avionics Circle, Bldg 620, Dayton, OH, 45433				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010. Sponsored in part by the Navy.					
14. ABSTRACT This paper considers the problem of space-time adaptive processing (STAP) in non-homogeneous environments where the disturbance covariance matrices of the training and test signals are assumed random and different with each other. A Bayesian detection statistic is proposed by incorporating the randomness of the disturbance covariance matrices, utilizing a priori knowledge, and exploring the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix. Specifically the Block-Toeplitz structure of the covariance matrix allows us to model the training signals as a multichannel auto-regressive (AR) process and hence, develop the Bayesian parametric adaptive matched filter (B-PAMF) to mitigate the training requirement and alleviate the computational complexity. Simulation using both simulated multichannel AR data and the challenging KASSPER data validates the effectiveness of the B-PAMF in non-homogeneous environments.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 4	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

where $\mathbf{A}^H = [\mathbf{A}^H(1), \mathbf{A}^H(2), \dots, \mathbf{A}^H(P)]$ denote the *unknown* multichannel AR coefficient matrix, and $\varepsilon_k(n)$ denote the $J \times 1$ temporally white but spatially colored noise vectors.

- **AS2 (Random Disturbance Covariance Matrix of Training Signals):** The noise vector $\varepsilon_k(n)$ is distributed as $\varepsilon_k(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$, and the spatial covariance matrix $\mathbf{Q}$ follows an inverse complex Wishart distribution with degrees of freedom μ and mean $\bar{\mathbf{Q}}$:

$$p(\mathbf{Q}) = \frac{|\mu - J| \bar{\mathbf{Q}}|^\mu}{\tilde{\Gamma}(J, \mu) |\mathbf{Q}|^{(\mu+J)}} e^{-(\mu-J) \text{tr}(\mathbf{Q}^{-1} \bar{\mathbf{Q}})}, \quad (3)$$

where $\tilde{\Gamma}(J, \mu) = \pi^{J(J-1)/2} \prod_{k=1}^J \Gamma(\mu - J + k)$ with Γ given by the Gamma function [24].

- **AS3 (Random Disturbance Covariance Matrix of Test Signal):** The noise vector in the test signal $\varepsilon_0(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_0)$, and $\mathbf{Q}_0$, given $\mathbf{Q}$, has a complex Wishart distribution with degrees of freedom ν and mean $\mathbf{Q}$

$$p(\mathbf{Q}_0|\mathbf{Q}) = \frac{\nu^J |\mathbf{Q}_0|^{\nu-J}}{\tilde{\Gamma}(J, \nu) |\mathbf{Q}|^\nu} e^{-\nu \text{tr}(\mathbf{Q}^{-1} \mathbf{Q}_0)}. \quad (4)$$

The multichannel AR process for the disturbances in both the test and training signals consists of two types of unknown parameters: one is the *deterministic* AR coefficient matrix $\mathbf{A}$, and the other is the *random* spatial covariance matrices $\mathbf{Q}$ and $\mathbf{Q}_0$. The available *a priori* knowledge is imposed on the mean of $\mathbf{Q}$, i.e., $\bar{\mathbf{Q}}$, which can be obtained from sources such as land-use maps, past measurements, etc [15]. It is said that the importance of the *a priori* knowledge $\bar{\mathbf{Q}}$ is controlled by parameter μ , while the heterogeneities, i.e., the statistical differences between the test and training signals, are determined by parameter ν . Most importantly, $\mathbf{Q} \neq \mathbf{Q}_0$ with probability one, which ensures the environment non-homogeneous [10].

III. BAYESIAN PARAMETRIC ADAPTIVE MATCHED FILTER

By assuming perfect knowledge on parameters $\mathbf{A}$ and $\mathbf{Q}_0$, the solution to the problem of interest is the classical parametric matched filter (PMF) [17]:

$$T_{\text{PMF}} = \frac{\left| \sum_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n) \mathbf{Q}_0^{-1} \tilde{\mathbf{x}}_0(n) \right|^2}{\sum_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n) \mathbf{Q}_0^{-1} \tilde{\mathbf{s}}(n)} \underset{H_0}{\overset{H_1}{\geq}} \gamma_{\text{PMF}}, \quad (5)$$

where γ_{PMF} denotes the PMF threshold subject to a selected probability of false alarm, and the whitened steering vector and test signal are obtained by using the true AR coefficient matrix $\mathbf{A}$

$$\tilde{\mathbf{s}}(n) = \mathbf{s}(n) + \sum_{p=1}^P \mathbf{A}^H(p) \mathbf{s}(n-p), \quad (6)$$

$$\tilde{\mathbf{x}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^P \mathbf{A}^H(p) \mathbf{x}_0(n-p). \quad (7)$$

The parametric AMF replaces the exact AR coefficient matrices $\mathbf{A}$ and the spatial covariance matrix $\mathbf{Q}_0$ in the PMF

statistic by their estimates (e.g., the maximum likelihood estimate (MLE) by using the training signals). For the non-homogeneous environment considered in this paper, due to the randomness of the spatial covariance matrix, we adapt a hybrid parametric AMF, which is denoted as the Bayesian PAMF (B-PAMF) by first obtaining the MLE of the deterministic AR coefficient matrix $\hat{\mathbf{A}}_{\text{ML}}$, then deriving a maximum *a posteriori* estimate (MAP) of the stochastic spatial covariance matrix $\hat{\mathbf{Q}}_{0,\text{MAP}}$ and finally replacing $\mathbf{A}$ and $\mathbf{Q}_0$ in the PMF statistic by their estimates.

A. MLE of $\mathbf{A}$

According to the signal model, the joint probability density function (pdf) of the training signals can be approximated (ignore the conditionality on the first P temporal vectors) as [18]

$$f(\mathbf{x}_1, \dots, \mathbf{x}_K | \mathbf{A}, \mathbf{Q}) = \left[\frac{1}{\pi^J |\mathbf{Q}|} e^{-\text{tr}(\mathbf{Q}^{-1} \Sigma(\mathbf{A}))} \right]^{K(N-P)},$$

where

$$\Sigma(\mathbf{A}) = \frac{1}{K(N-P)} \sum_{k=1}^K \sum_{n=P}^{N-1} \varepsilon_k(n) \varepsilon_k^H(n) \quad (8)$$

From AS2, we can remove the dependence of the above pdf on $\mathbf{Q}$ by integrating it over $\mathbf{Q}$:

$$\begin{aligned} f(\mathbf{x}_1, \dots, \mathbf{x}_K | \mathbf{A}) &= \int f(\mathbf{x}_1, \dots, \mathbf{x}_K | \mathbf{A}, \mathbf{Q}) p(\mathbf{Q}) d\mathbf{Q} \\ &= \frac{|\mu - J| \bar{\mathbf{Q}}|^\mu}{\pi^{JK(N-P)} \tilde{\Gamma}(J, \mu)} \int |\mathbf{Q}|^{-L} e^{-\text{tr}(\mathbf{Q}^{-1} \tilde{\Sigma})} d\mathbf{Q} \\ &= \frac{|\mu - J| \bar{\mathbf{Q}}|^\mu \tilde{\Gamma}(J, L - J)}{\pi^{JK(N-P)} \tilde{\Gamma}(J, \mu)} |\tilde{\Sigma}|^{-L+J} \end{aligned} \quad (9)$$

where $L = \mu + J + K(N - P)$ and $\tilde{\Sigma} = K(N - P) \Sigma(\mathbf{A}) + (\mu - J) \bar{\mathbf{Q}}$. Therefore, finding the MLE of $\mathbf{A}$ is equivalent to minimizing the determinant of $\tilde{\Sigma}$. Rewrite the matrix $\tilde{\Sigma}$ as

$$\begin{aligned} \tilde{\Sigma} &= K(N - P) \Sigma(\mathbf{A}) + (\mu - J) \bar{\mathbf{Q}} \\ &= \hat{\mathbf{R}}_{xx} + \hat{\mathbf{R}}_{yx}^H \mathbf{A} + \mathbf{A}^H \hat{\mathbf{R}}_{yx} + \mathbf{A}^H \hat{\mathbf{R}}_{yy} \mathbf{A} + (\mu - J) \bar{\mathbf{Q}} \\ &= \left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \right) \hat{\mathbf{R}}_{yy} \left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \right)^H \\ &\quad + \hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx} + (\mu - J) \bar{\mathbf{Q}}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \hat{\mathbf{R}}_{xx} &= \sum_{k=1}^K \sum_{n=P}^{N-1} \mathbf{x}_k(n) \mathbf{x}_k^H(n), \quad \hat{\mathbf{R}}_{yy} = \sum_{k=1}^K \sum_{n=P}^{N-1} \mathbf{y}_k(n) \mathbf{y}_k^H(n), \\ \hat{\mathbf{R}}_{yx} &= \sum_{k=1}^K \sum_{n=P}^{N-1} \mathbf{y}_k(n) \mathbf{x}_k^H(n), \end{aligned} \quad (11)$$

with $\mathbf{y}_k(n) = [\mathbf{x}_k^T(n-1), \dots, \mathbf{x}_k^T(n-P)]^T \in \mathbb{C}^{JP \times 1}$. Since $\hat{\mathbf{R}}_{yy}$ is nonnegative definite and the remaining terms $\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx} + (\mu - J) \bar{\mathbf{Q}}$ do not depend on $\mathbf{A}$, it follows from (10) that

$$\tilde{\Sigma} \geq \tilde{\Sigma}|_{\mathbf{A}=\mathbf{A}_{\text{ML}}} = \hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx} + (\mu - J) \bar{\mathbf{Q}}, \quad (12)$$

where the MLE of $\mathbf{A}$ is given as

$$\hat{\mathbf{A}}_{\text{ML}} = -\hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}. \quad (13)$$

When $\tilde{\Sigma}$ is minimized, the MLE $\hat{\mathbf{A}}_{\text{ML}}$ will minimize any nondecreasing function including the determinant of $\tilde{\Sigma}$.

B. MAP Estimate of $\mathbf{Q}_0$

The MAP estimate of $\mathbf{Q}_0$ requires the computation of the posterior distribution $f(\mathbf{Q}_0 | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K)$:

$$f(\mathbf{Q}_0 | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K) = \int f(\mathbf{Q}_0, \mathbf{Q} | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K) d\mathbf{Q}, \quad (14)$$

where

$$\begin{aligned} & f(\mathbf{Q}_0, \mathbf{Q} | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K) \\ & \propto f(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K | \mathbf{Q}_0, \mathbf{Q}) p(\mathbf{Q}_0 | \mathbf{Q}) p(\mathbf{Q}) \\ & \propto |\mathbf{Q}_0|^{\nu-J} |\mathbf{Q}|^{-(L+\nu)} e^{-\text{tr}(\mathbf{Q}^{-1}[\tilde{\Sigma} + \nu \mathbf{Q}_0])}. \end{aligned} \quad (15)$$

As a result, (14) can be calculated as

$$\begin{aligned} & \int f(\mathbf{Q}_0, \mathbf{Q} | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K) d\mathbf{Q} \\ & \propto |\mathbf{Q}_0|^{\nu-J} \int |\mathbf{Q}|^{-(L+\nu)} e^{-\text{tr}(\mathbf{Q}^{-1}[\tilde{\Sigma} + \nu \mathbf{Q}_0])} d\mathbf{Q} \\ & \propto |\mathbf{Q}_0|^{\nu-J} |\tilde{\Sigma} + \nu \mathbf{Q}_0|^{\mu+\nu+K(N-P)}. \end{aligned} \quad (16)$$

Taking the logarithm of the above equation, then taking the derivative with respect to $\mathbf{Q}_0$, and equating to zero, we have

$$\begin{aligned} & \frac{\partial \ln f(\mathbf{Q}_0 | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K)}{\partial \mathbf{Q}_0} \\ & \propto (\nu - J) \mathbf{Q}_0^{-1} - \nu (\mu + \nu + K(N - P)) [\tilde{\Sigma} + \nu \mathbf{Q}_0]^{-1} = 0, \end{aligned} \quad (17)$$

which suggests that, given $\mathbf{A}$, the estimate of $\mathbf{Q}_0$ is

$$\hat{\mathbf{Q}}_0 = \frac{(\nu - J)}{\nu(\mu + J + K(N - P))} \tilde{\Sigma}. \quad (18)$$

Replacing $\mathbf{A}$ with $\hat{\mathbf{A}}_{\text{ML}}$ of (13) in the above estimate (viz, $\tilde{\Sigma}$), the MAP estimate of $\mathbf{Q}_0$ is

$$\begin{aligned} \hat{\mathbf{Q}}_{0,\text{MAP}} &= \frac{(\nu - J)}{\nu(\mu + J + K(N - P))} \\ & \times \left[\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx} + (\mu - J) \tilde{\mathbf{Q}} \right]. \end{aligned} \quad (19)$$

It is seen that the MAP of $\mathbf{Q}$ is a linear combination of a standard estimate of $\mathbf{Q}$ as introduced in [18], [19] and the *a priori* knowledge $\tilde{\mathbf{Q}}$. This linear combination has been seen before for non-parametric approaches [10], [25].

C. Bayesian PAMF

With the ML estimate of $\mathbf{A}$ and the MAP estimate of $\mathbf{Q}_0$, the adaptive version of the PMF in the heterogeneous environment can be derived as

$$T_{\text{B-PAMF}} = \frac{\left| \sum_{n=P}^{N-1} \hat{\mathbf{s}}^H(n) \hat{\mathbf{Q}}_{0,\text{MAP}}^{-1} \hat{\mathbf{x}}_0(n) \right|^2}{\sum_{n=P}^{N-1} \hat{\mathbf{s}}^H(n) \hat{\mathbf{Q}}_{0,\text{MAP}}^{-1} \hat{\mathbf{s}}(n)} \underset{H_0}{\overset{H_1}{\geq}} \gamma_{\text{B-PAMF}} \quad (20)$$

where $\gamma_{\text{B-PAMF}}$ denotes the B-PAMF threshold subject to a selected probability of false alarm, $\hat{\mathbf{Q}}_{0,\text{MAP}}$ is given by (19), and the whitened steering vector and the whitened test signal are obtained by using $\hat{\mathbf{A}}_{\text{ML}}$ given by (13):

$$\hat{\mathbf{s}}(n) = \mathbf{s}(n) + \sum_{p=1}^P \hat{\mathbf{A}}_{\text{ML}}^H(p) \mathbf{s}(n-p), \quad (21)$$

$$\hat{\mathbf{x}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^P \hat{\mathbf{A}}_{\text{ML}}^H(p) \mathbf{x}_0(n-p). \quad (22)$$

From (20), on one hand, it is seen that the B-PAMF performs successive whitening, i.e., temporal whitening followed by spatial whitening, as opposed to joint spatio-temporal whitening across all JN dimensions of the Bayesian AMF in [10]. On the other hand, compared with the standard PAMF [17], the B-PAMF incorporates the *a priori* knowledge, i.e., $\tilde{\mathbf{Q}}$, and utilizes the heterogeneity parameter ν and the importance parameter of the *a priori* knowledge, i.e., μ into the estimate of the spatial covariance matrix $\mathbf{Q}$. Hence, it is allowed that the B-PAMF provides computational efficiency and mitigates training requirement, meanwhile exploiting the *a priori* knowledge and the heterogeneity to improve the performance of detection.

IV. NUMERICAL EXAMPLES

In this section, simulation results are provided to illustrate the performance of the B-PAMF. Specifically, we first test the B-PAMF by using simulated data which conforms to **AS1**, **AS2** and **AS3**, and then using the more challenging KASSPER dataset [15]. The disturbance signal is generated as a multichannel AR(2) process with AR coefficient $\mathbf{A}$ and a spatial covariance matrix $\mathbf{Q}$. The signal vector $\mathbf{s}$ corresponds to a uniform equispaced linear array with randomly selected normalized spatial and Doppler frequencies. The signal-to-interference plus noise ratio (SINR) is defined as

$$\text{SINR} = |\alpha|^2 \mathbf{s}^H \bar{\mathbf{R}}^{-1} \mathbf{s}, \quad (23)$$

where $\bar{\mathbf{R}}$ corresponds to the assigned AR coefficient matrix $\mathbf{A}$ and the mean of the training spatial covariance matrix $\tilde{\mathbf{Q}}$. For each Monte-Carlo trial, the spatial covariance matrix $\mathbf{Q}$ for the training signal is generated as an inverse Wishart distribution with mean $\tilde{\mathbf{Q}}$, and then, given $\mathbf{Q}$, the spatial covariance matrix $\mathbf{Q}_0$ for the test signal is generated as a Wishart distribution with mean $\mathbf{Q}$.

We focus here on performance comparison between the B-PAMF and the standard PAMF [17] in the non-homogeneous environment. Figs. 1(a)-1(c) show the probability of detection versus the SINR for the B-PAMF and the standard PAMF in cases of different values of μ and ν , when $P = 2$, $J = 4$, $N = 16$, $K = 1$, and $P_f = 0.01$. It is seen that, in all three cases, the B-PAMF outperforms the standard PAMF. Specifically, for a fixed value of μ , a larger value of ν , more homogeneous environments, results in slightly improved performance of detection of the B-PAMF, from Fig. 1(a) to Fig. 1(b). On the other hand, for a fixed value of ν , increasing μ means more importance of the *a priori* knowledge of $\tilde{\mathbf{Q}}$, which leads to wider performance gap between the B-PAMF and PAMF, as compared between Fig. 1(a) and Fig. 1(c).

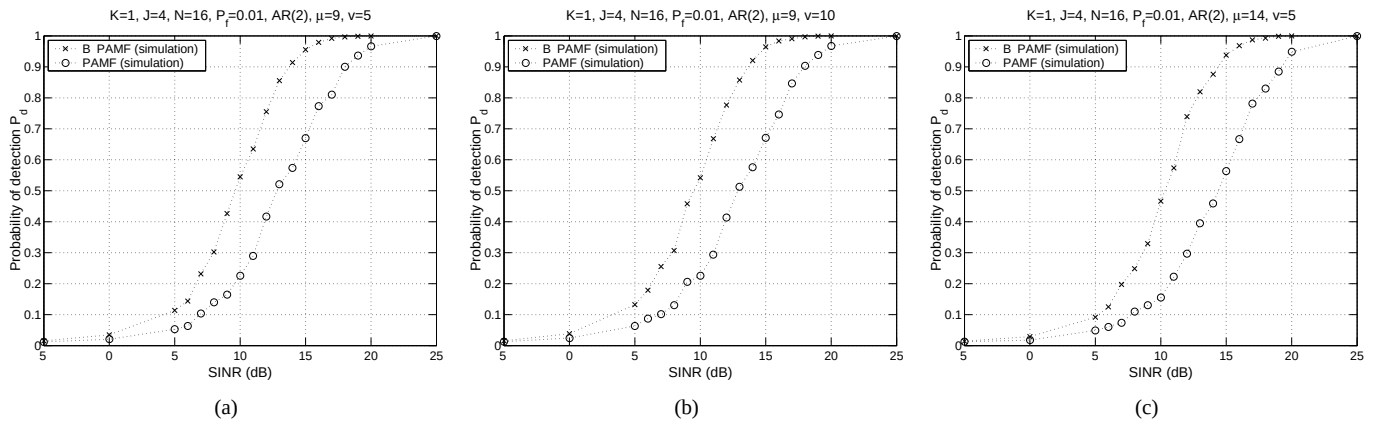


Fig. 1. Probability of detection versus SINR (a) when $P = 2$, $\mu = 9$, and $\nu = 5$; (b) when $P = 2$, $\mu = 9$, and $\nu = 10$; (c) when $P = 2$, $\mu = 14$, and $\nu = 5$.

V. CONCLUSION

A Bayesian parametric adaptive matched filter has been proposed by modeling the disturbances in the test and training signals as a multichannel AR process and simulating the heterogeneity between the training and test signals by introducing random disturbance covariance matrices. The B-PAMF admits successive temporal and spatial whitening, which reduces the computational complexity of the joint spatial-temporal whitening based adaptive detectors. The training requirement is also reduced. Simulation results validate the effectiveness of the B-PAMF.

REFERENCES

- [1] E. J. Kelly, "An adaptive detection algorithm," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 22, pp. 115–127, March 1986.
- [2] J. Ward, "Space-time adaptive processing for airborne radar," Technical Report 1015, Lincoln Laboratory, MIT, December 1994.
- [3] F. C. Robey, D. R. Fuhrmann, E. J. Kelly, and R. Nitzberg, "A CFAR adaptive matched filter detector," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 1, pp. 208–216, January 1992.
- [4] R. Klemm, *Principles of Space-Time Adaptive Processing*, The Institute of Electrical Engineers, London, UK, 2002.
- [5] S. Kraut and L. L. Scharf, "The CFAR adaptive subspace detector is a scale-invariant GLRT," *IEEE Transactions on Signal Processing*, vol. 47, no. 9, pp. 2538–2541, September 1999.
- [6] A. De Maio and S. Iommelli, "Coincidence of the Rao test, Wald test, and GLRT in partially homogeneous environment," *IEEE Signal Processing Letters*, vol. 15, pp. 385–388, 2008.
- [7] C. D. Richmond, "Statistics of adaptive nulling and use of the generalized eigenrelation (GER) for modeling inhomogeneities in adaptive processing," *IEEE Transactions on Signal Processing*, no. 5, pp. 1263–1273, May 2000.
- [8] E. Conte, M. Lops, and G. Ricci, "Asymptotically optimum radar detection in compound gaussian clutter," *IEEE Transactions on Aerospace Electron. and Systems*, vol. 31, no. 2, pp. 617–625, April 1995.
- [9] J. H. Michels, B. Himed, and M. Rangaswamy, "Performance of STAP tests in Gaussian and compound-Gaussian clutter," *Digital Signal Processing*, vol. 10, no. 4, pp. 309–324, October 2000.
- [10] O. Besson, J.-Y. Tournet, and S. Bidon, "Knowledge-aided Bayesian detection in heterogeneous environment," *IEEE Signal Processing Letters*, no. 5, pp. 355–358, May 2007.
- [11] A. De Maio, A. Farina, and G. Foglia, "Adaptive radar detection: a Bayesian approach," in *Proceedings of the 2007 IEEE International Conference on Radar*, Waltham, MA, April 2007, pp. 624–629.
- [12] S. Bidon, O. Besson, and J.-Y. Tournet, "The adaptive coherence estimator is the generalized likelihood ratio test for a class of heterogeneous environments," *IEEE Signal Processing Letters*, pp. 281–284, 2008.
- [13] S. Bidon, O. Besson, and J.-Y. Tournet, "A Bayesian approach to adaptive detection in non-homogeneous environments," *IEEE Transactions on Signal Processing*, no. 1, pp. 205–217, January 2008.
- [14] A. De Maio, A. Farina, and G. Foglia, "Knowledge-aided Bayesian Radar detector and their application to live data," *IEEE Transactions on Aerospace Electron. and Systems*, vol. 46, no. 1, pp. 170–182, January 2010.
- [15] J. S. Bergin and P. M. Techau, "High-fidelity site-specific radar simulation: KASSPER'02 workshop datacube," Technical Report ISL-SCRD-TR-02-105, Information Systems Laboratories, Inc., Vienna, VA, May 2002.
- [16] M. C. Wicks, M. Rangaswamy, R. Adve, and T. B. Hale, "Space-time adaptive processing: a knowledge-based perspective for airborne radar," *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 51–65, January 2006.
- [17] J. R. Román, M. Rangaswamy, D. W. Davis, Q. Zhang, B. Himed, and J. H. Michels, "Parametric adaptive matched filter for airborne radar applications," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no. 2, pp. 677–692, April 2000.
- [18] K. J. Sohn, H. Li, and B. Himed, "Parametric Rao test for multichannel adaptive signal detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 3, pp. 920–933, July 2007.
- [19] K. J. Sohn, H. Li, and B. Himed, "Parametric GLRT for multichannel adaptive signal detection," *IEEE Transactions on Signal Processing*, vol. 55, no. 11, pp. 5351–5360, November 2007.
- [20] K. J. Sohn, H. Li, and B. Himed, "Recursive parametric tests for multichannel adaptive signal detection," *IET Radar, Sonar and Navigation*, vol. 2, no. 1, pp. 63–70, February 2008.
- [21] P. Wang, K. J. Sohn, H. Li, and B. Himed, "Performance evaluation of parametric Rao and GLRT detectors with KASSPER and bistatic data," in *Proceedings of the 2008 IEEE Radar Conference*, Rome, Italy, May 2008.
- [22] Y. I. Abramovich, B. A. Johnson, and N. K. Spencer, "Two-dimensional multivariate parametric models for Radar applications, part i: Maximum-entropy extensions for Toeplitz-Block matrices," *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5509–5526, November 2008.
- [23] Y. I. Abramovich, B. A. Johnson, and N. K. Spencer, "Two-dimensional multivariate parametric models for Radar applications, part ii: Maximum-entropy extensions for Hermitian-Block matrices," *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5527–5539, November 2008.
- [24] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover, New York, NY, 1972.
- [25] P. Stoica, J. Li, X. Zhu, and J. R. Guerci, "On using a priori knowledge in space-time adaptive processing," *IEEE Transactions on Signal Processing*, no. 6, pp. 2598–2602, June 2008.