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MAINTAINABILITY DATA DECISION METHODOLOGY (MDDM)

June 2011

U.S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
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13. ABSTRACT (Maximum 200 words)

Organizations within the U.S. Army [i.e. Communications-Electronics Command (CECOM)] and other government organizations have the need to evaluate Maintenance Manpower requirements for systems (i.e. Power Generators, etc.) where fully developed maintenance data is NOT available. Maintenance Manpower requirements are computed by multiplying an estimated maintenance ratio (man-hours per usage) by a one year wartime usage, which results in a total number of recommended maintenance man-hours. Army organizations need to know how much maintenance ratio data needs to be collected until the sample can be used to generate maintenance manpower requirements.

AMSAA developed a maintainability data decisioning methodology (MDDM) which determines if enough sample maintenance data exists in order to infer the true fleet maintenance ratio (MR). This will allow the Army to make manpower requirement determinations. MDDM uses a systems aging model, parametric & nonparametric empirical bayes models, two stochastic inferencing and four coverage validation models using nonparametric & parametric bootstrapping, percentile method with bias correction & acceleration using jackknifing, Monte Carlo simulation, and other stochastic modeling techniques & processes.

AMSAA has applied MDDM to CECOM Power Generators. Accuracy and precision of the inference is what determines if enough data exists. A coverage validation model is used to measure accuracy and precision is measured by the size of the inference. MDDM is a decisioning process and can be applied to other Army weapon systems where the need exists.

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The author is:

John Nierwinski, Logistics Analysis Division.

LIST OF ACRONYMS

MDDM	- Maintainability Data Decisioning Methodology
CECOM	- Communications-Electronics Command
AWU	- Annual wartime usage
DPAMMH	- Direct production annual maintenance man-hours
MR	- Maintenance Ratio
AMMDB	- Army Maintenance Manpower Data Base
AMSAA	- Army Materiel Systems Analysis Activity
PEB	- Parametric empirical bayes
ML	- Maximum Likelihood
EB	- Empirical bayes
BCa	- Bias Corrected and Accelerated method

LIST OF SYMBOLS

$\hat{MR}$	Maintenance Ratio Estimate
$man-hours_j$	Man-hours per generator
$ophour_j$	Operating hours per generator
λ	Scale parameter in power law model
β_{PL}	Shape parameter in power law model
λ_j	True, unknown visit rate for a given generator
α	Shape parameter of a Gamma distribution
β	Scale parameter of a Gamma distribution
α_s	Significance level
α_2	Adjustment to maintain α_s significance
$\hat{z}_0$	The value of Bias Correction
$\bar{\Lambda}_w$	Random variable for weighted mean of visit rate
t_j	Operating hours per generator
M_w^2	Random variable for the weighted second moment of the visit rate
s_j or v_j	Number of visits per generator
$\Phi(\ )$	Standard normal cumulative distribution function
$\Phi^{-1}(\ )$	Inverse of $\Phi(\ )$
$\hat{a}$	Acceleration constant
$\hat{MR}^{(\alpha_s)}$	100* α_s th percentile from the B estimates of MR
$\hat{MR}^{(\alpha_2)}$	100* α_2 th percentile from the B estimates of MR
$z^{(1-\alpha_s)}$	100*(1- α_s)th percentile point of a standard normal distribution
$\hat{MR}_b$	Maintenance Ratio estimate for the bth bootstrap replicate
$\hat{MR}_{(i)}$	$\hat{MR}$ with the ith generator held out.
$\hat{MR}_{(*)}$	weighted average of $\{MR_{(1)}, MR_{(2)}, ..., MR_{(n)}\}$
B	Number of Bootstrap replications
O(1/n)	Second order accuracy
$O(\frac{1}{\sqrt{n}})$	First order accuracy

MAINTAINABILITY DATA DECISION METHODOLOGY (MDDM)

1. INTRODUCTION

Organizations within the U.S. Army [i.e. Communications-Electronics Command (CECOM)] and other government organizations have the need to evaluate Maintenance Manpower requirements for systems (i.e. Power Generators, etc.) where fully developed maintenance data is NOT available. Maintenance Manpower requirements are computed by multiplying an estimated maintenance ratio (man-hours per usage) by an annual wartime usage (AWU), which results in a total number of recommended maintenance man-hours – this is called direct production annual maintenance man-hours (DPAMMH). A maintenance ratio (MR) estimate ($\hat{MR} = \frac{\sum_{j=1}^{n} man-hours_j}{\sum_{j=1}^{n} ophours_j}$) is based on a random sample (without replacement) of n generators from a finite population, where the pair (man-hours and ophours) are associated with each generator. Army organizations need to know how much maintenance ratio data needs to be collected until the sample can be used to generate maintenance manpower requirements. The DPAMMH will reside in the Army Maintenance Manpower Data Base (AMMDB).

Army Materiel Systems Analysis Activity (AMSAA) developed a maintainability data decisioning methodology (MDDM) which determines if enough sample maintenance data exists in order to infer the true fleet MR. This will allow the Army to make manpower requirement determinations. The true fleet MR is unknown but is defined to be the sum of the man-hours divided by the sum of the operating hours for all of the Army generators in the fleet at the end of useful life. Here is what the sample of maintenance data looks like:

Table 1. Sample Maintenance Data.

serial #	man-hours	operating hours	visits
1	mh(1)	h(1)	v(1)
2	Mh(2)	h(2)	V(2)
.	.	.	.
.	.	.	.
.	.	.	.
n	Mh(n)	H(n)	V(n)

Along with the man-hours at each visit.

It is impossible to collect maintenance data on all generators in the fleet over their useful lives. Even the Army could collect on all of the generators; a manpower decision is needed way before these generators reach the end of useful life.

The Army's main goal is to make sure enough sample data exists to infer the fleet MR – ultimately they want to be sure enough manpower can support the mission, without wasting too much manpower. To assure enough manpower exists, MDDM determines, through a coverage validation model, that enough data exists to make an accurate inference. Additionally, to assure that not too much manpower exists, MDDM determines if enough data exists to make a precise inference, measured by the size of the inference.

This paper discusses and develops the MDDM methodology which aids and facilitates the manpower decisioning process. To illustrate the process, a sample of notional generators is used.

MDDM uses a systems aging model, generator-to-generator variation models, two stochastic inferencing and four coverage validation models using nonparametric & parametric bootstrapping, percentile method with bias correction & acceleration using jackknifing, Monte Carlo simulation, and other stochastic modeling techniques & processes.

2. DEVELOPMENT OF METHODOLOGY & APPLICATION

<u>Sample Data and Assumptions</u>

Let's take a random sample (without replacement) of n generators (man-hours [mh] and operating hours [ophours] for each vehicle) from a fleet of generators. Here is a notional sample of 14 generators:

Table 2. Generator Sample.

	mh	ophours	visits
	18.1	1114	7
	23.7	396	5
	9	268	3
	10.3	212	3
	9.7	789	3
	6.1	392	3
	6.9	112	2
	4.8	328	2
	9.6	560	1
	3	220	1
	3.3	220	1
	3.1	327	1
	2.3	357	1
	8.8	104	1
Total	118.7	5399	34
$\hat{MR} =$	0.0220		

A visit is defined to be when a generator was taken out of operation to be repaired for whatever reason. MDDM assumes ophours between visits for each unit are assumed to be exponentially distributed. Hence "no aging" for each generator. To test this assumption, we analyze each generator with at least 15 visits by using a systems aging model [1]. This aging model is based on the power law, which states that the expected number of visits by time t, i.e. N (t), is governed by the power law relationship:

$$N(t) = \lambda t^{\beta_{PL}}$$

Where λ , β_{PL} , t>0; λ is the scale parameter, and β_{PL} is the shape parameter.

If this "no aging per generator" assumption was not true then it would be impossible to plan for the correct manpower because the manpower requirement would either be constantly rising or falling.

The average number of man-hours per visit is assumed to experience no generator-to-generator variation, and is modeled with the best fitting distribution. For this sample, man-hours per visit are best fit by using a truncated lognormal distribution, based on a comparison of Chi-Square tests results (the smaller the Chi-Square statistic the better the fit).

Generator to Generator variation Models

The variation in true visit rate from generator to generator is determined based on the parent population of the true visit rates. If this parent is known to be a Gamma prior distribution then we will utilize the parametric empirical bayes (PEB) approach [2] to model the true visit rate (λ_j) variation from generator to generator.

These λ_j 's are assumed to be statistically independent realizations of a Gamma distribution with parameters α and β with mean α / β. Two methods are used to estimate these parameters using the PEB approach: Method of Moments and the Marginal Maximum Likelihood (ML). Refer to the Appendix of [2] for more detail on these methods.

The weighted moment estimators for α and β become [2]:

$$\hat{\alpha} = \frac{(\bar{\Lambda}_w^{\ 2})\sum_{j=1}^{N} t_j}{\sum_{j=1}^{N} t_j (M_w^2 - \bar{\Lambda}_w^{\ 2}) - N\bar{\Lambda}_w} \tag{1}$$

and

$$\hat{\beta} = \hat{\alpha} / \bar{\Lambda}_w . \tag{2}$$

The likelihood equations are [2]:

$$\alpha = \frac{\beta \sum_{j=1}^{N} [s_j / (t_j + \beta)]}{N - \beta \sum_{j=1}^{N} [1/(t_j + \beta)]} \tag{3}$$

and

$$N\ln(\beta)+\sum_{j=1}^{N}\ \sum_{i=0}^{s_j-1}\frac{1}{\alpha+i}-\sum_{j=1}^{N}\ln(t_j+\beta)=0 \qquad (4)$$

These equations (3) and (4) must be iteratively solved to obtain the ML estimates $\hat{\alpha}$ and $\hat{\beta}$. The estimates for α and β from the Weighted Method of Moments (1) and (2) may be used as starting values for the iteration routine to solve the likelihood equations.

If the parent distribution for the true visit rates in unknown then we will utilize the nonparametric technique called empirical bayes (EB) approach [3] to model the true visit rate (λ_j) variation from generator to generator.

The λ_j 's are generated by nonparametrically bootstrapping [4] a set of n λ's, where the λ's are the Lemon Krutchkoff (1969) iterated estimators repeated n times [3]. Nonparametric bootstrapping is defined to be sampling with replacement from the original set of n λ's. For example, if {$\lambda_1,\lambda_2,...,\lambda_n$} is the original set of n λ's, then the bootstrap set of n λ's, {$\lambda_2,\lambda_2,...,\lambda_n$} is one possible bootstrap sample. There are a total of n^n total possible bootstrap samples.

Based on our notional example of 14 generators, we see that the true visit rates are best modeled with the EB approach since the Poisson data does not fit the PEB assumptions of a Gamma prior distribution on the true visit rates.

Inferencing Models and Precision

Now, we want to use the sample of 14 generators and all of the information that was gathered (i.e. mh distribution and generator to generator variation) to infer the fleet MR. We want to be 90% (i.e. the decision maker assesses the confidence level) certain that we have an adequate MR estimate. Adequate implies estimate is at or above the fleet MR. To achieve this goal we need to construct a 1-sided 90% upper confidence bound (UB) for the fleet MR. This will assure us that we have enough manpower.

In order to create this 1-sided UB stochastic model, we build a sampling distribution of $\hat{MR}$'s using the original sample information and Monte Carlo simulation [5]. The first step is to obtain 14 λ's from the Gamma distribution, if the Poisson data fit the PEB. Note, if the data did not fit the PEB, then we use the EB and we would bootstrap the Lemon Krutchkoff λ's to obtain a bootstrap sample of 14. Next, we perform a Poisson process on each generator using λ_j and $ophour_j$ to obtain the # of visits, v_j – visits are a realization of a Poisson random variable with mean = $\lambda_j * ophour_j$. Finally we draw v_j man-hours for each generator from the fitted truncated lognormal. Now we obtain $\hat{MR}_1$. Repeat this entire step B (say 500+) times to obtain B estimates (bootstrap replications) of MR.

Now we apply the Bias Corrected and Accelerated (BCa) method [4] to this distribution of 500 estimates of MR.. The BCa method is basically an adjustment (for non-normal data) to the percentile points of the Percentile Method. The "BC" adjusts when the mean and median are not equal and the "accelerated (a)" variance stabilizes the distribution – hence both together try to normalize the distribution.

Let $\hat{MR}^{(\alpha_s)}$ indicates the 100* α_s th percentile from the 500 estimates of MR. This represents the percentile method. The upper bound using the BCa method is given by:

$$\hat{MR}^{(\alpha_2)}, \text{ where } \alpha_2 = \Phi(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha_s)}}{1-\hat{a}(\hat{z}_0 + z^{(1-\alpha_s)})}) \quad (5)$$

Here $\Phi(*)$ is the standard normal cumulative distribution function and $z^{(1-\alpha_s)}$ is the 100* $(1-\alpha_s)$ th percentile point of a standard normal distribution. For example $z^{(.95)}$ = 1.645 and $\Phi(1.645) = .95$.

The value of BC is derived by the proportion of bootstrap replications that is less than the original estimate $\hat{MR}$. Here is that value:

$$\hat{z}_0 = \Phi^{-1}(\frac{\#\{MR_b < MR\}}{B}) \quad (6)$$

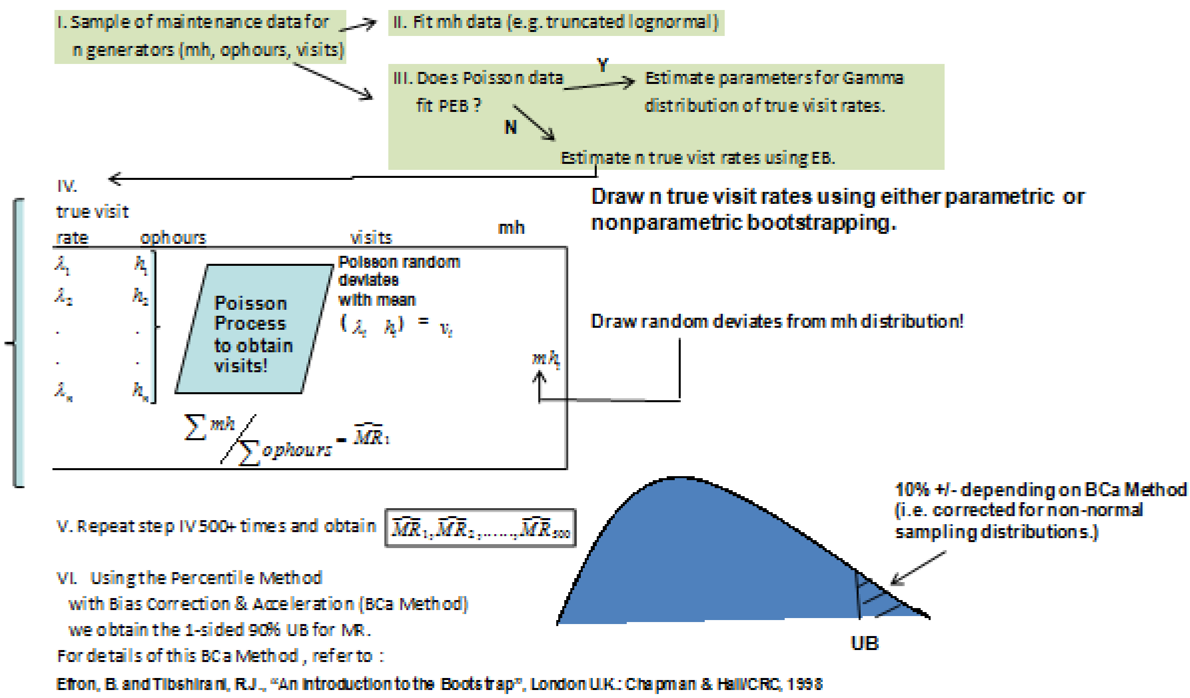
The acceleration constant is computed using jackknifing. Here it is:

$$a = \frac{\sum_{i=1}^{n}(MR_{(*)} - MR_{(i)})^3}{6\{\sum_{i=1}^{n}(MR_{(*)} - MR_{(i)})^2\}^{3/2}} \quad (7)$$

Where $MR_{(i)}$ is computed using the original sample with the ith generator deleted – this is jackknifing. $MR_{(*)}$ is the weighted average of $\{MR_{(1)}, MR_{(2)}, ..., MR_{(n)}\}$

Below is a flowchart of the entire process for the stochastic 1-sided UB inferenceng model.

Table 3. Inferencing Model Flowchart.



So using our sample of 14 generators and this inferenceing model, here are the stochastically created 1-sided 90% UB's for MR and the two metrics that are used to derive MR :

Table 4. Infercing Model Results.

Metric	Mean	UB	Precision
MR	0.0246	0.0319	0.30
Visit Rate	0.0071	0.0086	0.21
Man-hours	3.48	4.17	0.20

Our 2nd goal is to make sure the UB isn't too far to the right of the mean. Let's define precision to be the percent difference from the mean estimate to the UB (some applications define precision to be the absolute difference). Since it is difficult to access the precision for a multi-variable metric like MR, we examine the precision for the two single metrics that derive MR, since we have experience assessing precision for these. The decision maker determines what amount of precision they can tolerate. Based on past experience, 20% precision was recommended for the single metric applications.

As you can see from the inferencing results, precision for the visit rate is 21%, which is above 20%. Therefore, we do not have enough data to make a precise inference using a 20% precision decision requirement.

For this example we keep collecting data. However, if enough data was present to conclude an acceptable precision then we would need to make sure the 90% 1-sided CI is accurate. To validate this we proceed to the next section.

<u>Validation Coverage Models and Accuracy</u>

In order to accurately build this 1-sided UB stochastic model, we need to assure that the sample has "enough (# of generators and ophours)" data to achieve the 90% level. To validate this accuracy we use coverage models.

First let's define what we mean by coverage and accuracy. Coverage is defined to be the percentage of UB's that are greater than the true population MR, where each UB is constructed with some method at the 90% confidence level for a given random sample of n generators from the population. In other words, we need to run the inference method discussed in the prior section 500+ times (500+ samples drawn from a parametric or nonparametric population) to obtain 500+ inferences (i.e. 500+ UB's). These 500+ samples are not to be confused with the B (500+) iterations from the method. Note, the 500+ simulated populations are built based on the sample information. Then we determine how many UB's are greater than the true MR.

Accuracy is just a convergence rule for explaining the relative error of a 1-sided coverage. The rule focuses on the speed at which the relative error approaches 0. Second order accuracy is defined as the actual non-coverage probability intended to be α_s% for a 1-sided $(1-\alpha_s)$% CI, approaches the ideal of α_s% with error proportional to 1/n [6]. First order accuracy would approach the ideal of α_s% with error proportional to $\frac{1}{\sqrt{n}}$. This means that the relative error of the 1-sided coverage is of the order O(1/n) for second order accuracy and O($\frac{1}{\sqrt{n}}$) for first order accuracy. BCa is 2nd order accurate since it adjusts the percentile points based on the non-normal data. The percentile method is only 1st order accurate since it does not make any adjustments to the percentile points.

Using our sample of 14 generators and a validation coverage model, we obtain an acceptable coverage of 87%, when we ask for 90%. The absolute relative error was within our 5% requirement. Therefore, for this sample it is appropriate to use the inferencing model. Note, that we run this validation coverage model on a sample by sample basis. This makes sense since we want to make sure our particular sample (# of generators and ophours) will produce accurate results.

3. CONCLUSIONS

AMSAA has successfully applied and illustrated the MDDM process to a notional set of generators. It can be shown that increased generator to generator variation implies more sample data is needed to assure accuracy and precision. It can also be shown that decreased number of visits implies more sample data is needed to assure accuracy and precision.

MDDM is a decisioning process that determines if enough sample data is present to make an accurate and precise inference on the MR. MDDM is NOT a sampling problem or sequential problem with a target in mind.

Finally, if accuracy and precision are satisfied, then the aging risk needs to be considered before any the final decision is made. MDDM can be applied to other Army weapon systems where manpower decisions are required.

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