

Autoregressive Modeling with State-Space Embedding Vectors for Damage Detection under Operational and Environmental Variability

E. FIGUEIREDO, M. D. TODD, C. R. FARRAR AND E. FLYNN

ABSTRACT

A nonlinear time series approach is presented to detect damage in systems by using a state-space reconstruction to infer the geometrical structure of a deterministic dynamical system from observed time series response at multiple locations. The unique contribution of this approach is using a Multivariate Autoregressive (MAR) model of a baseline condition to predict the state space, where the model encodes the embedding vectors rather than scalar time series. A hypothesis test is established that the MAR model will fail to predict future response if damage is present in the test condition, and this test is investigated for robustness in the context of operational and environmental variability. The applicability of this approach is demonstrated using acceleration time series from a base-excited 3-story frame structure.

INTRODUCTION

Structural Health Monitoring (SHM) is the process of implementing a damage detection strategy for aerospace, civil, and mechanical systems. The general concept is typically one rooted in some (very generalized) form of pattern recognition, whereby hypothesis tests or generalized regressions are established between features extracted from measured data sets and some form of model or class of models, whether completely data-driven, physics-based, or a combination thereof.

One class of approaches in this paradigm has been rooted in state-space reconstruction, borrowed from the dynamical systems field. The time evolution of any system occurs in its state space. Under a given input, a trajectory will typically evolve towards a subset of the state space (the attractor), which has invariant properties [1]. Because one rarely has the ability to measure all the variables needed to describe the dynamics of a typical system, recourse is made to the so-called embedology theorems,

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which allow for qualitative reconstruction of state space from discrete measured data in a way that preserves all dynamic invariants [2]. Researchers have begun to exploit this reconstructed state space as a source for feature extraction in various SHM applications [3]. In this paper, a MAR model is formed on the reconstructed baseline dynamics of a system to predict the baseline state space, where the embedding vectors comprise the multiple variates that the MAR is trained to predict. This approach assumes that damage will change significantly the baseline trajectory of the dynamical system, and consequently, the MAR model with parameters estimated from the baseline system will fail to predict the damaged system response. This approach is tested using uni- and multi-variate embeddings. Even though a multivariate embedding approach destroys localization information associated with each discrete sensor response, it takes into account all available sensor network information simultaneously to produce a low-dimensional feature set for discrimination that encapsulates the full observation space.

The applicability of this approach is demonstrated using acceleration time series from a base-excited 3-story frame structure tested in laboratory environment. Damage is simulated through nonlinear effects introduced by a bumper mechanism that simulates a repetitive, impact-type nonlinearity. The nonlinearities are intended to be a small perturbation of an essentially stationary process, causing a nonlinear phenomenon called intermittency [1]. The damage detection is investigated for robustness in the context of operational and environmental variability, simulated by changing stiffness and mass conditions, based on the assumption that sources of variability usually manifest themselves as linear effects on measured data.

STATE SPACE RECONSTRUCTION AND TIME SERIES MODELING

Based on the embedding theorem of Takens [4], assuming a single observed time series $v_1, v_2, \dots, v_N$, one can reconstruct an m -dimensional state vector in the form of delayed versions of the time series,

$$\mathbf{x}_i = (v_i, v_{i+\tau}, \dots, v_{i+(m-1)\tau}), \quad (1)$$

where $i=1, \dots, n$ and $n = N - (m-1)\tau$. The time delay embedding depends on two parameters, the embedding dimension m and the lag or delay time τ , which have to be chosen properly in order to yield an equivalent representation of the original state space. Note that $m \geq 2D+1$, where D is the unknown dimension of the original state space. Thus, the trajectory matrix (or vector space) $\mathbf{X}$ is defined as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix}. \quad (2)$$

Broomhead and King [5] proposed the singular value decomposition (SVD) on a trajectory matrix of w -dimension state vectors ($w > m$) to estimate the embedding dimension m and assuming unit lag. An approach for damage detection can be constructed using the MAR model. In the context of state-space reconstruction, the MAR(p) model of order p is defined as follows

$$\mathbf{x}_i = \bar{\mathbf{x}} + \sum_{j=1}^p \mathbf{x}_{i-j} \mathbf{A}_j + \mathbf{e}_{x,i}, \quad (3)$$

where $\mathbf{x}_i$ and $\mathbf{e}_{x,i}$ are the i^{th} predicted state vector and additive Gaussian noise vector, respectively, $\bar{\mathbf{x}}$ is the mean vector of the variables, and $\mathbf{A}_j$ is an m -by- m matrix containing constant coupling parameters. The order of the MAR model is determined based on the Akaike's information criterion [6].

This approach is based on the assumption that when some source of damage affects the dynamical properties of a system, a MAR model with parameters estimated from the baseline system cannot accurately predict the attractor of the damaged system. The approach can be summarized in the following steps. First, as shown in Figure 1, a time series $v(t)$ from the baseline condition is embedded into a state space in order to establish the baseline and assumed undamaged vector space $\mathbf{X}$. Then, the MAR parameters $\mathbf{A}_j$ are determined through the multivariate least squares technique. Second, and for a similar input, the baseline MAR model is used to predict any new test vector space $\mathbf{Z}$ from an unknown system condition in the form of

$$\hat{\mathbf{z}}_i = \sum_{j=1}^p \mathbf{z}_{i-j} \mathbf{A}_j, \quad (4)$$

where the residual errors are given by $\mathbf{e}_{z,i} = \mathbf{z}_i - \hat{\mathbf{z}}_i$. Assuming that $\mathbf{A}_j$ contains the underlying information of the baseline system, a hypothesis test is established that the MAR model will fail to predict the attractor if damage is present and the dynamical properties of the new system have changed. Therefore, the residuals increase and, under the damage hypothesis test, the system $\mathbf{Z}$ is said to correspond to a different class. In this approach either the visualization of the predicted states or, in a very generalized form, the residual errors can be used as damage-sensitive features.

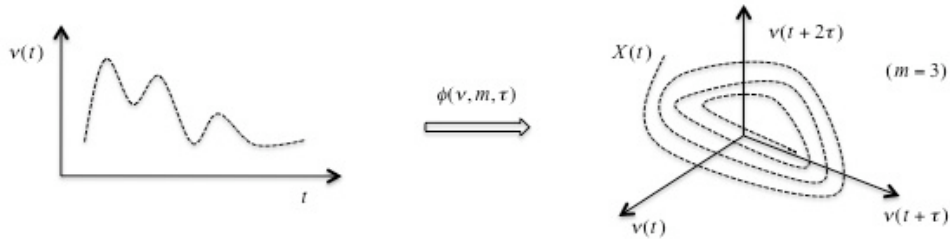


Figure 1. Schematic representation (for $m=3$) of the MAR model approach for the state space modeling.

A multivariate embedding to reconstruct the state space of the structure can be extended from the univariate case in Eq. (1) in the form of

$$\mathbf{x}_i = \left\{ v_i^1, v_i^2, \dots, v_i^l, v_{1+\tau_1}^1, v_{1+\tau_2}^2, \dots, v_{1+\tau_l}^l, \dots, v_{i+(m_1-1)\tau_1}^1, v_{i+(m_2-1)\tau_2}^2, \dots, v_{i+(m_l-1)\tau_l}^l \right\}, \quad (5)$$

where l corresponds to the number of sensor channels. This approach permits to combine structural response data measure at multiple locations into a global attractor. Note that the approach presented to the univariate case from Eqs. (1)-(4) is still valid for the multivariate one, where $M=m_1+m_2+\dots+m_l$ is the global embedding dimension.

EXPERIMENTAL PROCEDURE

The 3-story frame aluminum structure shown in Figure 1 has been used as a damage-detection test bed structure. The structure consists of columns and plates, assembled using bolted joints, and slides on rails that allow movement in the x-direction. At each floor, four columns are connected to the top and bottom plates, forming a 4-degree-of-freedom system. A center column is suspended from the top floor to simulate damage by inducing nonlinear behavior when it contacts a bumper mounted on the next floor. The position of the bumper can be adjusted to vary the extent of impacting that occurs at a particular excitation level. (This source of damage intends to simulate cracks, for instance.) The mass changes consisted of adding a 1.2 kg (approximately 19% of the total mass of each floor) to the first floor and to the base. The stiffness change was introduced by reducing one or more columns' stiffness by 87.5%. (These changes are intended to mimic operational and environmental effects.) An electrodynamic shaker provides a random lateral excitation to the base floor. Four accelerometers were attached at the centerline of each floor to measure the system's response. Acceleration time series for different structural state conditions were collected as shown in TABLE I. For each state, time data were acquired from one separate test. More details can be found in the references [7].

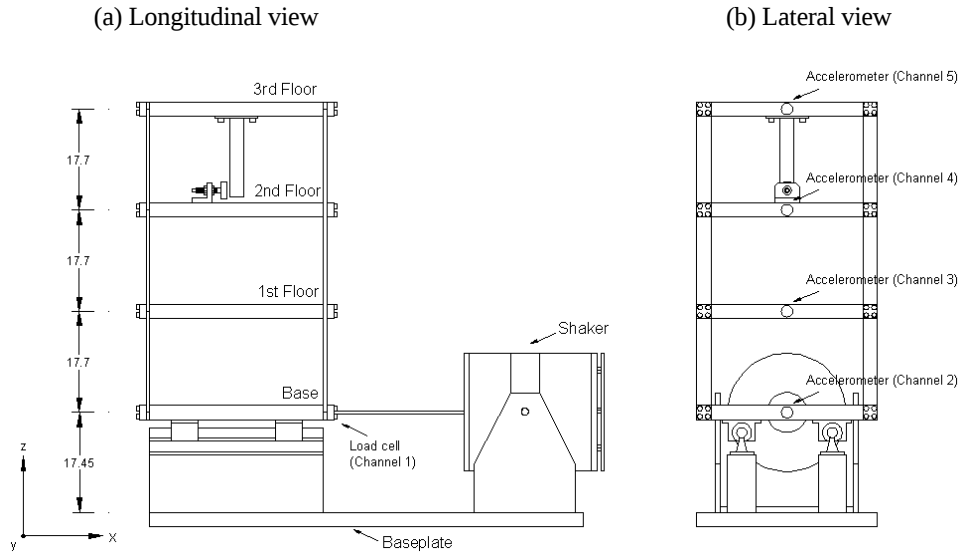


Figure 1. Basic dimensions of the base-excited 3-story test bed structure (all dimensions are in cm).

TABLE I. DATA LABELS OF THE STRUCTURAL STATE CONDITIONS

Label	State condition	Description
Condition#1	Undamaged	Baseline condition
Condition#2-3	Undamaged	Simulated operational effects (mass changes)
Condition#4-9	Undamaged	Simulated environmental effects (stiffness changes)
Condition#10-14	Damaged	Simulated damage (gaps: 0.20; 0.15; 0.13; 0.10; 0.05 mm)
Condition#15-17	Damaged	Simulated damage (gaps: 0.20; 0.20; 0.10 mm) together with simulated operational effects

ANALYSIS

The trajectory in the state space represents all the states that the system can assume for a specific input, and its shape can easily elucidate qualities of the system that might not be obvious otherwise. Figure 2 shows the predicted trajectory at Channel 5 of the baseline condition (Condition 1) and the trajectory of three other conditions, namely Conditions 7, 10, and 14 along with the baseline one in overlap format. The assumption of linear deterministic system implies that the existence of other forms of attractors indicates damage. The attractor of the baseline condition looks “noisy” and random. Even in a badly under-embedded state space (assumed $m=3$ for graphical representations), for the damaged conditions (10 and 17) the figure highlights state space distortions indicative of the nonlinearities induced when the suspended column hits the bumper. Furthermore, the distortions are indications that the representation seems to unfold the dynamics of the attractor even using an underestimated embedding dimension.

The average sum-of-square MAR errors for three different embeddings are plotted in Figure 3. The appropriate embedding was determined using the SVD technique. For the local embedding at Channel 5, the residuals increase significantly even when damage is present along with simulated operational and environmental variations (Conditions 15-17). On the other hand, the two multivariate embeddings assume distinct results. Clearly, for the global embedding (Figure 3c), the global dynamical attractor of the structure is not statistically reliable to detect the existence of damage. However, Figure 3b shows that the semi-global embedding with time series from Channel 4 and 5 can be used to classify the damaged conditions, even though it shows significant number of outliers in Condition 8 and 9 that can lead to undesirable false-positive indications of damage. Note that in all embeddings one can see a monotonic relationship between the level of damage and the residuals when operational and environmental variations are not present (Condition 10-14).

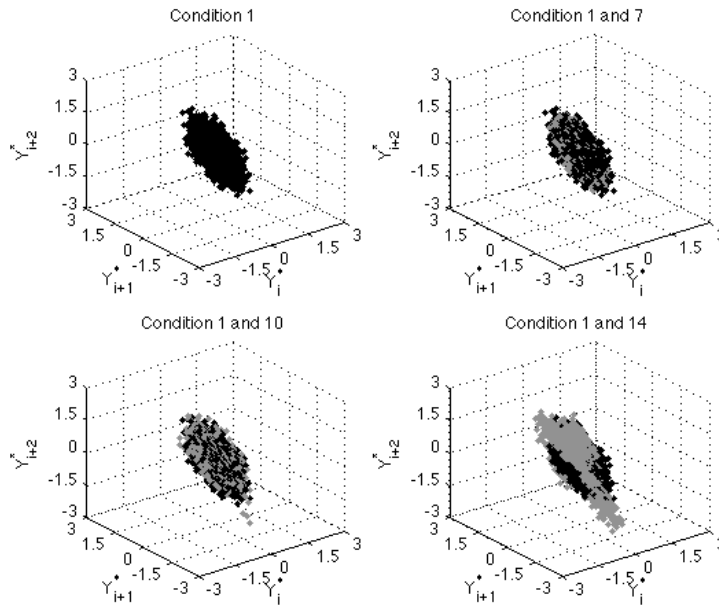


Figure 2. Predicted trajectory (black dots) of the baseline condition (Condition 1), and the predicted trajectories (gray dots) of Condition 7, 10, and 14 at Channel 5 ($m=3$, and $p=15$).

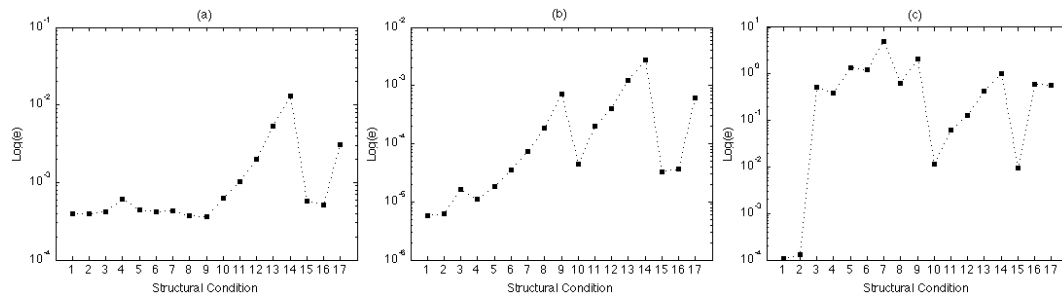


Figure 3. Average sum-of-square $MAR(p)$ errors in logarithm scale per structural condition based on (a) local embedding (Channel 5, $m=12$, and $p=15$); (b) semi-global embedding (Channel 4-5, $M=12$, and $p=8$); (c) global embedding (Channel 2-5, $M=12$ and $p=7$).

The results from both multivariate embedding indicates that the multivariate embedding amplifies the linear changes due to varying mass and stiffness and, consequently, average out the effects of damage in the form of singularities in the signal. Additionally, notice that the residual errors can have slightly differences from one test to another, because the performance of this approach depends highly on the number and intensity of random impacts that occur in each time series. These differences can be specially relevant for the low level of damage, such as Condition 10, 15, and 16 due to reduce number of impacts expected in each test.

CONCLUSIONS

The analysis in the predicted state spaces from a 3-story structure showed that the univariate embedding performs well. However, for multivariate time delay embedding, it was observed that the performance decreases with the number of channels used in the embedding. It amplifies the linear changes due to varying mass and stiffness and, consequently, averages out the effects of damage in the form of singularities in the signal. This possibly suggests that rather than global predictive power (error) as features, the MAR coefficients themselves and the optimum order might be a better comparative feature set.

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