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Efficient Resource Allocation for Broadcasting Multi-Slot Messages with Random Access with Capture

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***Abstract*—Optimal resource allocations for best effort delivery of broadcasting bursty multi-slots messages with random access are investigated. We consider broadcast multi-slot messages originating from all nodes in wireless random access networks of varying sizes, with receivers employing capture. Allocation of both access probabilities and transmission energy needed for sending various amounts of coded packets are investigated. The distributed transmitters must cooperate by only using the resources they are allocated, so as not to cause additional collisions that would diminish the performance of the other users. We evaluate resource allocation schemes in terms of message loss probability, throughput, and probability of satisfaction of specific delay constraints. It is demonstrated that optimal strategies for these metrics depend on the total offered traffic, the number of nodes, and the capture effect. It is shown how the optimal throughput can be implemented with finite delay. Furthermore, we show how even in large networks with the optimal allocation strategy, edge effects and capture significantly impact performance not only at the edges of the network, but also in the center of the network. Finally, we demonstrate how the addition of minimal feedback and further cooperation to the optimal best effort delivery strategy can significantly increase the probability of reception for all nodes.**

I. Introduction

The increasing demand for transmission of larger files such as images can result in the need to transmit bursty traffic in wireless networks. Such bursty traffic is not well suited to transmission schemes with fixed channel assignments such as TDMA. Random access protocols are good candidates for bursty traffic when there are many bursty users, each with an average low traffic rate. While protocols that reserve the channel such as CSMA/CA or DBTMA are beneficial for point-to-point transmissions, such protocols were not designed to handle delivery of a message to multiple destinations.

ALOHA type protocols could be used to handle bursty broadcast messages, but they generally suffer from collisions and low throughput, as well as delay at high traffic loads. Furthermore there has been relatively little study on the use of ALOHA for multi-slot messages, particularly when they are broadcast. Transmission of point-to-point multi-slot messages over multiple ALOHA channels was accomplished in [1] and [2] with an erasure code covering the multiple slots. In [3] a protocol for transmission of point-to-point multi-slot messages on a single channel was introduced.

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In this work, we handle best effort delivery of point-to-multipoint multi-slot messages, and investigate the case of ALOHA with capture. The difficulty with multi-slot messages is that for a number of applications if any single packet or slot is lost, the entire message is considered lost, which proves challenging when ALOHA is used owing to the large number of collisions. Therefore, we turn to erasure coding to add redundancy against collisions. Use of coding with random access was considered in small and moderate sized networks to maximize the throughput in [4] when all nodes are backlogged with packets. In contrast, we consider stochastic arrivals of packets at each node and moderate sized and large networks; we consider *best effort* delivery so that time sensitive newly arriving messages can be delivered without undue delay. Furthermore, we demonstrate how the amount of resources allocated to coding can be optimized for the capture effect.

If the entire coded message were to be transmitted in consecutive slots, little diversity would be gained by the coding. Therefore, we use packet level erasure coding, along with careful selection of the retransmission probability, to provide favorable tradeoffs between throughput and delay constraints. Our random spreading in time of coded packets introduces a *capture diversity*, that results from different coded packets of a single message receiving very different levels of interference from different interfering nodes. We demonstrate the spatial dependence of performance that results from the capture effect, and we utilize this dependence to further improve performance in the broadcast transmissions through use of a small amount of feedback.

In Section II we described the network and communication model used. Analysis of this model for no spreading of a message in time and for extreme spreading is presented in Section III. Intermediate levels of spreading are investigated with simulation in Section IV. A summary is provided in Section V.

II. Network and Protocol Model

We investigate the transmission of multi-slot messages when a slotted ALOHA protocol is used with capture. The challenge with multi-slot messages is that if a k slot message is sent, a node receives no useful throughput from that message unless it receives all k packets. Hence, we use erasure coding at the packet level and a random spreading of the k packets, as in [3]. Thus each k packet message is transmitted by sending n coded packets. If any k of the coded packets are received by a node, the message is successfully decoded; otherwise, it is not. A node is able to decode a message as soon as it receives k of the n coded packets.

Because in this work we consider broadcast transmissions in a network with a total of N nodes and the capture effect, if T nodes each transmit a coded packet simultaneously in a single slot, some of the $N-T$ non-transmitting nodes may capture a coded packet from one transmitting node, while other non-transmitting nodes may instead capture a coded packet from a different transmitter in that time slot.

For simplicity, a line of N equally spaced nodes is considered as an example in much of the paper. Other configurations are discussed in Section III. Each message is addressed to all of the other nodes. All nodes are subject to the same probability of message arrival, and in any given time slot multiple nodes may be transmitting. Upon arrival at a node, the first coded packet of a message is immediately transmitted, if there are no coded packets in the buffer awaiting transmission. If there are coded packets awaiting transmission in the buffer, the new message is placed at the end of the queue. Each of the remaining coded packets, after the first coded packet is transmitted, is transmitted successively in any given time slot with probability Pt. If $Pt = 1$, then the protocol is similar to unslotted ALOHA with a packet length of n, and the coded packets offer little diversity since it is likely that multiple coded packets will be lost in any collision. The same is true for large values of Pt less than 1. If Pt is too small, it will take a long time for a node to completely transmit a message.

We consider collisions as the only channel impediment, so that in the absence of concurrent transmission, every node can successfully receive from every other node. We have implemented a capture model in which the node will receive the packet from its nearest transmitting neighbor. In the event that two transmitting nodes are at equal distances from the receiver and that this distance is the shortest distance between the receiver and any transmitting node, the packet will be lost. If a node is transmitting, it is similarly not able to receive any other messages. Whenever a transmitter receives k of the n packets, that transmitter can decode the message and count the message as received.

III. Analysis for Extremes in Spreading Packets in Time

In this section we consider the two extreme limiting cases of the range of allocation strategies we consider. First, we consider transmitting coded packets, such that loss of one coded packet in a message is independent of loss of the other coded packets of the message. Next, we consider transmission of *uncoded* multi-slot messages with no spreading in time.

A. Independent Coded Packets

We explored, using equation (3), a range of k, n, and N values for the case when coded packets are independently

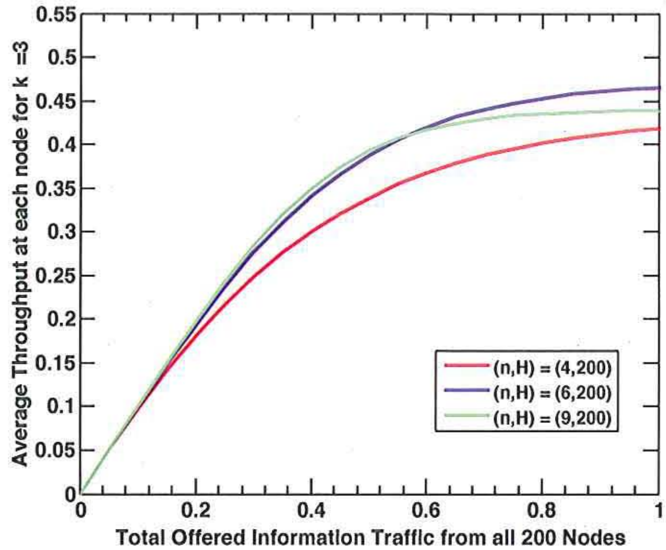

Fig. 1. Throughput in a 200 node network for 3 slot messages that are coded to 4, 6 and 9 slots. The coded packets are subject to independent losses.

lost, which as shown above, corresponds closely to the case of $Pt = .07$. Since this case yields the highest throughput, examination of the performance for this case when these parameters are varied will show the optimal throughput. The choice of $n = 6$ was found to be the optimal value of n for $k = 3$, yielding a maximum throughput of 0.47 at a total offered information traffic of 1.4. We illustrate this point in Figure 1 for a 200 node line network with several sample values of n of the many investigated. While transmission of more redundant packets ($n = 9$), is beneficial at low values of traffic, when the traffic becomes higher the additional coded packets cause more collisions, rendering the $n = 6$ more favorable at higher traffic levels. The lower values of n, such as $n = 4$ do not offer enough protection against loss of the whole message if a few coded packets are lost.

When $k = 2$, the optimal block code length found was $n = 3$, which yielded a maximum throughput of 0.54 at an offered traffic of 1.6. When $k = 4$, the value of $n = 9$ was optimal, yielding a maximum throughput of 0.44 at a total offered traffic of 1.1. Finally, for single-slot messages with $k = 1$, the optimal throughput of 0.97 was found at an offered traffic of 5.5. Due to the nearly perfect capture, it is beneficial to have the high values of offered traffic in these cases. Furthermore, the larger the message size, the lower the throughput due to the finite size block length and the loss of use of all coded packets when fewer than k are received. On the other hand, making the block length too long or using a rateless code would mean that most of the receivers, that is those that have already received k coded packets, would be obtaining no useful throughput during many of the redundant coded packet transmissions. These results are summarized in Table I, where the optimal coded message size n is given for each value of k. In addition, the highest achievable total throughput S is given, along with the corresponding value of total offered traffic G at which this maximum throughput is achieved.

TABLE I
OPTIMAL CODING AND MAXIMAL THROUGHPUT FOR LARGE NETWORKS WITH INDEPENDENT PACKETS IN EACH MESSAGE.

k	Best n	Highest S	G for Highest S
1	1	.97	5.5
2	3	.54	1.6
3	6	.47	1.4
4	9	.44	1.1

As N is increased from 10 to 20 to 100, significant improvement in throughput is seen at each increase in N, due to the fact that when there are more nodes, the transmit while receive collisions, as well as collisions from equidistant transmitters, become a smaller fraction of the total transmissions. As N is increased beyond a few hundred nodes, little further improvement is seen.

Channels with additional forms of losses can be incorporated in this model, as could alternative configurations of nodes, including those in two and three dimensions. Finally, we could incorporate multi-hop transmissions; this addition would most effectively be accomplished by generalizing the erasure coding to network coding.

Lastly, the findings here can provide insight to additional forms of random access, such as CSMA.

In Section IV-A we discuss the the throughput, delay, and loss characteristics when aggregated over all nodes in the network. In Section IV-B we consider the spatial variance of performance due to the capture effect and finite network size.

B. *Transmissions in Bursts*

We now consider the case of uncoded packets where each node transmits the entire multi-slot message upon its arrival at the node. In this case, there is no additional energy allocated to coding, and there is no cooperation, in that every node immediately transmits its entire message. Hence, with the capture effect for a k slot message, the probability of loss at any given node that has N_i interfering nodes closer to it than the transmitting node is:

$$Ploss \approx 1 - (1 - N_i \times G/N)^{2k-1}. \quad (1)$$

In particular in Figure 2, we plot the message loss probability for $k = 3$, $N = 21$, when we consider the transmitting node at the edge of a line of nodes. We have also plotted simulation results for comparison.

IV. SIMULATIONS

In this section, we consider a total of $N = 21$ nodes, $k = 3$, and $n = 6$. Values of Pt of 0.07, 0.15, and 1 are discussed as examples throughout this section. The simulations ran for 1,000,000 time slots. This number was selected so that

$$\frac{\sqrt{Varp}}{p} < .05. \quad (2)$$

The optimality of $n = 6$ for $k = 3$ for the channel model considered was discussed in Section III, as are other choices of these parameters.

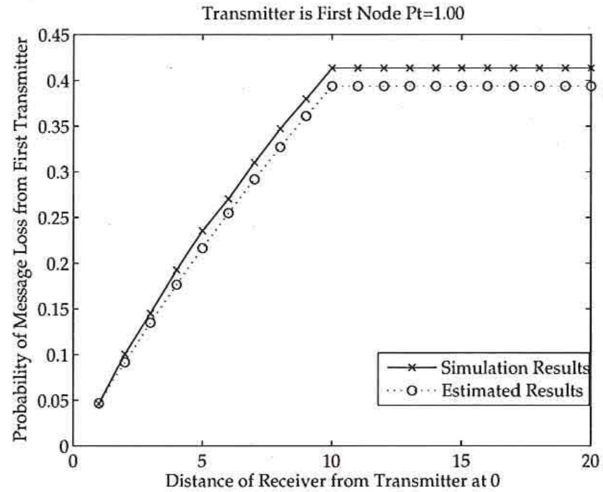

Fig. 2. Probability of loss for uncoded transmission.

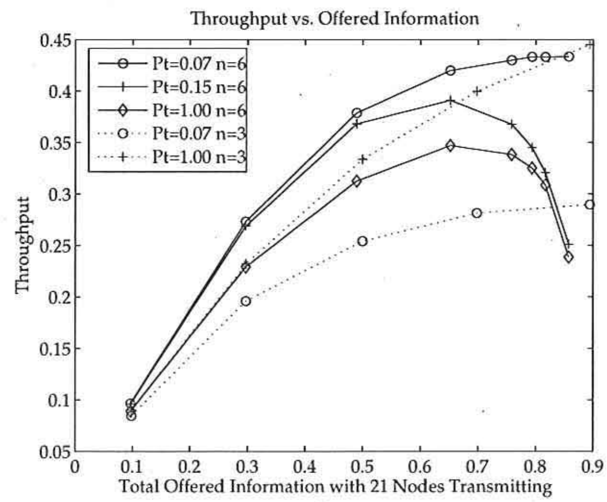

Fig. 3. Throughput received at a node from all other nodes vs. offered information traffic.

A message buffer of four messages per node is employed. The message buffer queues messages that arrive while another message has already begun transmitting. These messages would otherwise have created a queue drop. Increasing the size of the buffer beyond four messages had little impact on the number of queue drops, so all simulations discussed in this paper use a buffer size of four.

A. *Aggregate Performance*

Figure 3 plots the throughput received at a node as a function of the total offered information traffic in the network. This per node received throughput is averaged over all receiving nodes, which is all N nodes. The total offered information traffic is the product of the average number of coded packets transmitted by all N users in a time slot and k/n. The dotted curves represent no coding, whereas the solid curves represent the use of coding.

This figure illustrates that the ideal offered traffic value is

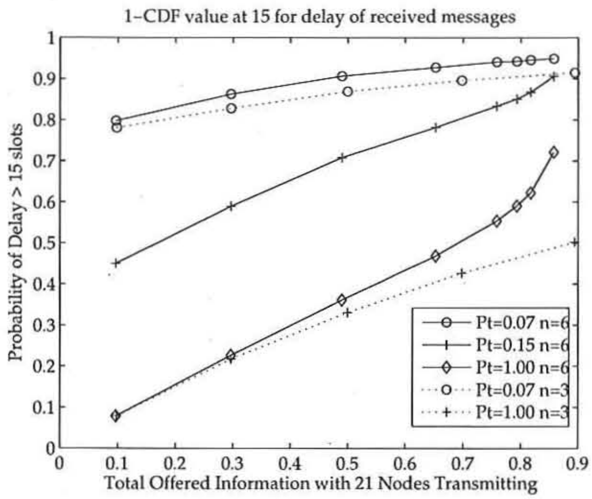

Fig. 4. CDF at 15 slots.

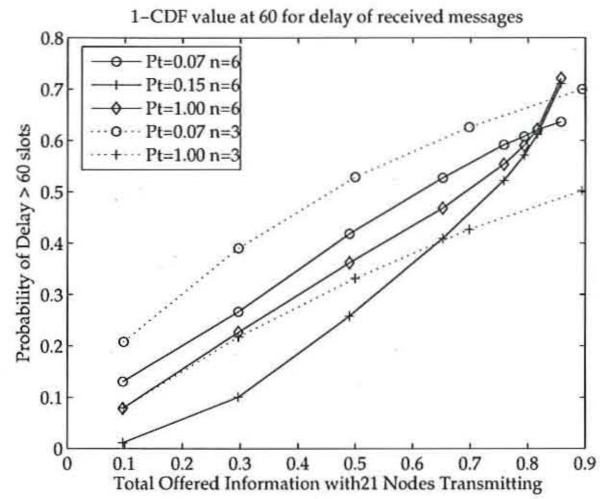

Fig. 5. Cumulative Distribution Function (CDF) at 60 slots.

between 0.6 and 0.7 for Pt values of 0.15 or 1 when coding is used and $N = 21$. Beyond this point the effective throughput in the system starts to decline. Higher levels of Pt prolong the appearance of non-monotonic behavior typically seen with the ALOHA instability.

An increase in throughput occurs when Pt decreases when coding is used, and the most favorable throughput is for $Pt = 0.07$. When Pt is smaller, the coded packets of a message are more spread in time, and if one coded packet of one message collides with a coded packet of a second message, it is less likely that subsequent coded packets of the two messages will also collide with each other than if Pt were larger. In the limit of very small Pt, we could consider the loss of different coded packets from a single message to be independent.

If all of the coded packets were independently subject to loss from collisions, then the per node average received throughput from all transmitting nodes would be given by

$$\frac{2}{N}\frac{k}{n}g_c\sum_{f=1}^{g-1}\sum_{g=2}^{N}\sum_{j=k}^{n}\binom{n}{j}\exp(-jN_i g_c)\times(1-\exp(-N_i g_c))^{n-j}, \tag{3}$$

where g_c is the per node offered coded traffic, and the number of interfering nodes N_i that each transmission is subject to is

$$N_i = \min(2\times(g-f), N-f). \tag{4}$$

Equation (3) is obtained from considering all pairs of nodes in the line, and assumes that the traffic from N_i nodes is Poisson distributed. This assumption matches the model used here when Pt is small, which is precisely when we expect the losses of coded packets from a single message to be the least correlated. Thus (3) shows the limiting behavior of independent losses on coded packets that we expect for very small Pt.

We next consider delay, and in particular we investigate what fraction of the nodes can satisfy specific delay constraints. When the delay constraint threshold is only five times the message length, in this case a 15 slot delay constraint, we see from Figure 4, which plots the cumulative distribution function (CDF) at 15, that the probability of delay increase as Pt increases. With all of these Pt values, the messages are more likely delayed than lost.

For larger values of the delay constraint threshold, the CDF graph resembles the probability of message loss graph. In Figure 5, this situation is shown for the CDF at 60 slots. The probability of message loss is plotted in Figure 6. The CDF shows the probability that the message is delayed more than twenty times the length of the message, which is also the approximate length of three TDMA cycles. This measurement takes into account messages that are never received. When looking at the delay that is greater than 60 slots, the values for $Pt = 1$ and $Pt = 0.15$ are dominated by the probability of message loss. As shown in Table 1, for those two throughput values the expected number of slots, in the absence of collisions, to send a full message out is well below 60. In these cases, a message not being received after 60 slots is most likely lost. For $Pt = 0.07$, the throughput is much lower leading to a larger number of slots required to ensure transmission of the full message. In this case, after 60 slots it is still possible that the transmitter has not finished sending all of the packets, causing the delayed messages to dominate the lost packets.

TABLE II
AVERAGE NUMBER OF SLOTS TO SEND MESSAGE IN ABSENCE OF COLLISIONS

Pt	Slots
1.00	2-5
0.15	14-35
0.07	26-65

As the offered traffic in the system increases, the probability of message loss also increases. Furthermore, larger values of Pt lead to more message loss when coding is used since larger values of Pt mean a higher likelihood of multiple coded packets from the same two messages colliding as they are more clumped in time. The dotted lines represent simulations in which n=3 which means that there is no coding used. It shows that when sending messages without coding, it is

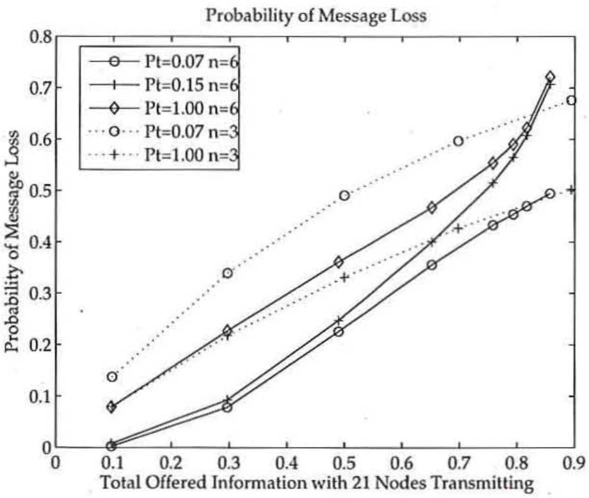

Fig. 6. Probability of message loss vs. offered information traffic.

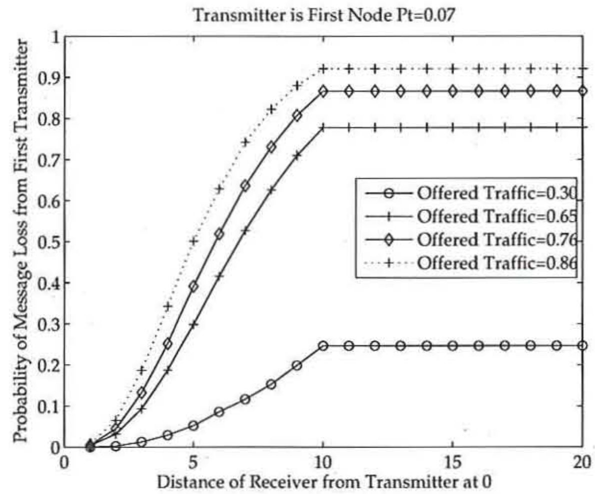

Fig. 7. Message loss vs. distance from the transmitting node.

more advantageous to clump the sending of the messages (i.e. increase the Pt value). Because the n coded packets of a message are randomly spread in time, it is less likely that k of the coded packets will be lost in a collision than when the packets are not spread. We dub this effect as "capture diversity" since at any given receiver, the different coded packets of a message will have independent attempts at being captured, if the coded packets are spread enough in time. Conversely, when coding is used, message loss is reduced by spreading out the sending of the packets by reducing the Pt value. The use of coding improves message loss until there are too many collisions and there is a sharp increase in message loss. The worst performance is to use a low Pt value and not employ coding, which corresponds to a random access protocol that sends a multi-slot message without coding, by metering out each packet over a long time period. When not using coding, for $N = 21$ and high traffic, it is best to send the entire message at once without spreading the packets in time; because if any one uncoded packet is lost, the whole message is lost anyway. Therefore, it is more favorable to have multiple slots of two uncoded messages collide rather than only single slots of these messages collide, in order to prevent the other slots of these messages from colliding with additional messages.

B. Spatial Dependence of Performance

We now explore the spatial dependence of the probability of message loss. The capture effect makes it more likely that nodes close to a transmitter will receive a message than nodes farther away. In addition, the fact that some nodes are at the edge of the network also impacts the spatial performance.

Figure 7 shows the probability that a message transmitted by the first node will be lost by each of the receivers for $Pt = 0.07$ for various levels of traffic. As shown in the figure, there is a saturation point at node 10 beyond which all future nodes are impacted by the same amount of message loss. Message loss occurs when another node closer to the receiver transmits in the same time slot as node 0, so receivers further away from the transmitter will receive more interference and therefore likelihood of message loss. This plateau point occurs due to edge effects. For example in this case, every node beyond node 10 has an equal number of potential interfering transmitters. To calculate the plateau point use the following equations:

if $Txer < \frac{1}{2}N$ **then**
$$P = round(\frac{N-Txer}{2}) + Txer$$
else
$$P = round(\frac{N}{2}) - round(\frac{N-Txer}{2})$$
end if

In this case N is the number of nodes in the system and P is the plateau point index. This figure also shows that as the offered traffic in the system is increased, the probability of message loss increases. Note: when the middle node is the transmitter there are *two* pivots, defined by each of the equations above, that both must acknowledge message receipt for feedback to be successful.

A similar profile is seen if mean delay is plotted as a function of distance from the transmitting node. More traffic leads to a higher mean delay, as expected. Mean delay increases rapidly with distance from the transmitting node until the plateau point is reached at which point the mean delay stays constant at longer distances due to edge effects.

We now consider packet receptions from transmissions made by the middle node. Figure 8 shows the probability of losing a message that is sent from the central transmitter. This figure shows the expected mirrored loss as the distance from the transmitting node is increased in either direction.

Here we have illustrated the spatial dependence of probability of message loss resulting from the combination of capture and edge effects in a one-dimensional network. The spatial dependence of offered traffic for a fixed level throughput resulting from capture and edge effects for single-slot messages were shown in [5].

We now propose utilizing the spatial dependence of packet loss that is induced by the combination of the capture effect and edge effects. Since in the model used here, with capture and collisions being the only source of packet loss, it is seen

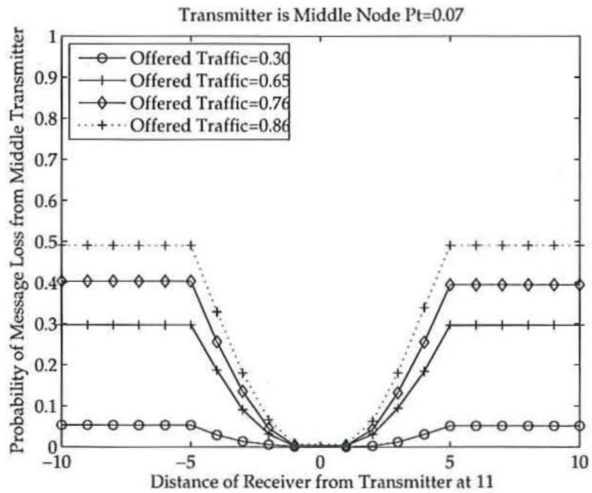

Fig. 8. Percent of Message Loss vs. Distance from Transmitter.

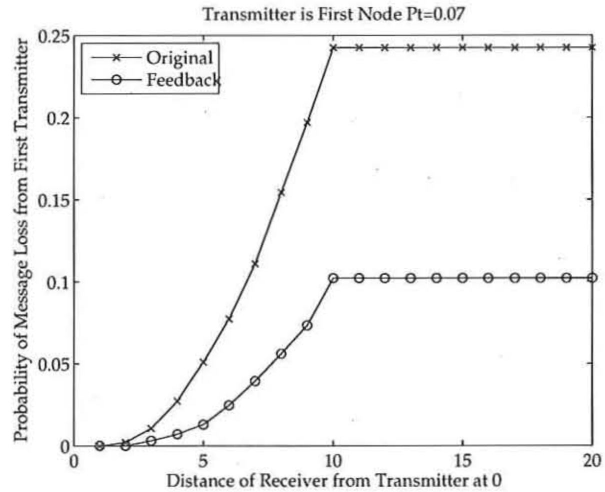

Fig. 9. Feedback to the transmitter at 0.

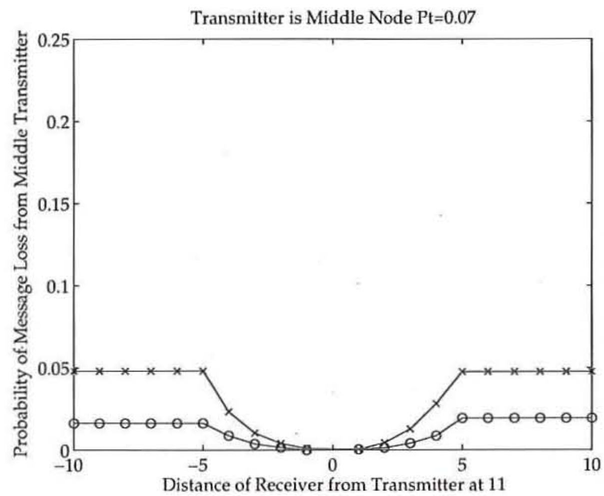

Fig. 10. Feedback to the transmitter at 0.

from Figure 7 that if any of the nodes between nodes 10 and 20 received the message, then they all received it. Therefore, we implement the following feedback protocol when coding and random spacing in time is used: When the receiver at the designated plateau point for each transmitter receives the message (that is any k coded packets), it sends an acknowledgement to the transmitting node. This acknowledgement signals to the transmitting node whether or not it has finished sending all n packets, and hence whether it can cease sending any additional packets for that message. The plateau point identifies the closest receiving node that will be impacted by every other potential interfering node. When this node has received the message, it can be assumed that every other node has as well. By enabling this feedback, we are able to reduce the overall traffic in the system and therefore reduce collisions and loss, as well as delay, of future messages.

Figure 9 shows the resulting decrease in message loss from this protocol for $N = 21, k = 3, n = 6$, and $Pt = 0.07$, for an offered traffic of .3. The upper curve represents no feedback when all n packets are transmitted, whereas the lower curve shows the performance with the feedback protocol. The difference between the two curves shows the significant reduction in loss provided by the feedback protocol. The savings from the feedback is greatest where the loss is greatest, which is beyond the plateau point. Here, the message loss is reduced by more than a factor of 2.

Similarly, the savings due to the feedback when the center node transmits are shown in Figure 10. Again, for the half of the nodes beyond the plateau point, the message loss is reduced by a factor greater than 2. The absolute value of the decrease in probability of message loss is only a few percent here, whereas it was 15% when the edge node transmitted, because the absolute value of the original message loss probability is smaller when the middle node transmits. Hence, when averaging over all transmitting and receiving nodes, the absolute value of the probability of message loss decreases a few percent when the feedback protocol is used. The savings from using the feedback protocol are greatest for the nodes most distant from the transmitting node.

V. Summary

The use of erasure coding, together with the random spreading in time of individual coded packets of a message to create "capture diversity", enables significant throughput gains in multi-slot messages transmitted with random access by multiple sources. It is shown how the amount of spreading can be selected to satisfy delay constraints, and that when coding is not used, spreading should not be used. Finally, it is shown that edge effects and capture result in an increase in message loss with distance from the transmitting node, until a saturation point is reached. Finally, we propose to use this phenomenon and demonstrate how it can enable consolidation of feedback and elimination of future unnecessary transmissions.

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