Complexity: A New Axiom for Structural Health Monitoring?

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ABSTRACT

The basic purpose of the paper is simple; having proposed a set of axioms or 'basic truths' regarding Structural Health Monitoring (SHM) in a previous paper, the authors would like to extend the set by the proposal for a new axiom. This axiom relates to an observation that the presence of damage in a structure or system usually results in increased complexity of measured responses or features. It is argued that this observation could lead to principled means of selecting effective features for SHM.

INTRODUCTION

The authors have previously suggested a set of axioms for Structural Health Monitoring (SHM) [1]. As usual in this context, the word 'axiom' is being used here to suggest a fundamental truth at the root of any SHM methodology.

The previously proposed axioms are:

Axiom I: *All materials have inherent flaws or defects.*
Axiom II: *Damage Assessment requires a comparison between two system states.*
Axiom III: *Identifying the existence and location of damage can be done in an unsupervised learning mode, but identifying the type of damage present and the damage severity can generally only be done in a supervised learning mode.*
Axiom IVa: *Sensors cannot measure damage. Feature extraction through signal processing and statistical classification are necessary to convert sensor data into damage information.*
Axiom IVb: *Without intelligent feature extraction, the more sensitive a measurement is to damage, the more sensitive it is to changing operational and environmental conditions.*
Axiom V: *The length and time scales associated with damage initiation and evolution dictate the required properties of the SHM sensing system.*

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14. ABSTRACT
The basic purpose of the paper is simple; having proposed a set of axioms or basic truths regarding Structural Health Monitoring (SHM) in a previous paper, the authors would like to extend the set by the proposal for a new axiom. This axiom relates to an observation that the presence of damage in a structure or system usually results in increased complexity of measured responses or features. It is argued that this observation could lead to principled means of selecting effective features for SHM.

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Axiom VI: *There is a trade-off between the sensitivity to damage of an algorithm and its noise rejection capability*.

Axiom VII: *The size of damage that can be detected from changes in system dynamics is inversely proportional to the frequency range of excitation.*

The authors would now like to add an additional axiom: *damage increases the complexity of a system*. Note that at this point *complexity* is left simply as a concept and not precisely defined because it can be assessed by many different methods ranging from very qualitative visual methods to complex quantitative measures. To begin, this paper will first provide heuristic evidence for this new axiom using examples where the increase in some form of complexity is evident in images or signals. Next, the discussion will focus on more quantifiable definitions of complexity that are tied to the concepts of probability distribution and entropy as defined in information theory [2]. The paper will conclude with a discussion on how the concept of complexity can be used to develop more rigorous approaches to feature selection.

COMPLEXITY IN SHM: A HEURISITIC DISCUSSION

If one presents a person with the pictures in Figure 1, and they are asked to classify this ship as damaged or undamaged, they will almost universally classify it as damaged. There are two reasons for this. The first is based on the fact that the person will usually frame an internalized comparison between all the visual examples of an undamaged structure they have so far experienced and they will note that the images presented deviate from this norm. The second reason - and this could apply even if the person has not seen an undamaged exemplar of the structure - relates to the 'appearance' of damage. Most engineered structures are designed to be 'smooth'; there are practical reasons for this e.g. minimizing drag, and there are aesthetic reasons. In any case, by the latter criterion, the person is likely to be making some qualitative assessment of *complexity*. For the ship in Figure 1, a person could perceive complexity in the very irregular nature of the ship's bow and note that few engineered structures have such irregular geometries. This concept of complexity can be extended to the case of corrosion as shown in Figure 1. Again, the corrosion adds *complexity* to the image of the girder through the increase in the number of edges or irregularities in this image. Engineers do not like to deal with such qualitative concepts and therefore more quantifiable means of assessing complexity are needed if such a concept is to be used in practice.

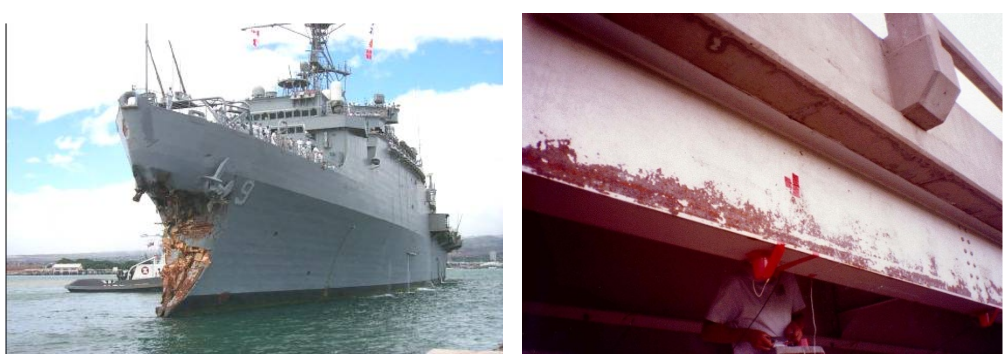


Figure 1. Damaged ship structure and corrosion on steel bridge girder.

QUANTIFYING COMPLEXITY THROUGH STATISTICS AND SIGNAL PROCESSING

A logical first step towards developing more quantifiable measures of complexity is to apply concepts of statistics and signal processing to data from the damaged and undamaged systems. To illustrate this concept, we will examine data from the test structure shown in Figure 2 and described in detail in [3]. Figure 3 shows the acceleration time history responses to a harmonic base input, measured on the top floor when the bumper is not present (undamaged) and when it is present (damaged). The response time history is seen to increase in complexity when damage is present through the irregular nature of the response. Normalized power spectra of these signals allow for a more quantifiable assessment of this complexity by identifying harmonics of the excitation frequency that are produced as a result of the impacting.

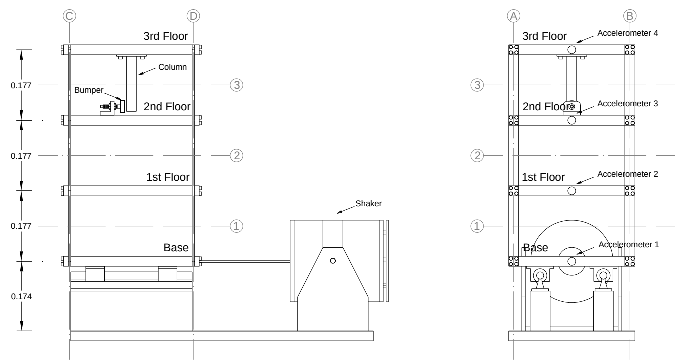


Figure 2. Diagram of test structure

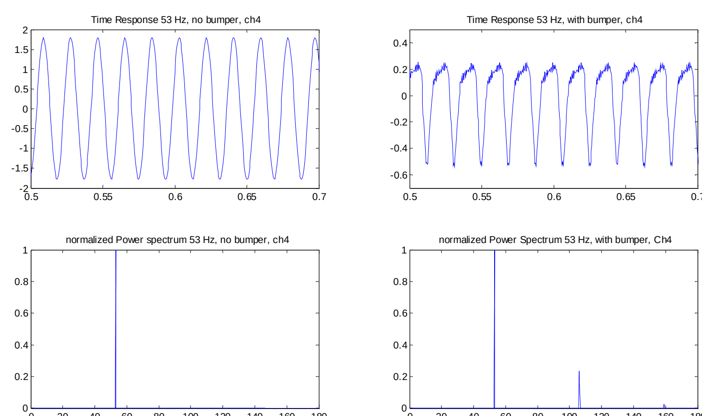


Figure 3. Response of the test structure in its undamaged state (left) and damaged state (right).

QUANTIFYING COMPLEXITY BY THE USE OF THE PROBABILITY DISTRIBUTION FUNCTION

To more directly quantify complexity through changes in the probability distribution function, lets again examine the test structure shown in Figure 2. This time the structure will be subjected to a random base input. Figure 4 shows the probability distribution functions for the undamaged and damaged systems. Clear distortions in the pdf are evident as a result of the damage where the pdf is starting to exhibit characteristics of a bimodal distribution. Additionally, statistics such as the skewness and kurtosis have changed significantly.

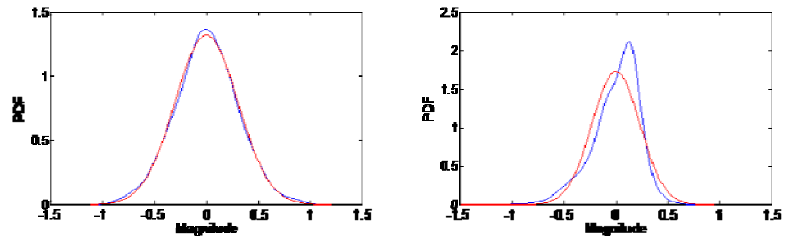


Figure 4. Changes in the probability distribution function that result from damage.

NUMERICAL EXAMPLE: A BILINEAR SYSTEM

The ideas of complexity will be further illustrated using synthetic response data acquired by numerical simulation of a simple dynamical system. The system of interest is a SDOF nonlinear (bilinear) oscillator specified by the equation of motion,

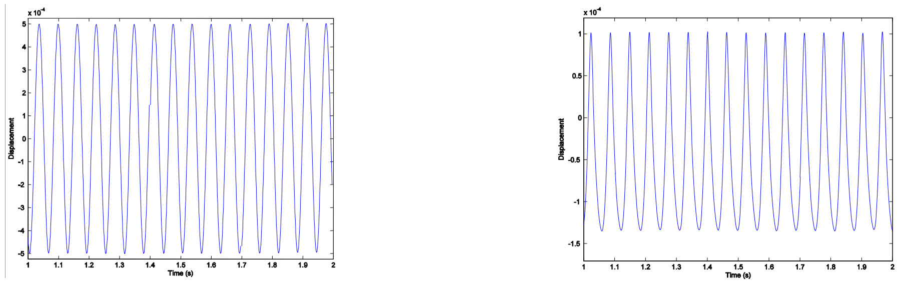
$$m\ddot{y} + c\dot{y} + k(y) = X\sin(\omega t) \tag{1}$$

where y is the displacement response to the harmonic excitation; m and c are the usual mass and damping coefficients and $k(y)$ is a nonlinear stiffness force given by,

$$\begin{aligned} k(y) &= ky & \text{if} \quad & k \leq d \\ k(y) &= kd + \alpha k(y-d) & \text{if} \quad & k > d \end{aligned} \tag{2}$$

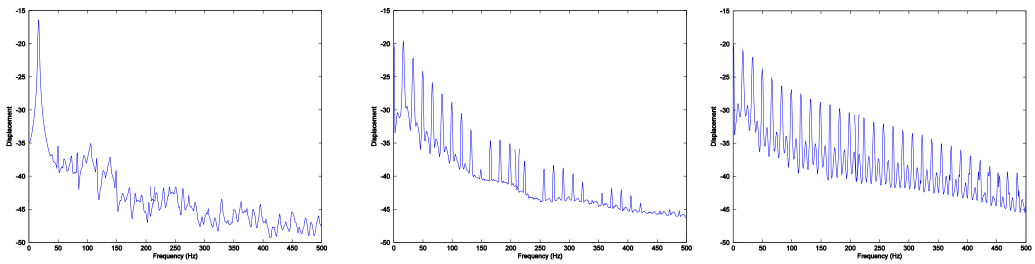
The parameter α governs the ratio of stiffness for the two linear regimes and the parameter d specifies the transition point between them. This system has some interest in terms of SHM is because it is arguably representative of the type of behavior observed when a breathing crack is present in a structure.

The discussion will be based on a phenomenon already discussed above, which is that the response of a nonlinear system to a simple harmonic excitation will contain components (harmonics) at multiples of the fundamental forcing frequency. The time data shown here were obtained by numerically integrating Eq. (1). The amplitude of excitation X was taken as 1 unit and the parameter choices: m = 1, c = 20, k = 10000 and d = 0.00005 were adopted. Different values of α were used in order to show the growth in complexity of the response as the severity of the nonlinearity increased. Figure 5 shows examples of the time response for the cases $\alpha = 1$ (linear, undamaged) and $\alpha = 10$ (nonlinear, damaged). Interesting, there does not appear to be a marked increase in structure or complexity for the more severe nonlinearity. In fact, the increased complexity is more visible in the frequency domain. Figures 6a and b show the spectra of the two signals from Figure 5; the presence of the harmonics in the nonlinear case is very marked. In fact, if the nonlinearity becomes even more severe, more structure appears; Figure 6c shows the spectrum corresponding to the case $\alpha = 100$. Peaks are beginning to appear in between the harmonics of the forcing frequency; these are the result of subharmonic generation.



(a) (b)

Figure 5. Response data from bilinear oscillator: (a) $\alpha = 1$, (b) $\alpha = 10$.



(a) (b) (c)

Figure 6. Response spectra from bilinear oscillator: (a) $\alpha = 1$, (b) $\alpha = 10$, (c) $\alpha = 100$

This example also allows one to make contact with a quantitative measure of complexity, or more properly, of information content. It is clear that the more complex pattern will require a greater amount of information in order to specify it than a simpler pattern. One way of encoding the information content of a pattern or signal is in terms of Shannon's entropy function [2]. Suppose that the state of a system is encoded in a variable x, whose values are governed by a probability density function $p(x)$. Shannon's entropy function or information entropy is given by,

$$S = -\int p(x) \log p(x)\, dx \tag{3}$$

The function has a minimum if all the probability is concentrated on a single state (total order) and a maximum if it is uniformly distributed over all states. The example considered here is not probabilistic, but it can be cast in a form where a probabilistic analogy is possible. Suppose that one were to normalise the spectrum (not the logarithmic version, but the linear version) by dividing by the total area under the spectrum; with a stretch of the imagination, one might say that the resulting function $p(\omega)$ represents the probability that an 'atom' of 'response energy' would be associated with a frequency ω. Now the situation in Figure 6a corresponds to 'total order', all (most of) the energy is associated with the forcing frequency; as the parameter α increases however, the energy is re-distributed across the spectrum and the order reduces. In order to illustrate this, the system in Eq. (1) was simulated with a range of α values from 1 to 10, and the 'spectral entropy' values were computed using Eq. (3). The result of this calculation is shown in Figure 7.

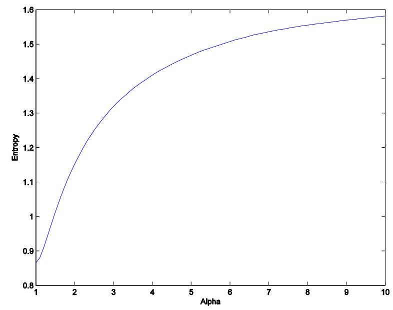


Figure 7. Spectral entropy measure as a function of the severity of the bilinear nonlinearity.

Note that the information content or complexity of the spectrum increases monotonically as a function of α, the severity of the nonlinearity. The entropy starts to reach a limiting value as the energy becomes distributed over all the harmonics in a more uniform fashion. Because of this monotonic behavior, the entropy is revealed as a useful feature for SHM purposes. (Not quite, the entropy increases with the nonlinear severity rather than with increased damage here. In the case of a breathing crack, the progressing variable would be the length of the crack rather than the open/closed stiffness ratio.) Features based on, or motivated by, information content have appeared at various points in the development of SHM research [4].

MEASURES OF COMPLEXITY

Almost all of the features proposed in the SHM literature, particularly those based on the assumption that damage will cause an initially linear system to exhibit nonlinear characteristics, are in some way assessing changes in the complexity of a structure [5,6]. This increase in complexity is assessed through some analysis of the systems measured response. Examples include the previously mentioned change in statistics, identification of harmonic generation, measures the continuity of a signal by parameters such as the Holder exponent, increase in residual errors associated with time-series models, and changes in wave propagation characteristics.

This axiom will impact SHM more directly if one can use it to define better features for a particular SHM application. By defining the type of damage that is of interest and then identifying how such damage increases the complexity of the structure and its associated dynamic response, one can begin to develop a principled approach to feature selection. With further development it is anticipated that this feature selection process can be directly linked to the definition of a damage observability criteria. Such rigorous procedures for feature selection and damage observability are essential developments that are needed to better transition SHM research to practice on *in situ* structures.

CONCLUSIONS

Anew Axiom for SHM has been proposed, based on the idea that *damage increases the complexity of a system.* The definition of complexity was intentionally left somewhat vague as it can be assessed in a variety of different manners ranging from very qualitative and heuristic definitions to much more quantifiable definition rooted in information theory. The definition will be dependent of the type of damage, the particular structure and the available data. Again, the term axiom is being used to imply that the statement is a "basic truth" for which there is evidence in the reported SHM literature. The authors believe this is a potentially powerful axiom because with further development it can lead to a more principled approach to feature selection and the definition of damage observability criteria.

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