

## **Novel Internal Wave Paradigm Strongly Nonlinear Scale Separated Interactions**

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### **LONG-TERM GOALS**

My long-term goal is to understand and predict spectral energy density of internal waves in the ocean, and the effects of internal waves on oceanic large scale motions.

### **OBJECTIVES**

First-Principle theory of strongly nonlinear wave-wave interactions for internal waves.

### **APPROACH**

Present study is based on the extension of the wave turbulence theory to the internal waves in the ocean. In particular, we will go beyond traditional wave turbulence assumptions of weak nonlinearity and resonant interactions. This study is characterized by strong synergy with the work by Dr. Kurt Polzin (WHOI).

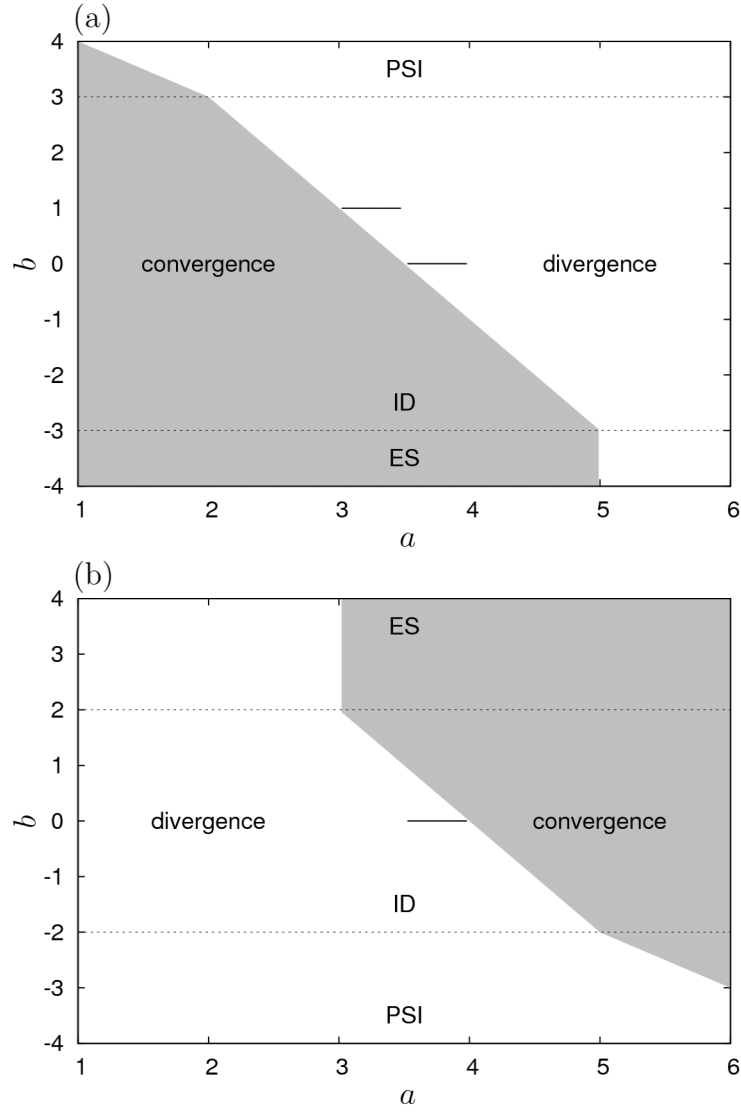
### **WORK COMPLETED**

- 1) During the previous year I have reported our results pertaining to near -resonant interactions. The main idea of that work is that inclusion of near-resonant interactions decreases effective nonlinearity of the internal waves, thus “healing” the kinetic equation, effectively leading for lower values of nonlinearity. To make this mathematical observation more practical, I have engaged in heavy numerical simulations to find “correct” value of the resonant surface smearing that appears in a kinetic equation. Such smearing is a result of nonlinear wave-wave interactions. These calculations can be drastically simplified by using variational approach. This work is ongoing, and I am optimistic that completion of analytical and numerical calculations will provide first principle calculations of “correct” width of internal waves resonances.
- 2) To characterize from first principle the effects of waves on large scale vortices and effects of large scale vortices on waves, it is advantageous to have a Hamiltonian description of the flow that allows both waves and vortices. I started to derive such a Hamiltonian structure, and hope to have results soon. This will provide a foundation needed for developing first principle wave-

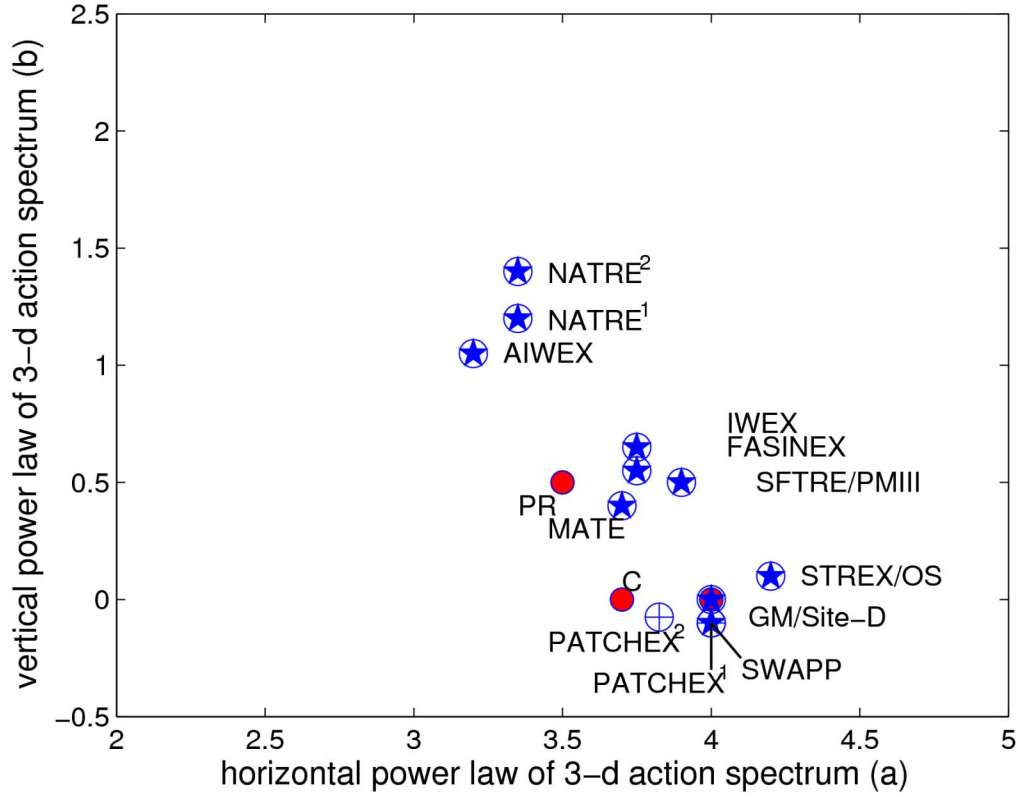
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vortex interactions theory. Other ingredients are kinetic like equations that take into account higher order terms, as described above.

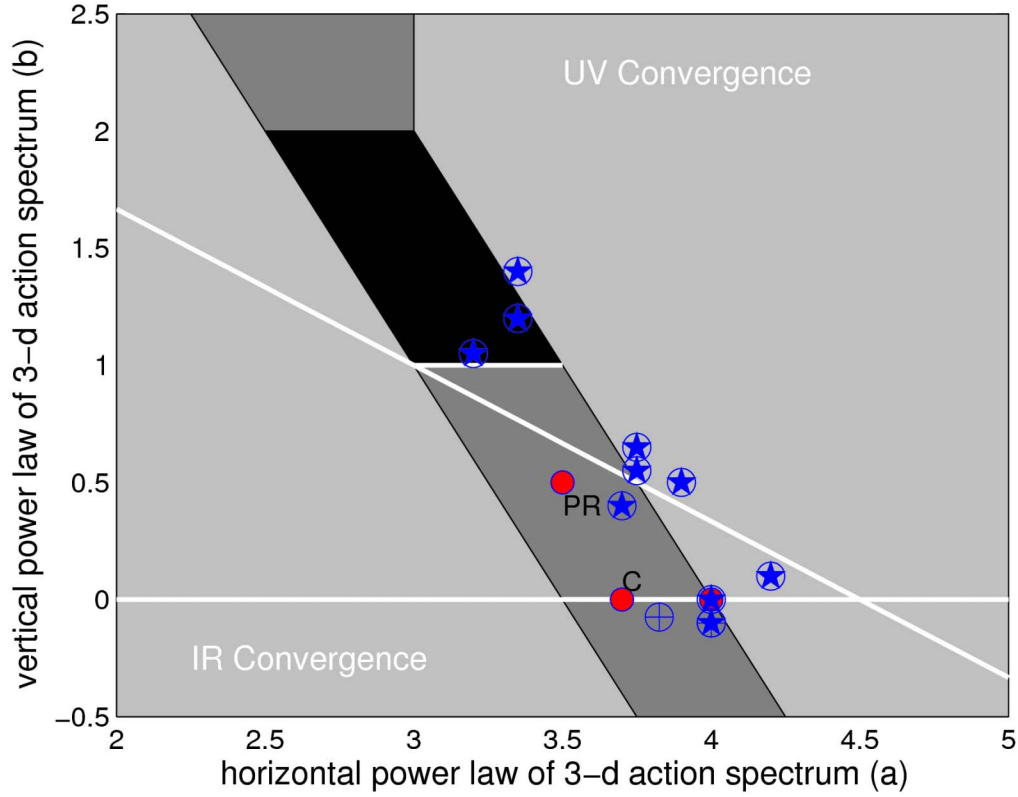
- 3) The rest of the report is devoted to the scale invariant solutions of the internal wave kinetic equation. Power-laws provide an easy and intuitive description of any complex systems, internal waves in particular. Therefore it is quite tempting to hope to find power-law solutions of the internal wave kinetic equations that we have derived in References [1-3]. In particular, we have assumed that the waveaction  $n_{k,m}$  where  $k,m$  are horizontal and vertical wavenumbers, can be found in the form,  $n_{k,m} = k^{-a} m^{-b}$ , where  $a,b$  are given fixed numbers. Before seeking steady solutions, however, one should find out whether the integrals in the kinetic equation converge. We have found that this is not the case, and that the kinetic equation *diverges* for almost *all* values of  $a,b$ . More specifically, Kinetic Equation may converge or diverges due to the interactions with the infinitely small wavenumbers. We call this Infa-Red limit. Kinetic Equation may converge or diverge due to the interactions with infinitely large wavenumbers. We call it Ultra-Violet (UV) limit. Picture ONE shows UV convergent area marked as “IR Convergence”, and IR convergent area marked as “IR Convergence”. The calculations of convergent/divergent areas were conducted analytically. Note that these areas do not intersect, except for a small segment of the  $b=0$  line. Interestingly, we found a convergent solution on that line  $(a,b) = (3.7,0.0)$ . Figure ONE shows these results, which are now published in Publication [3]
- 4) Therefore interactions of the internal waves, under traditional Resonant Interaction Approximation (RIA) are nonlocal, which implies that distant, nonlocal interactions in the wavenumber space dominate. This result put at rest any attempts to find “universal” spectrum of the internal wave kinetic equation in the Resonant Interaction Approximation.
- 5) We also analyzed the nature of the divergences of the kinetic equations, and ways to regularize them. Specifically, we proposed that divergencies coming from interactions with very large and very small wavenumbers may have different signs. Therefore in principle it is possible to have quasi-steady states of the kinetic equation that comes from such cancellations. We also rigorously rederived the ID stationary curves of the kinetic equation.
- 6) Together with Dr. Kurt WHOI we have completed an analyses of the major observational programs of the last four decades. These results are reported in Publications [2,3] below, and summarized on the Figure Two.
- 7) We have studied the correspondence between theory and experiments, i.e. combined Figure One and Figure Two in Figure Three. We see that there is significant correlation between boundaries of convergence/divergences. More specifically, many observational programs lies near the ID curves, and that all the experimental points correspond to spectra which is either both UV and IR divergent, or UV divergent. This is a highly synergistic activity that connects my research to the observational programs and provides sets of methods to characterize various observational programs.



**Figure One. Convergence/divergence regions of the kinetic equation.**  
**Divergence/convergence due to IR wavenumbers (a) and due to UV wavenumbers (b). The integral converge for the exponents in the shaded regions or on the segments ( $b=0$ ,  $7/2 < a < 4$ ) and ( $b=1$ ,  $3 < a < 7/2$ ). Dashed lines distinguish the domains where the indicated named triads dominate the singularity.**



**Figure Two. Observational Programs of the last four decades.** The filled circles represent the Pelinovsky--Raevsky (PR) spectrum  $(a,b) = (3.5,0.5)$ , the convergent numerical solution  $(a,b) = (3.7,0.0)$  and the GM spectrum  $(a,b) = (4.0,0.0)$ . Circles with stars represent estimates based upon one-dimensional spectra. There are two estimates obtained from two-dimensional data sets represented as circles with cross hairs. The figure contains 12 observational points from 10 observational programs. Note that PATCHEX1 is indistinguishable from SWAPP. Also note that one of the three filled circles (GM) coincides with the experimental point from Site-D.



**Figure Three.** *The observational points and the theories. The filled circles represent the Pelinovsky-Raevsky (PR) spectrum, the convergent numerical solution determined in Section 4c and the GM spectrum. Circles with stars represent power-law estimates based upon one-dimensional spectra. Circles with cross hairs represent estimates based upon two-dimensional data sets. Light grey shading represents regions of the power-law domain for which the collision integral converges in either the IR or UV limit. The dark grey shading represents the region of the power-law domain for which the IR and UV limits diverge and have opposite signs. The region of black shading represents the sub-domain for which both the IR and UV divergences have the same sign, i.e. when large contributions from interactions with very small and very large wavenumbers have the same sign. Overlain as solid white lines are the induced diffusion stationary states.*

## RESULTS

We Kinetic equation diverges for almost all scale invariant power laws, so that it is dominated by nonlocal interactions. We have shown that the Garrett and Munk spectrum of the internal waves in the ocean is NOT a steady state solution of the traditional kinetic equation, at least in the Resonant Interaction Approximation. This statement is at odds with traditional wisdom about wave-wave interactions in the ocean.

Boltzmann rate for the Garrett and Munk spectrum is too large for the system to be considered weakly nonlinear.

Inclusion of the near-resonant interactions makes a system of internal waves less nonlinear (more weakly nonlinear). This will lead to more self-consistent approach to wave-wave interactions.

## **IMPACT/APPLICATIONS**

Successful completion of this effort will lead to effective and simple internal waves parametrization schemes used for global ocean modeling.

## **REFERENCES**

- 1] Lvov, Y.~V., and E. G. Tabak, 2001: Hamiltonian formalism and the Garrett and Munk spectrum of internal waves in the ocean. *Phys. Rev. Lett.*, 87, 169501.
- 2] Lvov, Y.~V., and Tabak E.~G., 2004: A Hamiltonian Formulation for Long Internal Waves. *Physica D* 195 106--122.
- 3] Lvov, Y.~V., Polzin K.~L. and Tabak E., 2004: Energy spectra of the ocean's internal wave field: theory and observations. *Phys. Rev. Lett.* 92}, 128501.

## **PUBLICATIONS**

- [1] Yuri V Lvov, Kurt L Polzin, Naoto Yokoyama, "Resonant and Near-Resonant Internal Wave Interactions", submitted to the *Journal of Physical Oceanography*,
- [2] Kurt L. Polzin and Yuri Lvov "Toward Regional Characterizations of the Oceanic Internal Wavefield", submitted to "Review of Geophysics"
- [3] Yuri V Lvov, Kurt L Polzin, Esteban Tabak, Naoto Yokoyama, "Oceanic Internal Wave Field: Theory of Scale-invariant Spectra" , to appear in *Journal of Physical Oceanography*, 2010.