

# Modeling Wind Wave Evolution from Deep to Shallow Water

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Award Numbers: N000141010391, N0001411WX10010, N000141010453

## LONG-TERM GOALS

Ocean waves are an important aspect of upper ocean dynamics, in particular on the shallow continental shelves and in coastal areas. The long-term objective of this work is to advance modeling capability in such coastal areas by improving model representations of effects associated with nonlinearity, inhomogeneity, and dissipation.

## OBJECTIVES

The specific objectives of the present work are 1) to develop and implement an efficient and scalable approximation for the nonlinear quadruplet source term, 2) to develop and implement a generalized nonlinear source term that is accurate in water of arbitrary depth, 3) to develop and implement an improved nonlinear closure for triad nonlinear interactions in shallow water, and 4) improve representations of dissipation by wave breaking and wave-bottom interactions in shoaling waves.

| Report Documentation Page                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                    |                                     |                                                           | Form Approved<br>OMB No. 0704-0188                  |                                 |
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| 1. REPORT DATE<br><b>SEP 2011</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                    | 2. REPORT TYPE                      |                                                           | 3. DATES COVERED<br><b>00-00-2011 to 00-00-2011</b> |                                 |
| 4. TITLE AND SUBTITLE<br><b>Modeling Wind Wave Evolution from Deep to Shallow Water</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                    |                                     |                                                           | 5a. CONTRACT NUMBER                                 |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     |                                                           | 5b. GRANT NUMBER                                    |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     |                                                           | 5c. PROGRAM ELEMENT NUMBER                          |                                 |
| 6. AUTHOR(S)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     |                                                           | 5d. PROJECT NUMBER                                  |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     |                                                           | 5e. TASK NUMBER                                     |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     |                                                           | 5f. WORK UNIT NUMBER                                |                                 |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br><b>San Francisco State University, Department of Geosciences, San Francisco, CA, 94132</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                    |                                     |                                                           | 8. PERFORMING ORGANIZATION REPORT NUMBER            |                                 |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                    |                                     |                                                           | 10. SPONSOR/MONITOR'S ACRONYM(S)                    |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     |                                                           | 11. SPONSOR/MONITOR'S REPORT NUMBER(S)              |                                 |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br><b>Approved for public release; distribution unlimited</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                    |                                     |                                                           |                                                     |                                 |
| 13. SUPPLEMENTARY NOTES                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                    |                                     |                                                           |                                                     |                                 |
| 14. ABSTRACT                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     |                                                           |                                                     |                                 |
| 15. SUBJECT TERMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                    |                                     |                                                           |                                                     |                                 |
| 16. SECURITY CLASSIFICATION OF:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 17. LIMITATION OF ABSTRACT<br><b>Same as Report (SAR)</b> | 18. NUMBER OF PAGES<br><b>9</b>                     | 19a. NAME OF RESPONSIBLE PERSON |
| a. REPORT<br><b>unclassified</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | b. ABSTRACT<br><b>unclassified</b> | c. THIS PAGE<br><b>unclassified</b> |                                                           |                                                     |                                 |

## APPROACH

Modern, third-generation (3G) wave models are based on an action balance (or radiative transfer) equation, which describes the transport of wave energy (or action) through a slowly varying medium and time. In Lagrangian form (for convenience) this balance equation can be written as

$$\frac{dN(\mathbf{k})}{dt} = S_{in}(\mathbf{k}) + S_{ds}(\mathbf{k}) + S_{sc}(\mathbf{k}) + S_{nl}(\mathbf{k}) \quad (1)$$

where  $N(\mathbf{k})$  is the wave action at wavenumber vector  $\mathbf{k}$  and  $t$  is time. The forcing terms on the right-hand side are referred to as source terms and account for the input of energy by the wind ( $S_{in}$ ), spectral redistribution of energy through scattering by seafloor topography ( $S_{sc}$ ) or through nonlinear wave-wave interactions ( $S_{nl}$ ), and dissipation of wave energy ( $S_{ds}$ ) through e.g. breaking or bottom friction.

In this study we will develop and improve the source terms for nonlinear interactions  $S_{nl}$  and energy dissipation  $S_{ds}$ , to account for effects of finite depth and shallow water, and to ensure a consistent and smooth model representation of wave evolution from deep to shallow water.

### Nonlinearity

We will develop an efficient method for the evaluation of the nonlinear source term, allowing for greater efficiency and accuracy in operational use. To allow modeling of wave propagation from deep to shallow water, we will modify the nonlinear source term to account for changes in relative water depth [Janssen *et al.* 2006], and develop an improved closure approximation for nearshore wave propagation [Janssen, 2006].

### Dissipation

We will develop and test improvements to wave dissipation parameterizations through detailed comparisons against laboratory and field observations.

### Field data

We will make a number of existing field experimental data sets, collected by the PI's, available to the project team for the purpose of validation and calibration of new model developments.

## WORK COMPLETED

### Development of a Lumped Quadruplet Approximation (LQA)

To make evaluation of the quadruplet nonlinear source term more economical, while retaining accuracy, we are developing a scalable parameterization of nonlinear four-wave interactions based on the WRT method. This Lumped Quadruplet Approximation (LQA) clusters (lumps) contributions to the integrations over the interaction loci to limit the number of operations required to estimate the nonlinear source term (see figure 1).

We have extended existing routines for computing the non-linear transfer rate, as implemented in the SWAN and WAVEWATCH III models, to handle the clustering of incremental contributions to the total transfer rate.

A procedure has been developed to test the implementation (of LQA and other reduced versions of the WRT) where 1) the non-linear transfer rate is computed for fixed (often academic) spectra 2) growth curve experiments and field observations are reproduced, and 3) comparisons to existing DIA and multiple DIA (see Tolman 2010) implementations are made.

Ongoing developments include a comparison study to compare our implementation to other algorithms by other researchers (e.g. TSA by Resio and Perrie, GQM by Benoit & Gagnaire-Renoud, and RIAM by Tamura), and to incorporate a correct shallow-water asymptote, based on recent progress (Janssen and Onorato, 2007). This work is undertaken in collaboration with Peter Janssen (ECMWF) and Miguel Onorato (Turino University).

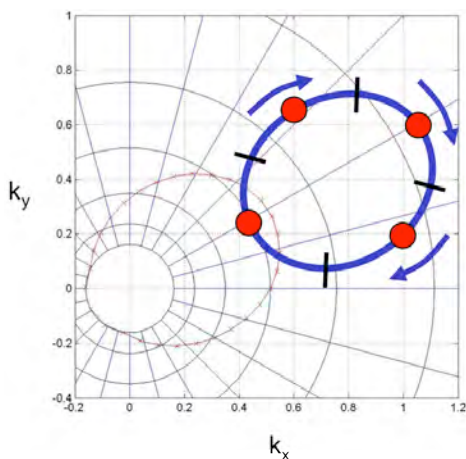


Figure 1 Illustration of lumped integration along integration paths. The complete path of the line integral in wavenumber space (solid blue line) is divided up into discrete sections (indicated by cross-lines), which represent the discrete interactions in the MDIA.

### Analysis of nonlinearity and dissipation in nonlinear shoaling waves.

We have completed a study to investigate nonlinearity and dissipation in shoaling waves. Through a detailed analysis of laboratory observations we have validated a one-point closure method, determined the spectral dissipation distribution, and investigated relaxation length scales in shoaling waves. These findings have been instrumental in the development and validation of a one-point closure approximation.

### Development of a shallow-water model based on a one-point closure

We have developed, implemented and validated a one-point closure approximation in a shallow-water wave model, using a quadratically-weighted dissipation formulation. The model has been validated against laboratory observations and Monte-Carlo simulations. This type of one-point closure approximation, although based on a quasi-empirical closure approximation, is an efficient alternative and suited for implementation in operational wave models.

### Development of a two-point closure approximation

To provide a benchmark stochastic model we have developed a new two-point closure approximation to test the overall potential of the stochastic modeling of three-wave interactions in weakly dispersive waves. Although this approach is computationally more intensive than a one-point approximation, it provides more physical insight into the nature of the closure approximation, and can be used to calibrate more approximate (one-point) models.

### Transport equations for wave correlators

To incorporate nonlinear dynamics in nearshore wave models, compatible transport equations for higher-order correlations (bispectrum) are needed. We have completed the development of a general theory that would form the basis for such transport equations. Thus far we have applied these ideas to inhomogeneous (but Gaussian) wave fields to incorporate coherent interference effects in focusing wave fields. The theoretical work will be extended to include three-wave correlations and is expected to form the basis for an isotropic description of nonlinear wave dynamics, compatible with the modeling framework of operational wave models.

## RESULTS

### Analysis and modeling of nonlinearity and dissipation in shoaling waves

Through a detailed analysis of laboratory data (Boers 1996) we have determined that dissipation in shallow water is approximately weighted as  $\sim \omega^2 F(\omega)$ , where  $F(\omega)$  is the flux spectrum and  $\omega$  is angular frequency. This finding is largely in agreement with findings by other researchers who used different methods of analysis (e.g. Kaihatu et al. 2007).

Further, we have successfully validated a quasi-empirical relaxation model through direct comparison of observed and model-predicted values of the nonlinear relaxation contribution  $\mu$  (see figure 2). This is an important finding since it suggests that a relatively simple and quasi-empirical parameterization of the relaxation length scales can capture the principal nonlinear dynamics in shoaling gravity waves.

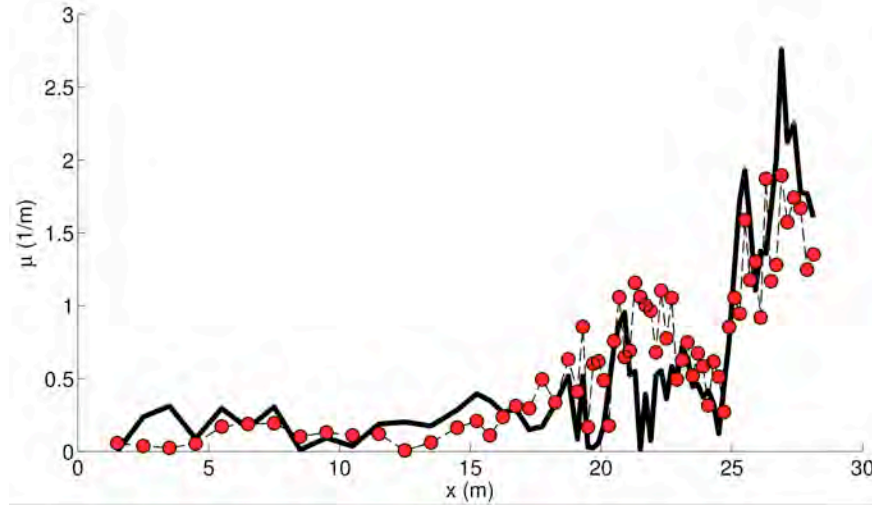


Figure 2. Comparison of observed (solid black line) and modeled with one-point approximation (red circles) evolution of nonlinear relaxation  $\mu_{pp}$  (self-self interaction at the peak of the spectrum) as a function of distance of the wavemaker ( $x$ ). The beach mimics a bar-trough profile with the still-water shoreline at about 30m from the wavemaker (Boers, 1996).

We have implemented a nonlinear shallow-water model with the proposed one-point closure approximation, and applied a squared weighting of the dissipation associated with depth-induced breaking (as found from the data analysis). The predicted evolution of nonlinear transfer (see figure 3), spectra, and higher-order bulk statistics (not shown), is in surprisingly good agreement with the observed nonlinear evolution. Preliminary validation of the model with field observations (not shown) confirm that the model captures the generation of harmonics in directionally spread waves over a beach with weak alongshore variations. Further validation with field data is ongoing.

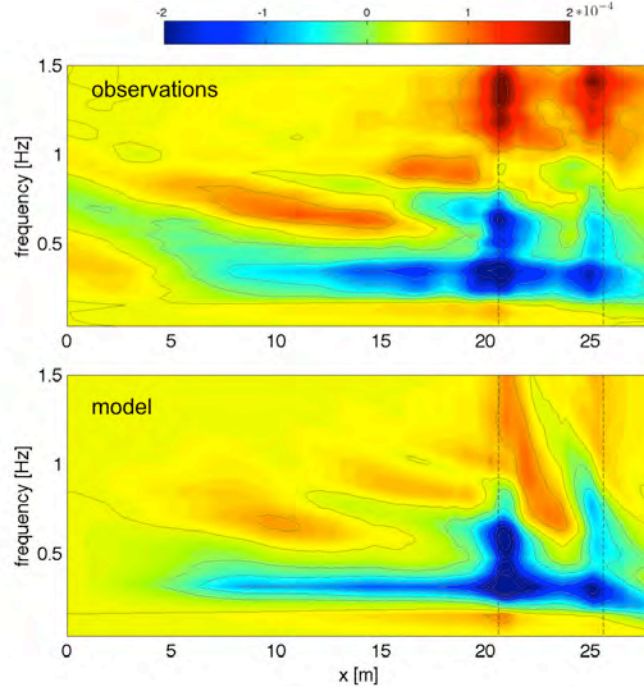


Figure 3 Comparison of observed (top panel, data Boers 1996) and modeled (bottom panel) evolution of nonlinear energy transfer ( $S_{nl}$ ). The agreement overall is very good, although nonlinear transfer of wave energy into the frequency range 1-1.5Hz is underestimated by the model in the nearshore ( $x > 20$  m).

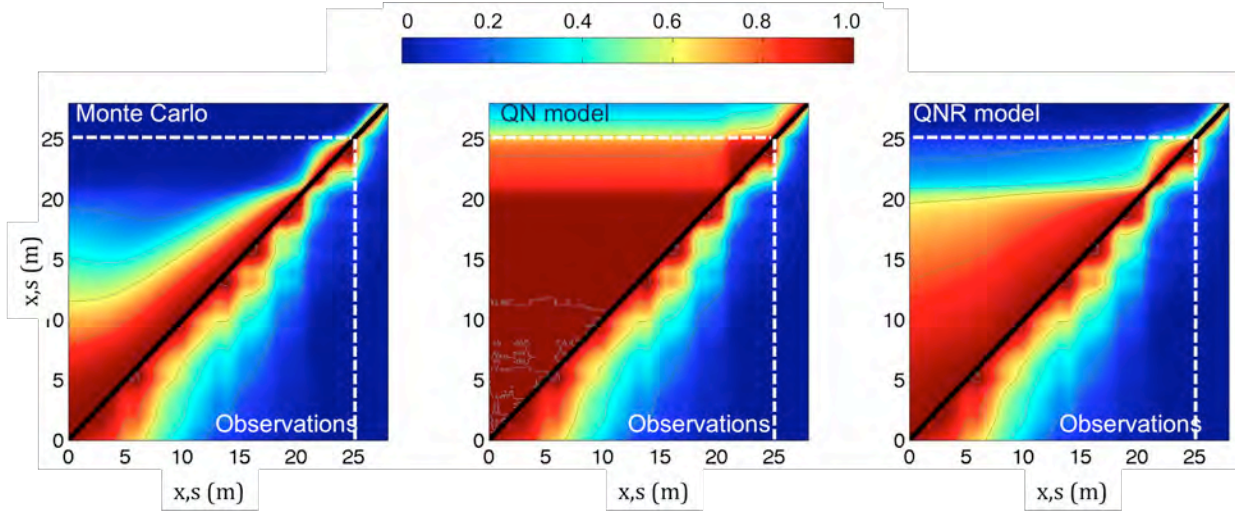


Figure 4. Comparison of  $|H_{pp}(x,s)|$  (self-self interaction at the peak of the spectrum) between observations (Boers, 1996) and Monte-Carlo simulations with the deterministic model (left panel), a quasi-normal approximation (middle panel), and the QNR model (equations (1) and (2)), shown in right panel. Although the relaxation characteristics for the QNR model are more realistic than those of the quasi-normal (QN) approximation (middle panel), the two-point method (which inherits the relaxation length scales from the deterministic model) is in significantly better agreement with observations (see left panel).

## Development of a two-point closure approximation

To serve as a benchmark, and to better understand the nonlinear dynamics in a stochastic shallow-water system, we have developed a two-point closure approximation, where the memory of the nonlinear wave system is derived from the relaxation scales implied by the underlying deterministic model. This approach requires the numerical evaluation of an additional transport equation for the spatial cross-correlations in the wave field (instead of just variance contributions). However, the resulting model does not require further empirical coefficients to close the system but naturally incorporates the effects of dispersion, dissipation, and nonlinearity (as present in the underlying deterministic model) on the relaxation scales of the three-wave interactions. This approach is more accurate than a one-point model (which invariably require some form of a quasi-empirical approximation, see figure 4) but because of the more extensive computational costs of this approach, it is expected to serve primarily as a benchmark model.

## Transport equations for inhomogeneous wave fields

To implement nonlinear interactions and non-Gaussian statistics in an operational wave model, isotropic transport equations for cross-correlations in the wave field are needed. We have developed a new transport equation for inhomogeneous (but Gaussian) wave fields, which forms a natural extension of the radiative transfer equation (see Smit & Janssen 2011). This approach has been shown to accurately resolve coherent interference in random waves (see figure 5), and will form a basis for the derivation and implementation of an isotropic model for the transport of three-wave correlations (bispectrum) through variable media.

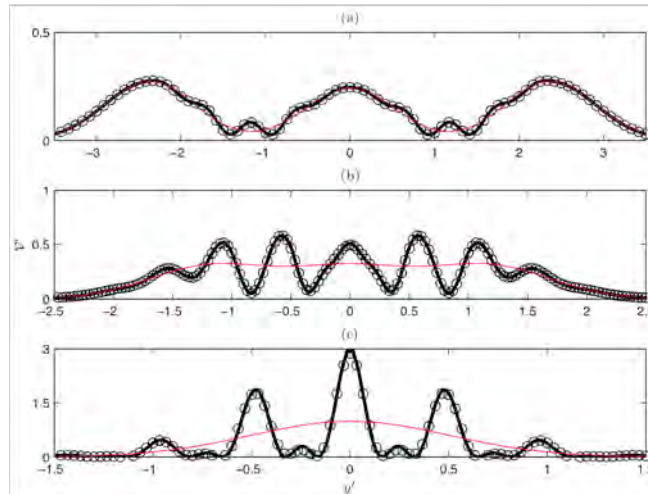


Figure 5 Cross-sections of normalized wave variance in a three-packet interference example. The wave variance is shown at discrete times (increasing in time from top to bottom). Comparison between exact model (Solid line), the coherent approximation derived in this work (circle markers) and the evolution according to quasi-homogeneous theory (thin red line). Note that the horizontal and vertical ranges vary between panels.



## IMPACT/APPLICATIONS

### Economic Development

The improvements to coastal wave prediction models as developed in this project, will contribute to various industries operating on the continental shelf and the coastal zone, such as fisheries, and shipping and offshore industry. Further, the availability of better wave prediction models will benefit coastal and ocean engineering companies e.g. in the design and operation of offshore and coastal structures, and the development of coastal management strategies.

### Quality of life

The improvements to coastal wave prediction models as developed in this project, will improve modeling capability of coastal circulation and transport processes, which will benefit coastal recreation (more reliable knowledge of wave heights, rip currents etc), coastal management, and help mitigate pollution hazards for humans (recreation) and coastal ecosystems.

## TRANSITIONS

### Economic Development

The developments in this project will be made available as open source software and as modules to widely used operational wave models. These models are used by NOAA and other agencies involved in coastal development and management, and by many coastal and ocean engineering companies.

### Quality of life

The software developed within this project will be disseminated in open source models used by local and federal agencies and companies involved in coastal recreation (surf prediction, rip currents, pollution), coastal management, and mitigation of coastal hazards.

## RELATED PROJECTS

The development of transport equations for cross-correlations in random waves also contributes to the study of coastal wave-current interaction as part of the Inlets and River Mouths DRI, funded by the Office of Naval Research.

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## PUBLICATIONS

Smit P. B. and T. T. Janssen, Coherent structures in random waves, *J. Fluid Mech.*, 2011. [submitted, refereed]