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Track-Initiation Probability for Multistatic Sonar Fields

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ABSTRACT

Expressions are derived for the track-initiation probability of a sonar field in terms of single-ping detection probabilities. Five types of network architecture—four main types and one variant—are treated. As well as providing the algebraic foundation for more detailed studies described in companion reports, several general results on the ASW performance of sonar networks are obtained.

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Track-Initiation Probability for Multistatic Sonar Fields

Executive Summary

This note is the first of a series of reports that aims to develop a simple and reliable description of the ASW performance of a multistatic sonar field. The extent of the mathematics may seem to belie the goal of simplicity—most of the note deals with the derivation of formulae for local track-initiation probability given single-ping detection probabilities—but in Section 3 we obtain the following simple results from the equations, using local track-initiation probability as the measure of performance:

- a demonstration that there is a performance gain in moving from a monostatic to a multistatic field, or in centralising the tracking, or both, and the indication that these are general features of sonar networks,
- the networking benefit of adding extra receivers, and how this multiplies the benefit of centralising the tracking, and
- for the monostatic field with centralised tracking, the performance gain available to a tracker that can initiate a track on a single cycle of pings over one that requires several.

For a system as complicated as multistatic sonar, conclusions of such simplicity and generality can be achieved only with assumptions and approximations. In the present analysis, we simplified the general expressions for track-initiation probability by adopting the (unrealistic) assumption that all source–receiver pairs have the same detection probability. (Other reports in this series do not use this assumption, but then one must specify the geographic layout of the sonar field. Companion reports deal with three classes of field layout.)

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1. Introduction

Quantitative analysis depends crucially on the metric employed; an inappropriate metric can easily lead to wrong conclusions. In sonar-system analysis for anti-submarine warfare, there is a tension between the metric most natural to operators and analysts working at the highest level—detection range—and the realities of acoustic propagation and ambient noise in the sea, which are best expressed in terms of detection probability. The detection range – detection probability nexus has recently been re-examined in the context of active sonar [1], resulting in a recommendation that the two be connected via the quantity 'cumulative track-initiation probability' P_{ti}. Reference 1 describes how to compute P_{ti} from single-ping detection probabilities p_{d}, and how thence to obtain detection ranges that are analytically useful. The method and example given in Reference 1 were developed with monostatic systems in mind. Multistatics raises additional complications, which it is the purpose of this note to address.

Reference 1 recommends use of cumulative track-initiation probability, but in this note we work with local track-initiation probability p_{ti}, for two reasons. First, computing a cumulative probability requires a scenario over which to accumulate the probability. This is too restrictive for the concept-development process that is the core of the present work. On the other hand (and this is the second reason), working with local probabilities allows the development of analytical formulae which are ideal for building intuition about the consequences of different concepts for operating a multistatic sonar field.

The next section reiterates the definition of local track-initiation probability and its calculation for monostatic sonar systems, discusses issues arising in moving to multistatics and presents derivations of formulae for p_{ti} under four concepts for operating a multistatic sonar field. Section 3 gives some numerical examples and relates the formalism used here with that of previous work. Section 4 summarises the work by way of conclusion.

2. Local Track-Initiation Probability

2.1 Definition

Although other types of track-initiation rule exist (e.g. [2,3]), fielded automatic tracking systems usually use rules of the form 'start tracking an object if it is detected m or more times in n consecutive opportunities'. Sometimes values of m and n are set by system designers; sometimes they are operator-selectable. For this type of rule, the 'local' track-initiation probability p_{ti} is:

the probability that a track is initiated after exactly n pings,

where the probability value depends, of course, on the value chosen for m.

The rest of this section concerns the application of this rule to various sonar systems, starting with a single sensor and then moving to geographically dispersed sensor fields potentially containing many sources and many receivers.

2.2 Single-Sensor Track-Initiation Probability

Consider a single sensor comprising a collocated sonar source and receiver. The usual rules for combining probabilities lead to the following general expression for the local track-initiation probability:

$$p_{\mathrm{ti}} = \sum_{l=m}^{n} \binom{n}{l} p_{\mathrm{d}}{}^{l} \left(1 - p_{\mathrm{d}}\right)^{n-l} , \tag{1}$$

assuming that the single-ping detection probability p_{d} is the same at each detection opportunity and that detection opportunities are statistically independent. (Equation 1 can be generalised if either of these assumptions do not hold, but the results are too complicated to be informative.)

This note mainly deals with the case m = 3 and n = 5, since these are common values in sonar systems. Substituting these values in Equation (1) gives

$$p_{\mathrm{ti}} = p_{\mathrm{d}}{}^{3}\left(10 - 15p_{\mathrm{d}} + 6p_{\mathrm{d}}{}^{2}\right). \tag{2}$$

Figure 1 shows the behaviour of p_{ti} for this and several other cases.[(a)]

Since tracks can be initiated by false detections as well as by detections of the target (or even a combination of false and target detections), it may seem reasonable to rewrite Equations (1) and (2) so that the argument of p_{ti} could be either p_{d} or a probability of false detection. However, expanding the problem space to include false detections necessarily introduces a new feature: data association. Starting a track requires more than just the occurrence of three detections in five pings (or satisfying whatever the tracking rule may be). The detections must also be associated by the tracker's data-association method; they must occur close enough to each other that the tracking algorithm recognises them as likely to have come from the same object. Target detections are clustered around the actual target position, so in most cases association will occur, since the size of the association gate is chosen to account for expected measurement errors. Therefore ignoring the data association requirement in the formulation of equations (1) and (2) is reasonable for the purposes

[(a)] The evaluation of Equation (1) rapidly becomes arithmetically tedious as n increases, particularly for mid-range values of m. Expressions like Equation (2) for the cases shown in Figure 1 are available from the authors.

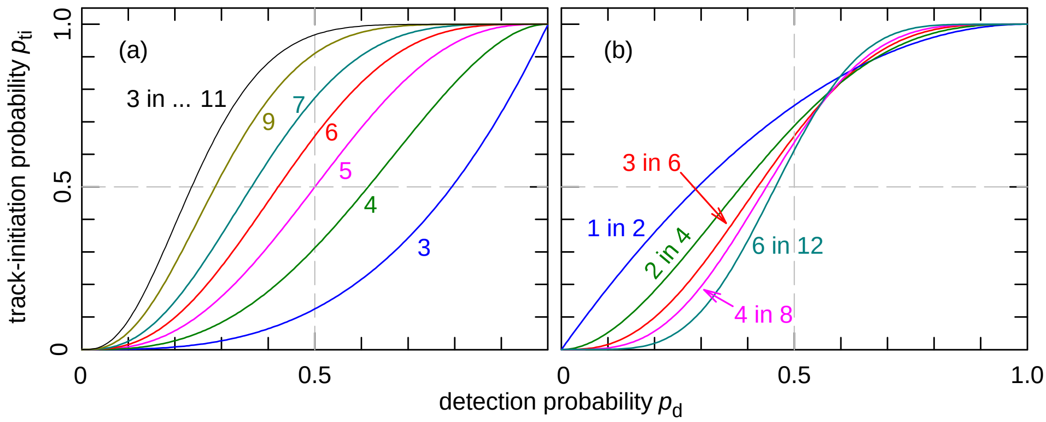

Figure 1: Dependence of single-sensor track-initiation probability on detection probability for various track-initiation rules: (a) for m = 3*, (b) for* m = n/2 *and* n *even*

of this study. It amounts to assuming that data association always occurs for target detections. On the other hand, false detections arising from random noise excursions lie at random locations in the sonar's field of view, so it is relatively rare for two of them to be close enough to be associated. Any assessment of a probability of false track initiation must therefore take into account the probability of data association. This cannot be achieved by merely replacing p_d in Equations (1) and (2) with some other probability; for standard tracking algorithms (e.g. [4]) use different sizes of association gate at different stages in the process. (For example, the gate is enlarged after a miss—a ping that fails to produce an associated detection). It means that the assumption of constant probability during the track-initiation process does not apply for false detections, even approximately.

2.3 Operating Concepts for Networked Multi-Sensor Fields

We seek a generalisation of Equation (1) applying to a network of geographically dispersed sensors. To achieve this, we must first settle issues arising from the presence of multiple sources and receivers. If the network has many sources, how should one count detection opportunities—is it n pings from one source, the next n pings from any source, or n pings of each source? We adopt the last, for reasons explained below. When the field has many receivers, does each perform its own tracking, or is detection information pooled at a central tracking node? Should the answer to this question depend on whether the field is operated monostatically or multistatically? These two questions cannot be settled as readily as the first; addressing them is the subject of a companion report [5]. The purpose of the present note is to derive the enabling equations.

To apply the 3-in-5 rule to a networked field with many sources, one must answer the first of the above questions: what constitutes 5 detection opportunities? Our view, that the answer should be 5 pings of every source in the field, was arrived at by identifying the characteristics of a 'baseline' sensor field—that is, one with no, or minimal, networking—and then analysing how these change as increasingly higher levels of networking capability are added. The argument is as follows.

The baseline system is a field of sonars that operate independently of each other. Each sonar comprises a source–receiver pair and each performs its own tracking based solely on detecting returning echoes of its own pings. We refer to this system as 'monostatic with distributed tracking'. A given sonar must ping at least 3 and up to 5 times before it can determine whether to start a track. Any data fusion that may occur consists only in passing information on initiated tracks.

The simplest first step in networking the baseline system is to consider passing detection information to a central tracker. To gain some advantage from this, we should continue to allow each sonar in the field to ping 5 times. That is, we take the point of view that the networking does not interfere with the detection processing in the sonars. Each sonar processes its acoustic data in the same manner whether the field is networked or not. The difference lies in how the resulting detections are handled. In effect, however, the track-initiation rule becomes 3-in-5J for a field comprising J sonars. This is the point of view adopted in an earlier study [6]. It not as great an advantage in practice as may at first appear for two reasons:

- In a geographically dispersed field, many sonars may be out of range of the target at a given time. This is the reason why one should not adopt '3 detections in the next 5 pings of any sonar'.
- Although the number of true detections rarely scales with the number of sonars in the field, the number of false detections almost always does. That is, the effect of centralising the tracking in a field of J sonars is approximately a J-fold increase in false-detection rate at the tracker, with a much smaller increase in the target-detection rate. The false-detection problem is the subject of many studies (e.g. [2,7–15]) and the data-fusion rule that minimises the initiation of false tracks has long been known. However, it requires knowledge of false and true detection probabilities at each ping, which are usually not available in practical situations. For this reason, we adopt the simplest data-fusion rule, known as the 'OR' rule: every detection recorded by any detector is passed to the central tracker.

We call such a network architecture 'multiple monostatic' or 'monostatic with centralised tracking'.

The next step would be to allow each receiver in the field to process echoes from any source, not just its own. Each ping then produces, in principle, K detection opportunities for a field of K receivers. If there are also J sources, each of which pings 5 times, then the track-initiation rule is effectively 3-in-5JK. Also, if there are 3 or more receivers in the field, then it is possible to initiate a track on a single ping.[b]

We term such an architecture 'multistatic with centralised tracking'. All the issues relating to a multiple monostatic field apply; for example, the false-alarm rate at the tracker is now magnified JK-fold approximately, compared with the baseline architecture. In an attempt to mitigate this, it has been suggested that a multistatic field could return to distributed

[b] Assuming that the tracking algorithm has the capability of allowing this. We return to this point in Section 3.3, where we evaluate the effect of one type of tracker limitation that prevents track initiation on a single ping.

tracking [2,10]. That is, each receiver operates multistatically but does its own tracking and reports only initiated tracks. With 5 pings from each of J sources, this returns the track-initiation rule to 3-in-5J. Hence, such a 'multistatic field with distributed tracking' should, at first sight, show similar sensitivity to the false-detection rate as a monostatic field with centralised tracking, provided it has a similar number of sources as the multistatic field.

2.4 Field Track-Initiation Probability — 3-in-5 Rule

In this subsection we convert the verbal descriptions of the various network architectures in the previous subsection into formulae for track-initiation probability as a function of single-ping detection probability. We restrict attention to the 3-in-5 track-initiation rule only and we assume that

- the detection probability is the same for all pings, and
- each ping is a statistically independent event.

2.4.1 Monostatic, Distributed Tracking (Baseline Sensor Field)

The baseline field comprises J monostatic sonars each performing its own tracking. The probability $p_{\mathrm{ti},j}$ that sonar j has started a track after 5 pings is obtained from its p_{d} values using Equation (2). The 'field track-initiation probability' $p_{\mathrm{ti,f}}$ is the probability that a sonar somewhere in the field has started a track after each has pinged 5 times. The usual rules for combining probability lead to

$$p_{\mathrm{ti,f}} = 1 - \prod_{j=1}^{J}\left(1 - p_{\mathrm{ti},j}\right). \tag{3}$$

This is by far the simplest case.

2.4.2 Monostatic, Centralised Tracking

Each receiver still listens only to echoes of pings from its own collocated source, but now detection information is pooled at a central tracking node. We still consider five pings from each source in the field, so as to match the operational situation with distributed tracking, but the central node starts a track on any three detections, each of which could come from any ping of any source.

The simplest way of evaluating the field track-initiation probability would appear to be in terms of the probability of a track not being initiated:

$$p_{\mathrm{ti,f}} = 1 - p(0) - p(1) - p_1(2) - p_2(2), \tag{4}$$

where, after five pings of each source in the field, $p(0)$ is the probability that there have been no detections anywhere, $p(1)$ is the probability of exactly one detection somewhere in the field, $p_1(2)$ is the probability of exactly two detections by the same receiver and $p_2(2)$ is the probability of exactly two detections, but by different receivers. These four probabilities exhaust all the ways of not starting a track after each source in the field has pinged five times. The first is straightforward:

$$p(0) = \prod_{j=1}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 , \qquad (5)$$

where $p_{\mathrm{d},j}$ is the detection probability for sensor j and J is the total number of sensors in the field.

To evaluate $p(1)$, label the sensor making the detection j'. The probability that it makes exactly one detection in five pings is

$$\binom{5}{1} p_{\mathrm{d},j'} \left(1 - p_{\mathrm{d},j'}\right)^4 . \qquad (6)$$

The combinatorial symbol appears in Equation (6) because the detection could have occurred on any of the five pings. We also require that no other sensor in the field makes a detection in any of its five pings. That is, the probability that only sensor j' has made a detection—and exactly one detection—after all sensors have pinged five times is

$$\binom{5}{1} p_{\mathrm{d},j'} \left(1 - p_{\mathrm{d},j'}\right)^4 \prod_{\substack{j=1 \\ (j \neq j')}}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 . \qquad (7)$$

This is turned into $p(1)$ by summing over j':

$$\begin{aligned} p(1) &= \sum_{j'=1}^{J} \binom{5}{1} p_{\mathrm{d},j'} \left(1 - p_{\mathrm{d},j'}\right)^4 \prod_{\substack{j=1 \\ (j \neq j')}}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 \\ &= \binom{5}{1} \sum_{j'=1}^{J} \frac{p_{\mathrm{d},j'}}{1 - p_{\mathrm{d},j'}} \prod_{j=1}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 \\ &= 5p(0) \sum_{j=1}^{J} \frac{p_{\mathrm{d},j}}{1 - p_{\mathrm{d},j}} , \end{aligned} \qquad (8)$$

where, for purely cosmetic reasons, the primes have been dropped from the variable of summation in the last line of Equation (8).

The quantity $p_1(2)$ is the probability that a given sensor makes two detections in five ping cycles, with no other sensors making any. It is evaluated in the same way as $p(1)$:

$$\begin{aligned} p_1(2) &= \sum_{j'=1}^{J} \binom{5}{2} p_{\mathrm{d},j'}^{\;2} \left(1 - p_{\mathrm{d},j'}\right)^3 \prod_{\substack{j=1 \\ (j \neq j')}}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 \\ &= 10p(0) \sum_{j=1}^{J} \left(\frac{p_{\mathrm{d},j}}{1 - p_{\mathrm{d},j}}\right)^2 . \end{aligned} \qquad (9)$$

The fourth probability also concerns the field making exactly two detections, but on different sensors. Let the sensors making the detections be labelled j' and j'' (with $j' \neq j''$, obviously). Then, analogously with Expression (7), the probability of sensors j' and j'' making exactly one detection each with no other sensor making a detection after all sensors have pinged five times is

$$\binom{5}{1} p_{\mathrm{d},j'} \left(1 - p_{\mathrm{d},j'}\right)^4 \binom{5}{1} p_{\mathrm{d},j''} \left(1 - p_{\mathrm{d},j''}\right)^4 \prod_{\substack{j=1 \\ (j \neq j', j \neq j'')}}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 . \tag{10}$$

To turn this into $p_2(2)$, it is necessary to note that the pair j', j'' is not ordered. It follows that, if both are summed over the full range, pairs will be counted twice. Hence

$$\begin{aligned} p_2(2) &= \binom{5}{1}^2 \sum_{j'=1}^{J-1} p_{\mathrm{d},j'} \left(1 - p_{\mathrm{d},j'}\right)^4 \sum_{j''=j'+1}^{J} p_{\mathrm{d},j''} \left(1 - p_{\mathrm{d},j''}\right)^4 \prod_{\substack{j=1 \\ (j \neq j', j \neq j'')}}^{J} \left(1 - p_{\mathrm{d},j}\right)^5 \\ &= 25 p(0) \sum_{j=1}^{J-1} \sum_{j'=j+1}^{J} \frac{p_{\mathrm{d},j}\, p_{\mathrm{d},j'}}{\left(1 - p_{\mathrm{d},j}\right)\left(1 - p_{\mathrm{d},j'}\right)} , \end{aligned} \tag{11}$$

where once again summation indices have been relabelled in the last line. This completes the derivation of the expressions needed to evaluate Equation (4).

2.4.3 Multistatic, Centralised Tracking

Each receiver records echoes of pings from any source and passes detection information to a central tracking node. The analysis is along the same lines as in Section 2.4.2, but now we must recognise four types of probability involving two detections. That is, the equivalent of Equation (4) is

$$p_{\mathrm{ti,f}} = 1 - p(0) - p(1) - p_1(2) - p_2(2) - p_3(2) - p_4(2), \tag{12}$$

where $p(0)$ and $p(1)$ are the probabilities of exactly zero or one detection in five pings of all sources, as before, $p_1(2)$ is the probability that a given source–receiver pair makes exactly two detections, $p_2(2)$ is the probability that the two detections involve the same source but different receivers, $p_3(2)$ involves the same receiver but different sources and, for $p_4(2)$, both sources and receivers are different for the two detections.

Let J be the number of sources in the field and K the number of receivers. The probability of zero detections by any receiver after all sources have pinged five times each is, similarly to the monostatic case,

$$p(0) = \prod_{j=1}^{J} \prod_{k=1}^{K} \left(1 - p_{\mathrm{d},jk}\right)^5 , \tag{13}$$

where $p_{\mathrm{d},jk}$ is the detection probability for the pair of source j and receiver k, which may or may not be collocated.

The derivation of $p(1)$ is also analogous to the monostatic centralised case. Let the detection be made by the pair of source j' and receiver k'. Then

$$p(1)=\sum_{j'=1}^{J}\sum_{k'=1}^{K}\binom{5}{1}p_{\mathrm{d},j'k'}\left(1-p_{\mathrm{d},j'k'}\right)^4\prod_{\substack{j=1\\(j\neq j'\wedge k\neq k')}}^{J}\prod_{k=1}^{K}\left(1-p_{\mathrm{d},jk}\right)^5$$

$$=\binom{5}{1}\sum_{j'=1}^{J}\sum_{k'=1}^{K}\frac{p_{\mathrm{d},j'k'}}{1-p_{\mathrm{d},j'k'}}\prod_{j=1}^{J}\prod_{k=1}^{K}\left(1-p_{\mathrm{d},jk}\right)^5$$

$$=5p(0)\sum_{j=1}^{J}\sum_{k=1}^{K}\frac{p_{\mathrm{d},jk}}{1-p_{\mathrm{d},jk}}, \tag{14}$$

where the condition on the double product in the first line indicates that one must skip the factor involving indices $j=j'$ **and** $k=k'$; that is, just one of the JK factors is omitted, exactly the one that is supplied in the second line. In the third line, $p(0)$ refers to Equation (13), not Equation (5); this is the case for the remainder of this subsection.

The quantity $p_1(2)$, in which the two detections are made by the same receiver–source pair is also derived analogously to the monostatic case, giving

$$p_1(2)=10p(0)\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\frac{p_{\mathrm{d},jk}}{1-p_{\mathrm{d},jk}}\right)^2. \tag{15}$$

The next case has the two detections being made by different receivers k', k'', but on pings from the same source j'. The probability of just these two detections and no other after five pings of all sources is, similarly to Expression (10),

$$\binom{5}{1}p_{\mathrm{d},j'k'}\left(1-p_{\mathrm{d},j'k'}\right)^4\binom{5}{1}p_{\mathrm{d},j'k''}\left(1-p_{\mathrm{d},j'k''}\right)^4\prod_{\substack{j=1\\((j\neq j'\wedge k\neq k')\\ \vee(j\neq j'\wedge k\neq k''))}}^{J}\prod_{k=1}^{K}\left(1-p_{\mathrm{d},jk}\right)^5, \tag{16}$$

where this time *two* factors must be omitted from the double product: those with $j=j'$, $k=k'$ and $j=j'$, $k=k''$. The probability $p_2(2)$ is obtained from this in the same manner as the monostatic case, giving (cf. Eq. 11)

$$p_2(2)=25p(0)\sum_{j=1}^{J}\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\frac{p_{\mathrm{d},jk}\,p_{\mathrm{d},jk'}}{\left(1-p_{\mathrm{d},jk}\right)\left(1-p_{\mathrm{d},jk'}\right)}. \tag{17}$$

When the two detections involve the same receiver k' and different sources j', j'', the expression can be derived in the same manner as Equation (17) by simply interchanging the roles played by sources and receivers, giving

$$p_3(2)=25p(0)\sum_{j=1}^{J-1}\sum_{j'=j+1}^{J}\sum_{k=1}^{K}\frac{p_{\mathrm{d},jk}\,p_{\mathrm{d},j'k}}{\left(1-p_{\mathrm{d},jk}\right)\left(1-p_{\mathrm{d},j'k}\right)}. \tag{18}$$

If J = K and every source has a collocated receiver, then $p_3(2)$ = $p_2(2)$, but this is not generally the case.

The final probability concerns two detections where both sources and receivers are different. Let one detection be made by the pair of source j' and receiver k', and the other by j'', k''. This produces only minimal change to Expression (16):

$$\binom{5}{1} p_{\mathrm{d},j'k'} \left(1-p_{\mathrm{d},j'k'}\right)^4 \binom{5}{1} p_{\mathrm{d},j''k''} \left(1-p_{\mathrm{d},j''k''}\right)^4 \prod_{\substack{j=1 \\ ((j\neq j' \wedge k\neq k') \\ \vee (j\neq j'' \wedge k\neq k''))}}^{J} \prod_{k=1}^{K} \left(1-p_{\mathrm{d},jk}\right)^5 , \tag{19}$$

with the omission of the two factors that have $j = j'$, $k = k'$ and $j = j''$, $k = k''$. We now require four sums to turn this into $p_3(2)$, and the issue of double counting again arises. This is avoided, as in Equation (17), by restricting the range of the second sum over receivers. (One could equally well restrict the second sum over sources, but one must not do both, for then some pairs would be omitted.) The result is

$$p_4(2) = 25p(0) \sum_{j=1}^{J} \sum_{\substack{j'=1 \\ (j'\neq j)}}^{J} \sum_{k=1}^{K-1} \sum_{k'=k+1}^{K} \frac{p_{\mathrm{d},jk}\, p_{\mathrm{d},j'k'}}{\left(1-p_{\mathrm{d},jk}\right)\left(1-p_{\mathrm{d},j'k'}\right)} . \tag{20}$$

2.4.4 Multistatic, Distributed Tracking

With this network architecture, receivers record echoes of pings from any source, but each does its own tracking. Once again we seek the probability of a track being started somewhere in the field after each source has pinged five times. There are only two types of probability involving exactly two detections, but now each receiver is treated separately. That is, the track-initiation probability for receiver k is

$$p_{\mathrm{ti},k} = 1 - p_k(0) - p_k(1) - p_{1,k}(2) - p_{2,k}(2). \tag{21}$$

The probabilities of zero and exactly one detection are similar to the previous ones:

$$p_k(0) = \prod_{j=1}^{J} \left(1-p_{\mathrm{d},jk}\right)^5 , \tag{22}$$

$$p_k(1) = 5p_k(0) \sum_{j=1}^{J} \frac{p_{\mathrm{d},jk}}{1-p_{\mathrm{d},jk}} , \tag{23}$$

where j enumerates sources. The quantity $p_{1,k}(2)$, for which both detections are made on pings from the same source, is also similar:

$$p_{1,k}(2) = 10p_k(0) \sum_{j=1}^{J} \left(\frac{p_{\mathrm{d},jk}}{1-p_{\mathrm{d},jk}} \right)^2 , \tag{24}$$

as is the expression for $p_{2,k}(2)$, for which the two detections involve different sources:

$$p_{2,k}(2) = 25 p_k(0) \sum_{j=1}^{J-1} \sum_{j'=j+1}^{J} \frac{p_{\mathrm{d},jk}\, p_{\mathrm{d},j'k}}{\left(1-p_{\mathrm{d},jk}\right)\left(1-p_{\mathrm{d},j'k}\right)} \; . \tag{25}$$

Finally, the field track-initiation probability $p_{\mathrm{ti,f}}$ is obtained from a combination of the track-initiation probabilities for each receiver:

$$p_{\mathrm{ti,f}} = 1 - \prod_{k=1}^{K} \left(1 - p_{\mathrm{ti},k}\right) . \tag{26}$$

2.5 Generalising to Other Track-Initiation Rules

The full generalisation of Equation (1) would produce equations applying to all values of *m* and *n*, not just 3-in-5. It is clear from the derivations above that the generalisation to *n* is straightforward: one replaces '5' with '*n*' everywhere. Thus, in the multistatic centralised case for example, Equations (13), (15) and (20) become

$$p(0) = \prod_{j=1}^{J} \prod_{k=1}^{K} \left(1 - p_{\mathrm{d},jk}\right)^{n} , \tag{27}$$

$$p_1(2) = \frac{n(n-1)}{2} p(0) \sum_{j=1}^{J} \sum_{k=1}^{K} \left(\frac{p_{\mathrm{d},jk}}{1-p_{\mathrm{d},jk}} \right)^{2} \tag{28}$$

and

$$p_4(2) = n^2 p(0) \sum_{j=1}^{J} \sum_{\substack{j'=1 \\ (j' \neq j)}}^{J} \sum_{k=1}^{K-1} \sum_{k'=k+1}^{K} \frac{p_{\mathrm{d},jk}\, p_{\mathrm{d},j'k'}}{\left(1-p_{\mathrm{d},jk}\right)\left(1-p_{\mathrm{d},j'k'}\right)} \tag{29}$$

respectively. On the other hand, generalisation to other values of *m* is more complicated. The method set out in Section 2.4 generalises, but it seems difficult to write simple general expressions because the number of terms in equations like Equation (12) varies with *m*, and indeed grows rapidly with increasing *m*. Each case must be derived separately.

3. Examples for the 3-in-5 Rule

3.1 One Receiver

The distinction between centralised and distributed tracking does not apply if the field contains one receiver only. It is a simple matter of algebra to show that the expression for $p_{\mathrm{ti,f}}$ in Section 2.4.1 becomes identical to that in Section 2.4.2 when K = 1. Similarly, the expressions for $p_{\mathrm{ti,f}}$ in Section 2.4.3 and 2.4.4 are also identical when K = 1. The same does not happen in the multistatic cases when one sets J = 1, reflecting the fact that tracking is a function of receivers, not sources; centralised tracking remains different from distributed tracking when there is only one source, provided there is more than one receiver.

3.2 Uniform Detection Probability

The equations in Section 2.4 assume that the detection probability for any source–receiver pair is the same for all 5 pings. In this subsection we go further and assume that it is the same for all source–receiver pairs. This highly artificial assumption is most unlikely to apply in any real situation, but adopting it allows us to look beyond the variation of p_{d} with range and focus on the effects of networking. It has been used by other authors for a similar purpose (e.g. [2,8,16]) and was used to generate Figure 4 in Reference 6.

Setting $p_{\mathrm{d},jk}$ (or equivalent) equal to a constant p_{d} in the formulae of Section 2.4 leads to the following expressions for the field track-initiation probability:

- *Monostatic distributed* —

$$\begin{aligned} p_{\mathrm{ti,f}} &= 1-\left(1-10p_{\mathrm{d}}^{3}+15p_{\mathrm{d}}^{4}-6p_{\mathrm{d}}^{5}\right)^{J} \\ &= 1-\left(1-p_{\mathrm{d}}\right)^{3J}\left(1+3p_{\mathrm{d}}+6p_{\mathrm{d}}^{2}\right)^{J} \end{aligned} \tag{30}$$

- *Monostatic centralised* —

$$p_{\mathrm{ti,f}} = 1-\left(1-p_{\mathrm{d}}\right)^{5J-2}\left[1+(5J-2)p_{\mathrm{d}}+\tfrac{1}{2}(5J-1)(5J-2)p_{\mathrm{d}}^{2}\right] \tag{31}$$

- *Multistatic distributed* —

$$p_{\mathrm{ti,f}} = 1-\left(1-p_{\mathrm{d}}\right)^{5JK-2K}\left[1+(5J-2)p_{\mathrm{d}}+\tfrac{1}{2}(5J-1)(5J-2)p_{\mathrm{d}}^{2}\right]^{K} \tag{32}$$

- *Multistatic centralised* —

$$p_{\mathrm{ti,f}} = 1-\left(1-p_{\mathrm{d}}\right)^{5JK-2}\left[1+(5JK-2)p_{\mathrm{d}}+\tfrac{1}{2}(5JK-1)(5JK-2)p_{\mathrm{d}}^{2}\right] \tag{33}$$

where, for the monostatic cases, J is the number of source–receiver pairs in the field and, for the multistatic cases, J is the number of sources and K is the number of receivers.

Setting $J = K$ allows comparison of monostatic and multistatic cases. Some examples are shown in Figure 2, each panel of which contains the single-sensor result (Eq. 2) as a reference and then, in order from right to left, monostatic distributed, monostatic centralised, multistatic distributed and multistatic centralised cases. For this track-initiation rule, the order is the same for all sizes of field. This has implications for a possible course of action in the situation where the false-detection rate is insupportably high with the multistatic centralised architecture. Recall the argument in Section 2.3:

- in the monostatic centralised architecture, the false-detection rate at the input to the track-initiation algorithm is approximately J times greater than that at the input of any one of the J trackers in the monostatic distributed architecture;
- in the multistatic distributed architecture, it is also approximately J times greater;
- in the multistatic centralised architecture, it is approximately JK times greater.

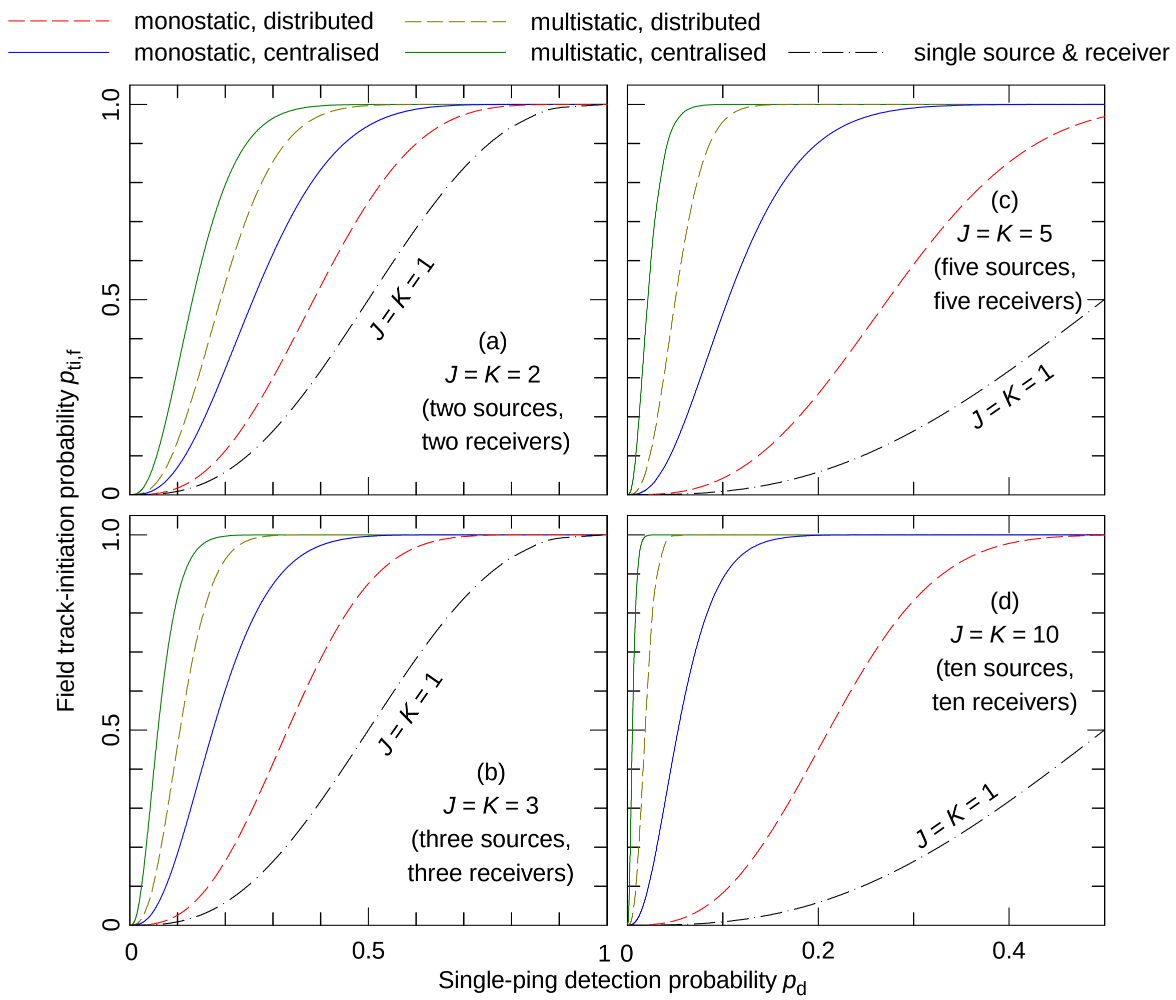

Figure 2: Effects of network architecture for fields of (a) 2, (b) 3, (c) 5 and (d) 10 sources and receivers. Full curves indicate cases with centralised tracking and broken curves cases with distributed tracking. The chain curve (same in each panel) shows the single-sensor case.

Hence, if the false-detection rate is too high when operating multistatic centralised and no other methods are available to reduce it, Figure 2 suggests that it is better to switch to multistatic distributed than to monostatic centralised; in other words, it is better to keep the multistatics than keep the centralised tracking.

Figure 3 shows some cases where numbers of sources and receivers differ. (These can only be multistatic.) As in Figure 2, the single-sensor result is shown in each panel to provide a reference. Cases with one receiver are uninteresting, and so are not shown, but cases with one source and many receivers are another matter. The panels of Figure 3 show a regular progression: with a small number of receivers, the distributed and centralised cases alternate as additional sources are added, but, as the number of receivers grows, the distributed-tracking architecture steadily falls behind so that, for ten receivers (Fig. 3d), centralising the tracking with just one source (full red curve in Fig. 3d) performs as well as three sources and distributed tracking (broken yellow curve).

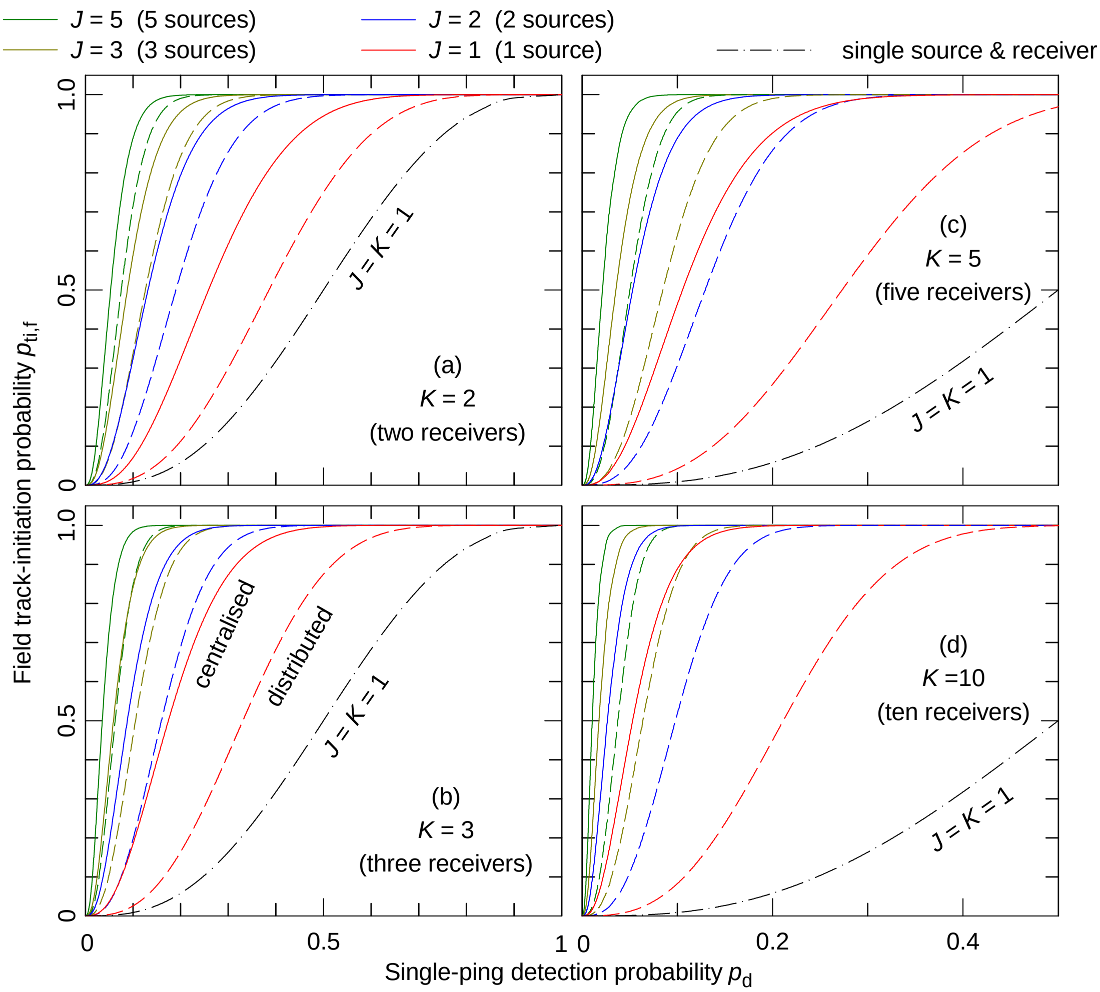

Figure 3: Effects of increasing the number of sources from 1 to 5 in multistatic fields of (a) 2, (b) 3, (c) 5 and (d) 10 receivers. Full curves indicate centralised tracking and broken curves distributed tracking. The chain curve (same in each panel) shows the case of a single source and single receiver.

3.3 Effect of a Tracking-Algorithm Limitation (Multiple Monostatic)

Tracking algorithms typically maintain a velocity estimate in order to predict where to expect the next detection, and some obtain and update the velocity estimate using detections at different times. Such trackers cannot initiate a track[c] on detections from one ping, no matter how many there may be. This was the case with the tracker used by us in a study of tracking performance in a multiple-monostatic sonar field [4,15,17], though for a slightly different reason. In that study, we effectively assumed that all sonars in the field ping simultaneously. Hence, one such 'ping cycle' does not provide the tracker with the necessary temporal information. In principle, tracking could start with three detections in

[c] This process is often called 'confirming a track' in the tracking literature. Similarly, 'detections' are frequently referred to as 'measurements', since the detection is usually accompanied by some type of measurement of the target's position.

two ping cycles, but this also would have required altering the track-initiation (i.e. confirmation) part of the standard algorithm, so we chose instead to demand a minimum of three ping cycles for track initiation. The consequence is a lower p_{ti} than the $p_{\text{ti,f}}$ values shown in Figures 2 and 3, since some events are now considered insufficient for track initiation, namely all those producing three or more detections in one or two ping cycles.

The expression for field p_{ti} in this situation is in fact just that used in Reference 6, since the quantity given by Equation (5) of that report is the probability of one *or more* detections from m receivers in one ping cycle. Combining Equations (5) and (6) of Reference 6, applying the 3-in-5 track-initiation rule and switching to the notation used herein gives, for a field of J monostatic sensors,

$$p_{\text{ti,f}} = \left[1-\prod_{j=1}^{J}\left(1-p_{\text{d},j}\right)\right]^3 \left\{10-15\left[1-\prod_{j=1}^{J}\left(1-p_{\text{d},j}\right)\right]+6\left[1-\prod_{j=1}^{J}\left(1-p_{\text{d},j}\right)\right]^2\right\}. \tag{34}$$

The further assumption of uniform detection probability (all p_{d}s equal), as in the previous subsection, reduces Equation (34) to

$$p_{\text{ti,f}} = 1-10\left(1-p_{\text{d}}\right)^{3J}+15\left(1-p_{\text{d}}\right)^{4J}-6\left(1-p_{\text{d}}\right)^{5J}. \tag{35}$$

Figure 4 compares Equation (35) (broken curves) to Equation (31) (full curves). As expected, the more general result—Equation (31)—always gives higher $p_{\text{ti,f}}$ values. The difference (inset to Fig. 4) shows the performance gain available to tracking algorithms that can initiate a track on one ping cycle, rather than three, for a 3-in-5 track-initiation rule. This performance gain refers to an increase in track-initiation probability—naturally,

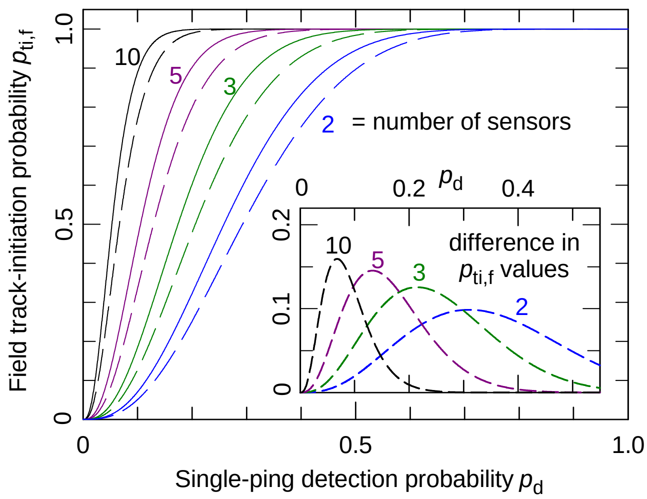

Figure 4: Effect of a tracking-algorithm limitation preventing track initiation on fewer than 3 ping cycles for the 3-in-5 track initiation rule, the monostatic centralised network architecture and fields comprising J = 2, 3, 5, and 10 *sensors: full curves – without the limitation; broken curves – with the limitation. The full curves are the same as the blue curves in Figure 2 and the broken curves are the same as the corresponding curves in Figure 4 of Reference 6. The inset shows a detail of the difference in* $p_{ti,f}$ *values for the two cases.*

given the metric employed—however there is an additional advantage in potentially initiating a track sooner: three ping cycles take longer to carry through than one.

4. Conclusion

This note is part of a high-level study of the performance of multistatic sonar networks for anti-submarine warfare. The main focus is technical—the derivation of formulae for local track-initiation probability given single-ping detection probabilities. To achieve this, we first must conceptualise the method of operating a sonar network. We identify four network types or 'architectures', and also look at the effect of varying one of them. The performance metric is local track-initiation probability $p_{\mathrm{ti,f}}$ of the field as a whole. Metric selection is very important; an inappropriate choice may well lead to wrong conclusions. The utility of track-initiation probability, both local and cumulative, in the context of anti-submarine warfare has been demonstrated previously [1,6].

We also apply sufficient approximations to reduce the $p_{\mathrm{ti,f}}$ formulae to simple algebraic expressions in single-ping p_{d}. Where they can be derived, algebraic relationships allow much greater breadth and depth of insight into the behaviour of a system than any other analytical approach, such as, for example, Monte-Carlo simulation. As an example, the present analysis displays:

- the extent to which the performance gains in moving from a monostatic to a multistatic field architecture and also in centralising the tracking are general features of sonar network architecture (Fig. 2)
- the networking benefit of adding extra receivers, and how this multiplies the benefits of centralising the tracking (Fig. 3), and
- for the monostatic centralised architecture, the performance gain available to a tracker that can initiate a track on a single ping cycle, rather than requiring several (Fig. 4).

On the other hand, these conclusions were obtained with the unrealistic assumption that all source–receiver pairs have the same detection probability. Companion reports give results of studies in which p_{d} varies with range for a variety of geographic field layouts [5,18,19].

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19. ABSTRACT

Expressions are derived for the track-initiation probability of a sonar field in terms of single-ping detection probabilities. Five types of network architecture—four main types and one variant—are treated. As well as providing the algebraic foundation for more detailed studies described in companion reports, several general results on the ASW performance of sonar networks are obtained.