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# Current and Future Development of a Non-hydrostatic Unified Atmospheric Model (NUMA)

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# Motivation and Goals

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Our goal is to construct numerical methods for non-hydrostatic mesoscale and global atmospheric models (for NWP applications); this is a unified model. The reason for this is economics - one (production) model is cheaper to support or at the very least, having the same dynamics simplifies a great many things (e.g. training future developers).

Our aim is to build a modeling framework with the following capabilities:

1. Highly scalable on current and future computer architectures (exascale computing and beyond and GPUs)
2. Flexibility to use a wide range of grids (e.g., statically and dynamically adaptive)
3. Global model that is valid at the meso-scale (i.e., non-hydrostatic)
4. NUMA Framework to include:
  - A suite of time-integrators (explicit, semi-implicit, fully-implicit)
  - Various numerical methods (SE/DG)
  - Various forms of the Governing Equations to determine what form is more accurate, efficient, robust.

# Talk Summary

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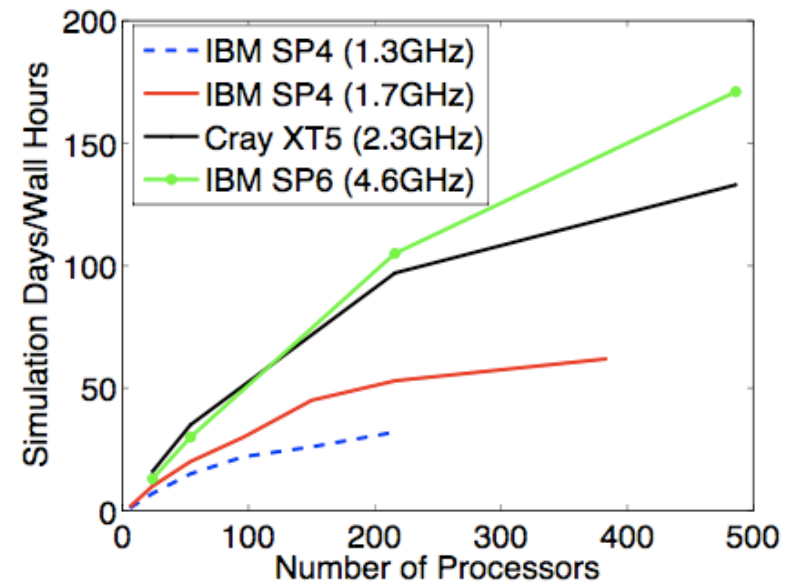
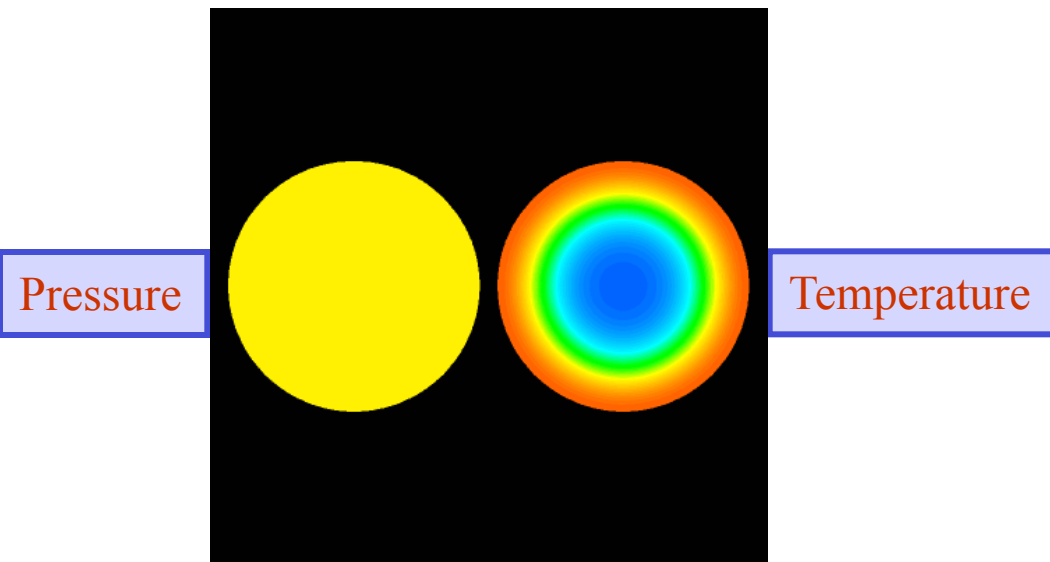
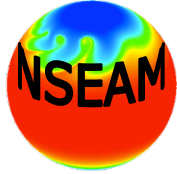
1. How do computer architectures affect the models?
2. Numerical Methods in New Dynamical Cores
3. How does resolution affect the model equations?
4. What Should we aim for in our New Models
5. Where We Plan to Head with NUMA

# Talk Summary

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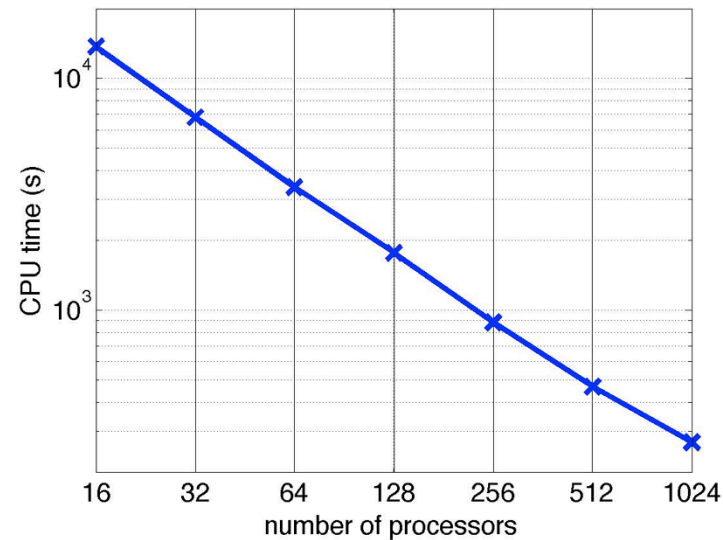
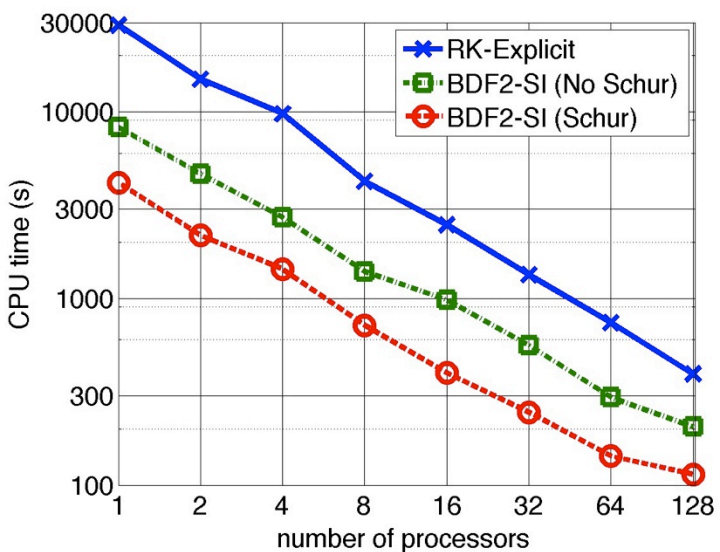
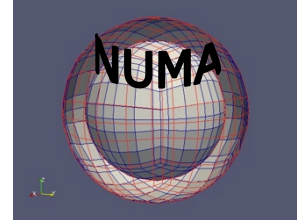
1. How do computer architectures affect the models?
  - From Terascale to Petascale/Exascale Computing
  - 10 of Top 500 are already in the Petascale range
  - Should also keep our eyes on GPUs (e.g., Mare Nostrum)
2. Numerical Methods in New Dynamical Cores
3. How does resolution affect the model equations?
4. What Should we aim for in our New Models
5. Where We Plan to Head with NUMA

# Performance of a Global Hydrostatic Model (T239 L42)

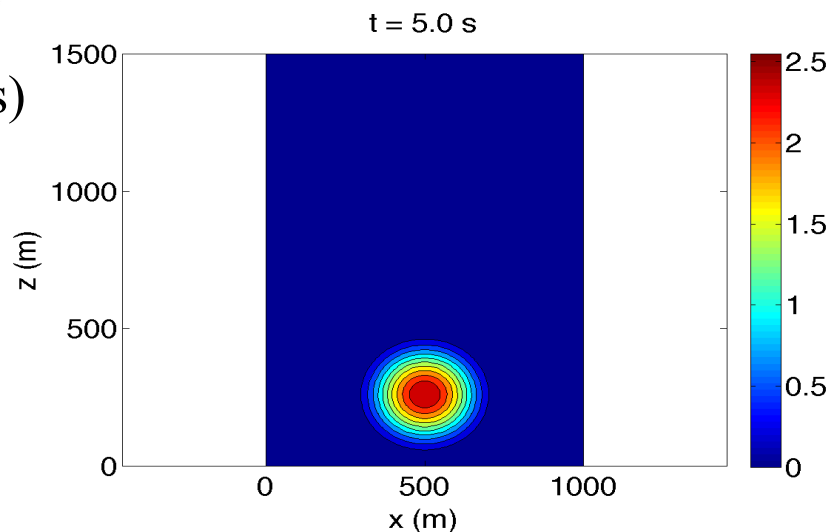


30 day simulation for a 3D global (hydrostatic) atmospheric model for a Baroclinic Instability (Giraldo QJRMMS 2005).

# Performance of a Global/Mesoscale Non-Hydrostatic Model



(2 Million Grid Points)



(7 Million Grid Points)

# Talk Summary

---

1. How do computer architectures affect the models?
2. Numerical Methods in New Dynamical Cores
  - Time-Integration is important (e.g., explicit, semi-implicit, fully-implicit)
  - Spatial Discretization methods is how we are able to take advantage of Parallel computers (i.e., domain decomposition of the physical grid)
  - What other properties should we strive for in our Numerical Methods?
3. How does resolution affect the model equations?
4. What Should we aim for in our New Models
5. Where We Plan to Head with NUMA



# Summary of Numerical Methods

- Proxy for Primitive Equations:

$$\frac{\partial q}{\partial t} = S(q) \longrightarrow S(q) = \frac{\partial q}{\partial x}$$

- $S(q)$  must be approximated discretely:  
Options are:

- Finite Difference Methods (Taylor Series)

$$q_{i+1} = q_i + \Delta x \frac{\partial q_i}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 q_i}{\partial x^2} + O(\Delta x^3)$$

- Galerkin Methods (Basis Function Expansions)

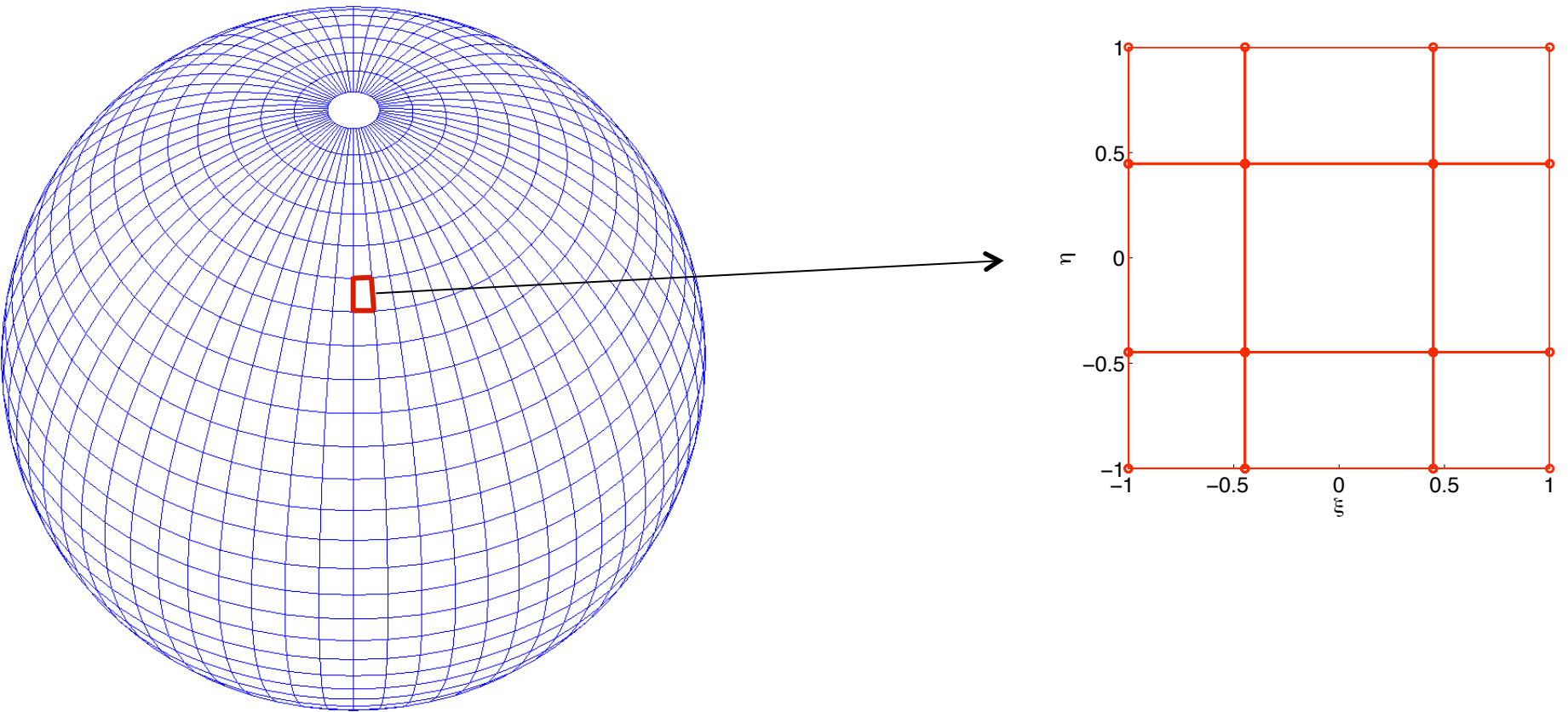
$$q_N(x, t) = \sum_{i=1}^{M_N} \psi_i(x) q_i(t)$$

- Godunov Methods (Control Volume Approach)

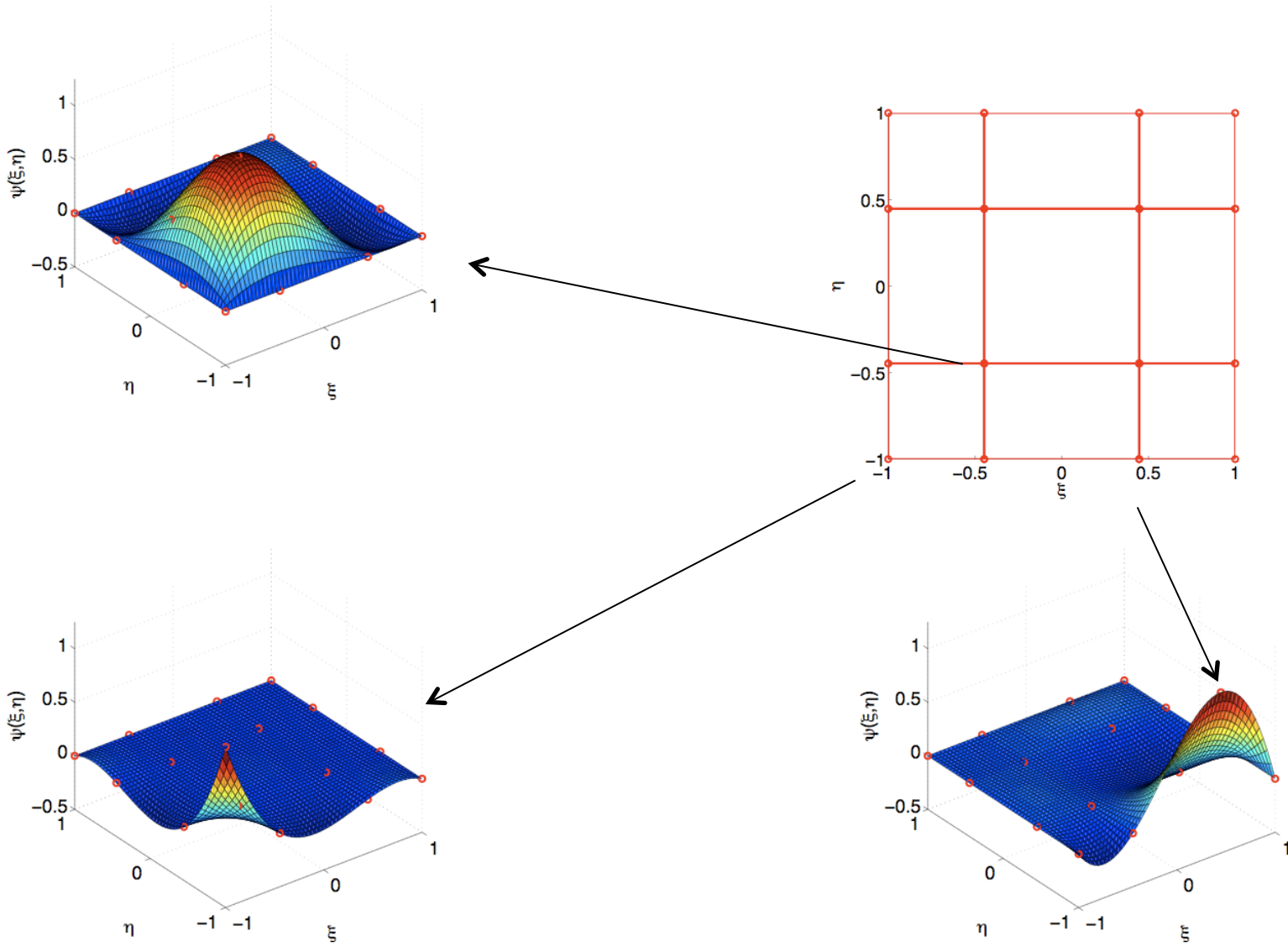
$$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{F} = 0 \longrightarrow \int_{\Omega_e} \psi \left( \frac{\partial q_N}{\partial t} + \nabla \cdot \mathbf{F}_N \right) d\Omega_e = 0$$

$$\int_{\Omega_e} \psi \frac{\partial q_N}{\partial t} d\Omega_e + \int_{\Gamma_e} \psi (\mathbf{n} \cdot \mathbf{F}_N) d\Gamma_e - \int_{\Omega_e} \nabla \psi \cdot \mathbf{F}_N d\Omega_e = 0$$

# Spectral Elements in a Nutshell



# Spectral Elements in a Nutshell

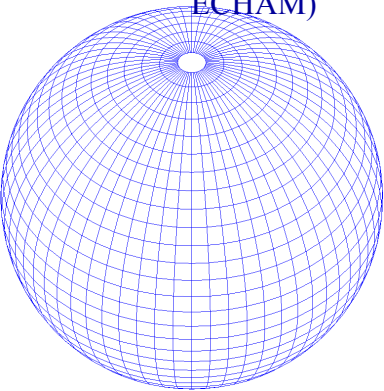


# With New Models Come New Grids

(conduit between dynamics and physics)

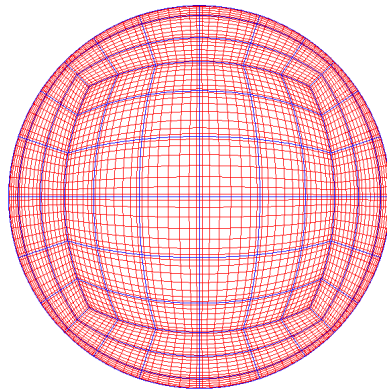
Lat-Lon

(IFS, GFS, NOGAPS, UM, CAM,  
ECHAM)

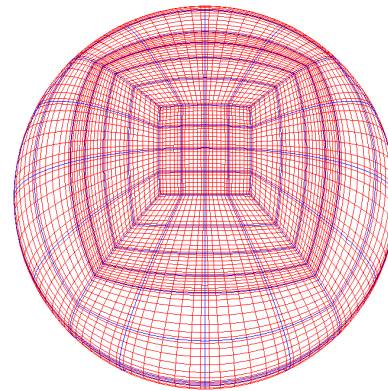


Hexahedral

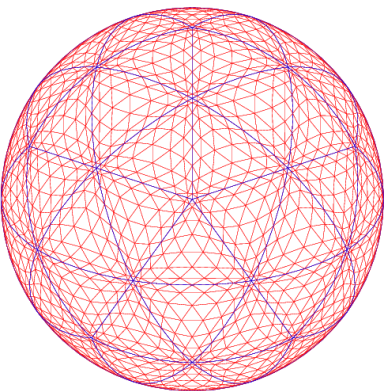
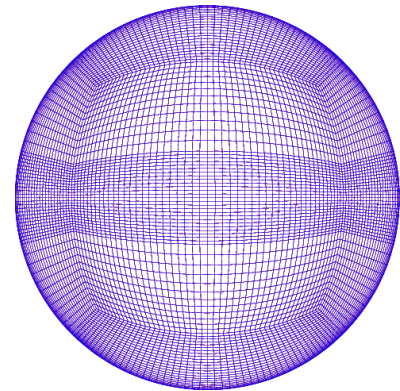
(HOMME, NSEAM, FV-Cube)



Telescoping

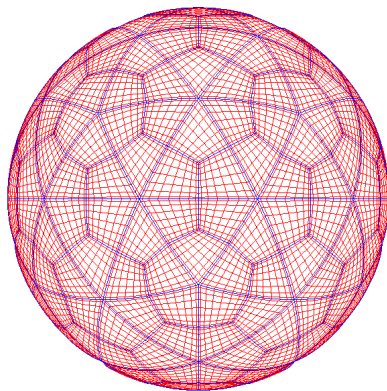


Banded

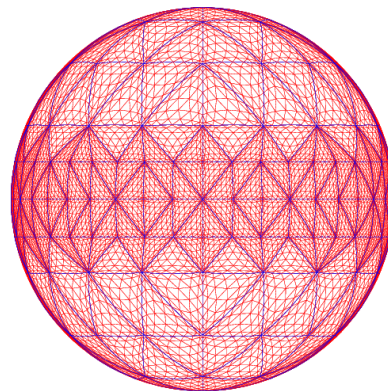


Icosahedral

(NICAM, ICON, FIM,  
MPAS)



Icosahedral



Adaptive

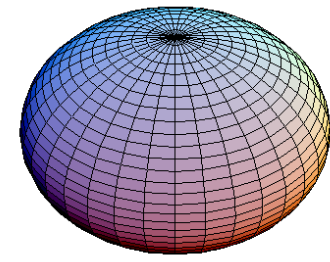


Adaptive

# Talk Summary

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1. How do computer architectures affect the models?
2. Numerical Methods in New Dynamical Cores
3. How does resolution affect the model equations?
  - Hydrostatic versus Non-hydrostatic models and regimes of validity of the governing equations
4. What Should we aim for in our New Models
5. Where We Plan to Head with NUMA



# Unified Equations Framework

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{U} = 0$$

(Mass)

$$\frac{\partial \mathbf{U}_{Horiz}}{\partial t} + \nabla \cdot \left( \frac{\mathbf{U} \mathbf{U}_{Horiz}}{\rho} + P \mathbf{I}_2 \right) = -f(\mathbf{k} \times \mathbf{U}_{Horiz}) - \nabla_{Horiz} \phi_A$$

$$\delta_H \left[ \frac{\partial W}{\partial t} + \nabla \cdot \left( \frac{\mathbf{U} W}{\rho} \right) \right] + \frac{\partial P}{\partial z} = -\frac{\partial \phi_A}{\partial z}$$

(Momentum)

$$\mathbf{U} = \rho \mathbf{u},$$

$$\Theta = \rho \theta,$$

$$\mathbf{u} = (u, v, w)^T,$$

$$\mathbf{x} = (x, y, z)^T,$$

$$\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)^T$$

$$\delta_{PI} \frac{\partial \Theta}{\partial t} + \nabla \cdot \left( \frac{\Theta \mathbf{U}}{\rho} \right) = 0$$

(Energy)

$$P = P_A \left( \frac{R\Theta}{P_A} \right)^\gamma$$

(Equation of State)

$$\theta = \frac{T}{\pi} \quad \text{and} \quad \pi = \left( \frac{P}{P_A} \right)^{R/c_p}$$

# When are these equations valid?

## 1. Non-hydrostatic Equations

- These equations are valid at all spatial scales; these are the most general form of the governing equations.
- The problem with this form is that they permit acoustic waves which are very fast and have little effect on the dynamical processes we are interested in.
- We know how to solve these equations well but requires sophisticated numerical methods (DG, semi-implicit, etc.)

## 2. Hydrostatic Equations

- These are very simple equations to solve but are not valid if you are interested in vertical acceleration. Because vertical acceleration is omitted, it then means that the vertically propagating acoustic waves are removed from the equations.
- This approximation is no longer valid below 10km resolution. Many NWP models are either at this limit or approaching it quickly (e.g., IFS, UM, Grapes).

## 3. Pseudo-Incompressible Equations

- It would be nice to keep the equations valid below 10km without having to deal with the vertically propagating acoustic waves; these equations are a likely candidate.
- No operational NWP or climate models currently use this approach, only research codes (e.g., EULAG). It would be interesting to explore this form of the equations.

# Talk Summary

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1. How do computer architectures affect the models?
2. Numerical Methods in New Dynamical Cores
3. How does resolution affect the model equations?
4. What Should we aim for in our New Models
  - E.g., Conservation, Scalability, High-order Accuracy, Adaptivity
5. Where We Plan to Head with NUMA



## 4. What Should We Aim For?

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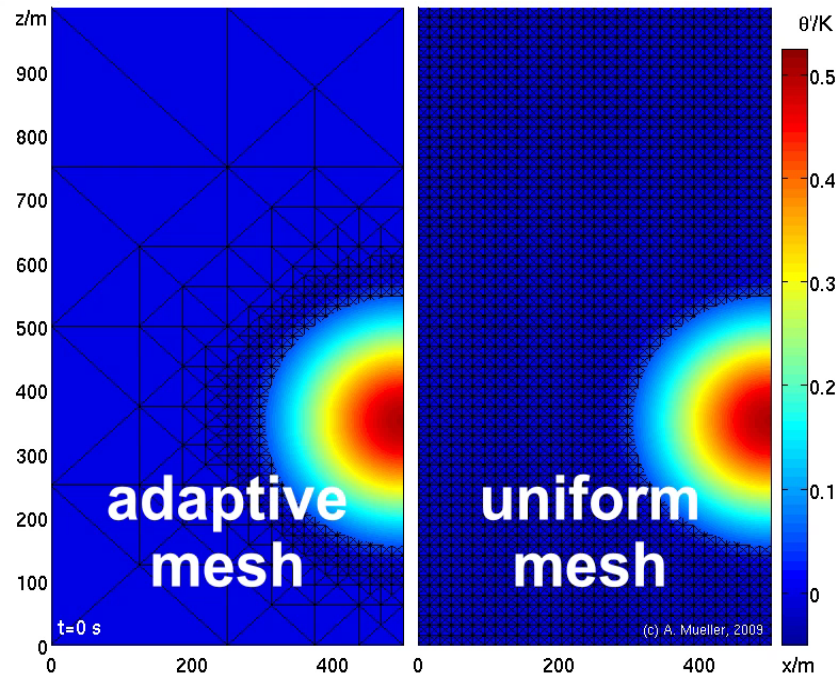
1. **Conservation** – Conservation of Mass and Energy are absolute musts; what else should we conserve?
2. **Scalability** – New models must be highly scalable because we will continue to get more processors
3. **High-Order Accuracy** – Accuracy is important, of course, but how do we measure this and what order accuracy is sufficient? This question is coupled to the accuracy of the physics, data assimilation, etc. From the standpoint of scalability, high-order is good (hp methods = on-processor work is large but the communication footprint is small). This is also a good strategy for exploiting MPI/Open MP Hybrid.
4. **Adaptivity** – Adaptive methods have improved tremendously in the past decade and it may offer an opportunity to solve problems not feasible a decade ago but we need to identify these applications (e.g., hurricanes, storm-surge modeling).

# Some Standing Issues for Adaptive Methods

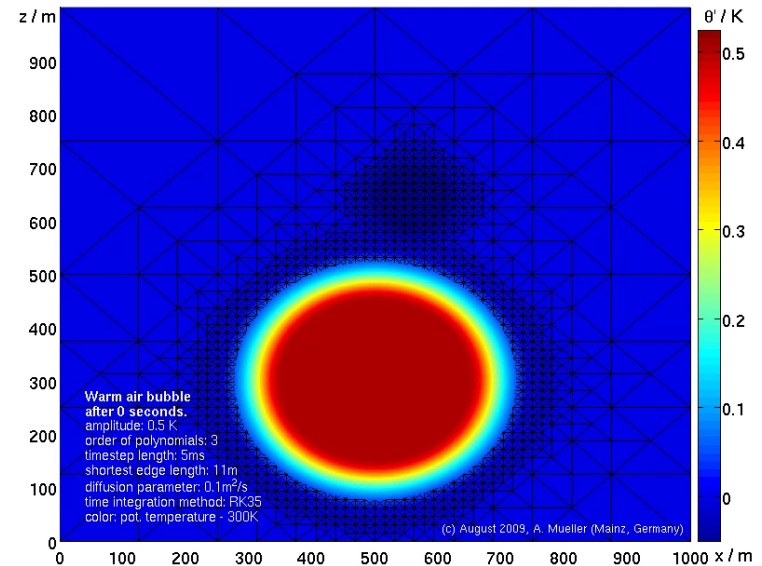
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- **Parallelization/Domain Decomposition:** Modifying the data structures dynamically slows the computations. E.g., the domain decomposition needs to be a direct by-product of the adaptive mesh generator. A good first candidate for AM is statically adaptive grids where the grid is modified and held fixed for the entire simulation. This must work well before moving onto dynamically adaptive grids.
- **Coupling of Dynamics and Physics:** Sub-grid scale parameterization is notoriously inconsistent meaning that changing the grid resolution changes the results. Also, the dynamics must use the “proper” approximations for the smallest scales. This means that both Atmospheric and Ocean codes should use the nonhydrostatic equations. This has direct effects on the time-integration strategies as faster waves (e.g., vertically propagating “acoustic” waves) must be considered in the choice of time-step.

# Non-hydrostatic Adaptivity Examples (Müller, Behrens, Giraldo, Wirth 2010)



**Rising Thermal Bubbles**



**Two (Warm/Cold) Thermal Bubbles**

# Talk Summary

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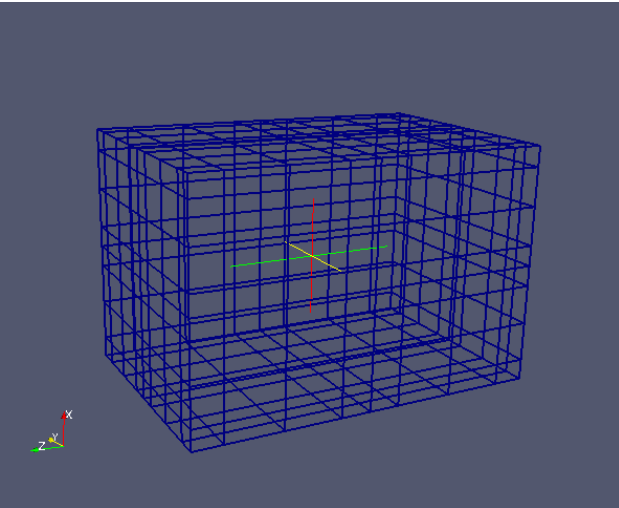
1. How do computer architectures affect the models?
2. Numerical Methods in New Dynamical Cores
3. How does resolution affect the model equations?
4. What Should we aim for in our New Models
5. Where We Plan to Head with NUMA
  - History of NUMA
  - Tested in Limited-Area Mode (Mesoscale applications)
  - Currently testing in Global Mode
  - Unstructured Grid capability should facilitate coupling to Unstructured Ocean (and Coastal Ocean) Models

# History of NUMA Model

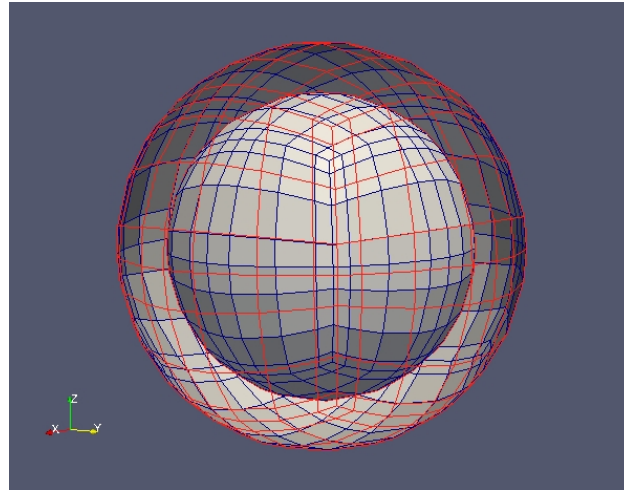
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- Early 2000s: Navy's Spectral Element Atmospheric Model (NSEAM)
  - Similar to SEAM/HOMME (see Mark Taylor's talk)
  - Hydrostatic Equations
  - Based on Spectral Elements
  - Based on cubed-sphere grid
  - Tested with NOGAPS physical parameterization
- Late 2000s: Prototype 2D (x-z slice) Mesoscale Atmospheric Model
  - Nonhydrostatic Equations
  - Based on Spectral Elements and Discontinuous Galerkin methods
  - Tested with simple moist physics (Kessler Physics)
- 2010: 3D Mesoscale Atmospheric Model (NUMA)
  - Nonhydrostatic Equations
  - Based on Spectral Element and Discontinuous Galerkin methods
  - Tested as Limited-Area model (NRBCs)
  - Currently being tested on the Sphere as a Global Model
  - Moist Physics will be included in the near future

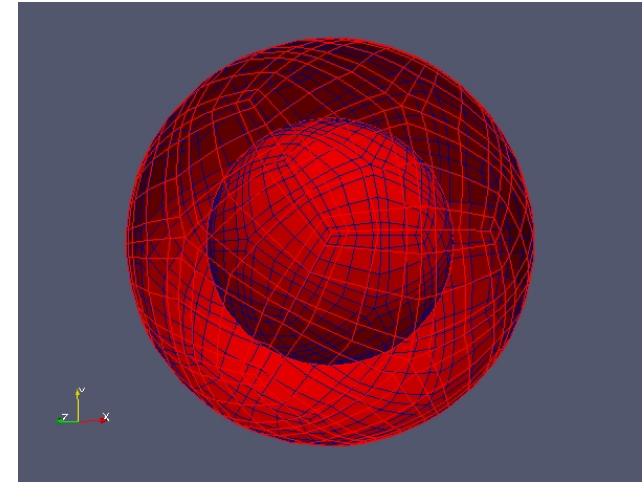
# Example of 3D Grids



**Mesoscale Modeling Mode**



**Global Modeling Mode  
(Cubed-Sphere)**



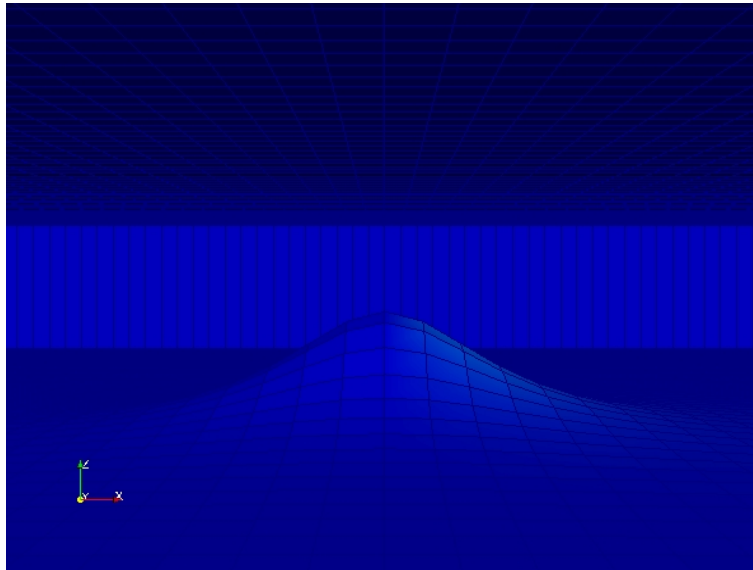
**Global Modeling Mode  
(Icosahedral)**

- NUMA runs in either Limited-Area or Global Mode.
- Currently, any grid can be used including completely unstructured grids.
- Parallel Domain Decomposition handled by METIS.

# Limited-Area Mode:

## Linear Hydrostatic Isolated Mountain

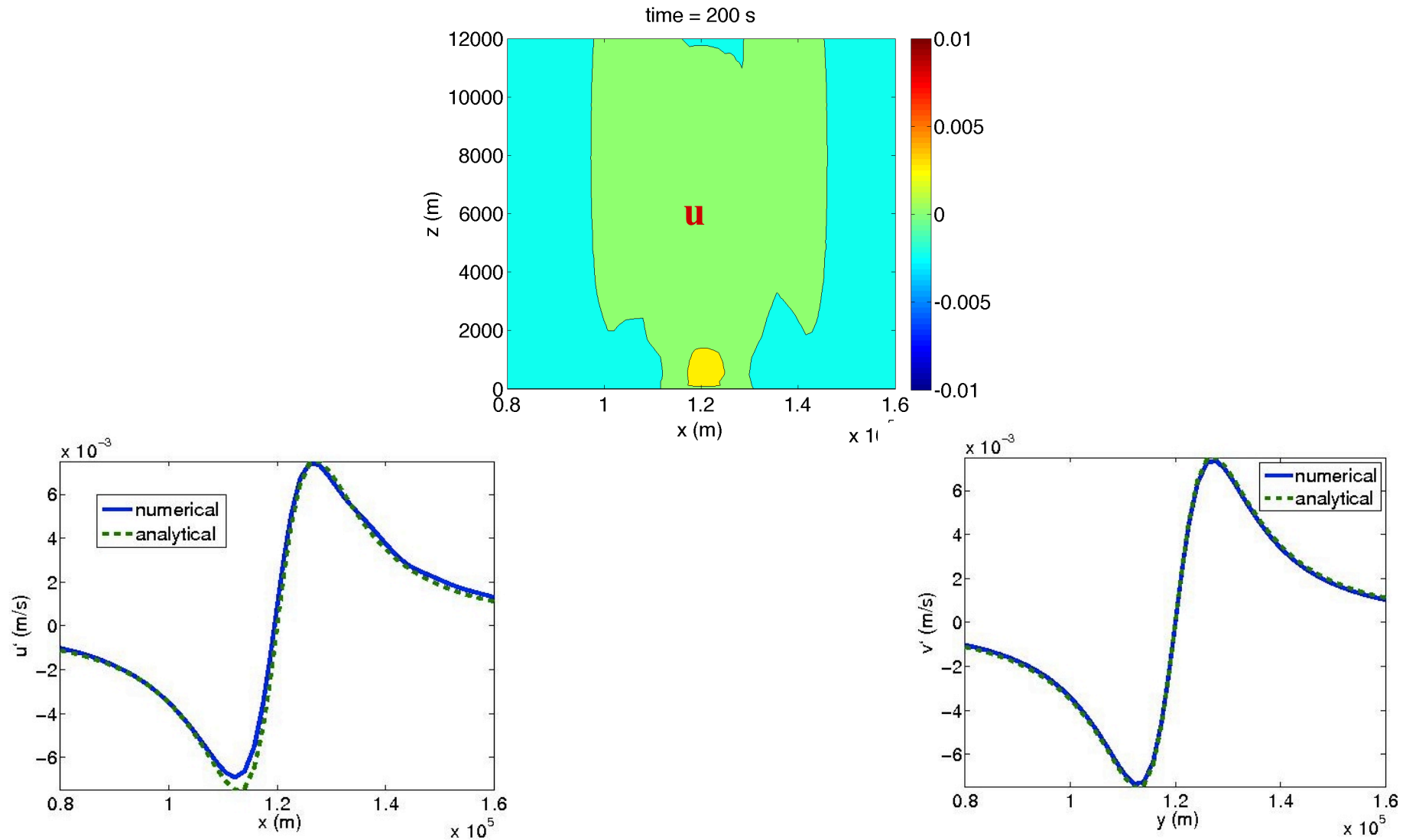
- Flow of  $U=20$  m/s in an isothermal atmosphere.
- LH Mountain: Solid of revolution of Witch of Agnesi: Mountain height = 1 m with radius 10 km.
- Absorbing (sponge) boundary condition implemented on lateral (4 sides) and top boundaries.



LH Isolated Mountain

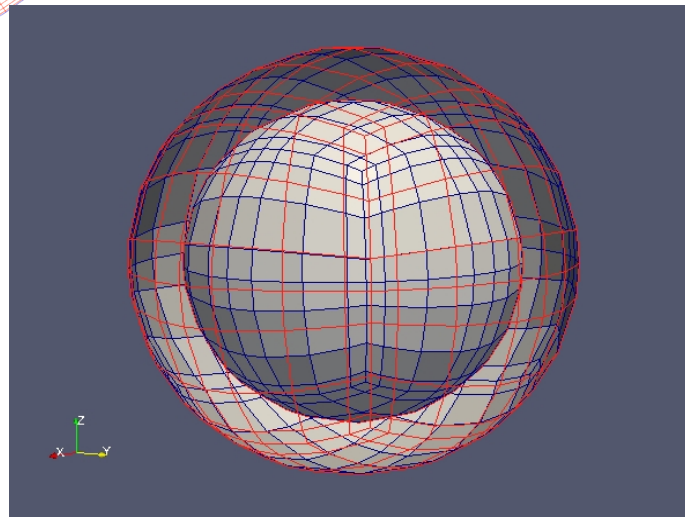
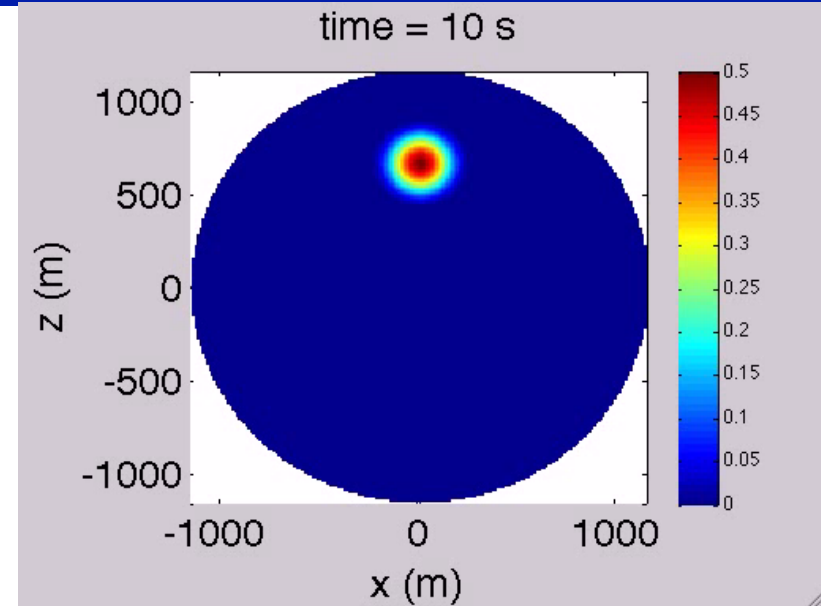
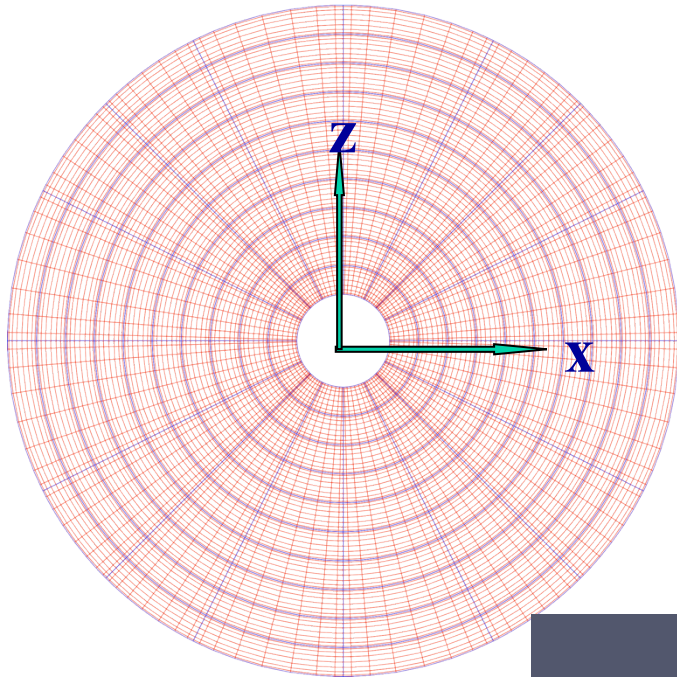
# Limited-Area Mode:

## Linear Hydrostatic Isolated Mountain





# Global Mode: Rising Thermal Bubble



# Summary

- The Spectral Element method allows for the construction of accurate, efficient, and flexible atmospheric (and possibly ocean) models.
- The scalability of this method has been shown previously in numerous other applications (see Gordon Bell Prizes awarded in CFD, Seismic Wave propagation, etc.).
- The geometric flexibility of SE facilitates the use of unstructured/adaptive grids (e.g., for tracking hurricanes)
- SE models have been shown to work well for the Hydrostatic Equations including Physical Parameterizations (Kim et al. JGR 2009, for MJO studies, Baer et al. MWR 2006, Mark Taylor's presentation)
- Although high-order, can be shown to be fully conserving and even positive-definite (recent work by Mark Taylor and Aime' Fournier JCP 2010).
- SE models have been shown to work well for the Non-hydrostatic Equations (Giraldo-Restelli JCP 2008, Restelli-Giraldo SISC 2009, Giraldo et al. SISC 2010)
- SE models have been shown to work well with simple physical parameterizations (Gabersek et al. MWR 2010)
- Collaborations between DOE (Sandia via Mark) and Navy (NPS/NRL) have already occurred but should strengthen these and extended to include various partners (e.g., numerics, physical parameterization, data assimilation, new grids, new applications)