

Rate-Dependent Homogenization based Continuum Plasticity
Damage Model for Dendritic Cast Aluminum Alloys

A Thesis

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ABSTRACT

The objective of this dissertation is to develop a computationally efficient rate-dependent homogenization based continuum plasticity damage model for macroscopic analysis of ductile failure in porous ductile materials containing brittle inclusions. The macroscopic model developed in this dissertation is an extension of the model proposed by Ghosh et al. [S. Ghosh, J. Bai, and D. Paquet. Homogenization-based continuum plasticity damage model for ductile failure of materials containing heterogeneities. *J Mech Phys Solids*, 57:10171044, 2009]. The overall framework of this rate-dependent HCPD model follows the structure of the anisotropic Gursen-Tvergaard-Needleman(GTN) type elasto-plasticity model for porous ductile materials. This model is assumed to be orthotropic in an evolving material principal coordinate system throughout deformation history. The viscoplastic behavior is modeled through an over-stress viscoplastic model. Anisotropy and viscoplastic parameters in the rate-dependent HCPD model are calibrated from homogenization of evolving micro-variables in representative volume element (RVE) of the microstructure. These parameters are dependent on microstructural features such as morphology and distribution of different phases. Micromechanical analyses for this purpose are performed by locally enhanced Voronoi cell finite element model (LE-VCFEM) [Hu, C., Ghosh, S., 2008. Locally enhanced Voronoi cell finite element model for simulating evolving fracture in ductile microstructures containing inclusions. *Int. J. Numer.*

Methods Eng. 76(12),1955-1992]. This work also introduces a novel rate-dependent void nucleation criterion for the macroscopic damage evolution due to the combined inclusion and matrix cracking happening in the underlying microstructure of the RVE. The results of the rate-dependent HCPD model are compared with the homogenized micromechanics (LE-VCFEM) results and show excellent agreement.

Dedicated to my family and friends.

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CHAPTER 1

INTRODUCTION

Metals and alloys containing heterogeneities e.g. particulates, precipitates, intermetallics, or voids in the microstructure are widely used in automotive, aerospace among other engineering systems. Figure 1.1 shows the micrograph of a Al-Si-Mg hypoeutectic cast aluminum alloy (AS7GU) used in the automotive industry. The microstructure comprises of (i) age-hardened aluminum matrix, strengthened by Mg/Si and Si precipitates, and (ii) a dispersion of brittle silicon particulates in the matrix. These heterogeneities can have adverse effects on failure properties like fracture toughness and ductility, and hence the need for including the morphology and distribution of these inclusions in the numerical model. The spatial distribution of the silicon inclusions and their morphology depends on the casting procedure used, and especially on the rate of heat extraction [1]. The solidification process tends to push particulates into the regions between the evolving secondary dendrite arms. The silicon inclusions are pinned in location once the eutectic temperature is reached. The secondary dendrite arm spacing (SDAS) of the microstructure depends on this solidification procedure. Cast alloys with small SDAS can be modeled readily, while alloys with large SDAS need to be modeled using a unique two-stage homogenization technique detailed in [2]. Ductile failure happens through micromechanical mechanisms

like particle fragmentation, interface debonding and matrix failure. Failure initiates by particle cracking, after which voids grow near the crack tips and subsequently coalesce to cause failure in the matrix. The nucleation and growth of the cracks are sensitive to local morphological parameters and constitutive parameters as shown in [3]. Damage evolution in microstructures are also sensitive to the applied strain rate as discussed in [32]. It is computationally expensive, if not impossible to simulate the entire microstructure with explicit representation of the heterogeneities, which has led to the development of the macroscopic rate-dependent homogenization based continuum plasticity damaged model. This macroscopic model has been designed to incorporate the morphological characteristics of the underlying microstructure, while saving computational time by working in a homogenized macroscopic scale.

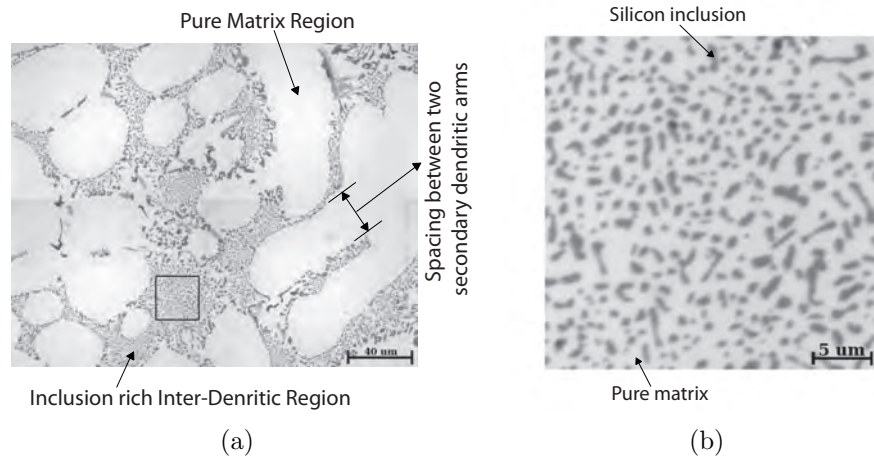


Figure 1.1: (a) Micrograph of a cast aluminum alloy AS7GU ($120 \mu\text{m} \times 96 \mu\text{m}$), (b) blow-up of the designated region in (a) ($28.4 \mu\text{m} \times 28.4 \mu\text{m}$).

1.1 Literature Survey

Various macroscopic constitutive models have been proposed to model heterogeneous material behavior and failure, based on morphological and phenomenological approaches. Most of the phenomenological models propose tensor or scalar damage variables whose evolution is governed by some evolution law, formulated from experimental observations. The general use of these phenomenological models is impeded by their lack of modeling the underlying physics and failure to explicitly account for the microstructure characteristics. On the other hand, models that incorporate morphological details solve boundary value problems of the representative volume element (RVE) to predict constitutive response at the macroscopic level. There are various analytical micromechanical models like those based on variational bounding methods e.g. [4, 5] and those based on effective medium approximations [6, 7]. These methods have limited capabilities for dealing with material nonlinearities, non-proportional load histories and complex morphologies. Multi-scale computational homogenization theories using asymptotic expansion [8, 9], where concurrent finite element analyses are executed at macro- and micro-scales with information transfer between them have been used for estimating averaged material properties of heterogeneous materials.

Ghosh and co-workers have combined the asymptotic homogenization method with the Voronoi cell finite element method (VCFEM), operating at the micro-scale for multi-scale analysis of deformation and damage in non-uniformly distributed microstructures in [10, 11, 12]. However, these approaches are computationally very expensive because of the detailed micromechanical analysis that needs to be performed at every integration point of macroscopic elements. To overcome the shortcomings of

simultaneous macro-micro modeling, [13] have developed the *homogenization based continuum plasticity-damage* or *HCPD* model for ductile materials containing heterogeneities that are undergoing ductile failure with evolving porosity. The HCPD model follows the Gurson-Tvergaard-Needleman or GTN models framework established in [14, 15, 16, 17] that account for void nucleation, growth and coalescence. GTN models where matrix anisotropy has been characterized by Hill’s quadratic yield criterion have been proposed in [18, 19]. Studies in [20] have shown that the yield surface of porous materials with rigid inclusions retain the form of the GTN yield surface. This dissertation deals with the extension of the constitutive model proposed in [13] into a rate-dependent framework. A specific focus of this dissertation is the proposal of a new and novel damage evolution law, which is also rate-dependent. This model can be used effectively in the macroscopic simulation of ductile fracture in a multi-scale framework, owing to the extremely reduced computational time while retaining the morphological characteristics of the underlying microstructure.

1.2 Thesis Overview

The thesis is organized as follows. Chapter 2 introduces the computational tools required for the development of the rate-dependent HCPD model. Major parts of the rate-dependent HCPD model are described in Chapter 3. Chapter 4 describes the procedure for calibration of the model parameters. Numerical validation and examples are provided in Chapter 5 and Chapter 6 lists the scope for future work and conclusion.

CHAPTER 2

TOOLS NEEDED FOR DEVELOPING THE RATE-DEPENDENT HCPD MODEL

This chapter introduces the computational tools utilized in developing the rate-dependent homogenization based continuum plasticity damage model. Each of the sub-section explains briefly about a tool and references are provided for further understanding.

2.1 Identification of the statistically equivalent RVE size

For the determination of the effective material properties of a microstructural domain, e.g. the inter-dendritic region of Figure 1.1(b), identifying the microstructural statistically equivalent RVE or SERVE is very important. A SERVE may be defined as the smallest volume element of the microstructure exhibiting the following characteristics [21, 22]:

- Effective constitutive material properties in the SERVE should be equivalent to properties of the entire microstructure, at least locally to within a prescribed tolerance.

- Distribution functions of parameters reflecting the local morphology, (local volume fraction, neighbor distance or radial distributions) in the SERVE should be similar to those for the overall microstructure within a prescribed tolerance.
- SERVE should be independent of the location in the local microstructure as well as of the applied loading direction.

Swaminathan et al. in [21, 22] have developed different techniques, like convergence of elastic stiffness tensor, two point correlation for anisotropy etc. for determination of SERVE size both without and with microstructural damage. The marked correlation function, first introduced by Pyrz [23] was used in [13, 2] to establish the SERVE size for porous ductile materials with a dispersion of heterogeneities. The marked correlation function $M(r)$ is a multivariate characteristic function of the microstructure that relates any state variable (eg. stresses, strains or their dependent functions) with the microstructural morphology and distribution. It is expressed as a ratio of the state variable dependent function $h(r)$ and the pair distribution function $g(r)$ as:

$$M(r) = \frac{h(r)}{g(r)} \quad (2.1)$$

where

$$h(r) = \frac{1}{2\pi r} \frac{dH(r)}{dr} \quad \text{and} \quad H(r) = \frac{1}{m^2} \frac{A}{N^2} \sum_{i=1}^N \sum_{k=1}^{k_i} m_i m_k(r) \quad (2.2)$$

Here m_i represents a mark associated with the i -th inclusion, r is a measure of the radial distance of influence, $m_k(r)$ corresponds to the mark associated with the k -th inclusion at a radial distance r and m is the mean of all marks. N is the total number of inclusions within the area A of the microstructural domain being analyzed and k_i is the number of inclusions that have their center within a circle of radius r centered at the i -th inclusion. The mark can be any chosen variable field in the microstructure

that has relevance the problem being pursued. The definition of the pair distribution function $g(r)$ in Equation (2.1) is given as

$$g(r) = \frac{1}{2\pi r} \frac{dK(r)}{dr} \quad \text{and} \quad K(r) = \frac{A}{N^2} \sum_{k=1}^N I_k(r) \quad (2.3)$$

The pair distribution function $g(r)$ characterizes the occurrence intensity of inter-inclusion distances. $I_k(r)$ is the number of inclusions excluding the one being considered that lie inside the circle of radius r . The definition of the second-order intensity function $K(r)$ is given in [21]. A high value of $M(r)$ indicates a strong correlation between entities in the microstructure. As discussed in [2, 13], $M(r)$ provides a good estimate of the size of the SERVE, for a local microstructure. It stabilizes to near-unity values at a characteristic radius of convergence r_0 . For $r \geq r_0$, $M(r) \approx 1$ and the local morphology ceases to have any significant influence on the state variables beyond this characteristic radial distance. The radius of convergence r_0 provides a good estimate for the SERVE size.

As discussed in [2, 21], a combination of three geometric parameters are used to calculate the SERVE size of the microstructure. The mark m_i associated with each inclusion is defined as a weighted average of local area fraction (LAF), inverse near neighbor distance (IND) and the number of near neighbors (NN) of that inclusion:

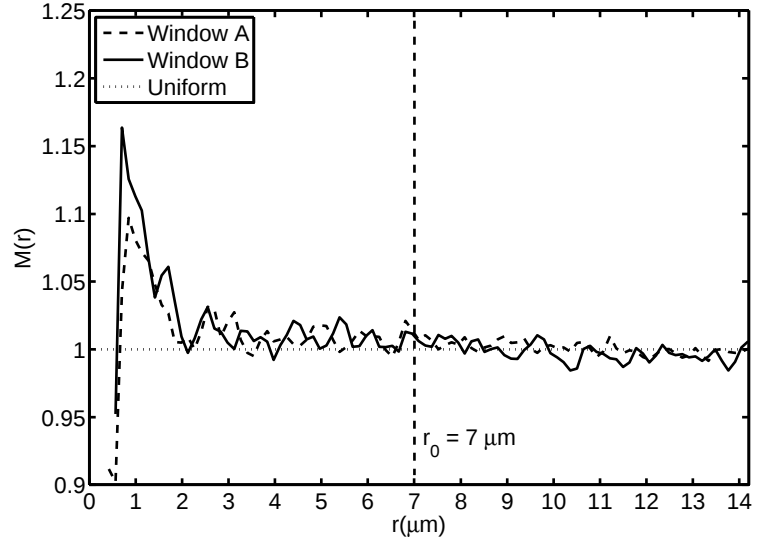
$$m_k = w_1 S_1^k + w_2 S_2^k + w_3 S_3^k \quad (2.4)$$

with

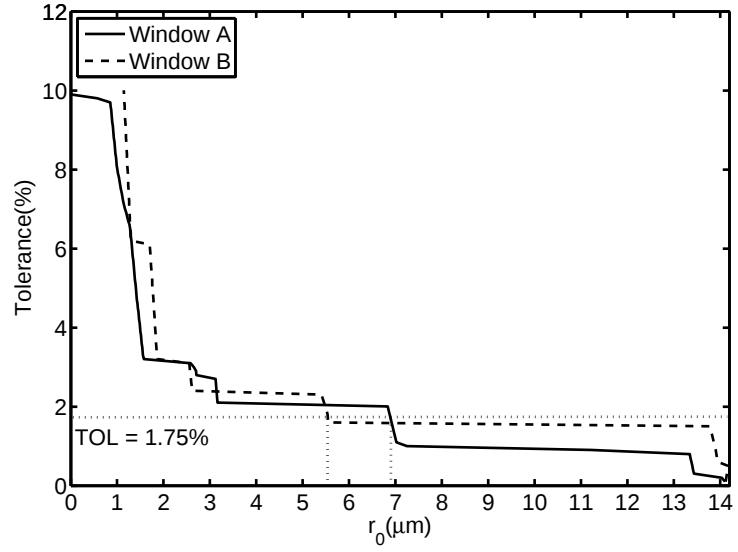
$$S_1^k = \frac{(LAF)^k}{\max_{1 \leq j \leq N} (LAF)^j} \quad ; \quad S_2^k = \frac{(IND)^k}{\max_{1 \leq j \leq N} (IND)^j} \quad ; \quad S_3^k = \frac{(NN)^k}{\max_{1 \leq j \leq N} (NN)^j}$$

where N is the total number of inclusions and $(LAF)^k$, $(IND)^k$, and $(NN)^k$ are the local area fraction, the inverse of the near-neighbor distance, and the number of near-neighbors of the k -th inclusion. Further details about calculating these quantities are given in [2]. From the micrograph shown in Figure 1.1(b), the marked correlation function in Equation (2.1) is calculated with the geometric mark of Equation (2.4). Weights $w_1 = 1.0$, $w_2 = 2.0$ and $w_3 = 1.0$ are used, as suggested in [21]. $M(r)$ distributions are plotted in Figure 2.1(a), showing the convergence to $M(r) = 1$ with varying r . In order to choose the tightest possible tolerance, the size of the SERVE corresponding to different acceptable tolerances for convergence are evaluated in Figure 2.1(b). This analysis suggests $r_0 \approx 7 \mu\text{m}$ based on the highest value for the two regions using a tolerance of 1.75%. Correspondingly, the SERVE size is estimated as $L_{SERVE} = 2 \times r_0 = 14 \mu\text{m}$.

In order to verify the location independence of the $14 \mu\text{m}$ SERVE, two different SERVE's are chosen from the microstructure and their respective homogenized responses are compared in rate-independent shear loading. The SERVE boundaries are created by periodically repeating the position of inclusions in the x and y directions, followed by drichlet tessellation [24, 25, 26]. The resulting SERVE's with the Voronoi cell mesh are shown in Figure 2.2. Their respective area fraction of inclusions are 18.31% and 18.45%, which are very close to the overall area fraction of the microstructural region of Figure 1.1(b) (18.62%). The elastic properties considered for micromechanical simulations are $E_{Si} = 165 \text{ GPa}$ and $\nu_{Si} = 0.27$ for the inclusions and $E_{Al} = 70 \text{ GPa}$ and $\nu_{Al} = 0.32$ for the matrix. The plastic hardening curve of the matrix is shown in Figure 2.3 and the initial yield stress is $\sigma_y = 88.5 \text{ MPa}$. The



(a)



(b)

Figure 2.1: (a) Marked correlation function for two different windows A and B using the mark of Equation (2.4), (b) radius of convergence r_0 based on the results of (a) as a function of the chosen tolerance.

averaged macroscopic stress-strain response in shear for the two SERVE's of Figure 2.2 is shown in Figure 2.4. The results for the two SERVE's match very well with each other. This justifies the choice of $L_{SERVE} = 14 \mu\text{m}$ and also proves the location independence.

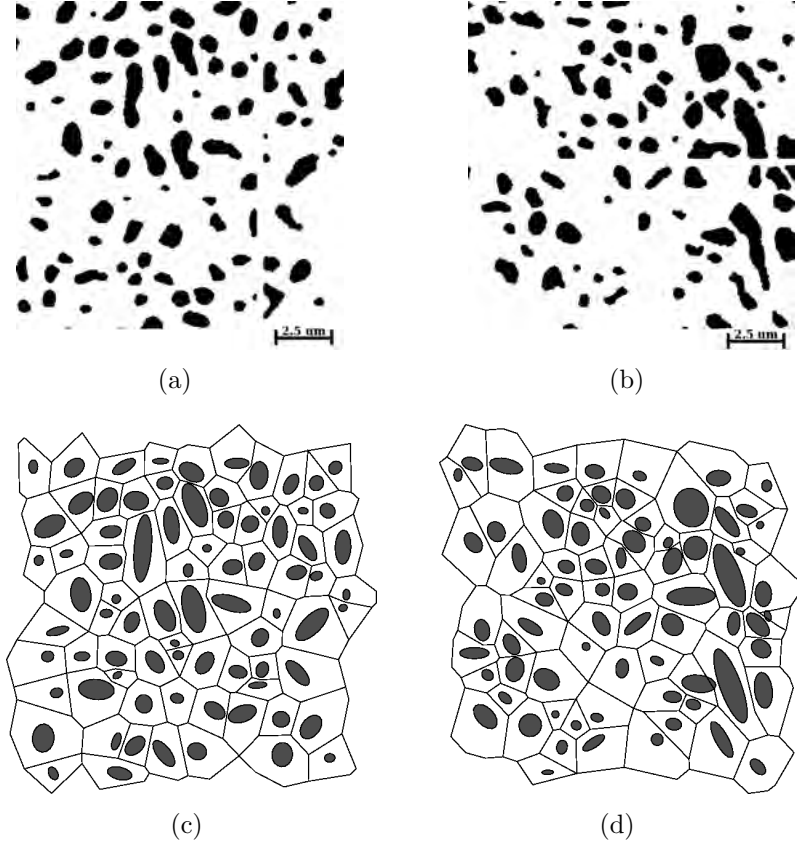


Figure 2.2: (a) Micrograph of the microstructural domain of SERVE A, (b) micrograph of the microstructural domain of SERVE B, (c) SERVE A with periodic boundary, (d) SERVE B with periodic boundary.

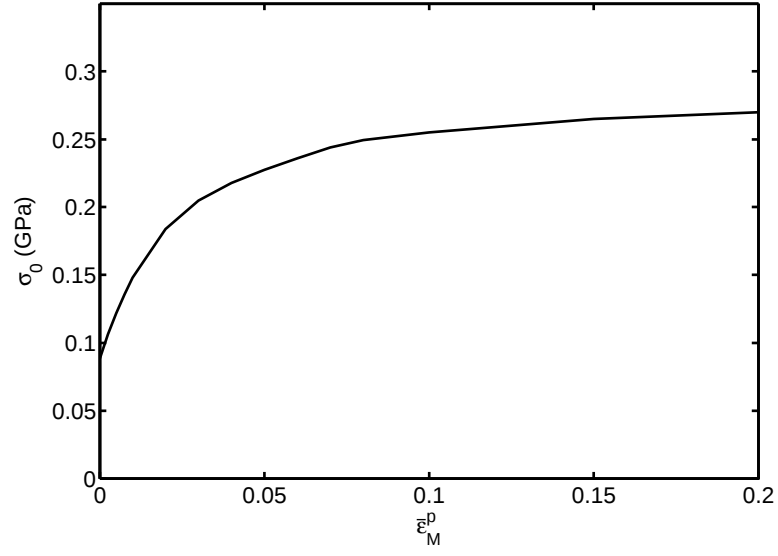


Figure 2.3: Hardening (stress-strain) curve for the aluminum matrix used for comparison of macroscopic behavior in shear for SERVE A and SERVE B of Figure 2.2.

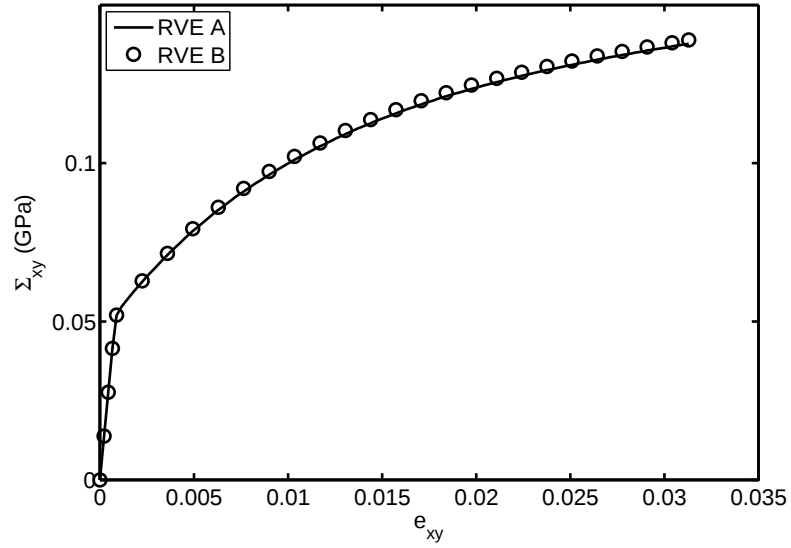


Figure 2.4: Comparison of the averaged macroscopic stress-strain response in shear of SERVE A and SERVE B of Figure 2.2.

2.2 Homogenization of micromechanical variables in RVE analysis

Development of the rate-dependent HCPD model requires evaluation of material properties from homogenized stresses, strains and other state variables. The asymptotic expansion homogenization method has been developed in [8, 9] for homogenized constitutive models of heterogeneous materials. Ghosh et al. have introduced a novel implementation of the asymptotic homogenization method, coupling macroscopic analysis with micromechanical VCFEM analysis of the RVE, in [11, 12, 27, 28]. The asymptotic homogenization method is implemented in conjunction with micromechanical elastic-plastic damage analysis of the RVE Y by LE-VCFEM in [13], which is used in the present work. The incremental form of the homogenized macroscopic stress-strain relation $\Delta \Sigma_{ij} = E_{ijkl}^H \Delta \bar{e}_{kl}$ is obtained by volume averaging stresses and strains over the RVE. For this, the LE-VCFEM generated microstructural variables are integrated as:

$$\begin{aligned}\Sigma_{ij} &= \frac{1}{Y} \int_Y \sigma_{ij}^\epsilon(y) dY \\ \bar{e}_{ij} &= \frac{1}{Y} \int_Y e_{ij}^\epsilon(y) dY\end{aligned}\tag{2.5}$$

where σ_{ij}^ϵ and e_{ij}^ϵ are the microscopic stress and strain tensor in the RVE. E_{ijkl}^H is the homogenized elastic-plastic tangent modulus in the macroscopic constitutive law. Additionally, the rates of macroscopic void volume fraction (f) and plastic work (W_p) are defined as

$$\dot{f} = \frac{1}{Y} \int_Y \dot{f}^\epsilon dY, \quad \dot{W}_p = \frac{1}{Y} \int_Y \sigma_{ij}^\epsilon \dot{e}_{ij}^{\epsilon p} dY\tag{2.6}$$

where $\mathbf{e}^{\epsilon p}$ is the microscopic plastic strain tensor in the RVE. For plane strain problems, components of E_{ijkl}^H can be obtained by averaging stress increments in response

to three separate imposed unit macroscopic strains increments $\Delta \bar{e}_{ij}$ and solving the boundary value problems for the RVE. Details of this process can be found in [11, 12, 27, 28].

2.3 Locally enhanced voronoi cell finite element model (LE-VCFEM)

Solutions of the micromechanical boundary value problem in the SERVE are necessary to provide data for the asymptotic expansion homogenization described in Chapter 2.2. The Voronoi locally enhanced cell finite element method or LE-VCFEM, developed by Ghosh et al. [29, 30, 31, 32] readily provides an accurate and extremely efficient means for micromechanical analysis of deformation and failure in arbitrary heterogeneous microstructures. Morphological non-uniformities in the heterogenities, like orientations, shapes, and sizes of inclusions are conveniently modeled by this method. In [32, 33] the VCFEM model has been extended for rate-dependent elastic-viscoplastic porous ductile material. Micromechanical analysis in the present paper uses the VCFEM model in [32, 33]. The inclusions are linear elastic, while the matrix constitutive relations follow the over-stress viscoplastic model model of Perzyna [34].

CHAPTER 3

RATE-DEPENDENT HOMOGENIZATION BASED CONTINUUM PLASTICITY DAMAGE MODEL

The rate-dependent homogenization based continuum plasticity damage model framework is based on the anisotropic Gurson-Tvergaard-Needleman model following the developements of [18, 19], with the rate-dependency modeled through an over-stress model as described in [32, 2]. The tools described in Chapter 2, namely asymptotic expansion homogenization (AEH), LE-VCFEM are used in the development of this model.

In this constitutive model, the total strain rate is assumed to be additively decomposed into an elastic and viscoplastic part as:

$$\dot{\mathbf{e}} = \dot{\mathbf{e}}^e + \dot{\mathbf{e}}^p \quad (3.1)$$

For small elastic strains, the rate of Cauchy stress $\dot{\boldsymbol{\Sigma}}$ is related to the elastic part of the strain rate tensor as:

$$\dot{\boldsymbol{\Sigma}} = \mathbf{C}^e : \dot{\mathbf{e}}^e \quad (3.2)$$

where $\mathbf{C}^e$ is a fourth order anisotropic elasticity tensor. The anisotropic yield function for the porous ductile materials containing inclusions is expressed in terms of the

deviatoric and hydrostatic components of stress tensor as:

$$\phi = \frac{\Sigma_{eq}^2}{Y_f^2(W_p)} + 2Q_1 f \cosh \left(\frac{3Q_2}{2} \frac{\Sigma^{hyd}}{Y_f(W_p)} \right) - 1 - (Q_1 f)^2 \geq 0 \quad (3.3)$$

where Σ_{eq} and Σ^{hyd} are the homogenized equivalent and hydrostatic stress respectively and f is the macroscopic void volume fraction. The homogenized equivalent stress is defined in the Equation (3.11) and the hydrostatic stress is $\Sigma^{hyd} = \frac{\Sigma_1 + \Sigma_2 + \Sigma_3}{3}$. Y_f is the homogenized yield stress of the heterogenous material without any voids, expressed as a function of the inelastic work W_p . The effect of strain work-hardening is incorporated by this yield strength $Y_f(W_p)$, whose functional form is determined from the homogenization process. The parameters Q_1 and Q_2 govern the void evolution in the macroscopic model and are calibrated from the homogenization of the response of the RVE. In case of rate-independence, the Equation (3.3) becomes $\phi = 0$. The rate-dependency is modeled with the over-stress viscoplastic model developed in [34]. In this model, the viscoplastic strain rate and the rates of plastic work and void volume fraction are expressed as:

$$\dot{\mathbf{e}}^p = \dot{\Lambda} \mathbf{N}, \mathbf{N} = \frac{\partial \phi^*}{\partial \Sigma} \quad (3.4)$$

where ϕ^* is given by Equation 3.9.

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation} \quad (3.5)$$

$$\dot{W}_p = \Sigma : \dot{\mathbf{e}}^p \quad (3.6)$$

The homogenized void growth law is the same as the one used in [13] and is given by $\dot{f}_{growth} = (1 - f)\dot{e}_{kk}^p$, which results from plastic incompressibility of the underlying matrix without voids. The void nucleation law, $\dot{f}_{nucleation}$ is one of the highlights of this dissertation and is explained in detail in Chapter 3.2. In Equations (3.4) , (3.5)

and (3.6) , $\dot{\Lambda}$ is the macroscopic viscoplastic multiplier. $\dot{\Lambda}$ is obtained by assuming that the Hill-Mandel micro-macro energy condition [35] governs the homogenization conditions for the porous ductile materials. The rate of dissipative energy in the porous matrix material with incusions can be expressed as:

$$\Sigma : \dot{\epsilon}^p = (1 - f)\sigma^*\dot{\epsilon}^p = \dot{\lambda}(1 - f)\sigma : \epsilon^p \quad (3.7)$$

where σ^* is the effective stress and $\dot{\epsilon}^p$ is the effective plastic strain rate in the matrix with incusions in the absence of voids. σ and ϵ^p are the stress and strain tensors in the matrix with incusions in the absence of voids. The viscoplastic multiplier in the matrix with incusions in the absence of voids is $\dot{\lambda} = \Gamma_0\Phi(F)$. Here, Γ_0 is the temperature dependent viscosity co-effecient. In [34] a power law expression, i.e. $\Phi(F) = \langle F \rangle^P$ has been discussed to adequately represent the behavior of most metals. $\langle \cdot \rangle$ is the MacCauley operator corresponding to the positive sign of the argument. The over-stress F is a measure of the excess stress over the rate-independent local yield strength Y_f (an internal state variable), i.e.

$$F = \Sigma^* - Y_f(W_p) \quad (3.8)$$

where $\Sigma^* = \sigma^*$ and can be obtained by setting

$$\phi^* = \frac{\Sigma_{eq}^2}{(\Sigma^*)^2} + 2Q_1f \cosh\left(\frac{3Q_2}{2} \frac{\Sigma^{hyd}}{\Sigma^*}\right) - 1 - (Q_1f)^2 = 0 \quad (3.9)$$

From the constitutive model discussed in [2] for non-porous matrix with incusions, the effective plastic strain rate in matrix with incusions in the absence of voids is derived as $\dot{\epsilon}^p = \dot{\lambda}$. From Equation (3.7) and (3.4) we can obtain the expression for the macroscopic viscoplastic multiplier, $\dot{\Lambda}$ as:

$$\dot{\Lambda} = \frac{(1 - f)\Sigma^*}{\Sigma : \mathbf{N}} \Gamma_0\Phi(F) \quad (3.10)$$

The set of Equations (3.1)-(3.6), (3.8)-(3.10) form the constitutive model for the rate-dependent macroscopic simulation of porous ductile material with inclusions, i.e. the rate-dependent HCPD model. The entire constitutive model described above is expressed in an evolving principal material co-ordinate system or PDCS, which takes care of the evolving anisotropy. This unique feature of the model is explained below.

3.1 Evolving Anisotropy

As discussed in [13, 2], initial macroscopic material anisotropy is due to the presence of heterogeneities, e.g. brittle inclusions, in the microstructure. Furthermore, this anisotropy evolves with deformation due to nonuniform and constrained plastic flow in micro-channels between heterogeneities. The equivalent stress Σ_{eq} in the expressions (3.3) and (3.9) is designed to accommodate both initial and evolving anisotropy. For plane strain problems, it is expressed using the 3D anisotropic yield function in Hill [36] as:

$$\Sigma_{eq}^2 = F(\Sigma_{22} - \Sigma_{33})^2 + G(\Sigma_{33} - \Sigma_{11})^2 + H(\Sigma_{11} - \Sigma_{22})^2 + C\Sigma_{12}^2 \quad (3.11)$$

The stress components (and other tensor variables) in the constitutive model are represented in the principal axes of material anisotropy. The material is assumed to remain orthotropic in this system throughout the deformation process. The use of this material coordinate system has been shown in [13] to capture the effects of non-proportional load and deformation histories with very good accuracy. The angle β , delineating the principal axes of anisotropy for plane strain analysis, is determined in every increment from the condition that the transformed tangent modulus $(E_{ijkl}^{\tan})'$ in this system remains orthotropic. This condition renders the terms coupling normal

and shear components of the tangent modulus to be equal to zero, i.e.

$$(E_{1112}^{\text{tan}})' = (E_{2212}^{\text{tan}})' = (E_{3312}^{\text{tan}})' = 0, \quad (3.12)$$

where $(E_{ijkl}^{\text{tan}})' = Q_{im}Q_{jn}Q_{kp}Q_{lq}E_{mnpq}^{\text{tan}}$ and

$$[\mathbf{Q}] = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Anisotropy parameters F , G , H , and C in Equation (3.11) are calibrated with respect to this principal coordinate system from homogenization. These parameters evolve due to the constrained plastic flow resulting from the presence of heterogeneities.

3.2 Rate-Dependent Void Nucleation Criterion

Ductile failure initiates with inclusion fragmentation and is followed by void evolution in the matrix. This microstructural mechanism manifests in the macroscopic rate-dependent HCPD model as void nucleation. The void nucleation model being developed in this dissertation is based on the work done by Ghosh et al. [13], where a strain-based rate-independent nucleation law is proposed and is proved to offer excellent agreement with the micromechanics VCFEM results. As discussed in [3, 32], the morphology and distribution of inclusions is exceedingly significant in governing the damage propagation in a porous ductile material with heterogeneities. Damage evolution is highly anisotropic as shown in Figure 3.1 and the void nucleation model proposed in this dissertation allows for the representation of this anisotropy with ease. In addition to the morphological characteristics, the loading rate also plays a significant role in the damage evolution in the microstructure. Paquet et al. [32, 33] have conducted a sensitivity study of the ductile fracture with respect to the loading rates and have concluded that the applied strain rates play an important role

on the ductile fracture of rate-sensitive heterogenous materials. As the strain rate increases, the stress in the inclusions are much higher than for lower strain rates, which results in the incusions cracking at a much lower strain. In addition to this, the matrix phase undergoes lower plastic deformation at higher strain rates, therefore reducing void growth and localization of damage near cracked inclusions. Strain to failure is thus determined by the competition between the two effects. This effect of rate-dependency is also included in the proposed void nucleation model.

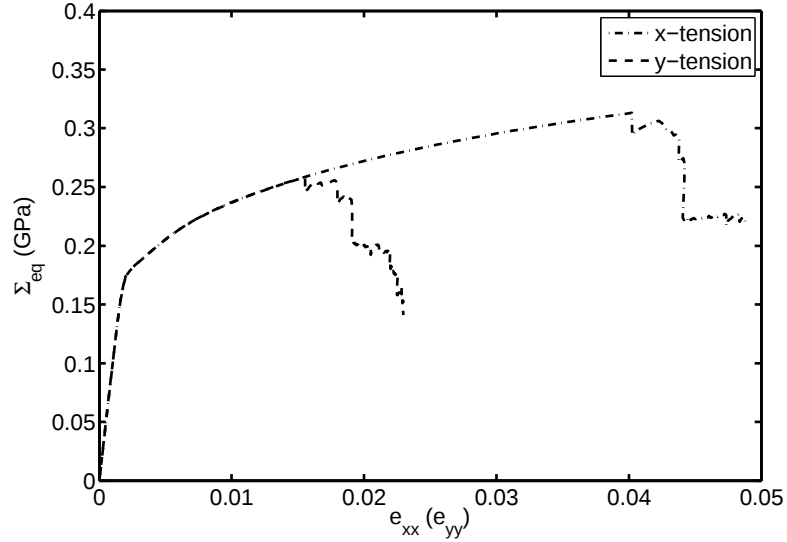


Figure 3.1: Anisotropy in damage evolution corresponding to mutually perpendicular loading of RVE 1 in Figure 4.3(a)

As in [13], the void nucleation model is based on the Weibull statistics-based probability of fracture $\bar{P}_{fr}$, which is parallel to the probability governing the cracking of inclusions in the microstructure as detailed in [13, 32]. The probability function

$\bar{P}_{fr}$ is written in terms of local strains as:

$$\bar{P}_{fr}(v, \hat{e}) = 1 - \exp \left[-\frac{v}{v_0} \left(\frac{\hat{e}}{e_0} \right)^m \right] \quad (3.13)$$

where e_0 and m are the Weibull parameters, which are functions of local strain rate $\dot{\hat{e}}$. $\hat{e}$ is an effective strain measure expressed as:

$$\hat{e} = \langle Ae_1 + Be_2 + Ce_3 \rangle \quad (3.14)$$

where e_1, e_2 and e_3 are the local principal strains and $\langle \ \rangle$ is the MacCauley operator corresponding to the positive sign of the argument. A, B and C are parameters that are calibrated from micromechanics VCFEM analysis of the RVE. The co-efficients A, B and C are functions of the angle of principal strain θ_p and provide an efficient method for modeling the damage anisotropy emerging from morphology and distribution of inclusions in the microstructure. A new state variable, area fraction of cracked particles ρ is introduced to acomodate instantaneous variations in local strain rates. The area fraction of cracked inclusions for constant strain rate deformation $\rho_c(\hat{e})$ is expressed in terms of the probability density function of the inclusion size $p(v)$ and the probability of fracture $\bar{P}_{fr}(v, \hat{e})$ as:

$$\rho_c(\hat{e}) = \int_0^\infty \frac{v}{v_0} p(v) \bar{P}_{fr}(v, \hat{e}) dv \quad (3.15)$$

If a discrete distribution of inclusions in a limited size RVE is to be considered, The probability density function $p(v)$ can be written as

$$p(v) = \sum_{i=1}^N \delta(v - v_i) p(v_i) \quad (3.16)$$

where $\delta()$ is the dirac delta function and N is the number of bins in the probability density function $p(v)$. Using Equation (3.16) in (3.15) gives:

$$\rho_c(\hat{e}) = \int_0^\infty \frac{v}{v_0} \left[\sum_{i=1}^N \delta(v - v_i) p(v_i) \right] \bar{P}_{fr}(v, \hat{e}) dv$$

$$\rho_c(\hat{e}) = \sum_{i=1}^N \int_0^\infty \frac{v}{v_0} [\delta(v - v_i) p(v_i)] \bar{P}_{fr}(v, \hat{e}) dv$$

and can be simplified to

$$\begin{aligned} \rho_c(\hat{e}) &= \sum_{i=1}^N \frac{v_i}{v_0} p(v_i) \bar{P}_{fr}(v, \hat{e}) \\ \rho_c(\hat{e}) &= \sum_{i=1}^N \frac{v_i}{v_0} p(v_i) \left(1 - \exp \left[-\frac{v_i}{v_0} \left(\frac{\hat{e}}{e_o} \right)^m \right] \right) \end{aligned} \quad (3.17)$$

where $p(v_i)$ is the probability of finding a inclusion of size v_i . If all the particles have the same size, i.e. $p(v) = 1$, the area fraction of cracked inclusion can be derived from Equation (3.17) as

$$\rho_c(\hat{e}) = 1 - \exp \left[-\left(\frac{\hat{e}}{e_o} \right)^m \right] \quad (3.18)$$

The evolution of the state variable, area fraction of cracked particles ρ is governed by the equation

$$\dot{\rho} = \frac{d\rho_c(\hat{e})}{d\hat{e}} \dot{\hat{e}} \tilde{k}^\star \quad (3.19)$$

Finally, the void nucleation for the rate-dependent HCPD model is given as

$$\dot{f}_{nucleation} = V_p \frac{d\rho_c(\hat{e})}{d\hat{e}} \dot{\hat{e}} \tilde{k}^\star \quad (3.20)$$

where

$$\tilde{k} = \frac{1 - \rho}{1 - \rho_c(\hat{e})}, \quad \tilde{k}^\star = \begin{cases} \tilde{k} & \text{if } \tilde{k} \geq 1 \\ 0 & \text{if } \tilde{k} < 1 \end{cases} \quad (3.21)$$

where $\tilde{k}^\star$ is a factor that takes care of the instantaneous change in strain rates. When there is an increase in local strain rate, the stress in the particles of the underlying microstructure increases and more particles start cracking. In order to incorporate this effect, the factor $\tilde{k}$ is included in the void nucleation law to increase the nucleation of voids and bring it up to the actual value, as shown in Figure 3.2. When the local strain rate decreases, all the particles that will crack eventually have already cracked

(because of the higher stress in higher strain rates) and therefore no nucleation is necessary till the appropriate value is reached, as shown in Figure 3.3. The value $\rho_c(\hat{e})$ in the denominator of Equation (3.21) is the area fraction of cracked particles at constant strain rate and is obtained from Equation (3.17) or (3.18). The parameter V_p in Equation (3.20) is calibrated from the volume fraction of cracked inclusions at given strain. The calibration procedure for the parameters e_0 , m , V_p , A , B and C is detailed in Section 4.5.

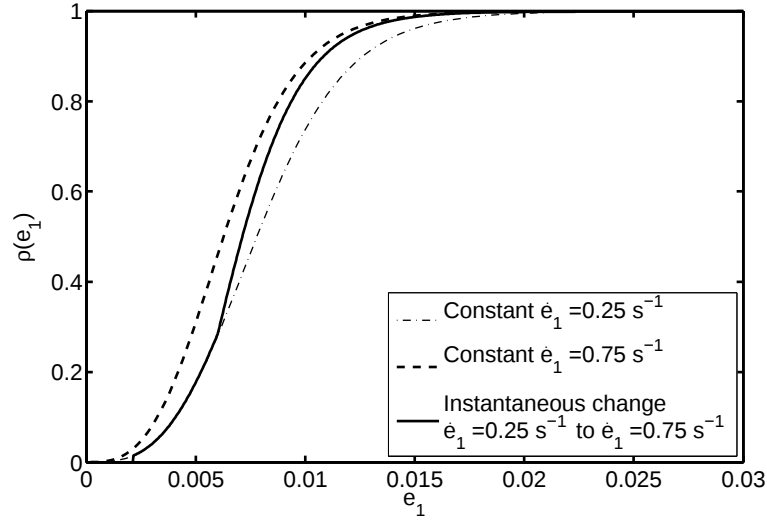


Figure 3.2: Area fraction of cracked inclusions ρ as a function of the principal strain e_1 of RVE 1 in Figure 4.3(a) for an instantaneous change in loading strain rate from 0.75 s^{-1} to 0.25 s^{-1}

3.3 Numerical implementation of the rate-dependent HCPD model

The numerical implementation of the macroscopic rate-dependent HCPD model follows the return mapping algorithm for rate-dependent plasticity developed in [37].

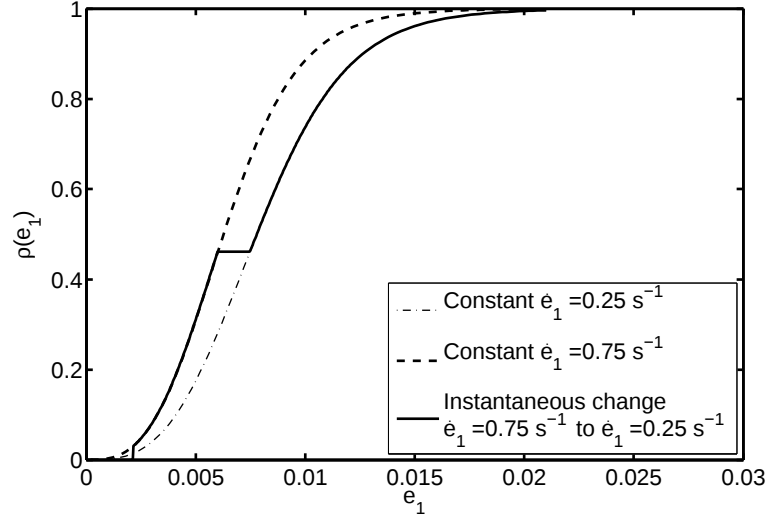


Figure 3.3: Area fraction of cracked inclusions ρ as a function of the principal strain e_1 of RVE 1 in Figure 4.3(a) for an instantaneous change in loading strain rate from 0.25 s^{-1} to 0.75 s^{-1}

The return mapping algorithm consists of an initial elastic predictor step, where the elastic response is assumed and the stresses are predicted. This is followed by the plastic corrector step which returns the stress to the updated yield surface. The numerical stress update algorithm is programmed in fortran and is used with *MSC Marc* through the *Uvscpl()* subroutine.

CHAPTER 4

CALIBRATION OF RATE-DEPENDENT HCPD MODEL PARAMETERS

The rate-dependent HCPD model parameters are calibrated from homogenized quantities such as macroscopic strain $\mathbf{e}$, plastic strain $\mathbf{e}^p$, stress Σ and plastic work W_p obtained from asymptotic expansion homogenization (AEH) of micromechanical variables from LE-VCFEM simulations. The parameters are calibrated in the material principal coordinate system following the procedure outlined for rate-independent materials in [13]. These are discussed next. The calibration procedure for the parameters of the void nucleation model is described in 4.5

4.1 $Y_f(W_p)$ and C in Equations (3.3, 3.11)

The flow stress in shear, $Y_f(W_p)$ describes the hardening behavior of the homogenized material. In order to calibrate the evolution of $Y_f(W_p)$, a rate-independent LE-VCFEM based micromechanical RVE analysis is conducted for shear deformation, with $e_{xx} = e_{yy} = 0$, $e_{xy} \neq 0$. The matrix material is assumed to be void free and no inclusion fragmentation is allowed. Micromechanical analysis is followed by homogenization, in which the macroscopic plastic work, stresses and strains are evaluated from the microstructural variables using equations (2.5) and (2.6). For pure

shear loading, the resulting macroscopic tensor is of the form $\Sigma_{xx} = \Sigma_{yy} = \Sigma_{zz} = 0$ and $\Sigma_{xy} \neq 0$. From the Equations (3.3) and (3.11), the yield stress is calculated as

$$Y_f(W_p) = \sqrt{3}\Sigma_{xy} \quad (4.1)$$

where the parameter C in equation (3.11) is set to 3. The yield stress Y_f is plotted as a function of W_p in figure 4.1.

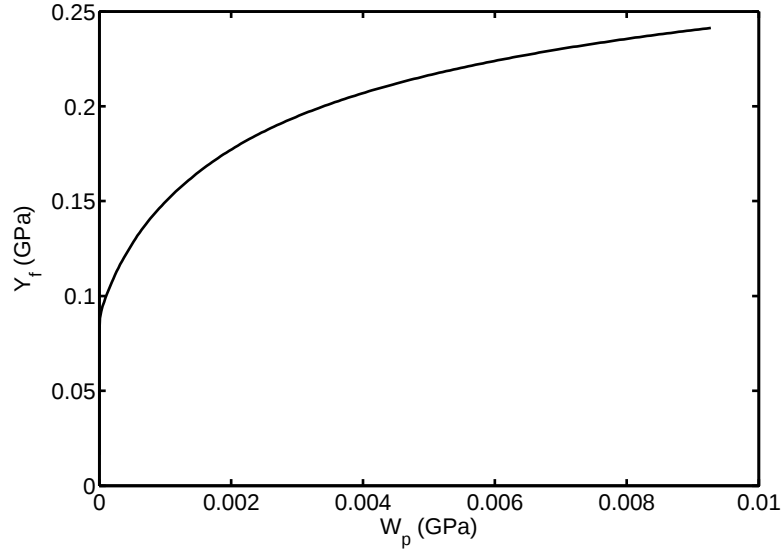


Figure 4.1: Yield stress in shear Y_f for the SERVE A shown in Figure 2.2(c)

4.2 Parameters F, G and H in equation (3.11)

For a given RVE, like the one shown in Figure 2.2(c), micromechanical LE-VCFEM simulations are performed for N different loading conditions, followed by homogenization. As with the calibration of Y_f , the simulations are rate-independent and no void evolution or inclusion cracking is allowed. Loading conditions are prescribed with different macroscopic strain ratios $e_{xx} : e_{yy} : e_{xy}$. A total of 18 different

loading conditions are used to calibrate the anisotropy parameters. At the end of each strain increment, the tangent stiffness E_{ijkl}^{tan} is evaluated and subsequently the material principal co-ordinate system is determined from Equation (3.12). Macroscopic stress tensor Σ and plastic work W_p are computed in the principal coordinate system using equations (2.5) for each load step. For a given value of plastic work W_p , the $Y_f(W_p)$ is obtained from the evolution curve such as Figure 4.1. This is done for all the load histories. The plastic work dependent anisotropy parameters $F(W_p)$, $G(W_p)$, $H(W_p)$ are then evaluated minimizing the least square residual of the rate-independent form of the Equation (3.11) as:

$$\min_{F,G,H} \sum_{i=1}^N [F(\Sigma_{22}^i - \Sigma_{33}^i)^2 + G(\Sigma_{33}^i - \Sigma_{11}^i)^2 + H(\Sigma_{11}^i - \Sigma_{22}^i)^2 + C(\Sigma_{12}^i)^2 - (Y_f^i)^2]^2 \quad (4.2)$$

For a given value of W_p , $N = 18$ points corresponding to the 18 different loadings are used. The parameters F , G , H are solved by an iterative algorithm with $C = 3$. The step is repeated for different values of W_p to obtain the evolution curves for F , G and H as a function of W_p . The evolution of the anisotropy parameters F , G and H for the RVE in Figure 2.2(c) is shown in Figure 4.2. While F and G reduce nonlinearly with W_p , H increases with W_p . More study on the evolution of the anisotropy parameters in different RVE's has been done in [13]. This study shows that relatively isotropic microstructures have $F \approx G$ and pronounced anisotropy results in $F \neq G$.

4.3 Viscoplastic parameters Γ_0 and p in Equation (3.10)

The homogenized viscoplastic properties in the rate-dependent HCPD model are calibrated from a set of loading conditions combining M applied strain rates for N different imposed strain ratios $e_{xx} : e_{yy} : e_{xy}$. These simulations are done subject to zero initial void in the matrix, no void evolution and no particle cracking. At each

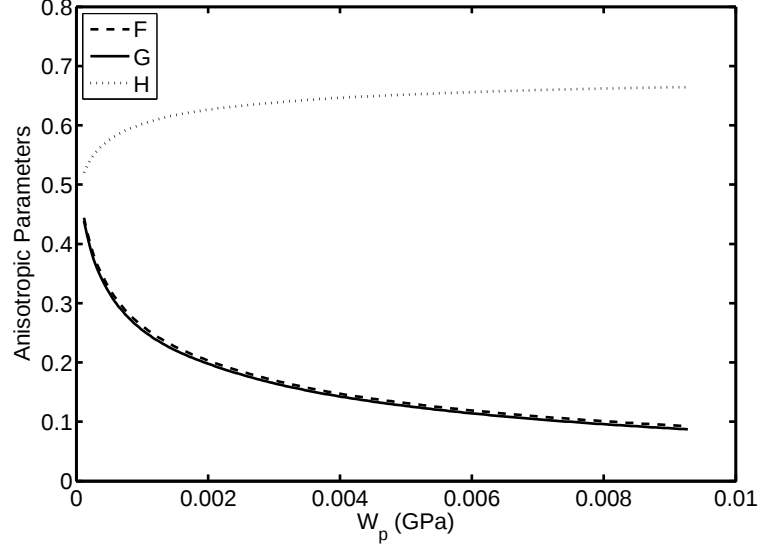


Figure 4.2: Evolution of anisotropy parameters F,G and H with plastic work for SERVE A shown in Figure 2.2(c)

increment, the principal coordinate system, the stress tensor Σ and the plastic work W_p are computed. The corresponding yield stress $Y_f(W_p)$ and anisotropy coefficients $F(W_p)$, $G(W_p)$, $H(W_p)$ are evaluated using the previously calibration results from Sections 4.1 and 4.2. The macroscopic viscoplastic properties are obtained by minimizing the square of the error between micromechanical analyses and macroscopic simulations using the rate-dependent HCPD model of Chapter 3, with the void nucleation and growth parts turned off. The error is defined as the Frobenius norm of the stress difference:

$$\min_{\gamma_0, P} \sum_{i=1}^N \sum_{j=1}^M \sum_{k=1}^{K_i} \|\Sigma_{\text{macro}}^{ijk} - \Sigma_{\text{micro}}^{ijk}\|_F^2 \quad (4.3)$$

where K_i is the number of applied increments in the simulations corresponding to the i -th load path. For the calibration of viscoplastic parameters, four different strain

rates in shear are applied, i.e. $N = 3$ and $M = 4$. The applied strain rates are $\dot{e}_{xy}^{(1)} = 0.01 \text{ s}^{-1}$, $\dot{e}_{xy}^{(2)} = 0.03 \text{ s}^{-1}$, $\dot{e}_{xy}^{(3)} = 0.06 \text{ s}^{-1}$ and $\dot{e}_{xy}^{(4)} = 0.10 \text{ s}^{-1}$.

4.4 Parameters Q_1 and Q_2 in Equation (3.3)

The parameters, Q_1 and Q_2 , which govern the void evolution in the macroscopic rate-dependent HCPD model are calibrated from a set of micromechanical simulations, now with void evolution enabled. Homogenization is performed on these micromechanical simulations for evaluating the coefficients Q_1 and Q_2 in Equation (3.3). The following steps are undertaken for this objective.

1. An rate-independent LE-VCFEM simulation of the RVE is conducted with an applied macroscopic shear strain, $e_{xx} = 0 : e_{yy} = 0 : e_{xy} \neq 0$ for plastic deformation and void evolution. The corresponding macroscopic stress tensor Σ and averaged void volume fraction f are evaluated and plotted as functions of the averaged matrix plastic work W_p . These averaged macroscopic quantities are obtained from homogenization as defined in Equation (2.6). Since the normal stresses $\Sigma_{xx} = \Sigma_{yy} = \Sigma_{zz} = 0$ for this loading, and also the hydrostatic part of plastic strain $e_{kk}^p = 0$, the void volume fraction does not change, i.e. $f = f_o$. The parameter Q_1 can be solved from the Equation (3.3) as:

$$Q_1 = \frac{1}{f_o} \left(1 - \frac{\sqrt{3}\Sigma_{xy}}{Y_f} \right) \quad (4.4)$$

2. The same set of numerical simulations as in Section (4.2) is again performed for the microstructural RVE with non-zero, evolving void volume fractions. For a given value of plastic work W_p , the parameter Q_2 is evaluated from known

values of F, G, H , Y_f and Q_1 by solving the minimization problem

$$\min_{Q_2} \sum_{i=1}^N \left[\frac{\Sigma_{eq}^2}{Y_f^2} + 2Q_1 f \cosh \left(\frac{3Q_2 \Sigma^{hyd}}{2 Y_f} \right) - 1 - (Q_1 f)^2 \right]^2 \quad (4.5)$$

where $N = 18$ is the total number of loadings.

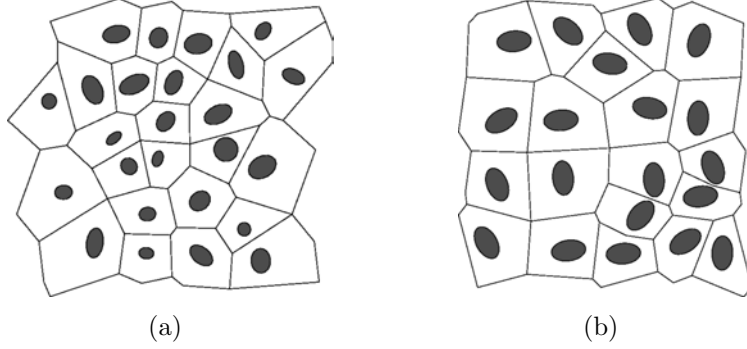


Figure 4.3: (a) RVE 1 with hardcore distribution, (b) RVE 2 with equal sized elliptical inclusions.

It has been shown by Ghosh et al. [13] that the calibrated values of Q_1 and Q_2 for the different RVEs exhibit only minimal dependence on plastic work and hence can be taken as constants. The mean and standard deviation of Q_1 and Q_2 for the two RVEs of Figures 4.3(a) and 4.3(b) are listed in Table 4.1. The standard deviations of Q_1 and Q_2 are very small compared to the mean values. and hence Q_1 and Q_2 for each RVE are taken as constants in the analyses, as concluded in [13]. The parameters Q_1 and Q_2 depend on the RVE they are calibrated from. The Q_1 values are quite different from the value in the pure matrix material ($q_1=1.5$). This difference is attributed to the effect of the inclusion volume fraction V_f . A sensitivity study has been conducted by Ghosh et al. [13] to relate the effect of the microstructure on the values of Q_1

and Q_2 . It has been identified that Q_1 and Q_2 have sole dependence on the volume fraction V_f of heterogeneities in the microstructure.

	Mean of Q_1	Standard Dev. of Q_1	Mean of Q_2	Standard Dev. of Q_2
RVE 1	1.70	0.0065	1.04	0.0179
RVE 2	1.77	0.0026	1.09	0.0064

Table 4.1: Mean and standard deviation of calibrated parameters Q_1 and Q_2 for different RVEs

4.5 Calibration of parameters in the void nucleation model

The calibration of parameters for the rate-dependent void nucleation criterion can be split into two distinct steps. The first step involves the calibration of the local strain-rate dependent parameters e_0 and m in the Equations (3.17) and (3.18). The second step involves the calibration of parameters A, B and C in Equation (3.14), which take care of the anisotropy in damage evolution due to the morphology and distribution of the inclusions. These parameters A, B and C are functions of the angle of principal strain θ_p . Rate-dependent LE-VCFEM simulations with void evolution and inclusion cracking along with the *rate-dependent HCPD* model analyses are conducted for calibrating all the required parameters.

4.5.1 e_0 and m calibration

The parameters e_0 and m in the Equations (3.17) and (3.18), defining the area fraction of cracked particles $\rho(e)$ are functions of local strain rate. These two parameters govern the rate-dependency of damage evolution. In order to calibrate e_0 and m , a total of K micromechanical LE-VCFEM simulations with strain rates spread

between the required range for the macroscopic HCPD model are performed. For the RVE 1 shown in Figure 4.3(a), a total of 10 LE-VPFEM simulations with strain rates between 0.01 s^{-1} and 0.75 s^{-1} were performed. All of the micromechanical simulations required for the calibration of e_0 and m are performed under an applied macroscopic strain ratio $e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$. For each of the K micromechanical simulations, e_0 and m are calculated, so that a best fit is obtained between the Equation (3.17) or (3.18) and micromechanics results for the area fraction of cracked particles. Figure 4.4 shows the micromechanics result for the area fraction of cracked particles at an applied strain rate of 0.1 s^{-1} as a function of the principal strain e_1 for the RVE 1 in Figure 4.3(a) along with the fitted curve of Equation (3.18) for the values $e_0 = 0.009739$ and $m = 3.11$. For this applied strain ratio of $e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$, the effective strain is, $\hat{e} = Ae_1$ and the parameter $A(\theta_p = 0^\circ) = 1$.

Once the calibration has been completed for all the K applied strain rates, the functional forms for the parameters e_0 and m can be obtained as a function of the local strain rate $\dot{\hat{e}}$. Figure 4.5 and 4.6 show the discrete calibrated values and the functional fit for the parameters e_0 and m calibrated from the RVE 1 in Figure 4.3(a). It can be observed in the Figure 4.5 and 4.6 that there are two distinct mechanisms taking place due to the rate-dependency. At lower strain rates, along with particle cracking, voids start to nucleate near the tip of the cracked inclusions and subsequently coalesce to cause matrix failure in the microstructure. This results in damage localization and failure of the material. On the other hand, at higher strain rates, the matrix starts to behave more elastic and therefore the void nucleation in the matrix and growth is reduced and no localization of damage happens in the microstructure.

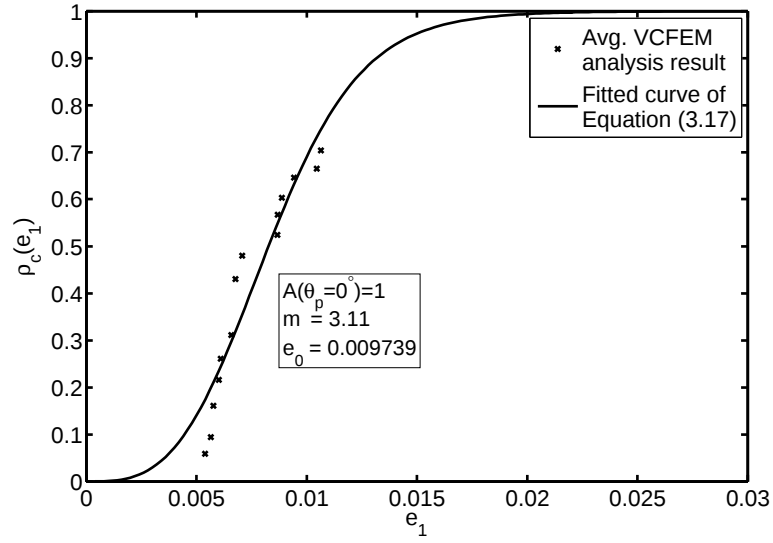


Figure 4.4: Area fraction of cracked inclusions from LE-VCFEM analysis of RVE 1 shown in Figure 4.3(a) at a strain rate of 0.1 s^{-1} along with the fit of Equation (3.17)

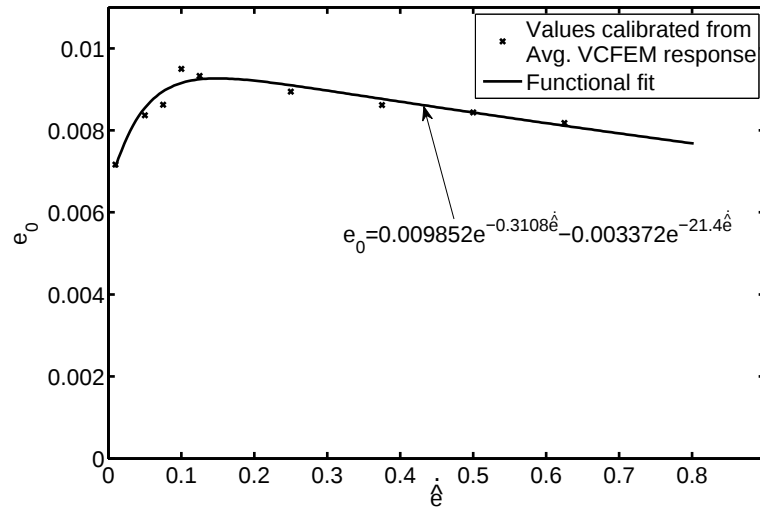


Figure 4.5: Evolution of parameter e_0 with the local strain rate $\dot{e}$ from micromechanics VCFEM results along with the functional fit

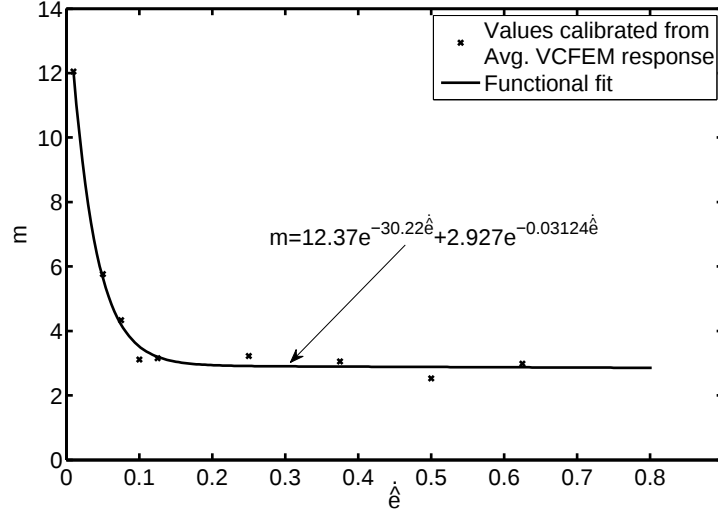


Figure 4.6: Evolution of parameter m with the local strain rate $\dot{\epsilon}$ from micromechanics VCFEM results along with the functional fit

4.5.2 A, B and C calibration

In order to calibrate the anisotropy parameters A , B and C , a total of four LE-VCFEM simulations at any strain rate, chosen between the limits used in Section 4.5.1 are needed. The four simulations with different loading conditions are

1. Tension tests in the y directions, with zero transverse strains ($e_{xx} = 0 : e_{yy} \neq 0 : e_{xy} = 0$),
2. Biaxial tension test ($e_{xx} = e_{yy} \neq 0 : e_{xy} = 0$),
3. Two simulations with strain ratios ($e_{xx} > 0 : e_{yy} < 0 : e_{xy} = 0$) and ($e_{xx} < 0 : e_{yy} > 0 : e_{xy} = 0$)

The anisotropy parameters A , B and C have the functional form of an ellipse, and at any given angle of principal strain θ_p , the value of A , B or C is the radius of the

ellipse. In the Section 4.5.1, the radius of the ellipse describing the parameter A at $\theta_p = 0^\circ$ is assumed to be 1. This value $A(\theta_p = 0^\circ) = 1$ is one of the axes of the ellipse describing A . The other axis of the ellipse is obtained from the y direction constrained tension test (1). For the RVE 1 shown in Figure 4.3(a), the area fraction of fractured inclusion ρ is plotted as a function of the principal strain e_1 in Figure 4.7. In this constrained y loading condition, value of A at $\theta_p = 90^\circ$ is evaluated by fitting either Equation (3.17) or (3.18) with the micromechanics result shown in Figure 4.7. The value of e_0 and m at the effective strain rate $\dot{\epsilon}$ are obtained from the functional forms calibrated in the Section 4.5.1. Figure 4.8 shows the functional form of the parameter A as a function of the angle of principal strain θ_p .

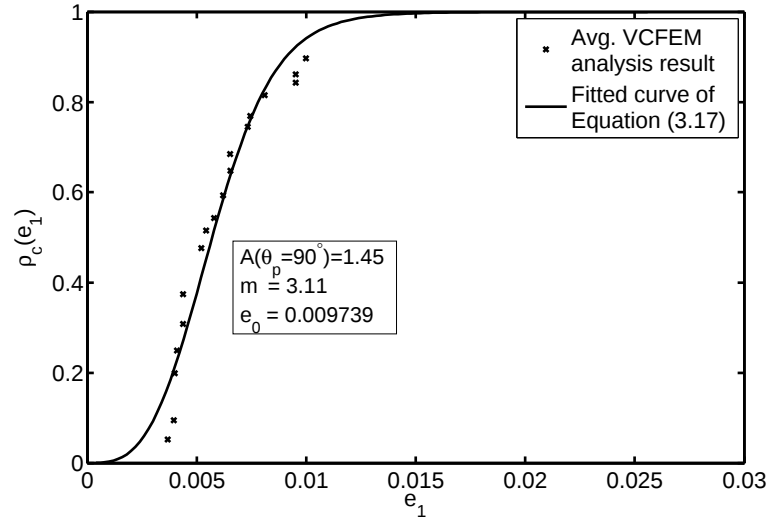


Figure 4.7: Area fraction of cracked inclusions for the y direction constrained tension LE-VCFEM analysis of RVE 1 shown in Figure 4.3(a) at a strain rate of 0.1 s^{-1} along with the fit of Equation (3.17)

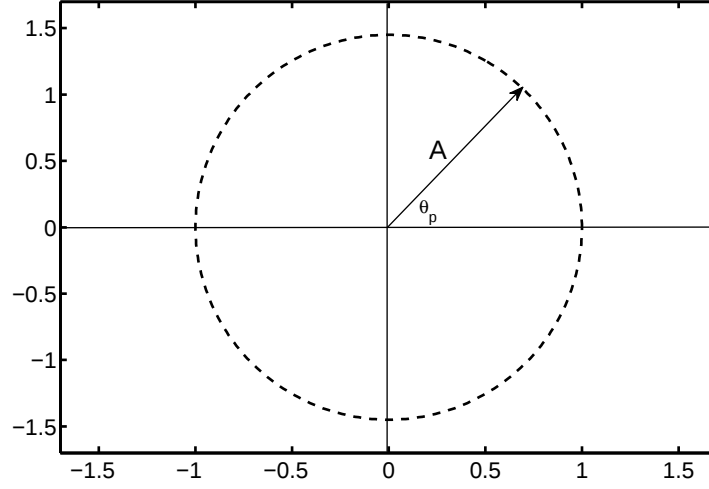


Figure 4.8: Void nucleation anisotropy parameter A as a function of the angle of principal strain θ_p

Similar to A , the major and minor axes of the ellipse describing the parameter B can be obtained from the biaxial tension test (2). Since for the biaxial tension test, the angle of principal strain $\theta_p = 0^\circ$ or 90° , only one of the axes of the ellipse needs to be determined. If the angle of principal strain is taken as $\theta_p = 0^\circ$, then $A(\theta_p) = 1$ and the effective strain is $\hat{\epsilon} = Ae_1 + Be_2$. Once again, either Equation (3.17) or (3.18) is fit with the micromechanics result for area fraction of cracked particles to evaluate the value of B at $\theta_p = 0^\circ$. The other axis of the B -ellipse can be obtained as

$$B(\theta_p = 90^\circ) = [A(\theta_p = 0^\circ) + B(\theta_p = 0^\circ) - A(\theta_p = 90^\circ)]$$

The other two loading conditions, $(e_{xx} > 0 : e_{yy} < 0 : e_{xy} = 0)$ and $(e_{xx} < 0 : e_{yy} > 0 : e_{xy} = 0)$ are used to calibrate the major and minor axis of the elliptical form of the

parameter C . The values of A, B and C for the RVEs 1 and 2 shown in the Figure 4.3 are given in the Table 4.2

	$A(\theta_p = 90^\circ)$	$B(\theta_p = 0^\circ)$	$B(\theta_p = 90^\circ)$	$C(\theta_p = 0^\circ)$	$C(\theta_p = 90^\circ)$
RVE 1	1.45	1.51	1.06	0.90	1.34
RVE 2	1.28	1.39	1.10	0.96	1.10

Table 4.2: Values of anisotropy parameters A, B and C at different angles of principal strain θ_p . The value of $A(\theta_p = 0^\circ) = 1$ for all RVEs

Subsequently the parameter V_p , which relates the volume fraction of cracked inclusions to the void nucleation rate in the rate-dependent HCPD model, is evaluated iteratively from several macroscopic numerical analyses, by minimizing the difference between the micromechanical and HCPD simulations :

$$\min_{V_p} \sum_{i=1}^N \sum_{j=1}^{M_i} \|\Sigma_{\text{HCPD}}^{ij} - \Sigma_{\text{micro}}^{ij}\|_F^2 \quad (4.6)$$

where N is the total number of simulations performed for the calibration of V_p and M_i is the number of increments in the i th micromechanical simulation.

CHAPTER 5

NUMERICAL EXAMPLES WITH THE RATE-DEPENDENT HCPD MODEL

The rate-dependent HCPD model is validated by comparing the results of the macroscopic finite element simulations with those obtained from the homogenization of the micromechanics LE-VCFEM results. An extensive validation of rate-independent HCPD model has been performed in [13], demonstrating the capability of the model for various proportional and non-proportional loading conditions. Since this rate-dependent HCPD model is an extension of the model proposed in [13], only the rate-effects and the novel rate-dependent void-nucleation model is validated and demonstrated in this dissertation. All of the macroscopic simulations were conducted in the commercial *MSC Marc* code, with a single *QUAD4* element with 4 integration points. The material properties considered for the micromechanics simulations are:

Ductile matrix: Young's modulus, $E = 72$ GPa, Poisson's ratio $\nu = 0.22$ and initial void volume fraction $f_0 = 0.01$. The post yield behavior for the matrix material without voids is governed by the yield curve shown in Figure 5.1. The yield stress is 175 MPa.

Brittle SiC inclusions: Young's modulus, $E = 320$ GPa, Poisson's ratio $\nu = 0.25$.

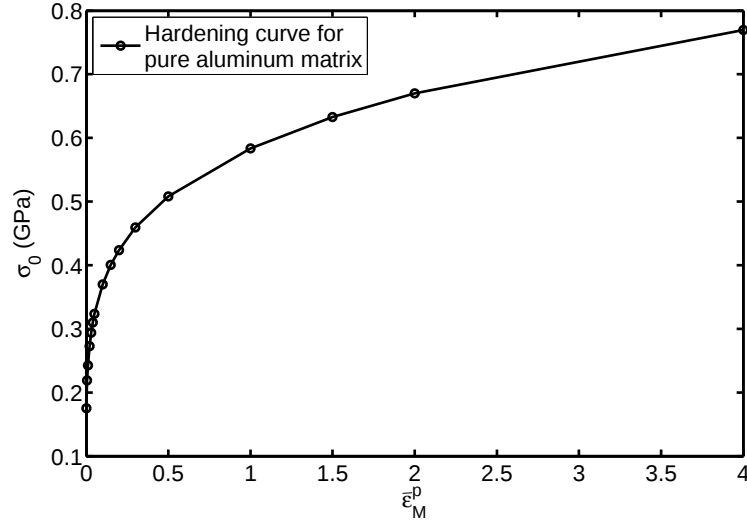


Figure 5.1: Hardening (stress-strain) curve for the aluminum matrix used for the micromechanics LE-VCFEM simulations of RVE 1 in Figure 4.3(a)

5.1 Viscoplastic response and void growth model

The RVE 1 shown in Figure 4.3(a), with 25 non-uniform elliptical inclusions of 10% volume fraction is used to validate the viscoplastic response of the macroscopic HCPD model, with and without void evolution. Four different micromechanics LE-VCFEM simulations with imposed shear loading, subject to different loading strain rates and initial void volume fraction f_0 are conducted. The homogenized shear stress Σ_{xy} from the micromechanics simulations are compared to the macroscopic stress components, from HCPD simulations in Figure 5.2 and show excellent agreement. Homogenized results from the LE-VCFEM simulation of the same RVE 1 subject to a strain ratio of $e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$ at a strain rate of 0.25 s^{-1} along with the HCPD macroscopic stress components are show in Figure 5.3. From both the shear

and restricted tension tests, excellent agreement can be observed between the HCPD model and the homogenized micromechanics solutions.

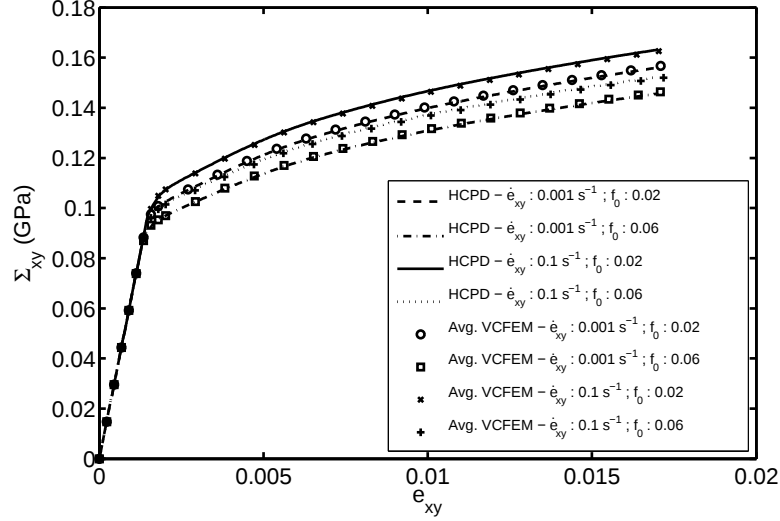


Figure 5.2: Macroscopic stress-strain response of the HCPD model and the homogenized micromechanical solutions for RVE 1 in Figure 4.3(a)

5.2 Void nucleation model

The void nucleation model is validated by simulating RVE 1 shown in Figure 4.3(a), with 25 non-uniform elliptical inclusions of 10% volume fraction. The plasticity parameters of this RVE is calibrated following the procedure described in Section 4.1 - 4.4. The nucleation parameters e_0 , m , A , B and C are calibrated following the procedure described in Section 4.5. The functional fit of the local strain rate dependent parameters e_0 and m for the RVE 1 is shown in Figure 4.5 and 4.6. The major and minor axes of the ellipses describing the damage anisotropy parameters A , B and C are given in Table 4.2. In order to validate the rate-dependent behavior

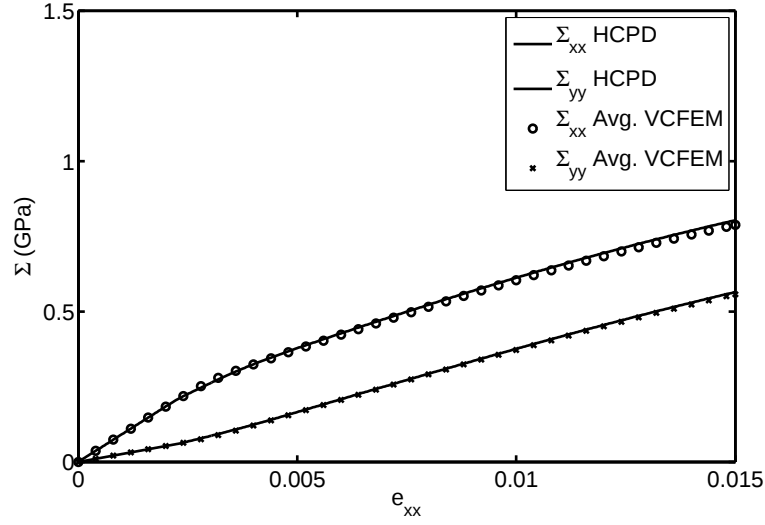


Figure 5.3: Macroscopic stress-strain response of the HCPD model and the homogenized micromechanical solution for RVE 1 in Figure 4.3(a) for strain ratio $e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$

of the nucleation model, four micromechanics LE-VCFEM simulations with RVE 1 are conducted, they are

1. Tension test in x direction with zero transverse strains ($e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$) at a loading strain rate of 0.25 s^{-1}
2. Tension test in x direction with zero transverse strains ($e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$) at a loading strain rate of 0.75 s^{-1}
3. Tension test in x direction with zero transverse strains ($e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$) at an initial loading strain rate of 0.25 s^{-1} , which is increased to 0.75 s^{-1} at 0.6% of the applied strain.

4. Tension test in x direction with zero transverse strains ($e_{xx} \neq 0 : e_{yy} = 0 : e_{xy} = 0$) at an initial loading strain rate of 0.75 s^{-1} , which is decreased to 0.25 s^{-1} at 0.6% of the applied strain.

The homogenized responses of the micromechanics simulations stated above is compared with their respective macroscopic rate-dependent HCPD model simulation results in Figures 5.4 and 5.5 and show very good agreement. For both macroscopic and LE-VCFEM simulations, the Von-Mises stress is plotted as a function of the applied strain. The micromechanics simulations show discrete drops in stress due to the inclusions cracking, however the macroscopic model does not exhibit this behavior due to the continuous function used in the nucleation model. The evolution of the area fraction of the inclusions cracked ρ when there is an instantaneous change in strain rate from 0.25 s^{-1} to 0.75 s^{-1} is show in Figure 3.2 and ρ when there is an instantaneous change in strain rate from 0.75 s^{-1} to 0.25 s^{-1} is show in Figure 3.3. The factor $\tilde{k}^*$ in Equation 3.19 takes care of the instantaneous change in local strain rate, accelerating and stalling the nucleation as required.

The ability of the nucleation model to capture material anisotropy in damage is validated by comparing the homogenized response from different micromechanics LE-VCFEM simulations subject to different loading conditions with their corresponding macroscopic HCPD response. Simulations are conducted for a number of applied macroscopic strain ratios ($e_{xx} : e_{yy} : e_{xy}$) at an applied strain rate of 0.1 s^{-1} . The values of parameters A, B and C are obtained from the calibrated elliptical forms at the angle of principal strain corresponding to the local strains. The Comparison between the micromechanics result and the rate-dependent HCPD model results are shown in Figure 5.6 and show good agreement.

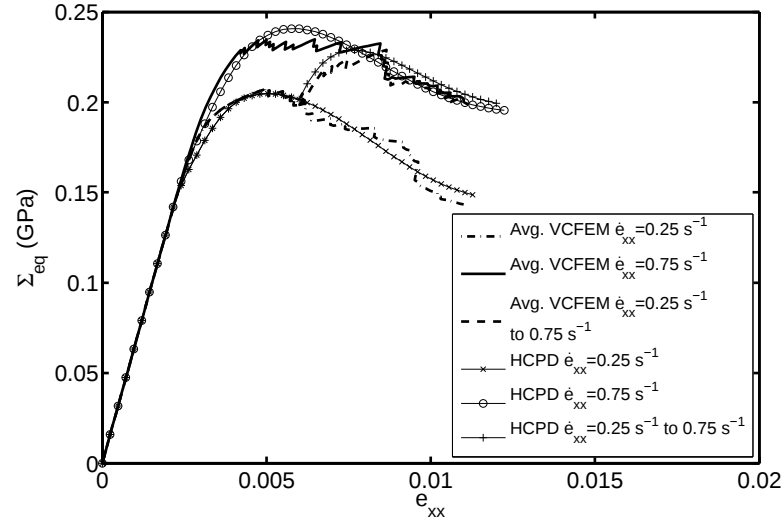


Figure 5.4: Comparison of macroscopic HCPD model simulation with the LE-VCFEM simulation of RVE 1 of Figure 4.3(a) subject to instantaneous change of strain rate from 0.25 s^{-1} to 0.75 s^{-1} at 0.6% applied strain.

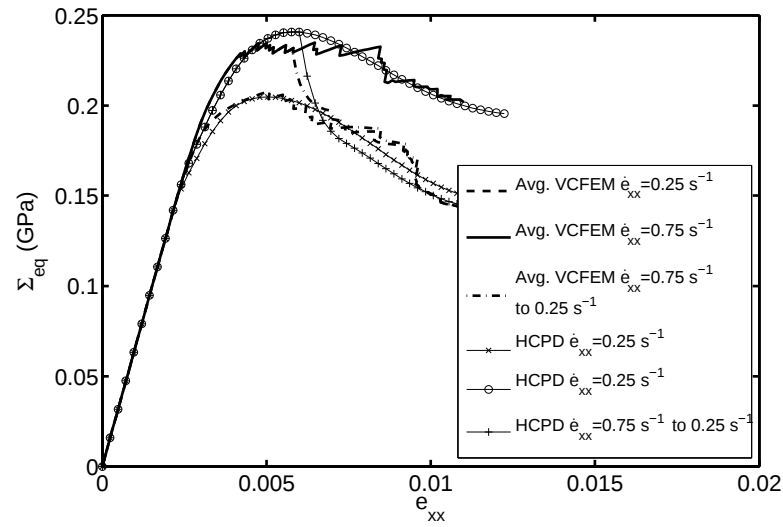


Figure 5.5: Comparison of macroscopic HCPD model simulation with the LE-VCFEM simulation of RVE 1 of Figure 4.3(a) subject to instantaneous change of strain rate from 0.75 s^{-1} to 0.25 s^{-1} at 0.6% applied strain.

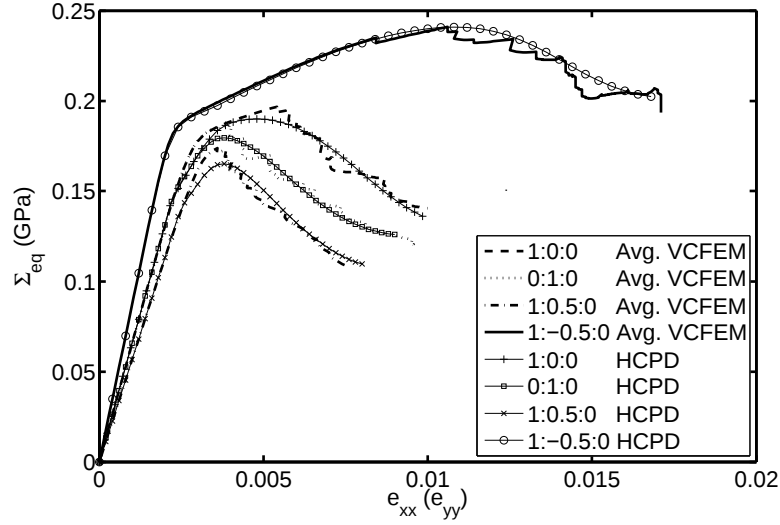


Figure 5.6: Macroscopic stress-strain response of the HCPD model and the homogenized LE-VCFEM solutions for RVE 1 in Figure 4.3(a) subject to different strain ratios $e_{xx} : e_{yy} : e_{xy}$

For validating the elliptical forms of the nucleation parameters A, B and C , a number of LE-VCFEM simulations with different loading ratios ($e_{xx} : e_{yy} : e_{xy} \neq 0$) were performed. The value of the parameter A is obtained from the ellipse shown in Figure 4.8 and from the homogenized micromechanics response, the parameter C is independently calibrated for different loading ratios. The independently calibrated values of the parameter C is plotted along with the elliptical form of C calibrated in Section 4.5 in Figure 5.7. From Figure 5.7, it is seen that the independently calibrated values are close to the elliptical form and thus validates the functional forms of the parameters A, B and C . Micromechanics simulations of RVE 1 for the two loading ratios $e_{xx} = -1 : e_{yy} = 0.5 : e_{xy} = 0.5$ and $e_{xx} = -1 : e_{yy} = 0.5 : e_{xy} = -0.5$ show no inclusion cracking. For these loading ratios, the value of A and C obtained from the ellipses calibrated in Section 4.5 used with Equation 3.14 and 3.17 gives a value of

zero for the area fraction of cracked inclusion ρ . Thus the anisotropy parameters are also capable of predicting loading conditions where no inclusion cracking happens.

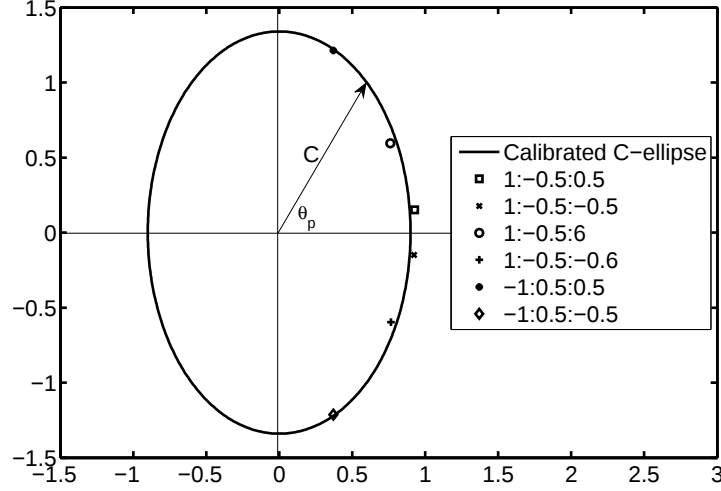


Figure 5.7: Parameter C for RVE 1 in Figure 4.3(a) along with the independently calibrated values of C for different strain ratios $e_{xx} : e_{yy} = 0 : e_{xy} = 0$

5.3 Dual-stage nested homogenization

The rate-dependent HCPD model, with the void nucleation and growth part switched off is used for the macroscopic simulation of large secondary dendrite arm spacing or SDAS cast aluminum alloy microstructures in [2]. Microstructures of these alloys are characterized by extremely inhomogeneous distribution of inclusions along the dendrite cell boundaries. Traditional single-step homogenization methods are not suitable for this type of microstructure due to the size of the representative volume element (RVE) and the associated computations required for micromechanical analyses. To circumvent this limitation, two distinct statistically equivalent RVE's are

identified, corresponding to the inherent scales of inhomogeneity in the microstructure. The homogenization is performed in multiple stages for each of the RVE's identified. Asymptotic expansion homogenization (AEH) and Self consistent homogenization (SCH) are used in this method. Anisotropy and viscoplastic parameters in for the different scales are calibrated from homogenization of micro-variables for the different RVE's as described in Section 4.1 - 4.3. The uniqueness of the nested two-stage homogenization is that it enables evaluation of the overall homogenized model of the cast alloy from limited experimental data, but also material properties of constituents like inter-dendritic phase and pure aluminum matrix. The capabilities of the HCP model are demonstrated for a cast aluminum alloy AS7GU having a SDAS of $30\text{ }\mu\text{.m}$ in [2].

CHAPTER 6

SUMMARY AND FUTURE WORK

6.1 Summary

In this thesis, an accurate and computationally efficient rate-dependent homogenization based continuum plasticity damage model is developed for the macroscopic analysis of ductile fracture in heterogeneous porous ductile materials. The rate-dependent HCPD model is an extension of the model developed by Ghosh et al. [13] and follows the structure of an anisotropic Gursen-Tvergaard-Needleman elastoporous-plasticity model for ductile materials. Material anisotropy is determined by the morphology of the microstructure, evolution of plastic deformation and damage. This anisotropy in plastic behavior is modeled with a set of anisotropy parameters, which evolve as a function of the plastic work. The entire rate-dependent HCPD model is expressed in an evolving material principal coordinate system, in which the material remains orthotropic throughout the deformation history. All of the model parameters are calibrated from the homogenization of microstructural variables obtained from LE-VCFEM simulations of the RVE.

The model also incorporates a unique rate-dependent void nucleation criterion that is capable of effectively simulating the loss of load carrying capacity of heterogeneous materials that happens because of inclusion cracking and void growth. The

model incorporates the effects of loading strain rates on damage by using two parameters which are a function of the local strain rate. The nucleation model also incorporates the anisotropy in damage evolution using three parameters A , B and C , which are rate-independent and can be calibrated at any loading strain rate. Numerical examples are conducted for a variety of loading conditions, like different strain rates, instantaneously varying strain rates and different loading ratios. In all the cases, the rate-dependent HCPD model results show excellent agreement with the homogenized micromechanics results. Above all the rate-dependent HCPD model has a huge advantage in terms of efficiency over explicit micromechanics simulations and hence is a very effective tool in making macroscopic damage predictions. These plasticity and nucleation capability is largely lacking in literature.

6.2 Future Work

Macroscopic analysis using the HCPD model has considerable efficiency advantage over explicit micromechanics simulations and hence the inclusion of such models in multi-scale framework for efficient analysis is inevitable. But it is also important to recognize the limitations of such homogenized models, especially the lack of accuracy and detail when simulating critical regions of microstructures containing cracks and highly localized damage evolution. Therefore implementation of the macroscopic rate-dependent HCPD model in a multi-scale framework, such as the one described in [28], followed by studies of error criterion and criterion for switching to lower scales is necessary.

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