

Minimum-Energy Multicast Tree in Cognitive Radio Networks

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Abstract

We address the multicast problem in cognitive radio networks, where secondary users exploit channels temporarily unused by primary users (*i.e.*, spectrum opportunities). The existence of a communication link between two secondary users depends not only on the transmission power of the secondary transmitter and the distance between these two users, but also on the occurrence of spectrum opportunities. This dependency on the occurrence of spectrum opportunities complicates the construction of an efficient multicast tree in cognitive radio networks. By taking into account this dependency, we propose a low-complexity approximation algorithm with bounded performance guarantee for constructing the minimum-energy multicast tree, which transforms the multicast problem into a directed Steiner tree problem. We also demonstrate this dependency by studying the impact of the traffic load of the primary network on the minimum-energy multicast tree.

I. INTRODUCTION

Multicast can provide better support for one to many communications than unicast or broadcast [1], and it thus has many potential applications in both civil and military domains, *e.g.* streaming media, Internet television, and delivery of situational awareness information and commands on the battlefield. One of the most significant problems in implementing the multicast is to construct an energy-efficient multicast tree, where several heuristic or approximation algorithms have been proposed in [2–4] for the conventional wireless networks.

Multicast in cognitive radio (CR) networks has received little attention. In a CR network, secondary users identify and exploit channels temporarily and locally unused by primary users without causing unacceptable interference to primary users [5]. When we calculate the energy consumption for the multicast trees in CR networks, we thus need to consider the energy used for sensing the availability of the channel as well as the energy used for transmissions. The

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14. ABSTRACT We address the multicast problem in cognitive radio networks, where secondary users exploit channels temporarily unused by primary users (i.e., spectrum opportunities). The existence of a communication link between two secondary users depends not only on the transmission power of the secondary transmitter and the distance between these two users, but also on the occurrence of spectrum opportunities. This dependency on the occurrence of spectrum opportunities complicates the construction of an efficient multicast tree in cognitive radio networks. By taking into account this dependency, we propose a low-complexity approximation algorithm with bounded performance guarantee for constructing the minimum-energy multicast tree, which transforms the multicast problem into a directed Steiner tree problem. We also demonstrate this dependency by studying the impact of the traffic load of the primary network on the minimum-energy multicast tree.				
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proportion of the sensing energy is significant especially when spectrum opportunities occur infrequently. In this case, much more energy is consumed for sensing the channel before each successful transmission.

Another elusive twist in constructing an efficient multicast tree for CR networks is that the occurrence of spectrum opportunities is affected by the transmission power of secondary users [6]. If a secondary user uses a high transmission power to reach a relatively large number of multicast nodes, it must wait for the opportunity that no primary receiver is active within its relatively large interference region, which happens less often. As discussed above, the secondary user will spend more sensing energy for the emergence of one opportunity. If, on the other hand, it uses a low power, it can only reach a small number of multicast nodes, and more transmissions, each relying on its own opportunities to emerge, are needed to reach all the multicast nodes. This tradeoff in choosing the secondary users' transmission power further complicates the construction of the minimum-energy multicast tree in a CR network.

To summarize, the construction of the minimum-energy multicast tree in a CR network depends not only on the topology of the secondary network which is essentially determined by the transmission powers of the secondary users, but also on the occurrence of spectrum opportunities which is determined by the transmission powers of the secondary users and the traffic load of the primary network [6]. As a consequence, the minimum-energy multicast tree under a low primary traffic load may not be optimal under a high primary traffic load.

By considering the impact of the occurrence of spectrum opportunities, we propose an approximation algorithm for constructing the minimum-energy multicast tree of a CR network in this paper. The basic idea of this algorithm is to formulate the multicast problem as a directed Steiner tree problem and then apply an approximation algorithm for the directed Steiner tree problem. The algorithm delivers a solution with a bounded approximation ratio $O(\log^2 |D|)$, where $|D|$ is the number of destinations. Let n be the number of secondary users and k their maximal degree¹, then the running time of the approximation algorithm is given by $O((2^k n)^{\log |D|} |D|^{2 \log |D|} + kn2^k)$. We also demonstrate the dependency of the minimum-energy multicast tree on the occurrence of spectrum opportunities by studying the impact of the traffic load of the primary network.

¹The degree of a secondary user is defined as the number of secondary users which are within its maximal transmission range.

II. NETWORK MODEL

We consider a secondary network overlaid with a Poisson distributed primary network, where both networks adopt a slotted transmission structure. Assume that the primary users are mobile and the relative positions of the secondary users are static.

A. Primary Network

At the beginning of each slot, the primary transmitters are distributed according to a two-dimensional Poisson point process X_{PT} with density λ_{PT} . To each primary transmitter, its receiver is uniformly distributed within its transmission range R_p . Here we have assumed that all the primary transmitters use the same transmission power and the transmitted signals undergo an isotropic path loss. Based on the displacement theorem [7, Chapter 5], it is easy to see that the primary receivers form another two-dimensional Poisson point process X_{PR} with density λ_{PT} . Note that the two Poisson processes X_{PT} and X_{PR} are correlated.

B. Secondary Network

In contrast to the case in a conventional ad hoc network, whether the communication is successful between two secondary users depends not only on the transmission power of the secondary transmitter and the distance between these two users, but also on the availability of the communication channel (*i.e.*, the presence of a spectrum opportunity). The latter is determined by the transmitting and receiving activities in the primary network as discussed below.

The notion of spectrum opportunity in a multicast setting is open for interpretation. Its basic composition, however, roots in the definition of spectrum opportunity in unicast [8]. Let A be the secondary transmitter and B its receiver (see Fig. 1). Under the disk signal propagation and interference model, there exists an opportunity from A and B if the transmission from A does not interfere with nearby *primary receivers* in the solid circle, and the reception at B is not affected by nearby *primary transmitters* in the dashed circle. The radius r_I of the solid circle at A , referred to as the interference range of the secondary users, depends on the transmission power of A and the interference tolerance of the primary receivers, whereas the radius R_I of the dashed circle (the interference range of the primary users) depends on the transmission power of the primary users and the interference tolerance of B .

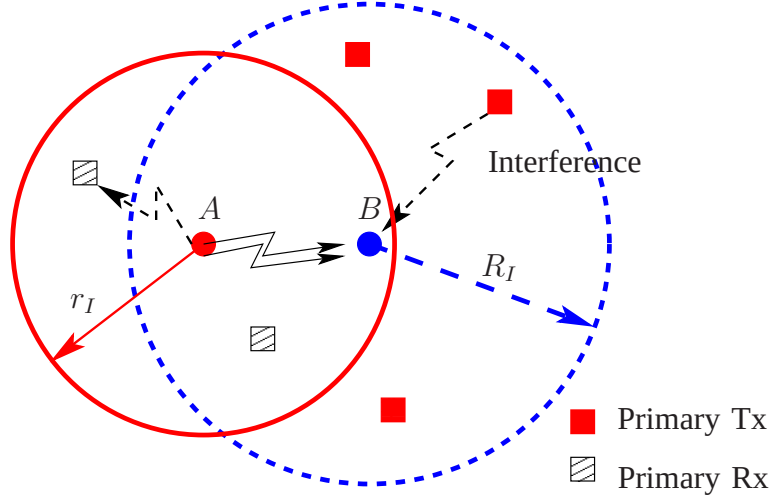


Fig. 1. Definition of spectrum opportunity.

It is clear from the above discussion that spectrum opportunities depend on both transmitting and receiving activities of the primary users. Furthermore, spectrum opportunities are *asymmetric*. Specifically, a channel that is an opportunity when A is the transmitter and B the receiver may not be an opportunity when B is the transmitter and A the receiver.

III. MINIMUM-ENERGY MULTICAST TREE

Let μ be the multicast source and D the set of destinations. Let T be a multicast tree rooted at μ in the secondary network, V_T the node set of T , and $V_{TL} \subseteq V$ the set of leaf nodes, then the objective and the constraint of the minimum-energy multicast tree problem is given by

$$\arg \min_T \left\{ \mathbb{E} \left[\sum_{\nu \in V_T - V_{TL}} e(\nu) \right] \right\} \text{ s.t. } e_{tx}(\nu) \leq e_M(\nu) \text{ for all } \nu \in V_T, \mu \in V_T, \text{ and } D \subset V_T,$$

where $e(\nu)$ denotes the energy used by node ν consisting of the sensing energy and the transmission energy, $e_{tx}(\nu)$ the energy used by node ν for each transmission, and $e_M(\nu)$ the maximal allowable transmission energy for node ν . The sum of $e(\nu)$ does not include V_{TL} because the leaf nodes in the multicast tree neither transmit nor detect the opportunity.

In general, the solution to the minimum-energy multicast tree should specify two parameters for each secondary node²: the transmission power and the group of its intended receivers. But

²If a secondary node is not included in the multicast tree, then its transmission power is zero and the group of its intended receivers is an empty set.

it can be shown in the proof of Theorem 1 that given the group of the intended receivers, the optimal transmission power is determined by the distance to the farthest intended receiver. It follows that the minimum-energy multicast tree is fully determined by the group of the intended receivers for each secondary node.

The approximation algorithm for the minimum-energy multicast tree is based on an auxiliary directed graph with edge weights. We will address the construction of the auxiliary graph G and the calculation of the edge weights in the following two subsections.

A. Construction of the Auxiliary Graph G

The auxiliary graph G is constructed in the following way. For each secondary node μ with degree k , we add $2^k - 1$ multicast nodes ν_i ($1 \leq i \leq 2^k - 1$) which represent all possible groups of its intended receivers. Then we connect the secondary node μ with each of its $2^k - 1$ multicast nodes by a directed edge and assign a positive weight to each edge (see Fig. 2). For the directed edge (μ, ν_i) , the assigned weight w_i is equal to the average energy required for transmitting successfully from μ to the group of the intended receivers denoted by ν_i . Finally, we connect each ν_i with its corresponding intended receivers by directed edges and assign zero weight to all those edges.

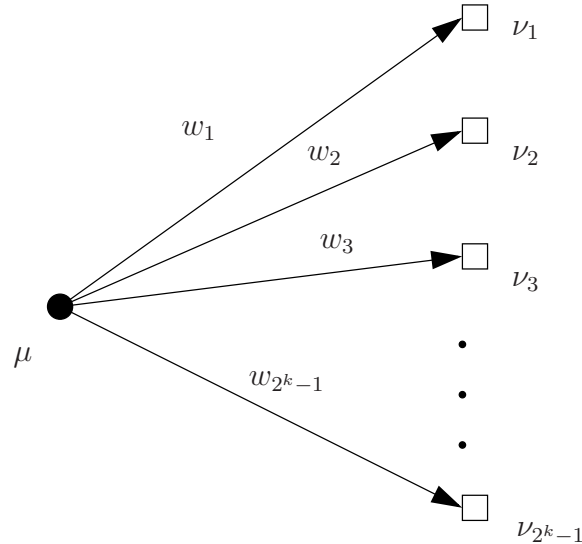


Fig. 2. Generation of multicast nodes for node μ in the secondary network.

Based on the above construction, we see that G is an edge-weighted directed graph, and it has

two types of nodes: user node and multicast node. The user nodes represent the secondary users, and the multicast nodes specify the transmission power and the group of intended receivers for the user nodes. Let V_G be the vertex set of G , and E_G the edge set of G . Given the source $\mu \in V_G$ and a destination set $D \subseteq V_G$, the directed Steiner problem $\mathcal{A}(\mu, D)$ is to find a tree T rooted at μ which spans all the nodes in D with the minimum weight. Then we have the following theorem about the relationship between the directed Steiner tree in G and the minimum-energy multicast tree in the secondary network.

Theorem 1: Given a multicast session in the secondary network, let μ be the source, and D the destination set. Define the weight of a tree in the auxiliary graph G as the total sum of the weights of the edges in the tree, and the weight of a multicast tree in the secondary network as the total sum of the average energy used by the nodes in the multicast tree. Then the directed Steiner tree rooted at μ which spans all the nodes in D has the same weight as the minimum-energy multicast tree in the secondary network.

Proof: Although the set of the transmission power that each secondary node can choose is continuous, it suffices to consider a finite subset of it in order to obtain the minimum-energy multicast tree. For a secondary node μ in the minimum-energy multicast tree, let $\mu_1, \mu_2, \dots, \mu_k$ be the secondary nodes within the maximum transmission range of μ which are listed in an ascending order of the distance from μ . Then under the disk signal propagation model, the optimal transmission power for μ must belong to $\{0, Cd_1^\alpha, Cd_2^\alpha, \dots, Cd_k^\alpha\}$, where C is a constant, d_i ($1 \leq i \leq k$) is the distance between μ and μ_i , and α path-loss exponent. Let $d_0 = 0$ and $d_{k+1} = \infty$. Suppose that the optimal transmission power takes some other value $p^* \notin \{0, Cd_1^\alpha, Cd_2^\alpha, \dots, Cd_k^\alpha\}$ with $Cd_i^\alpha < p^* < Cd_{i+1}^\alpha$ for some i . We can always replace p^* by Cd_i^α for the node μ , and it does not change the multicast tree but reduces the energy cost of the multicast tree. It follows that the minimum-energy multicast tree is fully determined by the group of intended receivers for each secondary node. Based on the construction of the auxiliary graph G , we thus conclude Theorem 1. ■

Based on Theorem 1, the problem of the minimum-energy multicast tree is transformed into a directed Steiner problem via the construction of the auxiliary graph G . By applying an approximation algorithm for the directed Steiner problem [9] to G (see Sec. IV), we can obtain an approximate solution to the problem of the minimum-energy multicast tree.

B. Calculation of the Edge Weights

Let ν denote the multicast node which is connected to a group of intended receivers $\{\mu_1, \mu_2, \dots, \mu_m\}$ for secondary node μ , then the weight w of the directed edge (μ, ν) is the expectation of the sum of sensing energy and transmission energy until the last one in the group $\{\mu_1, \mu_2, \dots, \mu_m\}$ receives successfully from μ , and it can be expressed as

$$w = \mathbb{E} \left[\sum_{t=1}^T (e_s + e_{tx} \mathbb{I}_{tx}(t)) \right], \quad (1)$$

where t is the slot index, T a stopping time which is the number of slots for μ transmitting successfully to all the receivers that are connected to ν , e_s the sensing energy per slot, e_{tx} the energy used for each transmission, and $\mathbb{I}_{tx}(t)$ an indicator function which is 1 if μ transmits in slot t and 0 otherwise.

Consider the following two transmission schemes for the secondary node μ : one is that μ transmits even if μ only sees the opportunity from μ to some of the receivers that are connected to ν ; the other is that μ does not transmit until μ sees the opportunity from μ to all the receivers that are connected to ν . If μ uses the first transmission scheme (referred to as sequential scheme), it may possibly transmit more than once, where each time it transmits to disjoint subgroups of receivers that are connected to ν . Although the calculation of (1) can be formulated into solving the absorbing time of a Markov chain in this case, the obtained expression is complicated and difficult to evaluate.

If μ adopts the second transmission scheme (referred to as simultaneous scheme), μ transmits only once to all the receivers that are connected to ν when μ sees the opportunity from μ to all the receivers. Then in this case, the edge weight w can be rewritten as

$$w = \mathbb{E}[T]e_s + e_{tx},$$

where T is the first slot that μ sees the opportunity from μ to all the receivers that are connected to ν . Due to the i.i.d. distribution of the primary network over slots, T is obviously a geometric random variable with parameter p_0 , where p_0 is the probability of having an opportunity from μ to all the receivers $\{\mu_1, \mu_2, \dots, \mu_m\}$ at any given time. Under the disk signal propagation model, we thus have that

$$\begin{aligned} w &= e_s/p_0 + e_{tx} \\ &= e_s/p_0 + Cd^\alpha, \end{aligned}$$

where C is a constant, d the distance between μ and its farthest intended receiver, and α the path-loss exponent.

The following proposition gives the expression for p_0 .

Proposition 1: Let λ_{PT} be the density of the primary transmitters. Let R_p and R_I denote the transmission range and the interference range, respectively, of the primary users. Given a secondary user μ and a group of its intended receivers $\{\mu_1, \mu_2, \dots, \mu_m\}$, let r_I denote the interference range³ of μ when its transmission reaches all the intended receivers. Choosing μ as the origin of the polar coordinate system, we have that the probability p_0 of having an opportunity from μ to all its intended receivers is given by

$$p_0 = \exp \left(-\lambda_{PT}\pi r_I^2 - \iint_{\mathcal{S}_U} \lambda'_{PT}(r) r dr d\theta \right), \quad (2)$$

where

$$\lambda'_{PT}(r) = \lambda_{PT} \left[1 - \frac{S_I(r, R_p, r_I)}{\pi R_p^2} \right],$$

$S_I(r, R_p, r_I)$ is the common area of two circles with radii R_p and r_I and centered r apart, and $\mathcal{S}_U$ is the union of the circles with radii R_I centered at each intended receiver μ_i ($1 \leq i \leq m$).

Furthermore, we have

$$\exp \left[-\lambda_{PT}(\pi r_I^2 + S_U) \right] < p_0 \leq \min \left\{ \exp \left(-\lambda_{PT}\pi r_I^2 \right), \exp \left(-\lambda_{PT}S_U \right) \right\}, \quad (3)$$

where S_U is the area of the region $\mathcal{S}_U$.

Proof: From the definition of unicast spectrum opportunity given in Sec. II-B, we know that a spectrum opportunity from μ to all its intended receivers $\{\mu_1, \mu_2, \dots, \mu_m\}$ occurs if and only if there are no primary receivers within distance r_I of μ and no primary transmitters within distance R_I of each receiver μ_i ($1 \leq i \leq m$).

Let $\mathbb{I}(\mu, d, \text{rx/tx})$ denote the event that there exists primary receivers/transmitters within distance d of a secondary user μ . Let $\overline{\mathbb{I}(\mu, d, \text{rx/tx})}$ denote the complement of $\mathbb{I}(\mu, d, \text{rx/tx})$. Then

³Since the minimum transmission power for successful reception is, in general, higher than the maximum allowable interference power, it follows that the transmission range r_p of μ is smaller than its interference range r_I , where r_p is set to be the farthest distance between μ and its intended receivers. Furthermore, under the disk signal propagation and interference model, we have $r_p = \beta r_I$ ($0 < \beta < 1$).

the probability p_0 of having an opportunity from μ to $\{\mu_1, \mu_2, \dots, \mu_m\}$ is given by

$$\begin{aligned} p_0 &= \Pr\{\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})} \cap \overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})}\} \\ &= \Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})} \mid \overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\} \Pr\{\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\}. \end{aligned} \quad (4)$$

Since the primary receivers admit a Poisson point process with density λ_{PT} , it follows that

$$\Pr\{\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\} = \exp(-\lambda_{PT}\pi r_I^2). \quad (5)$$

Given $\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}$, *i.e.*, there are no primary receivers within distance r_I of μ , based on Independent Thinning Theorem [7, Chapter 5], we have that the primary transmitters form an inhomogeneous Poisson point process with density specified by

$$\lambda'_{PT}(r) = \lambda_{PT} \left[1 - \frac{S_I(r, R_p, r_I)}{\pi R_p^2} \right].$$

Thus,

$$\Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})} \mid \overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\} = \exp\left(-\iint_{S_U} \lambda'_{PT}(r) r dr d\theta\right). \quad (6)$$

Plugging (5, 6) into (4) yields (2).

From (4), we can easily see that

$$\begin{aligned} p_0 &\leq \min\{\Pr\{\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\}, \Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})}\}\} \\ &= \min\{\exp(-\lambda_{PT}\pi r_I^2), \exp(-\lambda_{PT}S_U)\}. \end{aligned}$$

On the other hand, it follows from (6) that

$$\Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})} \mid \overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\} > \Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})}\}.$$

By considering (4), we have that

$$\begin{aligned} p_0 &> \Pr\{\overline{\mathbb{I}(\mu, r_I, \mathbf{rx})}\} \Pr\{\overline{\mathbb{I}(\mu_1, R_I, \mathbf{tx})} \cap \dots \cap \overline{\mathbb{I}(\mu_m, R_I, \mathbf{tx})}\} \\ &= \exp[-\lambda_{PT}(\pi r_I^2 + S_U)]. \end{aligned}$$

Thus, the inequality (3) is shown. ■

From (2), we can see that the computation of p_0 requires a double integral over an irregular region which is the union of several circles. Due to the complexity of the computation, we use the lower bound on p_0 given in (3) as an approximation of p_0 to obtain the edge weight w , *i.e.*,

$$w \approx \frac{e_s}{\exp[-\lambda_{PT}(\pi r_I^2 + S_U)]} + Cd^\alpha. \quad (7)$$

IV. APPROXIMATION ALGORITHM FOR DIRECTED STEINER TREES

In Sec. III, we transform the problem of the minimum-energy multicast tree into the directed Steiner problem by deriving a directed graph G with weighted edges from the secondary network. In this section, we present an approximation algorithm for the directed Steiner problem [9].

Consider a directed graph $G = (V_G, E_G)$ where V_G is the vertex set, E_G is the edge set, and each edge $e \in E_G$ is associated with a nonnegative weight $w(e)$. Given a source $\mu \in V_G$ and a destination set $D \subseteq V_G$, we recall that the directed Steiner problem $\mathcal{A}(\mu, D)$ is to find a tree T rooted at μ which spans all the nodes in D with the minimum weight. Let $k = |D|$ be the number of destinations. A slightly more general version of the directed Steiner problem is to find a tree T rooted at μ which spans *at least* m nodes in D with the minimum weight for some $m \leq k$, which is denoted by $\mathcal{A}(\mu, m, D)$.

1) *Preliminary Results:* Assume that the directed graph G is strongly connected, *i.e.*, for every pair of nodes μ and ν , there exist at least one directed path in G from μ to ν and at least one directed path in G from ν to μ . Then based on G , we construct a complete graph $\tilde{G}$ which has the same vertex set V_G as G , and we let the weight $w(\mu, \nu)$ of each edge (μ, ν) in $\tilde{G}$ equal the shortest path distance from μ to ν in G . Although $\tilde{G}$ contains more edges than G , the solution $\tilde{T}$ to the directed Steiner problem $\tilde{\mathcal{A}}(\mu, D)$ directly leads to the solution T to the directed Steiner problem $\mathcal{A}(\mu, D)$ if we replace the edges of $\tilde{T}$ that do not exist in G by the corresponding shortest paths in G .

Let $w(T)$ denote the weight of a tree T which is the total sum of the weights of the edges in T , and $k(T)$ the number of destinations contained in T , *i.e.*, $k(T) = |V_T \cap D|$. Then we define the density $d(T)$ of tree T as the ratio of the weight of T to the number of destinations contained in T , *i.e.*,

$$d(T) \triangleq \frac{w(T)}{k(T)}.$$

A tree is said to be an l -level tree if no leaf is more than l edges away from the root. It is shown in [10] that for all $l \geq 1$ there exists an l -level tree that provides a $k^{1/l}$ approximation to the general directed Steiner problem $\tilde{\mathcal{A}}(\mu, m, D)$, where $m \leq k$ is the minimum number of destinations required to be contained in the l -level tree. A trivial approximation when $l = 1$ is the combination of the edges of $\tilde{G}$ from the root μ to the m destinations which are closer to μ than the other $k - m$ destinations (in terms of the shortest path distance). This approximation to

$\tilde{\mathcal{A}}(\mu, m, D)$ gives an approximation to $\mathcal{A}(\mu, m, D)$ if we change the edges of $\tilde{G}$ that do not exist in G into the corresponding shortest paths in G . Let T_1 denote the above approximate solution to $\mathcal{A}(\mu, m, D)$ and T^* the directed Steiner tree for $\mathcal{A}(\mu, m, D)$ (i.e., the optimal solution). It can be easily shown that the

$$\frac{w(T_1)}{w(T^*)} \leq m.$$

2) *Algorithm:* This approximation algorithm is a special case of the approximation algorithm for the general directed Steiner tree problem $\tilde{\mathcal{A}}(\mu, m, D)$ [9]. The basic idea is to recursively obtain a $\lceil \log k \rceil$ -level tree $\tilde{T}$ which provides a $\log k(\log k - 1)k^{1/\log k}$ approximation to the directed Steiner problem $\tilde{\mathcal{A}}(\mu, D)$ and then transform $\tilde{T}$ into a tree T in the original directed graph G .

Approximate Algorithm for Directed Steiner Problem $\mathcal{A}(\mu, D)$

INPUT:

- (i) An edge-weighted directed graph $G = (V_G, E_G)$, where each edge $e \in E_G$ is associated with a nonnegative weight $w(e)$;
- (ii) A source $\mu \in V_G$ and a destination set $D \subseteq V_G$ with size $k = |D|$.

OUTPUT: A tree T rooted at μ that spans all the nodes in D .

STEP 0 If not all the destinations in D are reachable from μ , then **RETURN** ϕ .

STEP 1 Derive a complete graph $\tilde{G}$ from G . Specifically, for every pair of nodes μ and ν in G , if there exists a directed edge (μ, ν) from μ to ν in G , i.e., $(\mu, \nu) \in E$, we keep the edge (μ, ν) along with its weight in $\tilde{G}$; if not, i.e., $(\mu, \nu) \notin E$, we add an edge (μ, ν) in $\tilde{G}$ and assign the shortest path distance from μ to ν in G to the edge as its weight in $\tilde{G}$.

STEP 2 Invoke the function $F_I(\mu, k, D)$ which returns an I -level tree $\tilde{T}$ as an approximate solution to the directed Steiner problem $\tilde{\mathcal{A}}(\mu, D)$, where

$$I = \lceil \log k \rceil.$$

Given $m \leq |D|$, the function $F_i(\mu, m, D)$ for $i \geq 1$ is defined recursively as follows:

```

 $k \leftarrow |D|$  (the number of nodes in  $D$ ).
IF  $i = 1$ 
     $\tilde{T} \leftarrow$  the combination of the edges of  $\tilde{G}$  from  $\mu$  to the  $m$  nodes in  $D$ 
        which are closer to  $\mu$  than the other  $k - m$  nodes in  $D$ .
    RETURN  $\tilde{T}$ .
ELSE
     $\tilde{T} \leftarrow \phi$ .
     $D_{left} \leftarrow D$  (the set of destinations that are not contained in  $\tilde{T}$ ).
     $m_{left} \leftarrow m$  (the number of destinations that are needed by  $\tilde{T}$ ).
    WHILE  $m_{left} > 0$ 
         $T_{best} \leftarrow \phi$  and  $d(T_{best}) = \infty$ .
        FOR each vertex  $\nu \in V$  and each  $m'$  ( $1 \leq m' \leq m_{left}$ )
             $T' \leftarrow F_{i-1}(\nu, m', D_{left}) \cup \{(\mu, \nu)\}$ .
            IF  $d(T') < d(T_{best})$  (compare the density of the two trees)
                 $T_{best} \leftarrow T'$ .
         $\tilde{T} \leftarrow \tilde{T} \cup T_{best}$  (combine  $\tilde{T}$  and  $T_{best}$  into one tree).
         $m_{left} = m_{left} - |D_{left} \cap V(T_{best})|$ .
         $D_{left} = D_{left} - V(T_{best})$  ( $V(T_{best})$  is the vertex set of  $T_{best}$ ).
    RETURN  $\tilde{T}$ .

```

Notice that all the operations of the function $F_i(\mu, m, D)$ are performed on the complete graph $\tilde{G}$, not the original directed graph G .

STEP 3 Transform the tree $\tilde{T}$ in $\tilde{G}$ into a tree T in G by changing the edges of $\tilde{T}$ that do not exist in G into the corresponding shortest paths in G . The detailed procedure is shown as below.

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 $T \leftarrow \phi.$ 
FOR each edge  $e \in E(\tilde{T})$  ( $E(\tilde{T})$  is the edge set of tree  $\tilde{T}$ )
    IF  $e \in E(G)$  ( $E(G)$  is the edge set of graph  $G$ )
         $T \leftarrow T \cup e.$ 
    ELSE
         $L_e \leftarrow$  the shortest path in  $G$  corresponding to  $e.$ 
         $T \leftarrow T \cup L_e.$ 
RETURN  $T.$ 

```

This is the end of this algorithm.

Remark. If $\tilde{T}$ is an optimal solution to the directed Steiner problem $\tilde{\mathcal{A}}(\mu, D)$, then after we replace the edges of $\tilde{T}$ that do not exist in G by the corresponding shortest paths in G , we obtain an optimal solution T to the directed Steiner problem $\mathcal{A}(\mu, D)$. It is easy to see that T does not have duplicate edges of G , otherwise removing these additional edges yields a tree T' with a smaller weight than $\tilde{T}$ which leads to a contradiction. But since $\tilde{T}$ is only an approximate solution to the directed Steiner problem $\tilde{\mathcal{A}}(\mu, D)$, it is possible that during step 3, before adding edge e or the shortest path L_e corresponding to edge e to the tree T , either the edge e or some edge on the shortest path L_e is already included in the tree T . In this case, this duplicate edge is not added to the tree T after the combination of T and e or L_e . It implies that

$$w(T) \leq w(\tilde{T}).$$

On the other hand, it follows from the relation between the original directed graph G and the complete graph $\tilde{G}$ that $w(T^*) = w(\tilde{T}^*)$ where T^* and $\tilde{T}^*$ are the optimal solutions to the directed Steiner problems $\mathcal{A}(\mu, D)$ and $\tilde{\mathcal{A}}(\mu, D)$, respectively. We thus have that the approximation ratio of T is bounded above by the approximation ratio of $\tilde{T}$, i.e.,

$$\frac{w(T)}{w(T^*)} \leq \frac{w(\tilde{T})}{w(\tilde{T}^*)} \sim O(\log^2 k),$$

where $k = |D|$ is the number of destinations in D .

V. APPROXIMATION ALGORITHM FOR MINIMUM-ENERGY MULTICAST TREE

In this section, we present the approximation algorithm for the minimum-energy multicast tree and analyze its approximation ratio and time complexity.

A. Algorithm

As discussed in Sec. III, the approximation algorithm for the minimum-energy multicast tree relies on the approximation algorithm for the directed Steiner problem given in Sec. IV. The procedure of this approximation algorithm is detailed as below.

Approximation Algorithm for Minimum-Energy Multicast Tree

INPUT:

- (i) A secondary network where V denotes the set of nodes.
- (ii) The transmission range of each secondary node which is determined by the maximal transmission power for the secondary node.
- (iii) A source $\mu \in V$ and a destination set $D \subseteq V$ with size $k = |D|$.

OUTPUT: A multicast tree T rooted at μ that spans all the nodes in D .

STEP 1 Construct the auxiliary graph G .

1.1 Add the multicast nodes for each secondary node (see Sec. III-A).

1.2 Use (7) to compute the weight for each edge in G . Since there is no closed-form expression for S_U which is the area of the union $\mathcal{S}_U$ of several circles, we resort to the Monte Carlo method. Specifically, we generate N points uniformly distributed in a square with side length d_s which contains $\mathcal{S}_U$, and we count the number of points that fall into $\mathcal{S}_U$, denoted by N_S . Then we have

$$S_U \approx \frac{N_S}{N} d_s^2.$$

STEP 2 Apply the approximation algorithm for the directed Steiner problem given in Sec. IV to the auxiliary graph G and obtain an approximate Steiner tree T_G in G .

STEP 3 Transform the approximate Steiner tree T_G into a multicast tree T in the secondary network. The detailed procedure is shown in the following table.

```

 $T \leftarrow \phi.$ 
FOR each node  $\nu \in V(T_G)$  ( $V(T_G)$  is the vertex set of tree  $T_G$ )
    IF  $\nu \in V$  (i.e.,  $\nu$  is a user node, where  $V$  is the node set of the secondary network)
         $T \leftarrow T \cup \nu.$ 
    ELSE (i.e.,  $\nu$  is a multicast node)
         $\{\nu_1, \nu_2, \dots, \nu_m\} \leftarrow$  the group of intended receivers denoted by  $\nu.$ 
         $T \leftarrow T \cup \{\nu_1, \nu_2, \dots, \nu_m\}.$ 
         $\mu \leftarrow$  the parent of  $\nu$  in  $T_G.$ 
         $T \leftarrow T \cup (\cup_{i=1}^m (\mu, \nu_i)),$  where  $(\mu, \nu_i)$  is a directed edge from  $\mu$  to  $\nu_i.$ 
RETURN  $T.$ 

```

This is the end of the algorithm.

Remark. It is possible that for some user node ν in the approximate Steiner tree T_G , it has more than one child, that is, it has more than one group of intended receivers. Take two groups G_1, G_2 of intended receivers as an example, and the case of more than two groups of intended receivers can be easily extended. If $G_1 \cap G_2 = \phi$, it implies that the secondary node ν should first multicast to G_1 and then G_2 (a sequential way), or multicast to G_1 in odd slots and to G_2 in even slots (an alternating way). If $G_1 \cap G_2 \neq \phi$, then we can replace G_1 by $G_1 - G_1 \cap G_2$ and do the multicasting as the case of $G_1 \cap G_2 = \phi$. It is easy to see that this replacement of G_1 can only reduce the average energy cost of the resulting multicast tree, and it does not change the resulting multicast tree.

B. Approximation Ratio and Complexity

For the approximation ratio and the complexity, we have the following theorem.

Theorem 2: Let T^* be the minimum-energy multicast tree in the secondary network, and T the multicast tree given by the approximation algorithm in Sec. V-A. Define the weight $w(T)$ of a multicast tree T as the total sum of the average energy used by the nodes in the multicast tree. Then we have

$$\frac{w(T)}{w(T^*)} \leq \lceil \log |D| \rceil (\lceil \log |D| \rceil - 1) |D|^{-\lceil \log |D| \rceil}, \quad (8)$$

where D is the set of destinations and $|D|$ denotes the size of D .

Furthermore, the time complexity of the approximation algorithm for the minimum-energy multicast tree is $O\left((2^k n)^{\log |D|} |D|^{2 \log |D|} + kn2^k\right)$, where k is the maximal degree of the secondary nodes and n the number of secondary nodes.

Proof: Based on Theorem 1, we know that the weight $w(T^*)$ of the minimum-energy multicast tree T is equal to the weight $w(T_G^*)$ of the Steiner tree T_G^* . Moreover, when we transform the approximate Steiner tree T_G into the multicast tree T in Step 3, we have that $w(T) \leq w(T_G)$. Since $w(T_G)/w(T_G^*) \leq \log |D|(\log |D| - 1)|D|^{-\log |D|}$ (Theorem 4 in [9]), it follows that

$$\frac{w(T)}{w(T^*)} \leq \frac{w(T_G)}{w(T_G^*)} \leq w(T_G)/w(T_G^*) \leq \log |D|(\log |D| - 1)|D|^{-\log |D|}.$$

On the other hand, Step 1 takes $O(kn2^k)$ time since every user node has at most $2^k - 1$ multicast nodes, and each multicast node is connected to at most k intended receivers. Similarly, Step 3 can be done in $O(kn2^k)$ time. By noticing that the auxiliary graph G has $O(2^k n)$ nodes and setting $i = \log |D|$ in Theorem 4 [9], we have that Step 2 takes $O\left((2^k n)^{\log |D|} |D|^{2 \log |D|}\right)$ time. Thus, the time complexity of the approximation algorithm is $O\left((2^k n)^{\log |D|} |D|^{2 \log |D|} + kn2^k\right)$. ■

VI. SIMULATION RESULTS

In this section, we present several simulation results. We place 20 secondary nodes uniformly in a $500\text{m} \times 500\text{m}$ square such that they are connected if they use their maximal transmission power, and we fix their positions during the whole simulation. Other simulation parameters are given by: $r_p = R_p = 130\text{m}$, $r_I = R_I = 144\text{m}$, $e_{tx} = 3 \times 10^{-4}d^3$, and $e_s = 1$.

Since there is no comparable algorithm for cognitive radio networks, we choose the approximation algorithm proposed in [4] for conventional wireless networks and compare its performance with that of our approximation algorithm in cognitive radio networks. The percentage of the average energy saving of our approximation algorithm under different traffic load of the primary network (represented by the density of the primary transmitters) is shown in Fig. 3. The conventional approximation algorithm, which treats the secondary network the same as a primary network, transforms the multicast problem into an undirected node-weighted Steiner tree problem, and gives a multicast tree invariant to the primary traffic load. Conversely, our approximation algorithm takes into account the sensing energy, and produces the multicast trees

that adapt to the primary traffic load. We can see that our approximation algorithm can save up to 9 ~ 34% of the total energy, and the energy saving becomes more substantial as the primary traffic load increases.

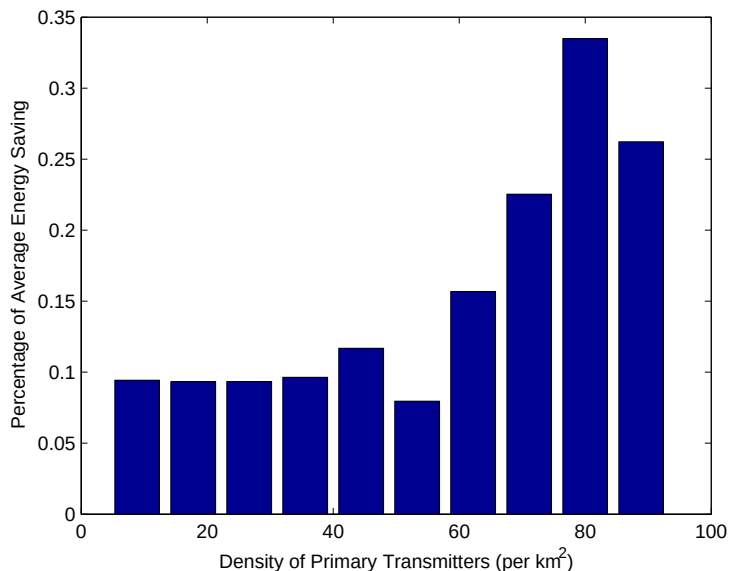


Fig. 3. Percentage of average energy saving vs. density of primary transmitters.

We also study the impact of the primary traffic load on the minimum-energy multicast tree. Fig. 4 shows the average energy cost of the sequential and the simultaneous schemes (see Sec. III-B) for a fixed multicast tree. We observe that the sequential scheme performs worse when the primary traffic load is low, whereas it performs better when the primary traffic load is high. Since the sequential scheme may probably lead to more than one transmission for each edge in the multicast tree, it usually costs more transmission energy than the simultaneous scheme. But on the other hand, it does not need to wait for all the intended receivers seeing the opportunity, and thus it costs less sensing energy than the simultaneous scheme. As the primary traffic load increases, its saving in the sensing energy gradually exceeds its extra cost in transmission energy.

VII. CONCLUSION

In this paper, we have presented a low-complexity approximation algorithm with bounded performance guarantee for constructing the minimum-energy multicast tree in cognitive radio

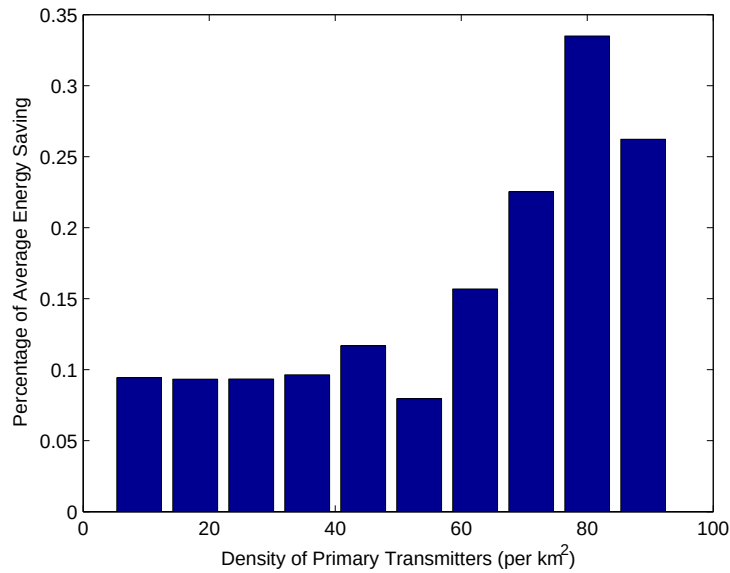


Fig. 4. Average energy cost of the sequential and the simultaneous schemes (in logarithmic scale) vs. density of primary transmitters.

networks. This approximation algorithm takes into account the energy used for sensing the spectrum opportunities, and its constructed multicast trees are adaptive to the traffic load of the primary network. Simulation results have demonstrated its substantial energy saving energy, compared with a well-known conventional approximation algorithm. Furthermore, we study the impact of the primary traffic load on the minimum-energy multicast tree. Simulation results have shown that the simultaneous transmission scheme is more suitable for light primary traffic load, while the sequential transmission scheme is desired for heavy primary traffic load.

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