Range Profile Specific Optimal Waveforms for Minimum Mean Square Error Estimation

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***Abstract*— Optimal waveforms for minimum mean square error range profile estimation are investigated. An idealized measurement and waveform adaptation process is developed that yields optimal scene and range specific waveforms. This process is idealized in that during each cycle of the process, a large number of dwells are required. As part of our method, a modified version of the Adaptive Pulse Compression (APC) estimation method is used to estimate the range profile after each dwell cycle. The proposed method is analogous to the APC method in that it yields a set of range specific optimal waveforms, while the APC method yields a set of range specific optimal pulse compression filters. In certain scenarios, the measurement and waveform adaptation process yields range profile estimates that are significantly better than those derived by the APC method alone.**

I. Introduction

In modern radar and other active sensors, a signal is transmitted, directed into the associated medium such as free space in radar, and reflected back to a receiver by any scatterers. The delays associated with reflected signals arriving at the receiver provide a spatial profile associated with the respective ranges to the scatterers.

In radar, the high-power narrow-pulse transmissions required to achieve high range resolutions are often limited by practical power constraints. To overcome such limits, transmitted radar signals often take the form of a phase- or frequency- modulated rectangular pulse to obtain high spatial resolution inversely proportional to the modulation bandwidth in a technique known as pulse compression [4]. The received signal consists of delayed, attenuated versions of the transmitted waveform. A filter matched to the transmitted waveform extracts the high-resolution spatial profile from the received signal.

For a solitary point scatterer in the presence of white Gaussian noise, the matched filter (i.e., matched to the transmitted signal and system noise) maximizes output signal-to-noise ratio (SNR) and the detectability of scatterers in noise. The matched filter correlates the delayed, attenuated versions of the transmitted waveform as they arrive at the receiver and provides the range and relative complex amplitudes of scatterers. The correlation of the matched filter with the waveform is the autocorrelation of the waveform. However, an inherent problem is the masking of small targets by large nearby targets due to range sidelobes that are present when using the standard matched filtering method.

Blunt and Gerlach [1] developed a method for range profile estimation, Adaptive Pulse Compression (APC), in which a filter adapted to the scene is derived for each range bin. Each filter is optimally adapted to its respective range cell based on the minimum mean-square error (MMSE) criterion [2]. The adaptation process for deriving optimal filters is iterative in the sense that a numerical solution method is iterative. However, it only requires the result of a single radar dwell as input.

The adapting of the signal processing with respect to the range bin allows the sidelobes of targets to be greatly reduced, thereby mitigating to a large extent the aforementioned problem of the masking of small targets by nearby larger targets. Specifically, APC has been demonstrated to mitigate autocorrelation sidelobes to the level of the noise and accurately estimate small targets in the presence of several larger targets. Comparisons of this Adaptive Pulse Compression (APC) method and the standard matched filter estimation showed it to be superior over a variety of stressing scenarios.

Despite the fact that APC derives optimal filters for each range cell, for radar scenes in which there are groups of closely spaced targets, even after APC processing there can be significant sidelobes near the group of targets that could potentially mask targets close to the cluster of targets. Such a radar scene could for example represent a littoral environment. In this paper, the APC method is extended by deriving optimal scene and range-specific waveforms. These optimal waveforms, in conjunction with optimal filter weights, have been found in simulation to further enhance the estimation performance for certain scenarios with clusters of targets.

The framework of APC is utilized here. The signal model, measurement model, and performance criterion are the same as those utilized in APC. Also as in APC, the waveforms are restricted to being coded waveforms with a fixed number of chips. Finally, the derivation for optimal waveforms is analogous to the APC derivation for optimal weights. Accordingly, the space of admissible waveforms over which optimization is done is such that for any particular waveform, the chips of the waveform do not all necessarily have the same amplitude. While the derivations are mathematically parallel to each other,

978-1-4244-5813-4/10/$26.00 ©2010 IEEE

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
MAY 2010		**00-00-2010 to 00-00-2010**

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Range Profile Specific Optimal Waveforms for Minimum Mean Square Error Estimation	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **U.S. Naval Research Laboratory,Radar Division,Washington,DC,20375**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010. Sponsored in part by the Navy.

14. ABSTRACT

Optimal waveforms for minimum mean square error range profile estimation are investigated. An idealized measurement and waveform adaptation process is developed that yields optimal scene and range specific waveforms. This process is idealized in that during each cycle of the process, a large number of dwells are required. As part of our method, a modified version of the Adaptive Pulse Compression (APC) estimation method is used to estimate the range profile after each dwell cycle. The proposed method is analogous to the APC method in that it yields a set of range specific optimal waveforms, while the APC method yields a set of range specific optimal pulse compression filters. In certain scenarios, the measurement and waveform adaptation process yields range profile estimates that are significantly better than those derived by the APC method alone.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**Same as Report (SAR)**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

the measurement and adaptation process required differs greatly.

As mentioned previously, in APC, the adaptation process, while iterative, only requires as input the result of a single dwell. This is because of the effect of changing a filter can be observed by simply processing the radar return again using the new filter. In contrast, the adaptation process for waveforms requires multiple dwells since the effect of the change in a waveform can only be observed by transmitting another dwell.

We have posited a joint measurement and adaptation process that assumes range cell specific waveforms are repeatedly tested against their respective range cells by transmitting them during the dwell cycle of the process. That is, if there are L range cells, then multiple sets of L waveforms are transmitted during each cycle of the process. This process is probably unrealistic except for certain specific situations, and as such is idealized. However, the waveforms derived using it are best-case waveforms, and thus given an indication of the upper bound on the estimation performance that can be achieved using joint waveform/filter adaptation. Additionally, this process could potentially be used as the basis of measurement processes for specific scenarios. For example, a modified process could be useful for a target tracking scenario where only a few range bins are of interest as in [3]. Another example could be a scenario where there is existing knowledge of the environment.

We point out that a modified version of APC is used to estimate the range profile as part of the measurement and waveform adaptation process. Specifically, the APC algorithm is used to estimate the range profile using the return from a different waveform for each range cell. The measurement and adaptation process was simulated for a few scenarios, and was found to yield estimation performance superior to that resulting from APC alone.

An agile waveform method has been proposed that derives optimal waveforms in heavy sea clutter in [3]. In [3], sidelobes of nearby scatterers are explicitly minimized at a particular location of interest, in the context of target tracking. Here, the sidelobe minimization is a by-product of the minimization of the mean-square error. Furthermore, in [3], the waveforms are restricted to constant amplitude waveforms, while here, less practical amplitude modulated waveforms are allowed.

II. Waveform Adaptation

Our notation is the same as that of [1], with minor exceptions as noted. For a more detailed exposition, please refer to [1]. The signal model assumed here is the same one that was defined in [1], except that we assume a range-specific waveform $\mathbf{s}(l)$ is used for each range $1 \leq l \leq L$. The range profile to be estimated is $\tilde{\mathbf{x}} = \begin{bmatrix} x(1) & x(2) & \ldots & x(L) \end{bmatrix}$ where $E[x(l)x(j)] = 0$ for $l \neq j$, $1 \leq l \leq L$, $1 \leq j \leq L$. The observation resulting from using waveform $\mathbf{s}(l)$ is assumed to be given by

$$\tilde{\mathbf{y}}(l) = \mathbf{A}^T(l)\mathbf{s}(l) + \tilde{\mathbf{v}}(l) \tag{1}$$

where $\tilde{\mathbf{v}}(l)$ is the measurement noise and $\mathbf{A}(l)$ is an $N \times N$ matrix where the jth row, $1 \leq j \leq N$, is given by $\begin{bmatrix} x(l-j+1) & x(l-j+2) & \ldots & x(l-j+N) \end{bmatrix}$. Denote the range profile estimate by $\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}(1) & \hat{x}(2) & \ldots & \hat{x}(L) \end{bmatrix}$. The estimate $\hat{x}(l)$ of $x(l)$ is of the form

$$\hat{x}(l) = \mathbf{w}^H(l)\tilde{\mathbf{y}}(l), \tag{2}$$

and consequently the estimation objective function to be minimized via choice of waveforms and adaptive weights is, as defined in [1],

$$\begin{aligned} J(l) &= E[|x(l) - \mathbf{w}^H(l)\tilde{\mathbf{y}}(l)|^2] \\ &= E[(x(l) - \mathbf{w}(l)^H\tilde{\mathbf{y}})(x(l)^* - \tilde{\mathbf{y}}^{\mathbf{H}}\mathbf{w}(l))] \\ &= E[(x(l)x(l)^* - \mathbf{w}(l)^H\tilde{\mathbf{y}}(l)x^*(l) - x(l)\tilde{\mathbf{y}}^H(l)\mathbf{w}(l) \\ &\quad +\mathbf{w}^{\mathbf{H}}(l)\tilde{\mathbf{y}}(l)\tilde{\mathbf{y}}^H(l)\mathbf{w}(l)] \\ &= E[x(l)x^*(l) - \mathbf{w}^H(l)\mathbf{A}^T(l)\mathbf{s}(l)x^*(l) \\ &\quad -\mathbf{w}^H(l)\tilde{\mathbf{v}}(l)x^*(l) - x(l)\mathbf{s}^H(l)\mathbf{A}^*(l)\mathbf{w}(l) \\ &\quad -x(l)\tilde{\mathbf{v}}(l)^H\mathbf{w}(l) + \mathbf{w}^H(l)\mathbf{A}^T\mathbf{s}(l)\mathbf{s}^H(l)\mathbf{A}^*\mathbf{w}(l) \\ &\quad +\mathbf{w}^H(l)\mathbf{A}^T(l)\mathbf{s}(l)\tilde{\mathbf{v}}^H(l)\mathbf{w}(l) \\ &\quad +\mathbf{w}^H(l)\tilde{\mathbf{v}}(l)\mathbf{s}^H(l)\mathbf{A}^*(l)\mathbf{w}(l) \\ &\quad +\mathbf{w}^H(l)\tilde{\mathbf{v}}(l)\tilde{\mathbf{v}}^H(l)\mathbf{w}(l)] \end{aligned}$$

Setting $\partial J(l)/\partial \mathbf{s}^H(l) = 0$ yields that for given $\mathbf{w}(l)$, the waveform $\mathbf{s}(l)$ that minimizes $J(l)$ satisfies

$$E[\mathbf{A}^*(l)\mathbf{w}(l)\mathbf{w}(l)^H\mathbf{A}^T(l)]\mathbf{s}(l) \quad = \quad E[x(l)\mathbf{A}^*(l)]\mathbf{w}(l)$$

Using the fact that the range cells are uncorrelated with each other yields

$$E[\mathbf{A}^*(l)\mathbf{w}(l)\mathbf{w}^H(l)\mathbf{A}^T(l)]\mathbf{s}(l) \quad = \quad \rho(l)\mathbf{w}(l)$$

where $\rho(l) = E[x^*(l)x(l)]$. By using this fact again, this equation becomes

$$(\sum_{n=-N+1}^{N-1} \rho(l-n)\mathbf{w}_n(l)\mathbf{w}_n^H(l))\mathbf{s}(l) = \rho(l)\mathbf{w}(l) \tag{3}$$

where $\mathbf{w}_n(l)$ contains the elements of the vector of weights $\mathbf{w}(l) = \begin{bmatrix} w_0(l) & w_1(l) & \ldots & w_{N-1}(l) \end{bmatrix}^T$ shifted by n samples and zero-filled, e.g. $\mathbf{w}_2(l) = \begin{bmatrix} 0 & 0 & w_0(l) & \ldots & w_{N-3}(l) \end{bmatrix}^T$ and $\mathbf{w}_{-2}(l) = \begin{bmatrix} w_2(l) & \ldots & w_{N-1}(l) & 0 & 0 \end{bmatrix}^T$. When $(\sum_{n=-N+1}^{N-1} \rho(l-n)\mathbf{w}_n(l)\mathbf{w}_n^H(l))^{-1}$ exists, the optimal waveform for range cell l can be expressed as

$$\mathbf{s}(l) = (\sum_{n=-N+1}^{N-1} \rho(l-n)\mathbf{w}_n(l)\mathbf{w}_n^H(l))^{-1}\rho(l)\mathbf{w}(l) \tag{4}$$

As was shown in [1], the filter $\mathbf{w}(l)$ that minimizes $J(l)$ for a given waveform $\mathbf{s}(l)$ is given by

$$\mathbf{w}(l) = \rho(l)(\mathbf{R} + \sum_{n=-N+1}^{N-1} \rho(l+n)\mathbf{s}_n(l)\mathbf{s}_n^H(l))^{-1}\mathbf{s}(l) \tag{5}$$

where $\mathbf{R} = E[\tilde{\mathbf{v}}(l)\tilde{\mathbf{v}}^H(l)]$. We note that $\rho(l)$ is estimated simply by $\rho(l) = \hat{x}^*(l)\hat{x}(l)$.

It is desired to find the waveform and filter/weights that minimize $J(l)$ for each range cell. However, as mentioned previously, the effect of updating a waveform can only be observed by actually transmitting the waveform. The measurement and adaptation process for deriving optimal waveforms is thus measurement intensive in contrast with APC, which, as it uses a common waveform for all range cells, only requires the result of a single measurement/radar dwell.

In our measurement and adaptation process, two cycles, a waveform update cycle (WC) and a filter update cycle (FC), are repeatedly performed one after the other. The filter remains fixed during the waveform update cycle (WC), and the waveform remain fixed during the filter update cycle (FC).

The waveform update cycle (WC) consists of several sets of dwells. During each set of dwells, each range cell is interrogated by a waveform specific to that range cell, resulting in the observation (1). After all range-specific waveforms in a set have been transmitted, the range profile estimate and waveforms are updated, and then this process repeats, i.e., another set of dwells is transmitted. The range profile estimate is updated by applying the current filter $\mathbf{w}(l)$ to the observation $\tilde{\mathbf{y}}(l)$ according to equation (2). The waveforms are updated by solving (3) (via matrix inversion) using the updated range profile estimate and the current range specific weights.

During the filter update cycle (FC), the weights are updated using APC, but with a waveform specific to each range cell, instead of a common waveform for all range cells. Specifically, in each cycle of the modified APC algorithm, the range profile estimate is first updated according to (2) for each $1 \leq l \leq L$, and then the weights $\mathbf{w}(l)$ are updated according to (5) for all $1 \leq l \leq L$.

Cycles of waveform updates (WC) followed by filter updates (FC) are repeated until the range profile estimate converges, which in practice is after only a few fixed number of cycles. Of course, each waveform update cycle consists of a large number of measurements/radar dwells. As in the APC method, the estimates $\hat{x}(l)$ calculated in either a waveform update cycle or a filter update cycle are bounded below in magnitude to prevent ill-conditioning in the matrix inversion.

III. Simulation Results

In this section, the adaptive measurement process described in the previous section is simulated. Two scenarios are simulated. In both scenarios, the true radar scene is combined with clutter as well as measurement noise for each dwell measurement. We note that our simulation of clutter is such that the clutter changes from measurement (dwell) to measurement, as the large number of measurements would make it unrealistic to assume fixed clutter over an entire set of measurements. Specifically, random clutter is generated anew for each measurement of the waveform cycle described above. The clutter and measurement noise levels are both set 65dB down from the peak of a unity amplitude target. For all scenarios, the measurement and adaptation process initially uses a 32 bit P4 code for the waveform and matched filter. In all scenarios, the phases of the targets are assumed to be random and uniformly distributed.

In one scenario, the radar scene assumed for the measurement and adaptation process consists of a "pedestal" of targets of the same magnitude, with random phases. For this scenario, the robustness of the optimal waveforms is investigated by applying the waveforms to a different scene in which the pedestal alternates between target and no target from range cell to range cell.

Fig. 1 shows the range profile estimate that results from the APC method alone. Fig. 2 shows the range profile estimate resulting from the measurement and waveform update process. From these figures, it is apparent that "sidelobes" from the cluster of targets is about 10dB higher for APC alone than for the joint waveform and weight adaptation process.

Fig. 3 shows the optimal waveforms for all range cells. Near the cluster of targets, the waveforms start to approach an impulsive waveform, but still have significant though reduced amplitudes away from their main spikes. Fig. 4 shows the optimal filters for all range cells. From this figure it can be seen that the filter weights are high near the edges of the target cluster, where the waveforms are quite "spiky" (see Fig. 3). Fig. 5 shows the filter responses for each range-specific optimal waveform-filter pair. The responses are very much like the desired "thumbtack" response, except near the edges of the target cluster. In these regions, a filter is orthogonal, via the adaptation, to a composite of the shifts of the waveform corresponding to the surrounding clutter environment. It is evident in Fig. 5 that the adaptive filters have shifted the range sidelobes of the waveform/filter pairs according to the surrounding environment. The optimization of waveforms eases the requirement of the adaptive filter weights for a particular range cell to mitigate interference from the surrounding environment.

Fig. 6 shows the range profile estimate resulting from using the waveforms and filters derived for the "solid" cluster of targets for a cluster of targets with spaces between the targets. In this case, the waveform-filter pairs apparently perform well away from the target cluster, where the radar scene is largely unchanged from the scene used for optimization. In particular, a small target has been added to the right of the cluster of targets, and is visible even though it was not included in the original radar scene used for optimization. However, the pairs do not perform as well within the spaces in the target cluster, since in the optimization scene the spaces were targets.

Fig. 7 shows the range profile estimate resulting from utilizing for all range cells the waveform-filter pair that is optimal for a location to the left of the cluster where an additional small target has been added. Apparently, this waveform-filter pair is not particularly good for a number of range cells, due to the range-specific nature of the pair. Fig. 8 shows the range profile estimate using the waveform that is optimal for the small target, but allowing the filters to be optimized over all range bins using APC. In this case, the performance is significantly improved over Fig. 7. Fig. 9 shows the magnitude of the waveform used in Fig. 7 - 8. As can be seen, this waveform on one hand is quite similar to an impulsive waveform, but on the other hand it still has significant amplitude away from its peak.

Another scenario simulated involves a cluster of targets of various amplitudes, again with random phases. Fig. 10 shows the range profile estimate resulting from APC alone. Fig. 12 shows the range profile estimate resulting from the joint measurement and waveform-filter adaptation process. Again, the joint process results in a range profile estimate that is better than that of the APC method alone. Fig. 11 shows the range profile estimate resulting from a modified measurement process in which only the waveforms are optimized. The weights used are the original matched filter weights, except that they are normalized with respect to each range specific waveform. In this case, the resulting range profile estimate appears to have about the same accuracy as the APC estimate. However, the peak signal levels are inaccurate due to the fact that the waveforms are, after each iteration, normalized to have a maximum unity amplitude.

IV. Conclusion

An measurement and adaptation process that jointly updates range specific waveforms and filters to optimize them with respect to the minimum mean square error criterion was developed. The process was simulated for a few scenarios, and was found to yield significant improvement over APC alone. The robustness of the resulting optimal waveform-filter pairs was examined by testing them against scenarios derived by altering the scenario for which the pairs were optimized. Although idealized, the process developed could potentially be adapted for specific scenarios of interest, such as littoral scenarios with pre-existing environmental knowledge, or target tracking, in which only a very limited number of range bins are of interest. Finding optimal waveforms within the class of phase coded, constant amplitude waveforms, using the APC framework, is a topic for future research.

V. ACKNOWLEDGMENTS

We would like to acknowledge Aaron Shackelford, Thomas Higgins, and Larry Leibowitz for helpful discussions, comments, and suggestions. This research was supported by NRL base funding.

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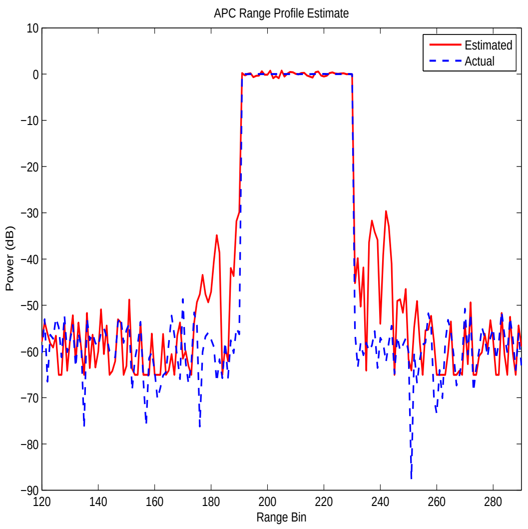

Fig. 1. APC range profile estimate for a cluster of targets

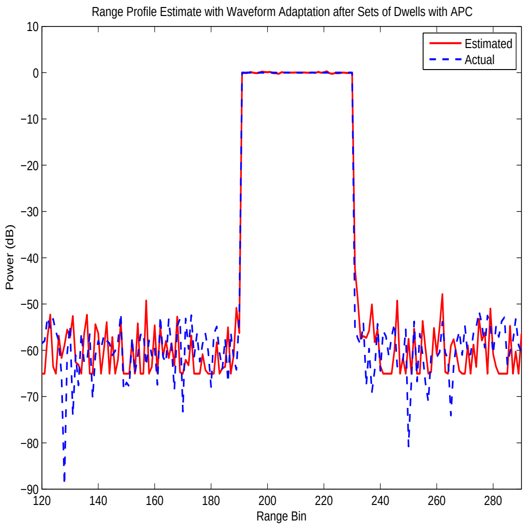

Fig. 2. Range profile estimate resulting from measurement and adaptation process after several cycles of sets of dwells followed by APC

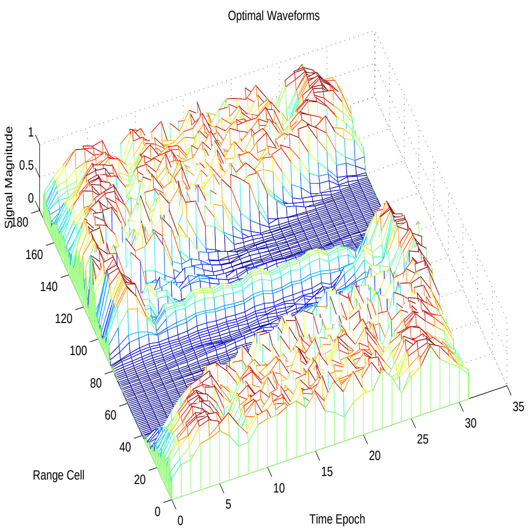

Fig. 3. Optimal waveforms for pedestal target scene

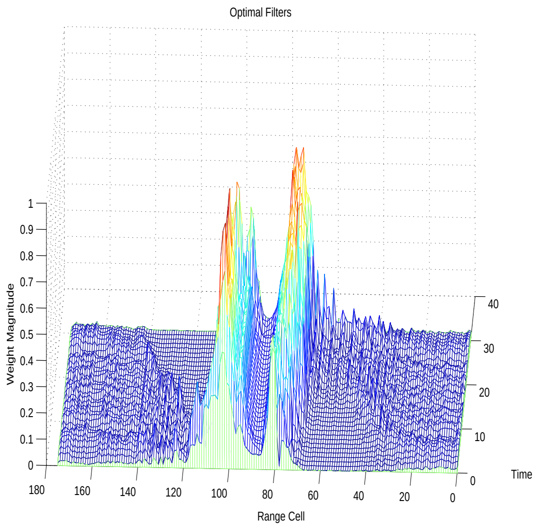

Fig. 4. Optimal filters for pedestal target scene

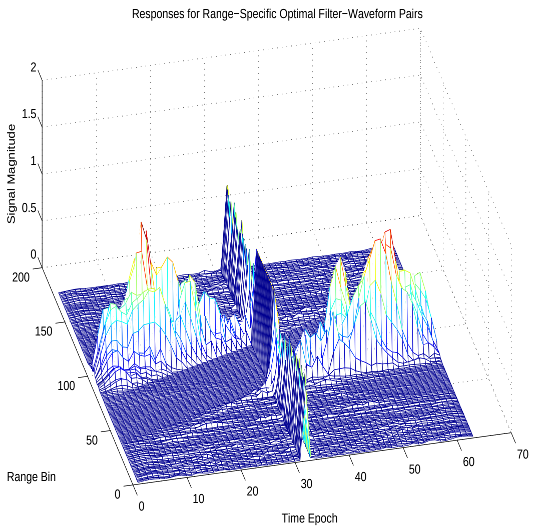

Fig. 5. Filter responses for optimal waveform-filter pairs

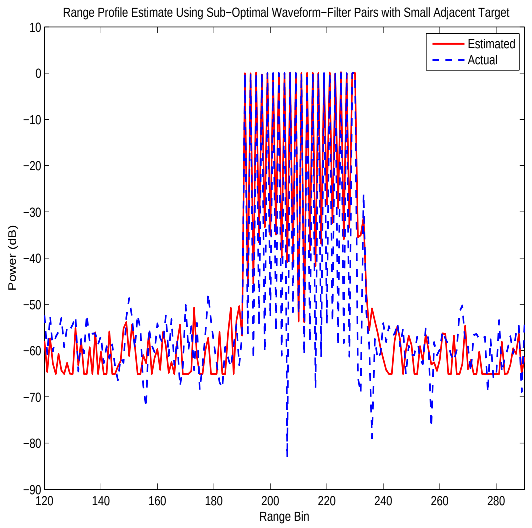

Fig. 6. Range profile estimate resulting from using sub-optimal waveform-filter pairs and APC processing, with a small adjacent target

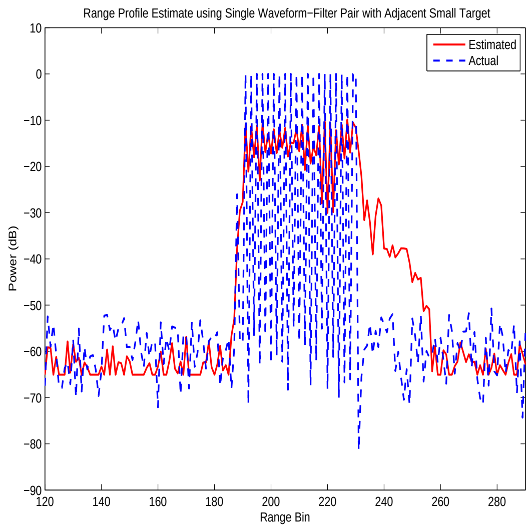

Fig. 7. Range profile estimate resulting from using a single waveform-filter pair for all range cells, with an adjacent small target

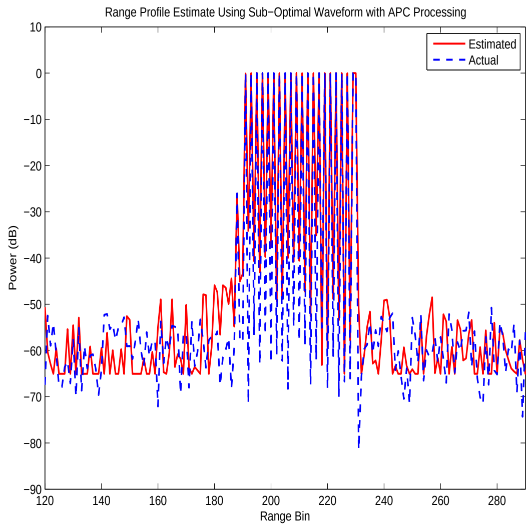

Fig. 8. Range profile estimate resulting from using a single waveform with APC processing

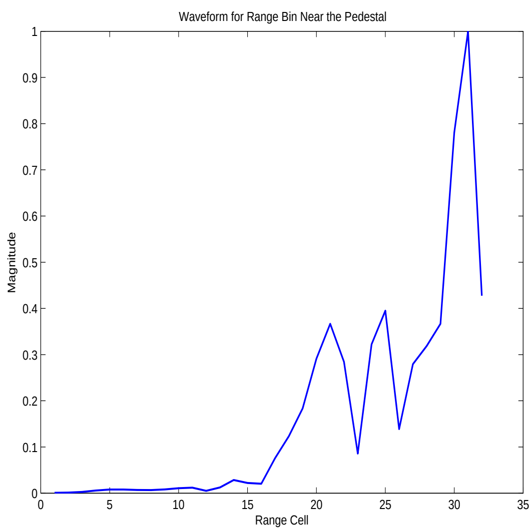

Fig. 9. Waveform for range bin near the pedestal

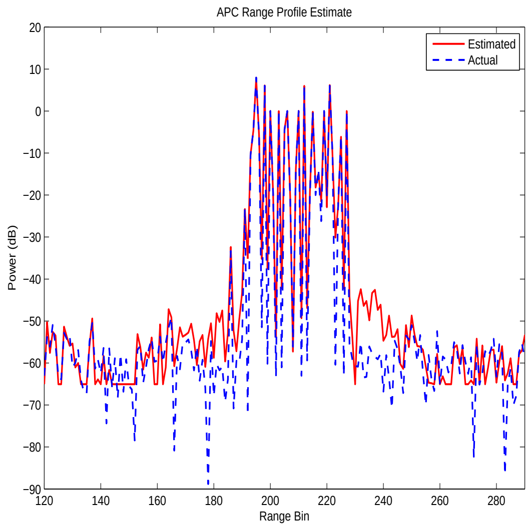

Fig. 10. APC range profile estimate for multiple target scene

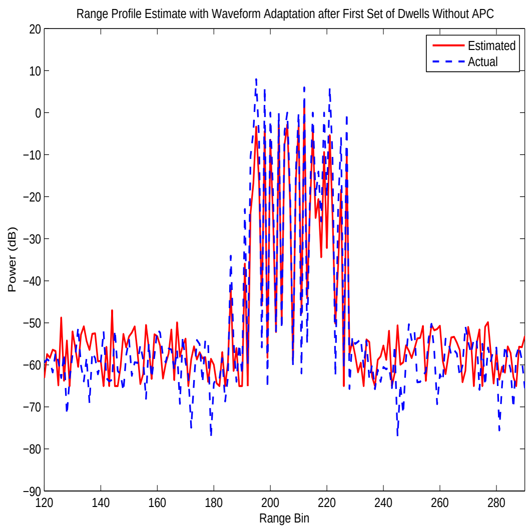

Fig. 11. Range profile estimate obtained by a single matched filter after one set of waveform adaptation dwells (without APC) for multiple target scene

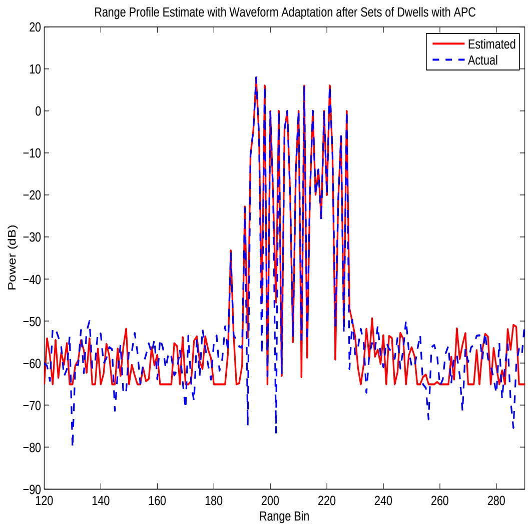

Fig. 12. Range profile estimate resulting from measurement process consisting of cycles of sets of dwells followed by APC for scene with multiple targets