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Report Title

Final Report on STIR Project on "Modeling and Analysis of Stochastic Dynamics and Emergent Phenomena in Swarm Robotic Systems Using the Fokker-Planck Formalism"

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FTE Equivalent:	0.15
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<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Subramanian Ramakrishnan	0.81
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Total Number:	1

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Manish Kumar	0.03	No
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Modeling and Analysis of Stochastic Dynamics and Emergent Phenomena in Swarm Robotic Systems Using the Fokker-Planck Formalism

Abstract

This research lays down a mathematical framework within which a system of interacting entities, such as a swarm robotic system, operating in a dynamic and uncertain environment can be analyzed for its properties such as stability, robustness, and emergent behavior. The framework, based upon stochastic differential equations and the Fokker-Planck formalism, allows the calculation of transient and steady state probability densities of the states of these systems. The framework has been applied to two scenarios related to swarm systems. The first scenario relates to a robotic swarm, the dynamics of which is inspired by ant foraging. The article develops a continuous time reaction-diffusion model based on Keller-Segel model of bacterial chemotaxis and establishes a connection with the Fokker-Planck formalism. The model encapsulates the coupled dynamics of density of agent population with that of pheromone laying, following, and evaporation in a 2D environment and carries out a preliminary numerical stability analysis of the solutions. In addition, the authors also present a distributed control law in continuous time that combines gradient following for pheromone concentration as well as food scent with random motion seen in ants.

The second scenario relates to a system of robots interacting via non-linear potentials and experiencing random excitations. The proposed framework allows calculation of probability density functions of states of the robots, such as positions and velocities, that helps analyze the stability of system and study its response to random excitations. The framework allows verification of the notion that random effects in swarm systems actually help them achieve robust self-organization and impart adaptive behaviors in changing dynamic conditions. Although this notion has existed for a long time, a comprehensive mathematical framework to study the effect of randomness and to actively design randomness to achieve desirable properties has been absent in literature. The paper formulates the dynamics of two robots interacting via a non-linear potential and experiencing random forces. The notion of adaptability is achieved by using a bistable interaction potential where the two stable points represent the two desirable configurations of robots. Adaptation, in this case, refers to the ability of the system to change from one configuration to the other due to change in environmental conditions. Bistable interaction potential is implemented by introducing a quadratic potential (called control potential) to a Morse potential. The analytical framework presented in this paper allows the study of the effect of varying noise intensity on the ability of the system to switch from one stable configuration to the other and thus presents a novel approach to the control of swarm systems aimed at extracting desirable emergent behavior from such complex systems.

I. INTRODUCTION

Advances in communication, computation, and sensing systems have provided unprecedented capabilities to generate, process, and communicate vast amounts of information. From the perspective of military applications, these advances have the potential to induce a paradigm shift in the military Command and Control (C2) by providing a new technological framework for network centric warfare capability. Highlights of the framework include novel mechanisms for sharing information and enhanced real-time situational awareness across different operational groups, resulting in increased agility, coordination, speed of command, and synchronization in C2. From the standpoint of basic research, the technological innovations described above are critically dependent on fundamental advances in the area of complex networked systems. However, as affirmed by a recent National Research Council report by Committee on Network Science for Future Army Applications [2], the current state of knowledge in this field lacks the tools to that can accurately predict the properties of large scale networked systems. In particular, a comprehensive framework comprising rigorous mathematical and computational approaches to model,

predict, and control the behavior of such systems is lacking in the present time. In this context, analytical approaches using ordinary differential equations in control-theoretic frameworks originally developed for simple physical systems have met with only limited success in the case of complex networked systems primarily because of two fundamental reasons: (1) the inability to handle the higher dimensionality that characterizes most complex systems, and (2) the incapacity to effectively address inherent dynamic uncertainties. Furthermore, salient characteristics of complex networked systems include their ability to adapt, self-organize, and exhibit other emergent behavior which often fails to reveal itself in an analysis of the individual sub-systems using traditional methods. In summary, complex networked systems are certain to play pervasive roles in future army applications and the lack of a robust analytical framework to model, analyze, and control their unique dynamical behavior represents a significant challenge from both the theoretical and applications perspectives.

Motivated thus, this research focuses on a class of complex networked systems namely swarm robotic systems typically represented by a large number of robots in mutual, potentially nonlinear, interaction whilst collectively exhibiting emergent, group behavior. Swarm systems have gained considerable attention among the robotics community primarily due to three reasons: i) robustness of these systems against failure and in the presence of uncertainty, ii) emergence of global behavior with global performance characteristics that cannot be achieved by the resources available at the level of the individual unit, and iii) scalability properties due to the unique features of local interactions and decision making processes. Major challenges in developing analytical frameworks include nonlinearities in the interactions, typically high dimensionality of the state space of the full system comprised of several subsystems, and uncertainties that are characteristic of the sensing, communication, and actuation processes. Additionally it is well known that swarm robotic systems exhibit adaptive, self-organizing and emergent behavior. However, currently, the ability to predict such global behavior from the analysis of local subsystems and their dynamical interaction as well as the capability to control the emergent behavior to achieve desired performance levels are conspicuously absent.

In this context, paradigms from statistical mechanics which deals with complex systems with many degrees of freedom and several states offer immense potential. Of particular note is the Fokker-Planck formalism that aims to determine probability density functions for the system evolution and hence can be used to determine mean values of dynamical variables rather than to obtain exact trajectories. In the presence of uncertainty, the system of ordinary differential equations representing a dynamical system is replaced by a stochastic differential system, the simplest example of which is the Langevin equation. Fundamental results in stochastic analysis provide a canonical method of obtaining a partial differential equation satisfied by the probability density function associated with the Langevin equation (see, for instance [7]). All available information about the dynamical response of a system driven by random excitation is encapsulated in the probability density function which identically satisfies this deterministic partial differential equation - the Fokker-Planck equation ([7], [26], [29], [18]). Hence solving this equation allows complete statistical characterization of all dynamical random variables associated with the system thereby categorically solving the problem. While the analysis of dynamical uncertainty using partial differential equations satisfied by probability density functions is a classical technique that has found success in many modern applications such as nanoelectromechanical systems [20], [24], the Fokker-Planck formalism has not yet been comprehensively investigated in the context of swarm robotics systems and this serves as the primary motivation for the approach taken in this research.

In broader terms, the framework adopted in this report has immense potential to open up various exciting avenues of research in complex networked systems. Of particular interest are phenomena that arise uniquely due to the synergetic influence of noise and nonlinearity and are being actively pursued in the context of dynamical systems. A case in point is the counterintuitive phenomenon of stochastic resonance wherein the addition of noise to certain coupled nonlinear systems leads to coherently amplified response [6]. Yet another instance is the influence of stochastic forcing on complex systems such as coupled microresonator arrays where the effects of noise on interesting phenomena such as energy localizations and noise induced enhanced response have recently been studied analytically using the Fokker-Planck approach [23], [21],

[22]. Whether such phenomena arise in swarm robotic systems and if they do, whether they could be harnessed to practical advantage are all exciting open questions that can be pursued within the proposed framework.

The rest of the report is set as follows. In Section 2, we approach the dynamics and control of a robotic swarm motivated by an analytical model of ant foraging in mathematical biology. A continuous time reaction-diffusion model obtained from a system of Langevin equations that essentially yields a Fokker-Planck equation for the robot density is presented. The model, which yields equations similar to the Keller-Segel model of chemotaxis, encapsulates the coupled dynamics of the density of agent population with that of pheromone laying, following, and evaporation in two spatial dimensions. Next, we carry out a preliminary numerical stability analysis of the solutions obtained from the model. As a highlight of our treatment, we also present a distributed control law in continuous time that combines gradient following for pheromone concentration as well as food scent with random motion seen in ants. In addition, we apply recent analytical results on stability to uniform solutions of the model. In Section 3, we begin by applying the Fokker-Planck formalism to a system of robots interacting via non-linear potentials and experiencing random excitations. We then focus on the dynamics of two robots interacting via the deterministic Morse potential and driven by white noise. The notion of adaptability is achieved by using a bistable interaction potential where the two stable points represent the two desirable configurations of robots. Adaptation, in this case, refers to the ability of the system to change from one configuration to the other due to change in environmental conditions. Bistable interaction potential is implemented by introducing a quadratic potential (called control potential) to the Morse potential. The mathematical tools developed in this paper allows the study of the effect of varying noise intensity on the ability of the system to switch from one stable configuration to the other. Concluding remarks are collected along with a discussion of directions of future research in Section 4.

II. CONTROL LAWS FOR A ROBOTIC SWARM EMULATING ANT FORAGING BEHAVIOR

A significant challenge in swarm robotics is the design and control of a robotic swarm capable of adaptive behavior dictated by local communication in uncertain environments. Despite the problem being a formidable one, the potential pay off is attractive as robotic swarm systems play a fundamental role in a broad spectrum of cutting edge technological applications including space exploration, nano-scale drug delivery, large-scale surveillance, and search and rescue operations. In all these applications one essentially deals with a system comprising a large number of robots working towards achieving prescribed objectives. The dynamics of the system is at once nonlinear and stochastic due to the inevitable uncertainties in the environment as well as the complexity of the interactions between the individual elements. The success of robotic swarm systems as a technological paradigm is ultimately dependent on our ability to extract desirable adaptive behavior (such as self-organization) from such complex dynamical systems in the absence of external control interference.

Given their remarkable propensity to adapt to rapidly changing environments, the dynamics of biological systems provide fundamental insights in this context. Of particular interest is the modeling of robotic swarm inspired by the behavior of ant colonies [13], [11], [17]. For instance, ants are capable of choosing the best nest sites out of multiple possibilities during migration and reproductive swarming. Furthermore, ant colonies efficiently establish shortest distance trails to food sources that minimize foraging time. This is achieved by the deposition of a chemical known as pheromone by ants returning to the nest from profitable food sources. Successive foragers from the nest are motivated to follow the pheromone trails and in turn they reinforce the trail by depositing more of the chemical depending on the continued attractiveness of the food source. Various analytical models have been proposed to study the social behavior of biological systems in general and ant colonies in particular (see, for instance [8], [16]). Furthermore, the constructive role played by noise in collective decision making during foraging has been reported in interesting recent work [4].

In this section, we discuss the analysis and control of a swarm of mobile robots based on an analytical model [25] inspired by ant foraging behavior. Specifically, we present a distributed control law in continu-

ous time that combines gradient following for pheromone concentration as well as food scent with random motion seen in ants. In addition, we provide a continuous time model for pheromone laying in a two dimensional environment. Extensive simulation studies confirm emergent behaviors seen in ant systems such as trail formation and convergence to single food site. Furthermore, we also discuss the effect of randomness on robustness of convergence to a single food site. It is noted that, in contrast to previous grid based approaches, a continuous robot model is employed here. To the best of our knowledge, generating ant-like behavior in a swarm of mobile robots using continuous control laws has not been reported in the literature and is a highlight of the present work.

A. Analytical Model

The two dimensional dynamics of a swarm of mobile robots dictated by pheromone concentrations may be essentially captured in a two component reaction-diffusion model comprising coupled evolution equations for the robot density $a(r, t)$ and the pheromone density $b(r, t)$, where $r \in R^2$ represents position in two dimensional environment and t represents time. The model described here is a modified version of the one first discussed in [28]. The robot dynamics evolves on a two dimensional surface described by a potential $U(r, t)$ which is the sum of contributions from the environment U_{env} and the pheromone deposit U_{ph} . Indeed the robots are able to alter U_{ph} locally by the production of pheromone and the explicit functional relationship between the pheromone density and the potential is postulated to be $U_{ph} = -gb(r, t)$ where g is a dimensional constant. The environmental potential U_{env} encodes information about desirable locations in the space, and in the context of ant foraging can be considered as potential whose gradient will drive the ants to the food sites. Assuming localized pheromone production at a rate q at location R_i , allowing for the pheromone to independently diffuse over the surface with a diffusion coefficient D_b as well as to evaporate linearly at a rate γ , the evolution of $b(r, t)$ may be modelled as a reaction-diffusion equation given by:

$$\frac{\partial b(r, t)}{\partial t} = -\gamma b(r, t) + q \sum_{i=1}^N \delta(r - R_i(t)) + D_b \nabla^2 b(r, t). \quad (1)$$

where R_i represents the position of robot i (i ranges from 1 to N), δ is the Dirac delta function and ∇^2 stands for the Laplacian in cartesian co-ordinates. The first term in Eqn (1) represents the evaporation of pheromone, the second term represents deposition of pheromone at location R_i and the third term represents diffusion of pheromone.

The stochastic motion of robot i in the potential $U(r, t)$ under the influence of white noise process dW of intensity σ and damping coefficient β may be represented by the first order system of stochastic differential equations given by:

$$\begin{aligned} \dot{R}_i &= v_i; \\ \dot{v}_i &= -\beta v_i - \nabla U(r, t)|_{R_i} + \sigma dW. \end{aligned} \quad (2)$$

where ∇ represents the gradient with respect to position r . In the Einstein-Smoluchowski limit[28], the inertia term in Eqn.(2) may be neglected to obtain:

$$\beta \dot{R}_i = -\nabla U(r, t)|_{R_i} + \sigma dW. \quad (3)$$

Substituting the explicit expression for the potential $U(r, t)$ in terms of $b(r, t)$ into Eqn.(3), and considering U_{env} to be uniformly distributed (when no extra information regarding the environment is available), one obtains:

$$\dot{R}_i = -\frac{g}{\beta} \nabla b(r, t)|_{R_i} + \frac{\sigma}{\beta} dW. \quad (4)$$

Now, the discrete description of the individual robots is replaced by a continuum description characterized by the robot density $a(r, t)$. Accordingly, the pheromone growth term (the second term in the RHS of Eqn(1)) is replaced by:

$$q \sum_{i=1}^N \delta(r - R_i(t)) \rightarrow qNp(r, t) \rightarrow qa(r, t), \quad (5)$$

where $p(r, t)$ represents the probability density of finding a robot at point r at time t . Using this description, the Fokker-Planck equation for the robot density $a(r, t)$ corresponding to the stochastic differential equation (4) can be derived as:

$$\frac{\partial a(r, t)}{\partial t} = \frac{g}{\beta} \nabla \cdot (-a(r, t) \nabla b(r, t)) + D_a \nabla^2 a(r, t), \quad (6)$$

where D_a stands for the diffusion coefficient corresponding to the robot density $a(r, t)$, and $\nabla \cdot$ represents the divergence operator. Finally, introducing the robot density (as obtained in the expression (5)) into the evolution equation for the pheromone density Eqn. (1), we obtain:

$$\frac{\partial b(r, t)}{\partial t} = -\gamma b(r, t) + qa(r, t) + D_b \nabla^2 b(r, t). \quad (7)$$

Equations (6) and (7) represent the coupled evolution equations for the robot density and the pheromone density and as such form the analytical basis of the model. It may be noted that the coupled Equations (6) and (7) represent the Keller-Segel model for bacterial chemotaxis that models the spatio-temporal evolution of population of bacteria under influence of chemicals that they deposit and external environmental stimuli. The model, first proposed by Keller and Segel in 1970 [12], has been researched much in fields of mathematics, physics and mathematical biology [19], [5], [9], [3], [10], [1], [27], [30]. However, its direct implementation to obtain robot control laws, to the best of authors' knowledge, has been absent.

B. Numerical Stability Analysis

It is of interest to examine the stability of solutions to the coupled system of equations 6 and 7 with respect to the various co-efficients. Of particular significance is the behavior of solutions with respect to variations in magnitude and sign of g as well as changes in the rate of pheromone evaporation γ . We note that positive values of g imply an attractive pheromone potential and will result in a highly localized swarm. On the other hand, a negative value of g indicates a repulsive pheromone potential profile. The spatial distribution of pheromone obtained from numerical simulations of Eqns. 6 and 7 at the end of time $t = 200$ units for different values of g and γ are provided in Figures 1(a)-1(d). The corresponding robot distributions are provided in Figures 2(a)-2(d). As observed from Fig.1(a), for the case of negative values of g and no evaporation ($\gamma = 0$), the pheromone density shows a highly ordered pattern of spatial diffusion at the end of the simulation. Correspondingly, from Fig.2(a), the robots show a high degree of spatial dispersion. However, in the presence of a relatively small evaporation rate ($\gamma = 0.1$) the robots begin to show a tendency to converge (see Fig.2(b)) whereas the pheromone is much more even distributed (Fig.1(b)). However, for positive values of g , which may be understood as a strongly attracting potential, and relatively low evaporation rate ($\gamma = 0.1$), the pheromone density is highly concentrated around a point (Fig.1(c)). Accordingly, the robots cluster around this point (Fig.2(c)) in a stable configuration. Interestingly, a high evaporation rate ($\gamma = 10$) under the same conditions appears to induce instability in the cluster (see Figs. 1(d) and 2(d)). This can be explained by the fact that due to high evaporation rate, the effect of pheromone on dynamics of robots is lesser pronounced than the effect of randomness (or diffusion) on the dynamics. Hence, the robots appear to disperse. These numerical results indicate that the dynamical stability of the configurations emerging from the analytical model are critically dependent on the physical parameters of the system of governing equations.

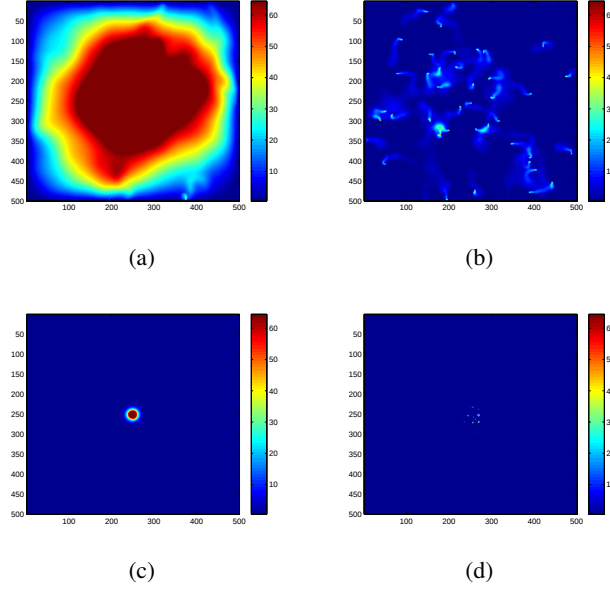


Fig. 1. Spatial pheromone distribution at $t = 200$ for:(a) $g=-5; \gamma=0$ (b) $g=-5; \gamma=0.1$ (c) $g=5; \gamma=0.1$ (d) $g=5; \gamma=10$

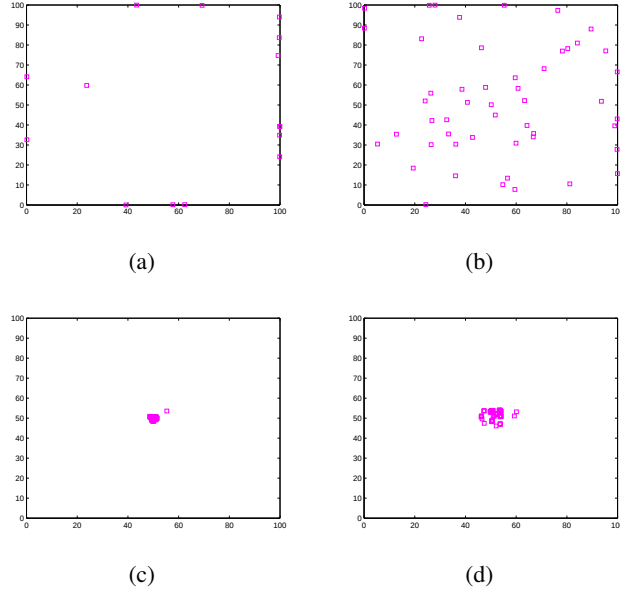


Fig. 2. Spatial robot distribution at $t = 200$ for:(a) $g=-5; \gamma=0$ (b) $g=-5; \gamma=0.1$ (c) $g=5; \gamma=0.1$ (d) $g=5; \gamma=10$

C. Stability of Uniform Steady State Solutions

As mentioned earlier, stability of solutions of Keller-Segel model has been well studied in literature especially with respect to blow up or spike formation of solutions. Of particular interest is the steady state solutions of the Keller-Segel equations that result into a uniform density of chemicals (pheromone) and bacteria (robots or ants). There has been some recent research carried out on this aspect that have aimed at obtaining relationships between parameters of Keller - Segel model such as diffusion coefficients of pheromone and ants, and pheromone evaporation and deposition rate. In this respect, readers are referred to papers by Payne and Straughn [19] and Solis et. al. [30].

A preliminary study was carried out to verify our robot control model using the results published in the above two papers. In order to verify the results, various parameters were chosen that satisfied

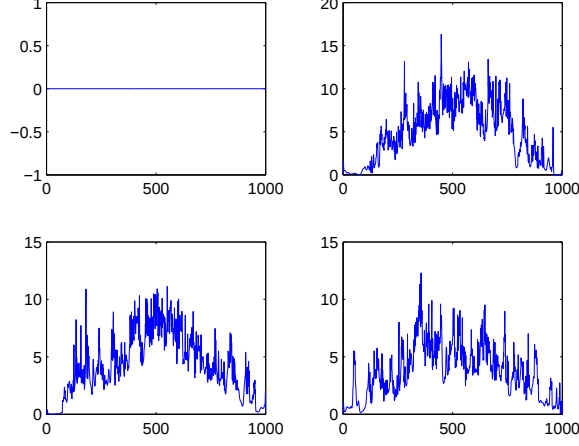


Fig. 3. Distribution of pheromone (1D case) at: (a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

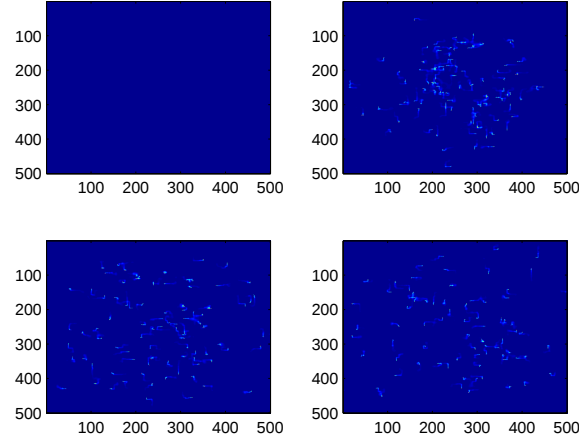


Fig. 4. Distribution of pheromone (2D case) at: (a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

the conditions for steady state uniform solutions reported in the above literature. Figure 3 shows the evolution of pheromone density in one-dimensional space (y-axis represents pheromone concentration and x-axis represents the 1D space) for different time instants. It can be seen that as time elapsed the spatial distribution of pheromone approached towards a uniform distribution. The results are as well valid for the 2-dimensional case as are shown by Figures 4 and 5 that respectively represent the evolution of pheromone and robots. It can be seen that robots start off initially from a clustered position and get distributed uniformly as time elapses. The simulation results validate the derivation of individual robot control law as stochastic differential equation given by (4) corresponding to the Fokker Planck equation given by (6).

D. Discrete Ant Behaviors

While the two component reaction-diffusion model presented earlier provides a reasonable analytical basis for ant foraging, in order to better reproduce the phenomenon in simulations, we need to consider some of the discrete behaviors exhibited by ant colonies while foraging. Of particular interest are the following two types of behaviors: i) search for food; and ii) transportation of food. In the "search for food" behavior, an ant's motion is governed by three factors: a) environmental potential gradient (∇U_{env})

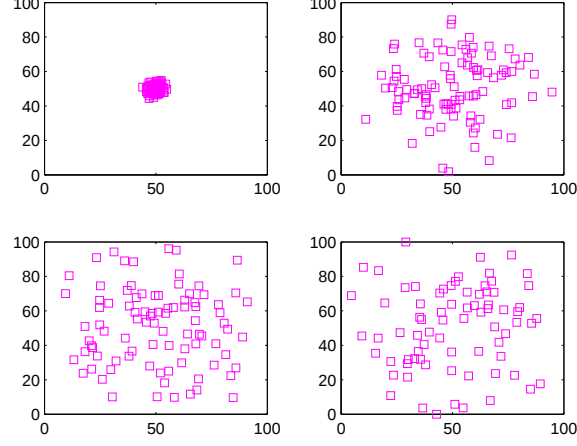


Fig. 5. Distribution of robots (2D case) at: (a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

which can be construed as a gradient generated by the smell of food; b) gradient of pheromone (∇U_{ph}) laid by other ants; and iii) inherent randomness which allows the ant to explore. The dynamics of motion of ants engaged in this behavior is given by Equation (4). Furthermore, the intensity of randomness is dependent on whether an ant has picked any gradient on either environmental potential or pheromone potential. If an ant has not picked the gradient, then it will have certain specific amount of randomness in its motion. This randomness allows the ant to explore the space and search for the food site till it picks up the gradient. This kind of randomness behavior is consistent with behavior seen in actual ants.

During the "transportation of food" behavior, the ant carries the food back to the nest while laying the pheromone on the path. In accordance with actual ant system behaviors, the ants are assumed to move with a constant velocity in straight line from the food site to the nest.

E. Simulation Results: The Base Case

In this section we present the simulation results for the base case of very low noise intensity. Randomness is introduced into the dynamics by passing the effective potential $U(r, t)$ through a white Gaussian noise channel using the MATLAB function 'awgn'. This is equivalent to adding white noise to the effective potential. The noise intensity is modulated by specifying the signal-to-noise ratio (SNR) of the channel. For the base case, a relatively high value of $SNR = 10E6$ is considered and therefore the dynamics is almost deterministic in this case. The simulation is run for time $t = 200$ units with a time step of 0.04. The nest is represented by the red star (with grid co-ordinates (55, 20)) whilst the blue stars indicate the two food sources with grid co-ordinates (30, 50) and (70, 50) respectively. U_{env} is considered to be uniform for the simulations. The food source located at (70, 50) is considered to be superior because of its proximity to the nest. Figures 6(a)-6(d) represent snapshots of the traffic and time $t = 0$, $t = (\frac{1}{3}) 200$, $t = (\frac{2}{3}) 200$ and $t = 200$ respectively. The formation of trails is quite clear from the simulation results. In addition, snapshots of the pheromone density in both the trails at each of the time instances indicated above are also furnished in Figures 7(a) - 7(d) using the color code indicated in the figures. It is interesting to note the disparity in traffic intensity between the food sources with the superior food source being the clear favourite. This is further demonstrated in Figure 8 wherein the number of robots visiting each of the food sources is plotted as a function of time.

F. Simulation Results: $SNR=2$

As a step towards studying the effects of noise, in this section we present simulation results for the case of high intensity noise ($SNR=2$) influencing the dynamics once a robot has picked up a gradient. Similar

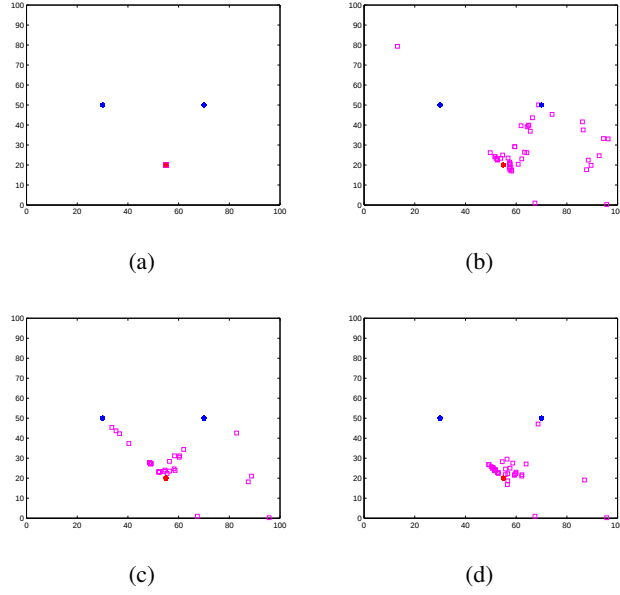


Fig. 6. Configuration of robots (Base Case: SNR = 10E6) at:(a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

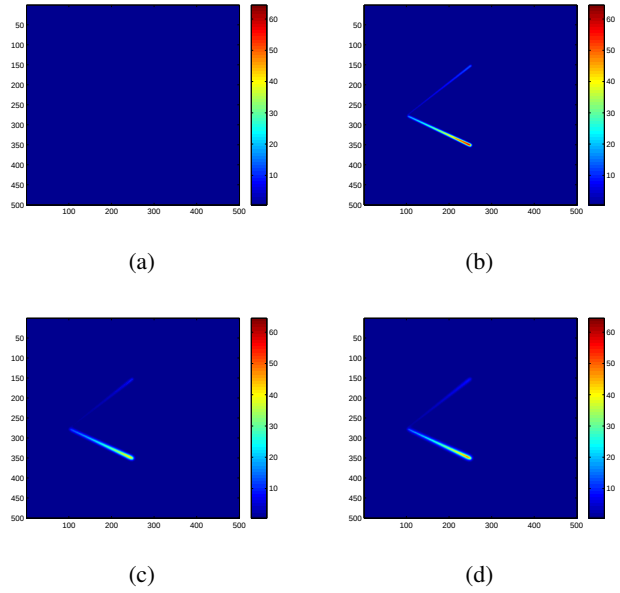


Fig. 7. Trails to the food sources as demonstrated by pheromone density (Base Case: SNR = 10E6) at:(a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

to the base case, Figures 9(a) to 9(d) represent snapshots of the traffic and time $t = 0$, $t = \left(\frac{1}{3}\right) 200$, $t = \left(\frac{2}{3}\right) 200$ and $t = 200$ respectively. The corresponding trails are depicted in Figures 10(a)-10(d) and the number of robots visiting the two food sources as functions of time are plotted in Figure 11.

G. Effects of Varying Noise Intensity

To better understand the effect of varying the intensity of noise on the traffic, in this section we present results for the time variation of the number of robots visiting the two food sources for the cases of SNR=3, SNR=10, SNR=100 and SNR=1000 in Figures 12(a), 12(b), 12(c) and 12(d) respectively. Comparing the base case (Figure 8) with the cases of higher noise intensity (Figures 11 and 12) we first note that

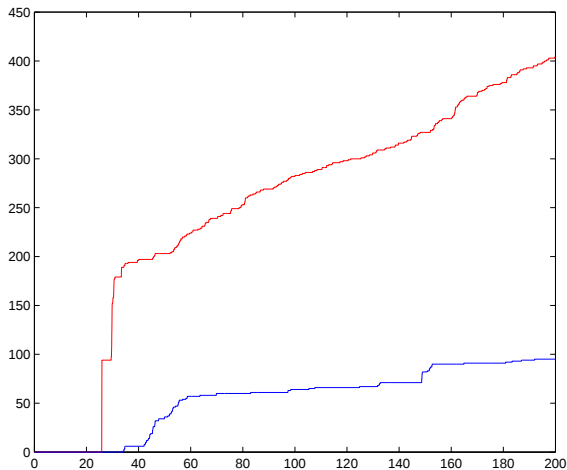


Fig. 8. Number of robots visiting Food Source 1 (red) and Food Source 2 (blue).

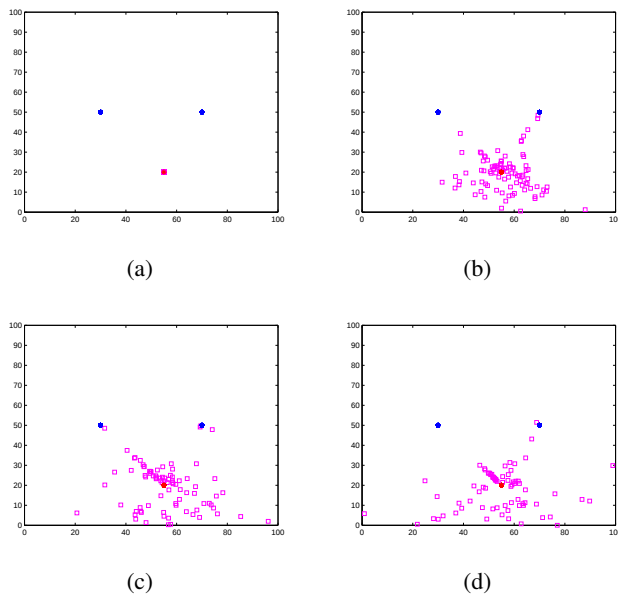


Fig. 9. Configuration of Robots for SNR=2 at:(a) $t=0$ (b) $t=67$ (c) $t=133$ (d) $t=200$

addition of noise does not fundamentally alter the preference of the better food source with food source 1 continuing to be favored. However, in the base case of very low intensity noise the traffic to food source 1 shows an abrupt increase at around $t = 25$ while an increase in traffic towards food source 2, albeit to a lesser extent, occurs between $t = 40$ and $t = 60$. One observes a qualitative difference in this feature as the noise intensity is increased. As can be seen from Figures 11 and 12, at higher intensity of noise, the traffic to both the sources picks up at much earlier times and also shows growth almost proportional to the time elapsed.

III. FOKKER-PLANCK EQUATION FOR MORSE POTENTIAL

In this section, we present the Fokker-Planck formalism within which a swarm robotic system can be modeled and analyzed. The swarm, comprising a system of N coupled robots undergoing collective motion in which robot i experiences a total interaction potential $U(q_i)$ involving attractive and repulsive

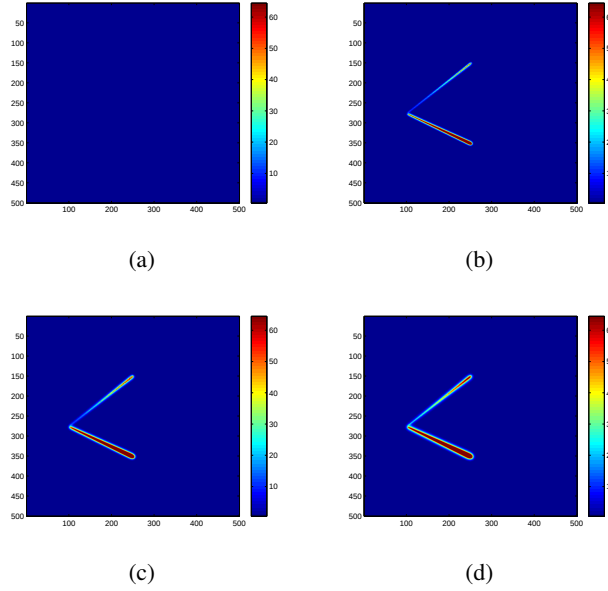


Fig. 10. Trails to the food sources as demonstrated by the pheromone density for SNR=2 at:(a) t=0 (b) t=67 (c) t=133 (d) t=200

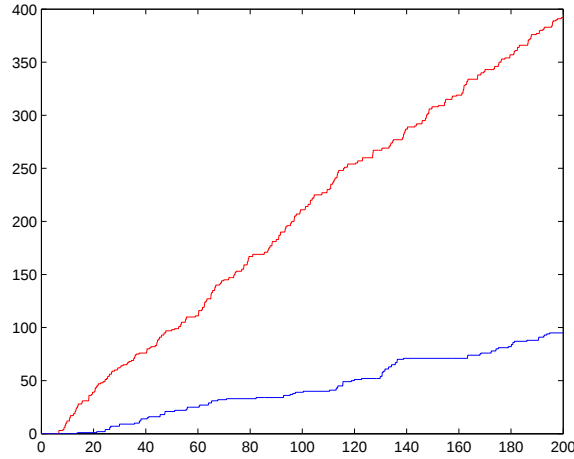


Fig. 11. Number of robots visiting Food Source 1 (red) and Food Source 2 (blue) for case SNR=2.

forces resulting from pairwise potential $U(q_{ij})$, a control input u_i , dissipative force $f(p_i)$ and white noise process dW of strength σ , may be represented as:

$$\dot{q}_i = p_i; \quad \dot{p}_i = u_i = f(p_i) - \nabla_i U(q_i) + \sigma dW_i. \quad (8)$$

Artificial potential functions have been used frequently [15], [14] for modeling of intractions in swarm robotic systems. Compare the above system of equations (8) with a standard Ito SDE, $d\bar{x} = \bar{A}dt + \tilde{B}d\bar{W}$, where $\bar{x}$ is the state vector, $\bar{A}$ is the drift vector and $\tilde{B}$ is the diffusion matrix. The Fokker-Planck equation for the probability density P_i governing the dynamics of robot i may be derived as:

$$\frac{\partial P_i}{\partial t} = -\frac{\partial}{\partial x_j} [A_j P_i] + \frac{1}{2} \sum_{j,k} \frac{\partial^2}{\partial x_j \partial x_k} \left[(BB^T)_{jk} P_i \right] \quad (9)$$

Equation (9) provides a probabilistic description of the dynamics of each element of the swarm. One of the objectives of this research to study the influence of noise on emergent behavior of swarm robotic systems.

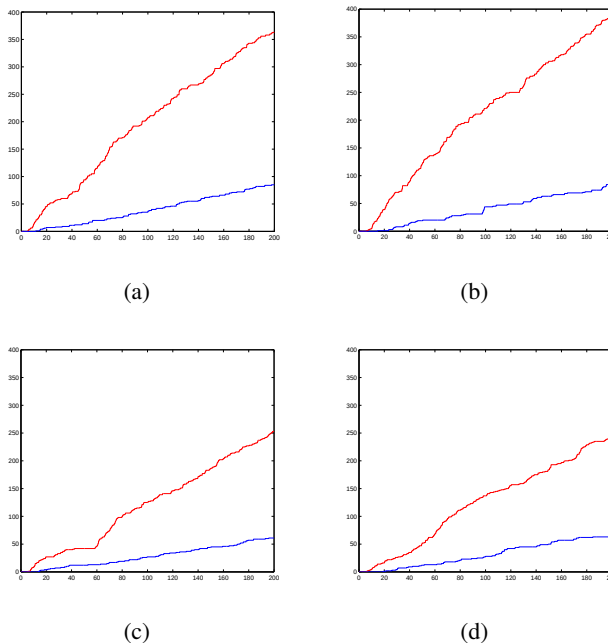


Fig. 12. Number of robots visiting Food Source 1 (red) and Food Source 2 (blue) for: (a) SNR=3 (b) SNR=10 (c) SNR=100 (d) SNR=1000

Contrary to common intuition, noise can sometimes play a constructive role in nonlinear systems. In particular, as evidenced by the phenomenon of stochastic resonance, the effects of noise may be harnessed to advantage in applications [6]. In interesting recent work, Meyer and co-workers have suggested that hitherto unobserved adaptive behavior in ant colonies can be explained using noise induced transitions between stable equilibrium points of a double-well potential [4].

Inspired by the above biological example, we carried out preliminary research to study the noise induced transitions in the robotic swarm using a two-robot system. The central idea is to introduce a control potential to the regular pairwise interaction potential resulting in an altered effective potential. Altering the resultant interaction potentials using control interventions, as proposed, results in new equilibrium points for the resultant effective potential. Concomitantly, the stationary densities obtained from the Fokker-Planck equations will have peaks around these equilibrium points. The control objective of extracting desirable adaptive behavior will be realized by driving the system to these equilibrium points in the probabilistic sense.

As a proof of concept for the ideas introduced above, we now consider a two robot system, wherein the mutual interaction is represented by the Morse potential given by the following equation:

$$U_{int} = D_e (1 - e^{-a(r-r_e)})^2 \quad (10)$$

A harmonic oscillator potential as the control intervention is given by:

$$U_{con} = b_e (r - r_c)^2 \quad (11)$$

where r represents the inter-robot distance. The resultant effective potential is given by $U = U_{int} + U_{con}$. The Morse potential is a well known potential for modelling inter-robot interaction and Figure 13 shows a plot of Morse potential with a minimum at inter-robot distance of three (3) units. Addition of the harmonic oscillator potential results in an extra "well" as can be seen in Figure 14 at inter-robot distance of 9.5. Hence, the resultant potential has two minima: one at inter-robot distance of 3 and another at inter-robot distance of 9.5.

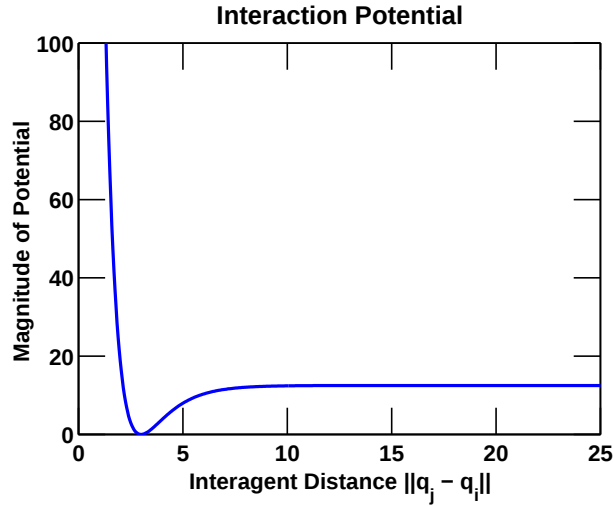


Fig. 13. Morse potential plotted against inter-robot distance

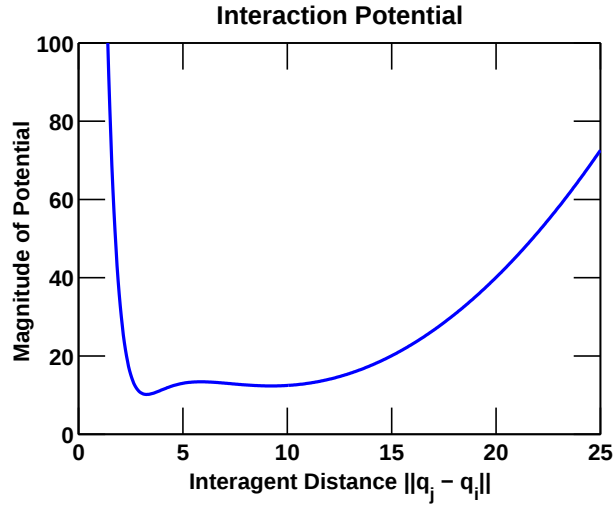


Fig. 14. Resultant effective potential plotted against inter-robot distance

Numerical simulation of the two-robot system undergoing the dynamics given by Equation 8 under the influence of combined potential and noise, as demonstrated in Figure 15, clearly shows the transition of the system between equilibrium states for a noise intensity of 25. The key points to be noted here are: (1) new equilibria, corresponding to desirable emergent behavior, can be introduced using control potentials and (2) optimal levels of noise intensity may be used to control transitions of the system between equilibrium states.

Finally, we derive the Fokker - Planck equation corresponding to the stochastic differential equation governing the dynamics of the two-robot system under the influence of an effective potential given by the sum of Morse potential and harmonic oscillator potential as:

$$\frac{\partial P}{\partial t} = -y \frac{\partial P}{\partial r} + [cy + 2aD_e (e^{-a(r-r_e)} - e^{-2a(r-r_e)}) + r] \frac{\partial P}{\partial y} + cP + \frac{\sigma^2}{2} \frac{\partial^2 P}{\partial r^2} \quad (12)$$

Here, $y = \dot{r}$, and P represents the probability density of r and y , $P(r, y)$. Steady state solutions to the above Fokker-Planck equation may be utilized to analytically characterize the transitions between equilibria that were numerically demonstrated earlier in this section. On the other hand, while Fokker-Planck equations for nonlinear systems are notoriously hard to solve, approximate techniques such as

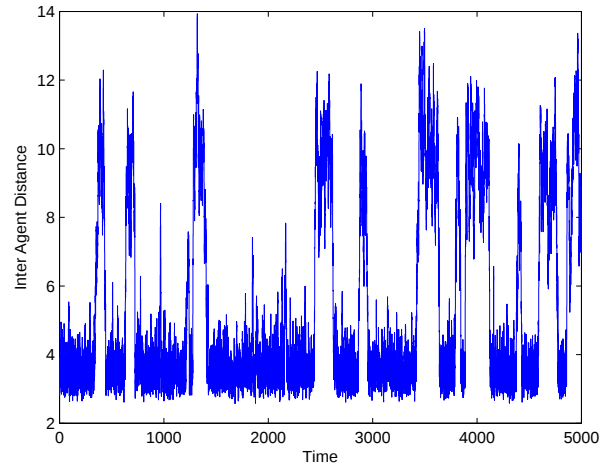


Fig. 15. Inter-agent distance plotted against time

the method of moment approximations or cumulant expansions may be used to obtain the time evolution of averaged dynamical quantities from the Fokker-Planck equation. These investigations are currently underway and will be reported in future publications.

IV. CONCLUSIONS AND FUTURE WORK

In Section 2 of this report, we presented continuous time control laws for robots based upon the Keller-Segel model of bacterial chemotaxis designed to emulate ant foraging behavior. The motion control law was based on gradient following that attempted to minimize an effective potential which is the sum of an environmental potential and the potential arising due to pheromones. Furthermore, the control law incorporated a random component that allowed the robots to stochastically explore the spatial domain for food sources. A continuous time model of pheromone dynamics that includes pheromone deposition, diffusion, and evaporation was also presented. The discrete control law formulated on the basis of correspondence of Fokker-Planck equation to stochastic differential equation was validated based upon solutions of Keller-Segel model reported in literature. Finally, a set of discrete behaviors inspired by foraging ant colonies was incorporated. Exhaustive simulations demonstrate two of the emergent behaviors seen in ant foraging : i) formation of trails, and ii) increased traffic along the trail that leads to the superior food source. In addition, preliminary results on the influence of noise intensity on the dynamics were also presented. The simulation results suggest that, interestingly, higher intensity noise encourages traffic towards the better food source in a manner almost directly proportional to the time elapsed. Several interesting directions of future research emerge from these results. For instance, generalizations of the Keller-Segel model that take into account the qualitative differences between the dynamics exhibited by foraging ants and food carrying ants are important topics for further study. On the other hand, while there exists a vast amount of mathematical literature pertaining to singular behavior of solutions to the Keller-Segel model and stability of solutions, a detailed study of the implications of the results for the design and control of swarm robotic systems is absent at the current time. The initial steps in this direction taken by the present report need to be pursued in further detail. Important open questions on the influence of noise intensity on swarm dynamics also need to be addressed comprehensively.

In Section 3 of this article, we presented a direct approach to studying the dynamics of a collection of interacting entities starting with the Langevin equations and then deriving the Fokker-Planck equation. The novel idea of extracting desirable emergent behavior by altering the effective potential via the introduction of control potentials was introduced and applied to the case of a two-agent system. It was shown that a control potential of the harmonic oscillator type, when added to a Morse potential, resulted in a new

equilibrium point for the system. Furthermore, transition between the two equilibrium points for an optimal level of noise intensity was shown numerically. The Fokker-Planck equation for this scenario was derived and an analytical characterization of the state transitions is currently under investigation. In addition, the influence of noise intensity on the dynamics is being studied using the method of moment approximations, the starting point of which is the Fokker-Planck equation derived above. A central issue in the use of discrete particle models in swarm robotics is the smooth passage from such models to continuum models. It is of great interest to investigate the effect of control potentials on emergent behavior in the context of continuum models. These and allied questions are the subject of ongoing research and will be reported in future publications.

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