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# **Nonhomogeneity Detection in CFAR Reference Windows Using the Mean-to-Mean Ratio Test**

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## **ABSTRACT**

A new method designated as the mean-to-mean ratio (MMR) test is proposed for the detection of nonhomogeneities in a radar's Constant False Alarm Rate (CFAR) reference window. No a priori knowledge of the nonhomogeneity topology is assumed. Analysis using the Monte-Carlo method based on Rayleigh clutter and Swerling I target models is presented. Target-like interferences which seriously degrade the detection performance of the cell-averaging CFAR detector can be detected with a higher probability by the MMR test.

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## **Executive Summary**

In radar Constant False Alarm Rate (CFAR) signal processing, the cell-averaging CFAR (CA-CFAR) is the most popular algorithm employed in practical radar detection. As the CA-CFAR detection performance degrades seriously when the reference window used in the estimation of the mean noise level is contaminated by nonhomogeneous samples, many modifications have been proposed. Each of these modified CFAR algorithms has its own advantages and drawbacks, depending on the topology of the nonhomogeneity. They all, nevertheless, share the same design methodology in that an attempt is made to censor out the inappropriate reference samples. Since censoring operations result in a reduced number of reference samples, a higher detection loss is inevitable. Therefore, a censoring operation should only be performed when it is absolutely necessary.

The focus of this report is on detecting the presence of nonhomogeneous samples in the reference window prior to censoring, which is an important test that receives less attention in the literature. Based on the existence of rare events, a nonhomogeneity detection scheme designated as the mean-to-mean ratio (MMR) test is proposed. No a priori knowledge of the nonhomogeneity topology is assumed. Results obtained from Monte-Carlo simulations based on Rayleigh clutter and Swerling I target models are presented.

When being implemented in parallel with a CA-CFAR detector, target-like samples that are not detected by the CA-CFAR and yet have a deleterious effect on CA-CFAR performance can be detected with higher probabilities by the MMR test.

## Author

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Tri-Tan Van Cao graduated from Flinders University, South Australia with a Bachelor of Biomedical Engineering (1998), and a Ph.D. in Electrical Engineering (2002) specialising in Nonlinear Automatic Control. He has been with DSTO since 2002, working mainly in the area of radar *constant false alarm rate* (CFAR) detection.

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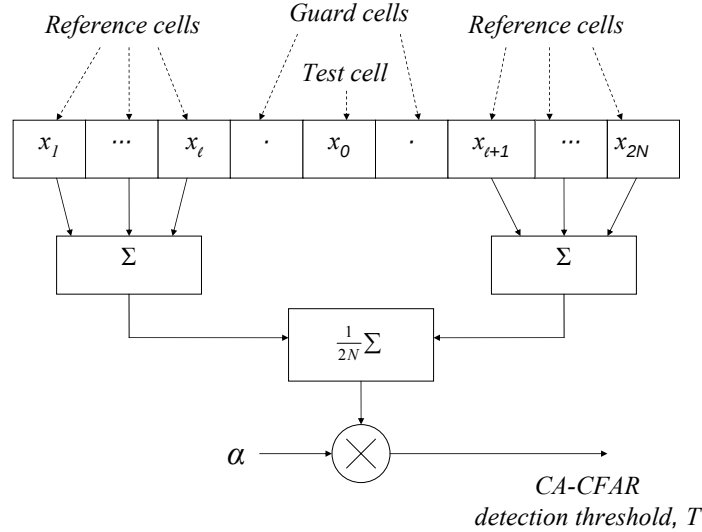
# 1 Introduction

A radar detection process involves testing whether the signal level in the resolution cell under test exceeds a detection threshold. In modern radar systems, the detection threshold is adaptively adjusted according to the background clutter and noise levels using a *constant false alarm rate* (CFAR) processor [1].

The most basic form of the adaptive threshold processor is the well-known *cell-averaging* CFAR (CA-CFAR) [2]. As shown in Figure 1, a CA-CFAR processor receives input from the square law detected video range samples (also known as *range cells*) which consists of the sample  $x_0$  in the *test cell* (where a decision on target presence or absence is to be made) and  $2N$  *reference* samples  $x_1, x_2, \dots, x_{2N}$  in the neighbourhood of the test cell. A few immediate neighbouring cells known as *guard cells* are excluded to prevent possible power spill-over from the test cell. A decision on whether a target is present or absent in the test cell is performed by verifying the following two alternative hypotheses:

$$x_0 \underset{H_0}{\overset{H_1}{\gtrless}} T, \quad (1)$$

*i.e.*, either hypothesis  $H_1$  of target presence is declared to be true if the sample  $y$  in the test cell is greater than an adaptive threshold  $T$ , or hypothesis  $H_0$  of target absence is verified otherwise. The threshold  $T$  is formed by multiplying the interference estimate (which is the sample mean of the  $2N$  reference samples) with a constant  $\alpha$  (the value of which is determined by the required false alarm rate).



**Figure 1:** Formation of a CA-CFAR detection threshold.

Under the condition that the sample in each reference cell is *independent and identically distributed* (*iid*) and is governed by the exponential distribution, the performance of the

CA-CFAR processor is optimal (in the sense that the detection probability is maximised for a given false alarm rate) when the number of reference cells is large [2]. However, there are many detection problems associated with the CA-CFAR algorithm if the assumption of identical statistics of the reference cells is not valid [3]. In practice, there are two common situations when such an assumption no longer holds: (i) there is a clutter edge (*e.g.*, at the border of land and sea), where the energy of interference changes, and (ii) there is an outlier, *e.g.*, a clutter spike, an impulsive interference, or another interfering target. These can result in the masking of weaker targets near stronger targets, excessive false alarms at clutter transitions, and missing of targets near clutter edges.

In order to adapt to the presence of multiple targets and clutter power transitions within the CFAR window, there are two main streams of approaches in the CFAR literature.

The first stream focuses on the modifications of the conventional CA-CFAR [4]. In general, these modifications can be classified into two groups, depending on whether or not the algorithms rely on the ordering of the reference samples for sample selection. The group of CFAR algorithms that do not use rank ordering includes the *smaller-of* CFAR, which is designed to improve target detection in the presence of multiple targets by splitting the reference window into a leading part and a lagging part and then selecting the part with a smaller sample sum for threshold computation [5]; the *greater-of* CFAR which is designed to minimise the false alarm rate at a clutter edge (by selecting the part with a greater sample sum) [6]; the *excision* CFAR in which those samples with amplitudes greater than an excision threshold will not be used for detection threshold computation [7], [8], [9]; the *switching* CFAR where the sample in the test cell is used to select appropriate reference data [10], [11], *etc.* The group of CFAR algorithms that rely on rank ordering includes the *order statistic* CFAR, where the interference estimate is given by the amplitude of the  $k^{th}$  ordered reference sample [12]; the *censored mean level detector* CFAR, where the  $K$  largest ranked samples are discarded and the remaining samples are used for interference estimation via the cell averaging method [13]; the *trimmed mean* CFAR where the smallest  $2N$  ranked samples are also discarded in addition to the  $K$  largest ranked samples [14], *etc.* Each of the modified CFAR algorithms, however, has its own advantages and drawbacks, depending on the operating environment and the statistical model of both target and clutter returns. They all, nevertheless, have the same design methodology in that a censoring operation is made to eliminate inappropriate reference samples in a nonhomogeneous environment.

The second stream aims to either (i) combine the individual algorithms proposed in the first stream in order to use their advantages in certain situations, or (ii) detect the presence of nonhomogeneity in the CFAR window prior to applying suitable CFAR processing. For instance, in [15], based on the detection of clutter power change in the CFAR window, an adaptive censoring algorithm performed on a cell-by-cell basis and ordered reference samples was proposed. A somewhat similar cell-by-cell censoring algorithm without ordered reference samples was discussed in [16]. These two algorithms do not rely on the distribution of the nonhomogeneous samples in the reference window. In [17], a heterogeneous clutter estimation algorithm based on a combination of hypothesis testing and maximum

likelihood estimation procedures was proposed. In [18], a method for the detection of clutter power transition in a CFAR window using the Mann-Whitney test was analysed. In [19], the reference samples were first used to compute a second-order statistic and the leading-lagging mean ratio. These computed data were then used to tailor the selection of appropriate CFAR detectors. An underlying assumption in these works is that there is homogeneity between two clutter changes.

Recently, a new approach for detecting nonhomogeneity in a CFAR window based on the detection of rare events is proposed in [20]. This detection scheme is designated as the *mean-to-mean ratio* (MMR) test, which is simple for implementation since no rank ordering operation is required. Target-like interferences which seriously degrade the detection performance of a CA-CFAR can be detected with a higher probability by the MMR test. This report is a solid consolidation of the work presented in [20]. The new materials include a comparison with five other nonhomogeneity detection methods.

The focus of this report is on detecting the presence of nonhomogeneous samples in the reference window prior to censoring, which is an important test that receives less attention in the literature. No a priori knowledge of the nonhomogeneity topology is assumed. The report is structured as follows. The statistical models for targets and interferences in radar detection is introduced in Section 2, while the nonhomogeneity detection problem is formulated in Section 3. In Section 4, the rare event nonhomogeneity detection algorithm using the MMR test is proposed. The results obtained from Monte-Carlo simulations are then presented in Section 5, followed by the discussion in Section 6.

## 2 Target and Interference Models

Consider a generic CFAR processor that receives the sample  $x_0$  from the test cell where a decision on target presence or absence is to be made and  $2N$  reference samples from the CFAR window  $\Psi = \{x_1, x_2, \dots, x_{2N}\}$ . The following Swerling I targets in a Rayleigh background model are considered [14].

Let  $\Omega$  be the set consisting of the sample  $x_0$  in the test cell and  $2N$  reference samples in the CFAR window  $\Psi$ , *i.e.*,

$$\Omega = \{x_0; \Psi\} = \{x_0, x_1, x_2, \dots, x_{2N}\}, \quad (2)$$

All samples in  $\Omega$  are assumed to be statistically independent, and the amplitude of each sample is described by the following exponential probability density function (pdf):

$$p_z(z) = \frac{1}{\lambda} \exp\left(-\frac{z}{\lambda}\right), \quad z \geq 0, \quad (3)$$

where:

- $\lambda = \lambda_0$  if the sample is thermal noise only where  $\lambda_0/2$  is the thermal noise power;

- $\lambda = \lambda_0(1 + \sigma)$  if the sample contains a target return with an average signal-to-noise ratio (SNR) of  $\sigma$ ;
- $\lambda = \lambda_0(1 + C)$  if the sample contains a clutter return with an average interference-to-noise ratio (INR) of  $C$ .

As both clutter and target returns share the same pdf and the same power model, to facilitate the notation, nonhomogeneity due to either secondary targets or clutter returns is referred to as target-like nonhomogeneity.

### 3 The Nonhomogeneity Detection Problem

The problem of nonhomogeneity detection in  $\Omega$  is to verify the following two alternative hypotheses:

- $H_{00}$ : all samples in  $\Omega$  are thermal noise only, or
- $H_{11}$ : there is at least one target-like sample in  $\Omega$ .

**Remark.** Note that a censoring operation is only necessary when hypothesis  $H_{11}$  is true. Furthermore, it is only when there are at least two target-like samples in  $\Omega$  that a censoring operation has to be performed. This can be explained as follows. Suppose that there is only one target-like sample in  $\Omega$ , then there are only two possibilities: either (i) the sample in the test cell is the target-like sample while other samples in the CFAR window are noise samples; or (ii) the sample in the test cell is noise only while one of the samples in the CFAR window is target-like. In case (i) no censoring operation is required since all samples in the CFAR window are noise only, whereas in case (ii) target is absent in the test cell and therefore there is no detection loss if a censoring operation is not performed.

## 4 The Mean-to-Mean Ratio Test Algorithm

In this section, the mean-to-mean ratio test is proposed and the corresponding target-like detector design procedure is presented.

### 4.1 The Mean-to-Mean Ratio Test for Target-like Detection

Let  $\mu$  be the mean of the samples in  $\Omega$ , *i.e.*,

$$\mu = \frac{1}{2N+1} \sum_{k=0}^{2N+1} x_k. \quad (4)$$

Sorting  $\Omega$  into the following two subsets:

$$\Omega_0 = \{x \in \Omega : x \leq \mu\} \quad (5)$$

$$\Omega_1 = \{x \in \Omega : x > \mu\} \quad (6)$$

*i.e.*,  $\Omega_0$  consists of the small samples that are not greater than their mean while  $\Omega_1$  consists of the large samples that are greater than their mean. Let  $\mu_0$  and  $\mu_1$  be the means of the samples in  $\Omega_0$  and  $\Omega_1$ , respectively.

Consider the following mean-to-mean ratio (MMR) test:

$$\frac{\mu_1}{\mu_0} \geq T, \quad (7)$$

where  $T$  is a constant greater than 1.

Denote  $E_{MMR}(N, T)$  as the event that the set  $\Omega$  survives the MMR test (7), *i.e.*,

$$E_{MMR}(N, T) = \{\text{Inequality (7) is true}\} \quad (8)$$

Denote the probability that event  $E_{MMR}(N, T)$  occurs when hypothesis  $H_{00}$  is true as:

$$F_{MMR} = \text{Prob} \left[ \frac{\mu_1}{\mu_0} \geq T \middle| H_{00} \right] \quad (9)$$

Let  $\epsilon$  be a small positive number, for instance, equal to a CA-CFAR false alarm rate, and  $T_\epsilon$  be a positive constant. Set  $T_\epsilon$  implicitly according to the following equation:

$$F_{MMR} = \text{Prob} \left[ \frac{\mu_1}{\mu_0} \geq T_\epsilon \middle| H_{00} \right] = \epsilon \quad (10)$$

Equation (10) means that: if hypothesis  $H_{00}$  is true, then  $E_{MMR}(N, T_\epsilon)$  is an event of probability  $\epsilon$ . Using the equivalent statements in mathematical logic: "if A then B" is equivalent to "if not B then not A" [21], this is equivalent to: if  $E_{MMR}(N, T_\epsilon)$  is not an event of probability  $\epsilon$ , then hypothesis  $H_{00}$  is not true, *i.e.*, the fact that event  $E_{MMR}(N, T_\epsilon)$  occurs more frequently than the specified false alarm rate  $\epsilon$  indicates that there is at least one target-like sample in  $\Omega$ .

In practice, it is not necessary to wait until event  $E_{MMR}(N, T_\epsilon)$  occurs many times to declare that  $H_{11}$  is true. Instead, at the first instance test (7) is passed with  $T = T_\epsilon$ , the presence of at least one target-like sample in  $\Omega$  can be deduced, since the probability that all samples in  $\Omega$  are thermal noise only is very small and equal to the CA-CFAR false alarm rate  $\epsilon$  as evident in (10).

Denote the probability that test (7) is passed with  $T = T_\epsilon$  when hypothesis  $H_{11}$  is true as:

$$P_{MMR} = \text{Prob} \left[ \frac{\mu_1}{\mu_0} \geq T_\epsilon \middle| H_{11} \right] \quad (11)$$

The applicability of test (7) in deducing the presence of nonhomogeneity in  $\Omega$  lies in the fact that for a properly designed threshold  $T = T_\epsilon$ ,  $F_{MMR}$  is very small under hypothesis  $H_{00}$  while  $P_{MMR}$  is very large under hypothesis  $H_{11}$ . This point will be elaborated further in the following sections.

In summary, for a specified false alarm rate  $\epsilon$ , an MMR detector has only one parameter  $T_\epsilon$  to be designed. The design of  $T_\epsilon$  is as follows.

## 4.2 MMR Detector Design Procedure

- Step 1. Given the size of  $\Omega$ , plot  $F_{MMR}$  as given in (9) over a range of  $T$  (*e.g.*,  $T \in [1, 20]$ ) using Monte-Carlo method.
- Step 2. Select  $T_\epsilon$  based on the required false alarm rate  $\epsilon$  and the result of Step 1.

Once the threshold  $T_\epsilon$  has been found, the nonhomogeneity detection algorithm is as follows.

## 4.3 The MMR Detection Algorithm

Given the set  $\Omega$  as in (2) and the threshold  $T_\epsilon$ :

- Step 1. Sort the samples of  $\Omega$  as described in (5) and (6);
- Step 2. If the MMR test (7) is passed with  $T = T_\epsilon$ , then the presence of at least one target-like sample in  $\Omega$  is declared with a false alarm rate of  $\epsilon$ .

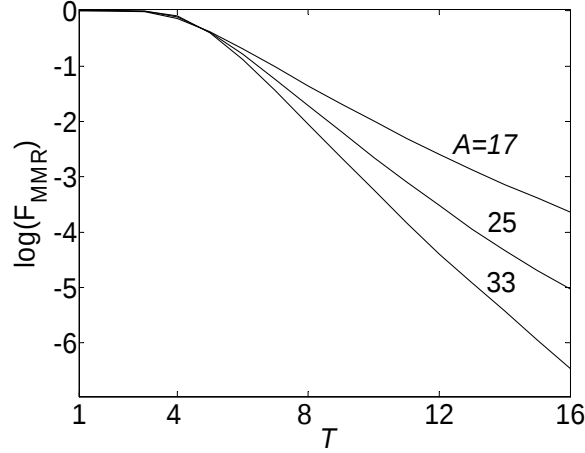
# 5 Results

In this section, the use of the MMR test in detecting the presence of target-like samples in a CFAR window is presented. Suppose that the MMR test is implemented in parallel with a CA-CFAR detector which uses a CFAR window of  $2N$  reference cells. The design of the corresponding MMR test is first demonstrated, and its performance is then examined based on three scenario studies. The purpose of these simulation studies is to demonstrate that the MMR test complements the strength of a CA-CFAR detector, i.e., target-like samples in the reference window, which are blind to the CA-CFAR and seriously degrade CA-CFAR performance, can be detected with higher probabilities by an MMR detector.

Assume that target detection is performed over a range profile that consists of 128 range gates in total. Consider three CA-CFAR reference window sizes  $2N=16, 24$ , and  $32$ . Corresponding to these CA-CFAR window sizes, the sizes of the set  $\Omega$  are  $A = 2N + 1=17, 25$ , and  $33$ , respectively.

## 5.1 Design an MMR Detector

- Step 1. Using Monte-Carlo simulation based on the signal model described in Section 2 with  $10^8$  trials at each data point,  $F_{MMR}$  given in (9) is plotted in Figure 2 for  $T \in [1, 16]$ .
- Step 2. Thresholds for the MMR test with different sizes of  $\Omega$  and different representative false alarm rates can be read from Figure 2 and are shown in Table 1.



**Figure 2:** False alarm rates of the MMR detector for different sizes of  $\Omega$ .

In order to demonstrate the use of the MMR test, suppose that  $2N = 32$  is the CA-CFAR window size of interest, and the required false alarm rate is  $\epsilon = 10^{-6}$ . From Table 1, the MMR threshold corresponding to  $A = 2N + 1 = 33$  at  $F_{MMR} = 10^{-6}$  is  $T_\epsilon = 15.09$ .

**Table 1:** Thresholds for the MMR test.

|           |           | Size of $\Omega$ , $2N + 1$ |       |       |
|-----------|-----------|-----------------------------|-------|-------|
|           |           | 17                          | 25    | 33    |
| $F_{MMR}$ | $10^{-4}$ | 17.75                       | 13.13 | 11.31 |
|           | $10^{-5}$ | 23.44                       | 15.92 | 13.18 |
|           | $10^{-6}$ | 31.04                       | 19.21 | 15.09 |

Let  $P_{CA}$  be the CA-CFAR detection probability of the primary target in the test cell. In the following scenario studies,  $P_{MMR}$  given in (11) is computed using Monte-Carlo simulations with  $10^6$  trials at each data point, whereas  $P_{CA}$  is computed using the closed-form formula presented in [14].

The following three scenarios are studied. Scenarios 1 and 2 represent the case when the detection of the primary target is interfered with by other targets in the CFAR window. In Scenario 1, the primary target and the interfering targets are assumed to have the same signal strength. In Scenario 2, the primary target and the interfering targets are assumed to have different signal strengths. Scenario 3 represents the case when detection of the primary target is interfered with by the presence of a clutter edge.

## 5.2 Scenario 1

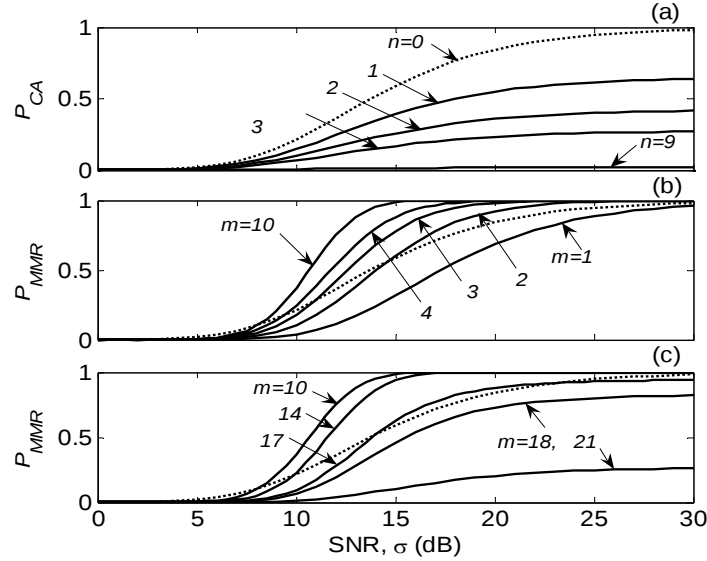
- The test cell contains a (primary) target of SNR  $\sigma$ .

- There are  $n$  target-like samples in the reference window  $\Psi$ , each of which has the same SNR  $\sigma$  as that of the primary target. The set  $\Omega$  then consists of  $m = n + 1$  target-like samples.

$P_{CA}$  of the CA-CFAR detector using  $2N = 32$  reference samples at false alarm rate  $\epsilon = 10^{-6}$  is shown in Figure 3(a). It is evident that as the number of target-like samples in the reference window increases,  $P_{CA}$  decreases significantly and totally collapses when  $n \geq 9$ .

For the same scenario, when the designed MMR detector is implemented in parallel with the CA-CFAR detector,  $P_{MMR}$  given in (11) is shown in Figure 3(b). As the number of target-like samples in the reference window increases from  $n=0$  to 9, the number of target-like samples in  $\Omega$  increases from  $m = n + 1 = 1$  to 10. Unlike  $P_{CA}$ ,  $P_{MMR}$  improves and reaches its best performance at  $m=10$ . The dotted curve marks the homogeneous  $P_{CA}$  (*i.e.*, with  $n=0$ ).

When  $m$  continues to increase above 10,  $P_{MMR}$  begins to decrease as shown in Figure 3(c). Even when half of the CFAR window is filled with target-like samples ( $m=17$ ),  $P_{MMR}$  is still higher than the homogeneous  $P_{CA}$  (marked by the dotted curve) in certain SNR range.

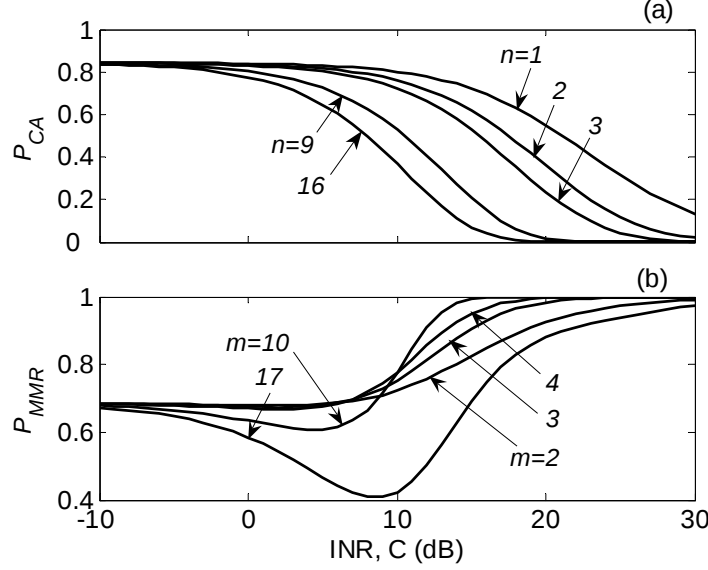


**Figure 3:**  $P_{CA}$  and  $P_{MMR}$  with  $2N = 32$  at  $10^{-6}$  false alarm rate for Scenario 1.

### 5.3 Scenario 2

- The test cell contains a (primary) target of SNR  $\sigma=20$  dB.
- There are  $n$  target-like samples in the reference window  $\Psi$ , each of which has the same INR  $C \in [-10 \text{ dB}, 30 \text{ dB}]$ .

As shown in Figure 4(a), the primary target detection probability  $P_{CA}$  decreases when the number of secondary targets and their INR increase. The critical region is between 10 dB and 20 dB in which  $P_{CA}$  curves undergo the steepest roll-down. Unlike  $P_{CA}$ ,  $P_{MMR}$  only decreases slightly when  $C \in [-10 \text{ dB}, 10 \text{ dB}]$  and then increases when  $C \geq 10 \text{ dB}$  as evident in Figure 4(b). As shown by the  $m=17$  curve in Figure 4(b), even when target-like samples of the same INR  $C=9 \text{ dB}$  occupy up to half of the reference window, the MMR detector still gives a 40% probability of detecting the presence of target-like samples in  $\Omega$ .



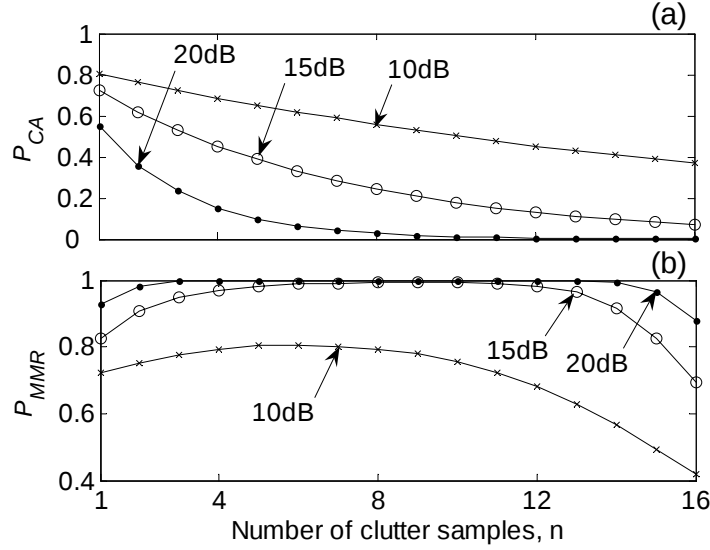
**Figure 4:**  $P_{CA}$  and  $P_{MMR}$  with  $2N = 32$  at  $10^{-6}$  false alarm rate for Scenario 2.

## 5.4 Scenario 3

Detection during clutter transition is now investigated. Consider the scenario in which the CA-CFAR window slides along the range dimension and its right-hand side reaches a clutter region. The number of target-like samples now increases from 1 to 16, *i.e.*, until the right-half CA-CFAR window is totally submerged in the clutter. Assume that:

- The test cell contains a (primary) target of SNR  $\sigma=20 \text{ dB}$ .
- There are  $n$  clutter samples ( $n \in [1, 16]$ ) in the reference window  $\Psi$ , each of which has the same SNR  $C$ .

Figure 5 shows  $P_{CA}$  and  $P_{MMR}$  for 3 clutter power levels, namely,  $C=10 \text{ dB}$ ,  $15 \text{ dB}$ , and  $20 \text{ dB}$  over the interval  $n \in [1, 16]$ . As shown in Figure 5(a), as the clutter power  $C$  increases from 10 dB to 20 dB,  $P_{CA}$  worsens. On the contrary,  $P_{MMR}$  enhances as evident in Figure 5(b). Especially when the whole right-half of the CFAR window is occupied by clutter samples (*i.e.*,  $n = 16$ ) with clutter power  $C = 15 \text{ dB}$  and  $20 \text{ dB}$ , the MMR detector gives a nonhomogeneity detection probability of 70% and 90%, respectively, while the CA-CFAR primary target detection collapses below 10%.



**Figure 5:**  $P_{CA}$  and  $P_{MMR}$  with  $2N = 32$  at  $10^{-6}$  false alarm rate for Scenario 3.

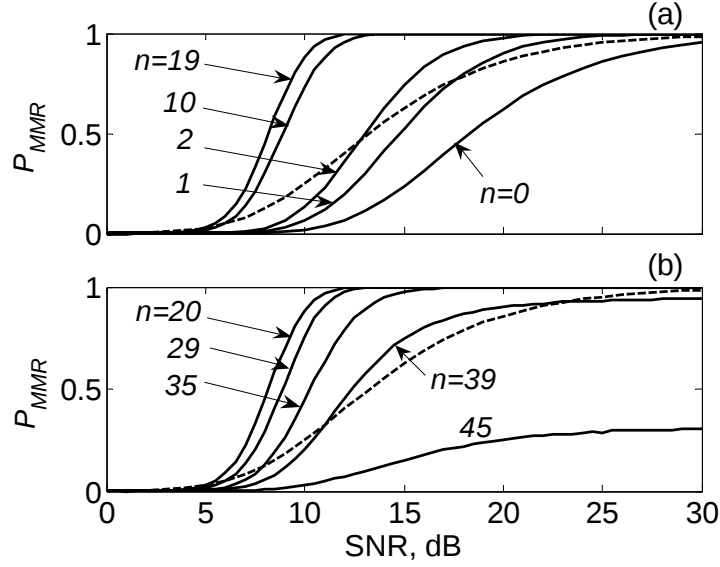
In summary, the MMR test complements the strength of a CA-CFAR detector, in the sense that target-like samples in the reference window, which are blind to the CA-CFAR and seriously degrade CA-CFAR performance, can be detected with higher probabilities by an MMR test.

## 5.5 The MMR Test Using Large Reference Window

The use of the MMR test with CA-CFAR reference window size  $2N = 32$  can only detect the presence of target-like samples that occupy up to half of the reference window. In this section, the use of the MMR test with larger reference window sizes is examined.

Consider two reference windows with  $2N=64$  and 128. Using Monte-Carlo simulations with  $10^8$  trials, the thresholds for the MMR test at  $F_{MMR} = 10^{-6}$  are  $T_{\epsilon}(64) = 10.41$  (for reference window 64) and  $T_{\epsilon}(128) = 8.16$  (for reference window 128).

As shown in Figure 6 for the case  $2N = 64$ ,  $P_{MMR}$  is still comparable to  $P_{CA}$  in the presence of 39 target-like samples. In Figure 7 for the case  $2N = 128$ ,  $P_{MMR}$  is still better than  $P_{CA}$  in the presence of 90 target-like samples. It is observed that the MMR test is better when applied to a larger reference window in the sense that the presence of a larger number of target-like samples can be detected. However, the uncertainty in the whereabouts of those target-like signals in the reference window is also larger. Note that the MMR test only detects the presence of target-like signals, while which samples are target-like is unknown.



**Figure 6:**  $P_{MMR}$  with  $2N = 64$  at  $10^{-6}$  false alarm rate for Scenario 1.

## 6 Discussions

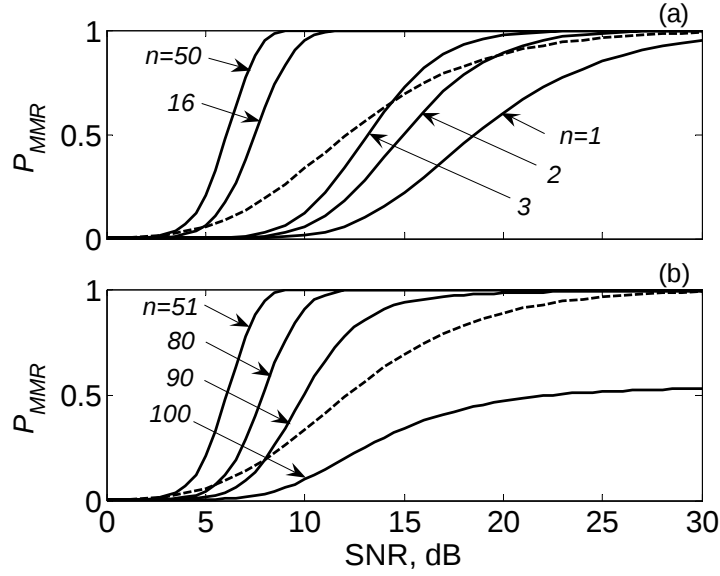
In this section, performance of the MMR test is explained, followed by a comparison with other existing nonhomogeneity detection methods.

### 6.1 The MMR Test

The key point of the MMR nonhomogeneity detection is the increase in the mean  $\mu$  of the samples in  $\Omega$  due to target-like samples. Such an increase can be detected by observing the gap between the mean  $\mu_1$  of the large samples in  $\Omega_1$  and the mean  $\mu_0$  of the small samples in  $\Omega_0$  as dictated by the MMR thresholding test (7).

When there are no target-like samples in  $\Omega$ , there is an approximately equal proportion of the number of the smaller samples in  $\Omega_0$  and the number of the larger samples in  $\Omega_1$ . The ratio  $\mu_1/\mu_0$  is also small since it is the ratio of the mean of the larger noise samples to the mean of the smaller noise samples. In the presence of a few target-like samples with significant SNR (for instance,  $\sigma > 15$  dB), the mean  $\mu$  of the samples in  $\Omega$  increases considerably due to the total sum of the power of the target-like samples. Therefore, fewer but considerably larger samples are sorted to  $\Omega_1$ , and more small samples are sorted to  $\Omega_0$ . The result is that the ratio  $\mu_1/\mu_0$  increases significantly, leading to a much better detection probability  $P_{MMR}$ .

As the target-like samples occupy more than half of the CFAR window ( $m \geq 17$  in Figure 3(c)), the set  $\Omega$  becomes more ‘homogeneous’ in the sense that it now contains more large target-like samples than the small noise samples. As a result, there is a high probability that some of these large target-like samples are sorted to  $\Omega_0$ . This leads to an



**Figure 7:**  $P_{MMR}$  with  $2N = 128$  at  $10^{-6}$  false alarm rate for Scenario 1.

increase in the mean  $\mu_0$ , which in turn makes the ratio  $\mu_1/\mu_0$  smaller. Therefore,  $P_{MMR}$  deteriorates.

## 6.2 Comparison with Other Nonhomogeneity Detectors

In this section, performance of the MMR test is compared with those of representative nonhomogeneity detection algorithms reported in [16], [15], [17], [18], and [19].

In [16], Barbooy et. al. proposed an algorithm for detecting and censoring of nonhomogeneous samples as follows.

- The sum of  $2N$  reference samples is formed:

$$S_{2N} = x_1 + x_2 + \cdots + x_{2N} \quad (12)$$

- Each reference sample is then compared with a threshold  $b_1$  computed as:

$$b_1 = \alpha_0 S_{2N} \quad (13)$$

Samples which exceed this threshold are discarded from the sum and a new sum is formed from the rest of the reference samples. A new threshold  $b_2$  is formed by multiplying this new sum with a new multiplier  $\alpha_1$ .

- The thresholding procedure then continues until no reference samples survive the subsequent thresholding tests.

For this algorithm, the nonhomogeneity detection probability is determined by the probability that at least one reference sample survives the first thresholding test. The

reason is that if none of the reference samples survives the first thresholding test, then the procedure ends.

In [15], Hinomas et. al. proposed a similar nonhomogeneity detection and censoring algorithm in which the detection of nonhomogeneous samples is performed on a cell-by-cell basis using a maximum likelihood estimation method. The difference is that in the Hinomas algorithm, the reference samples are first sorted in ascending order based on their amplitudes before the application of the detection operation.

Consider a reference window of  $2N = 16$  samples, and assume that there are 3 target-like samples in the reference window. For Scenario 1 with a false alarm rate of  $10^{-4}$ , the nonhomogeneity detection probabilities given by the Barbois detector, the Hinomas detector, and the MMR detector are shown in Figure 8. It is evident that the MMR test gives the highest nonhomogeneity detection probabilities when the target-like SNR is greater than 10 dB. Although not shown here, it is found that, when the number of target-like samples increases further from 3 to 8 (i.e., occupying up to half of the CFAR window), the MMR detection probability increases while the Barbois and Hinomas detection probabilities decrease. Note that none of these three algorithms rely on the assumption that the target-like samples are confined to one side of the test cell.

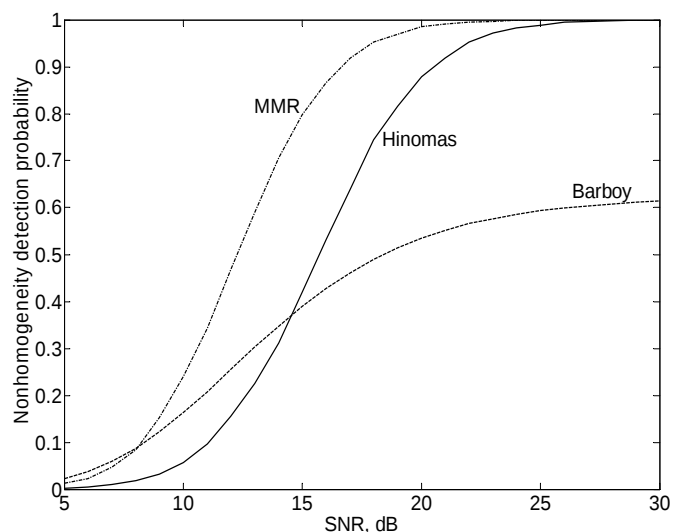
In terms of computational complexity, the MMR detector is simpler for implementation in comparison with the Hinomas detector since no sample ordering is required in the MMR algorithm. There are three sample means to be computed in the MMR algorithm ( $\mu$ ,  $\mu_0$ , and  $\mu_1$ ), while the Barbois detector requires the computation of only one sample mean. However, the MMR detector has only one stage detection, while the Barbois detector relies on an iterative procedure.

Compared with other nonhomogeneity detection algorithms such as those proposed in [17] (heterogeneous clutter estimation algorithm), in [18] (using the Mann-Whitney test), and in [19] (second-order statistic with leading-lagging mean ratio), an advantage of the MMR algorithm is that the assumption of homogeneity between two clutter changes is relaxed.

In summary, the MMR algorithm gives better nonhomogeneity detection performance, is simple for implementation since no sample ordering is required, and does not require that target-like samples are confined to only one side of the test cell. The information that is not given by the MMR detector is the exact locations of the target-like samples within the CFAR window. However, once the MMR test is passed, the samples in the set  $S_1$  can be deduced as target-like samples. This topic will be elaborated in a later publication.

## 7 Conclusions

In this report, a new detection method designated as the MMR test is proposed to detect the presence of target-like samples in CFAR reference windows. Unlike other existing CFAR algorithms that attempt to censor the potentially contaminated samples, the proposed MMR test focus on the detection of nonhomogeneity itself prior to the application of any censoring operation. Based on Rayleigh noise and Swerling I target models, it is demonstrated that the contaminated reference samples which seriously degrade the CA-CFAR performance will be detected with much higher probabilities using the MMR test.



**Figure 8:** Comparison of three nonhomogeneity detectors at false alarm rate  $10^{-4}$ , with three target-like samples in the reference window.

In other words, the proposed MMR detectors have a performance which complements that of the CA-CFAR detector in the presence of signal contamination. Such a characteristic has not been achieved by any other existing CFAR detectors.

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| 19. ABSTRACT<br>A new method designated as the mean-to-mean ratio (MMR) test is proposed for the detection of nonhomogeneities in a radar's Constant False Alarm Rate (CFAR) reference window. No a priori knowledge of the nonhomogeneity topology is assumed. Analysis using the Monte-Carlo method based on Rayleigh clutter and Swerling I target models is presented. Target-like interferences which seriously degrade the detection performance of the cell-averaging CFAR detector can be detected with a higher probability by the MMR test. |  |                              |                                                                                                                                 |                                        |  |