

A Simulation and Optimization Methodology for Reliability of Vehicle Fleets

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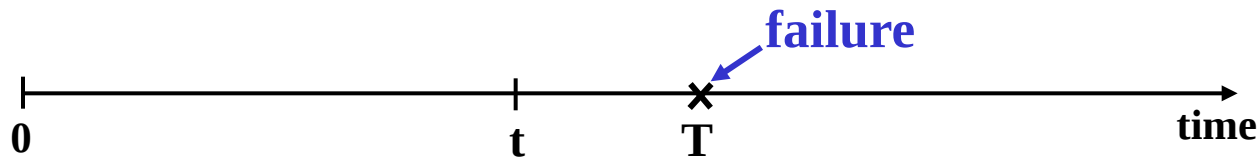
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Overview

- What is reliability ??
- Basics of reliability methods for **repairable** and **non-repairable** systems
- Estimation of PDF of Time Between Failures (TBF) using limited, censored data
 - “Frequentist” approach (Method 1)
 - Bayesian updating approach (Method 2)
 - ✓ “Enhances” data with expert opinion

What is Reliability?

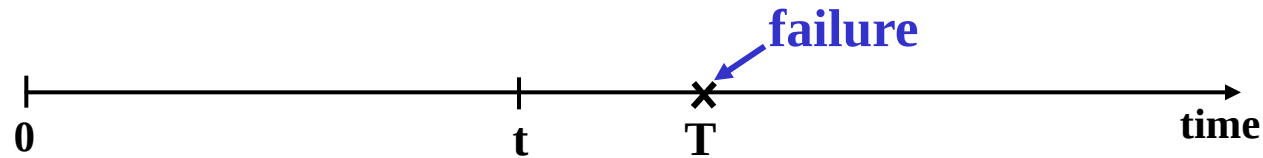
Reliability at time t is the probability that the system **has not failed** before time t .



$$R(t) = P(T > t) = 1 - P(T \leq t)$$

Reliability Basics for Non-Repairable Systems

Reliability of Non-Repairable Systems



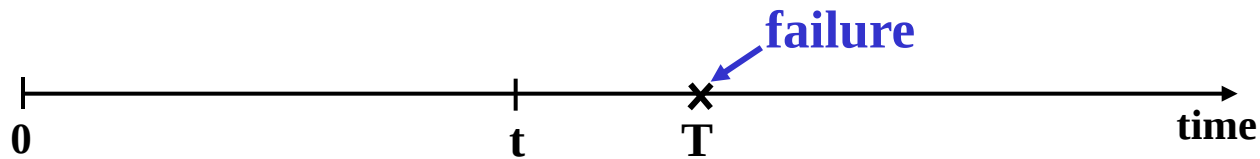
$$R(t) = P(T > t) = 1 - P(T \leq t) \Rightarrow \boxed{R(t) = 1 - F(t)} \quad (1)$$

Failure Rate $\nearrow$

$$\lambda(t) = \frac{P(t < T \leq t + dt / T > t)}{dt} = \frac{P(t < T \leq t + dt)}{dt * P(T > t)} =$$

$$= \frac{F(t + dt) - F(t)}{dt * R(t)} \Rightarrow \boxed{\lambda(t) = \frac{f(t)}{R(t)}} \quad (2)$$

Reliability of Non-Repairable Systems



$$R(t) = 1 - F(t) \Rightarrow \frac{dR}{dt} = -f(t) \Rightarrow \frac{dR}{dt} = -\lambda(t)R(t) \Rightarrow$$

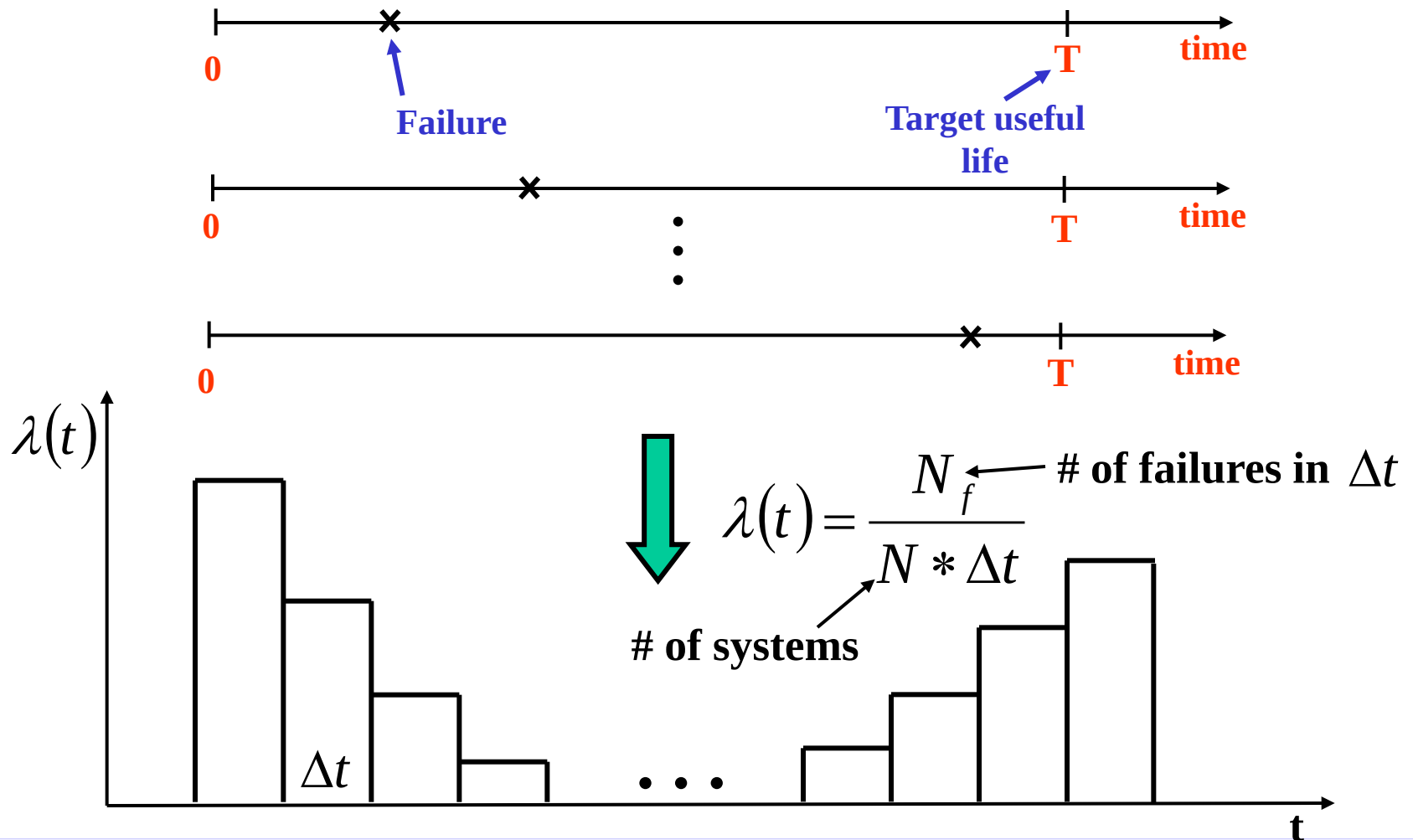
due to (2)

$$\Rightarrow \frac{dR}{R} = -\lambda dt \Rightarrow d(\ln R) = -\lambda dt \Rightarrow \ln\left(\frac{R(t)}{R(0)}\right) = -\int_0^t \lambda dt \Rightarrow$$

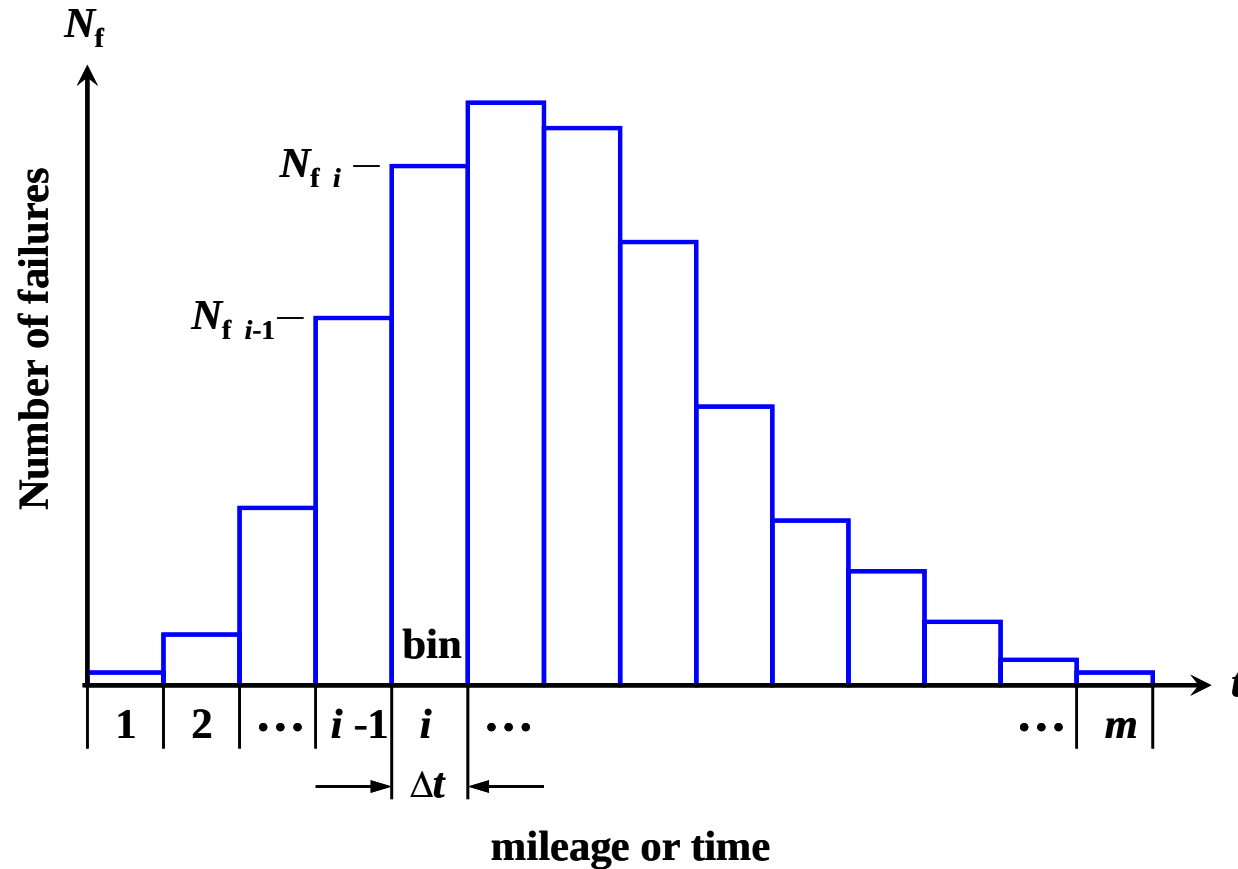
$$\Rightarrow R(t) = \exp\left[-\int_0^t \lambda dt\right]$$

All we need is the failure rate

Reliability of Non-Repairable Systems



Reliability of Non-Repairable Systems



$$N_f = \sum_{i=1}^m N_{f_i}$$

$$\lambda_i = \frac{f_i}{1 - F_i} = \frac{f_i}{1 - \sum_{j=1}^{i-1} \frac{N_{f_j}}{N_f}} = \frac{N_{f_i}}{\left(N_f - \sum_{j=1}^{i-1} N_{f_j} \right) \Delta t}$$

Reliability of Non-Repairable Systems

$$H_i = \sum_{j=1}^i \lambda_j \Delta t$$

$$R_i = 1 - F_i = 1 - \sum_{j=1}^{i-1} \frac{N_{f_i}}{N_f}$$

$$R_i = e^{-H_i}$$

Reliability Calculation

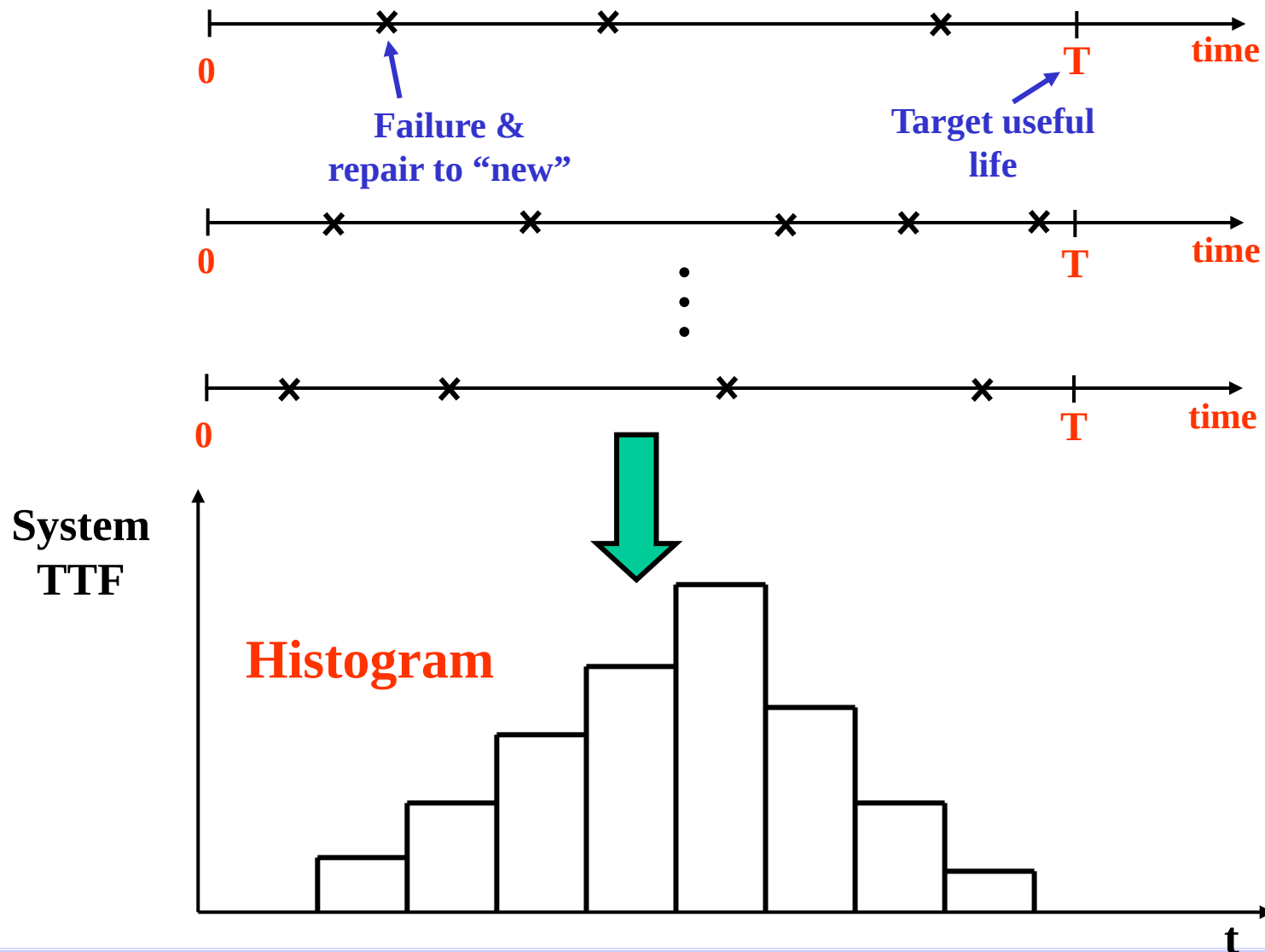
All we need for calculating the reliability of a system (non-repairable** or **repairable**) is the system PDF of time to failure (TTF)**

We use :

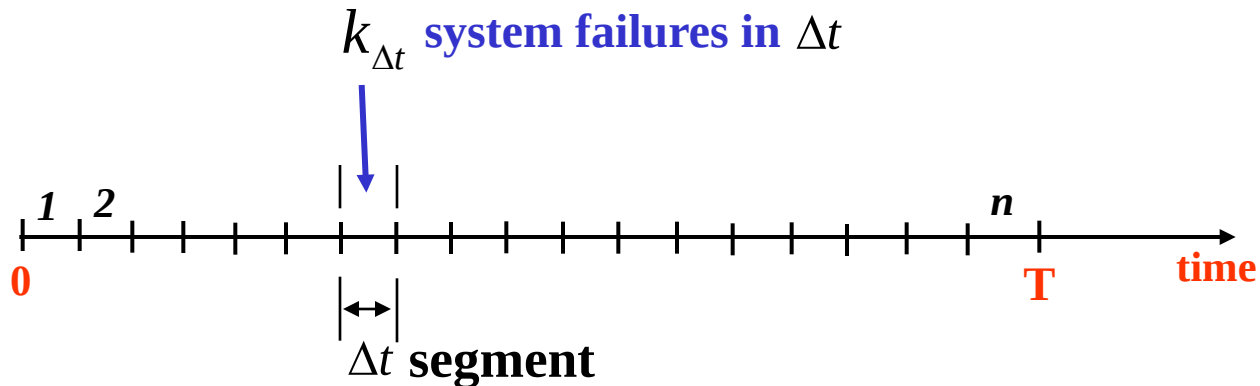
- **Data to estimate the PDF of TTF **for each component****
- **Monte Carlo simulation to estimate the PDF of TTF for the **system****

Basics of Reliability Methods **(Repairable Systems)**

Reliability of Repairable Systems



Reliability of Repairable Systems



$p = \frac{k_{\Delta t}}{N}$: Probability of failure in Δt (**very small** if $\Delta t \rightarrow 0$)

$\nearrow$
of systems (vehicles) in fleet

If p is independent of Δt segment $\Rightarrow$ { Homogeneous Poisson Process (**HPP**)

If p depends on Δt segment $\Rightarrow$ { Non-Homogeneous Poisson Process (**NHPP**)

Homogeneous Poisson Process (HPP)

pn : Expected (average) # of failures / system in $(0, T]$

↑
Probability of
failure in Δt

Define :

$$\lambda T = pn$$

λ : Average # of failures per system per unit time
(failure rate, hazard rate, intensity rate,
repair rate)

Homogeneous Poisson Process (HPP)

Reliability Calculation for HPP at $T = n\Delta t$

❖ Assume statistically independent events at each Δt

$$\begin{aligned} R(T) &= (1-p)(1-p)\cdots(1-p) = (1-p)^n \\ &= \left(1 - \frac{\lambda t}{n}\right) \left(1 - \frac{\lambda t}{n}\right) \cdots \left(1 - \frac{\lambda t}{n}\right) = \left(1 - \frac{\lambda t}{n}\right)^n = (1 - \lambda \Delta t)^n \\ &= e^{-\lambda T} \text{ if } \Delta t \rightarrow 0 \end{aligned}$$

Non-Homogeneous Poisson Process (NHPP)

Reliability Calculation for NHPP at $T = n\Delta t$

❖ Assume statistically independent events at each Δt

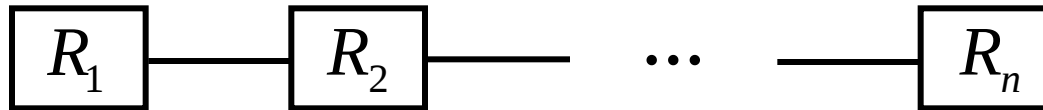
$$\begin{aligned} R(T) &= (1 - p_1)(1 - p_2) \cdots (1 - p_n) \\ &= (1 - \lambda_1 \Delta t)(1 - \lambda_2 \Delta t) \cdots (1 - \lambda_n \Delta t) \\ &= \exp \left[- \int_0^T \lambda(t) dt \right] = e^{-H(T)} \quad \text{if } \Delta t \rightarrow 0 \end{aligned}$$

Same formula with non-repairable systems

Cumulative Hazard Rate

Reliability Calculation

Series Systems



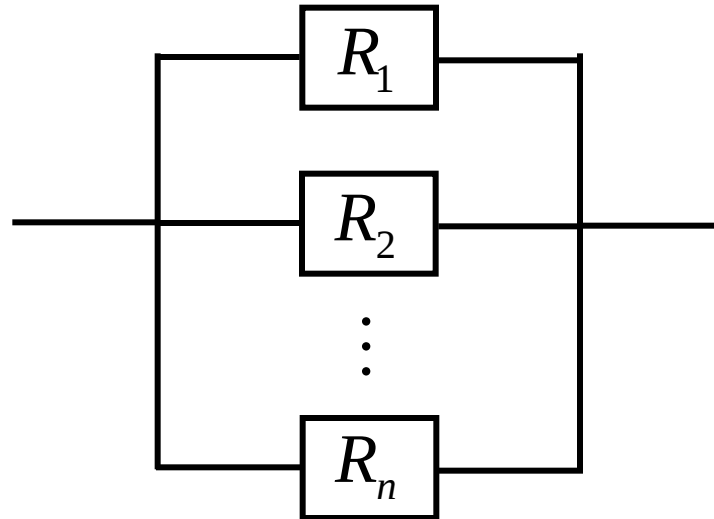
$$\begin{aligned} R &= R_1 * R_2 * \dots * R_n = \\ &= e^{-\lambda_1 t} * e^{-\lambda_2 t} * \dots * e^{-\lambda_n t} = e^{-\lambda t} \end{aligned}$$

where :

$$\lambda = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

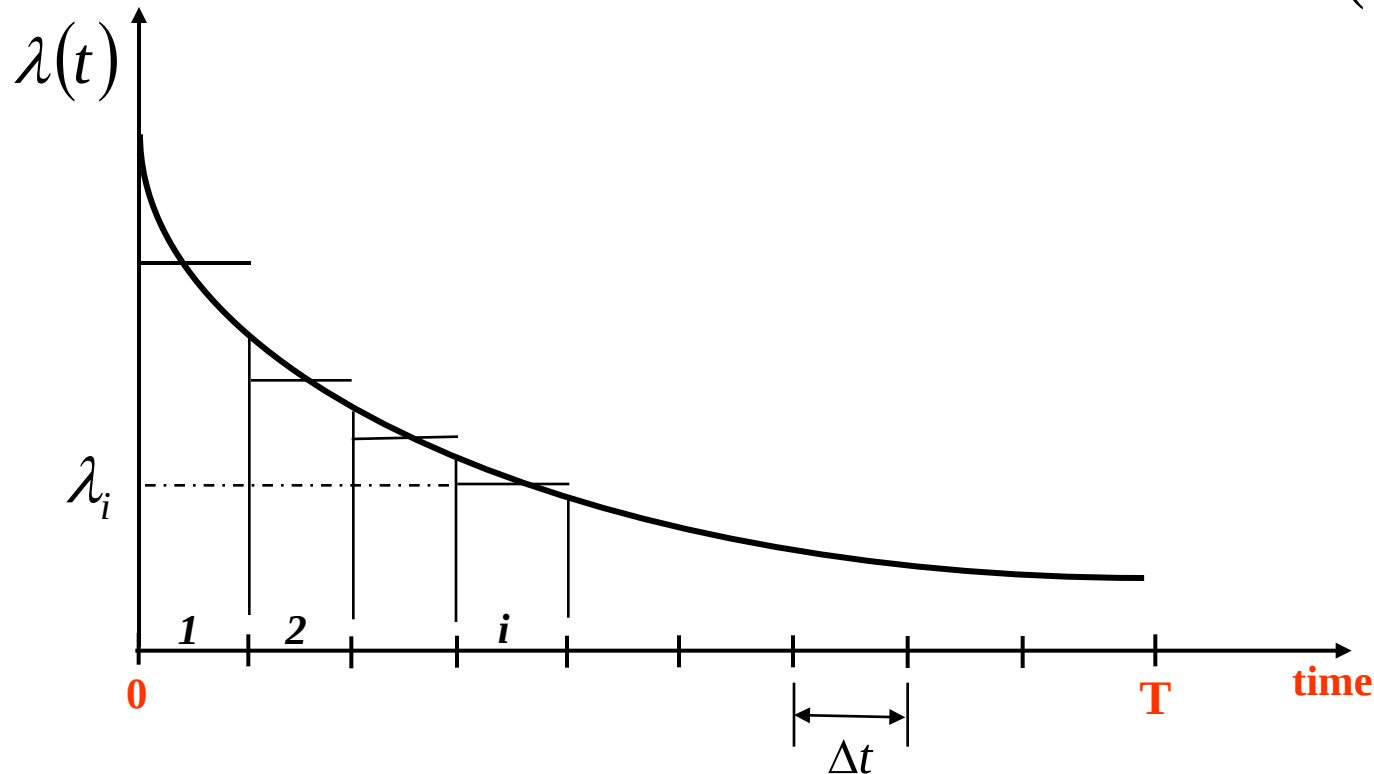
Reliability Calculation

Parallel Systems



$$R = 1 - (1 - R_1) * (1 - R_2) * \dots * (1 - R_n)$$

Calculation of Hazard Rate $\lambda(t)$



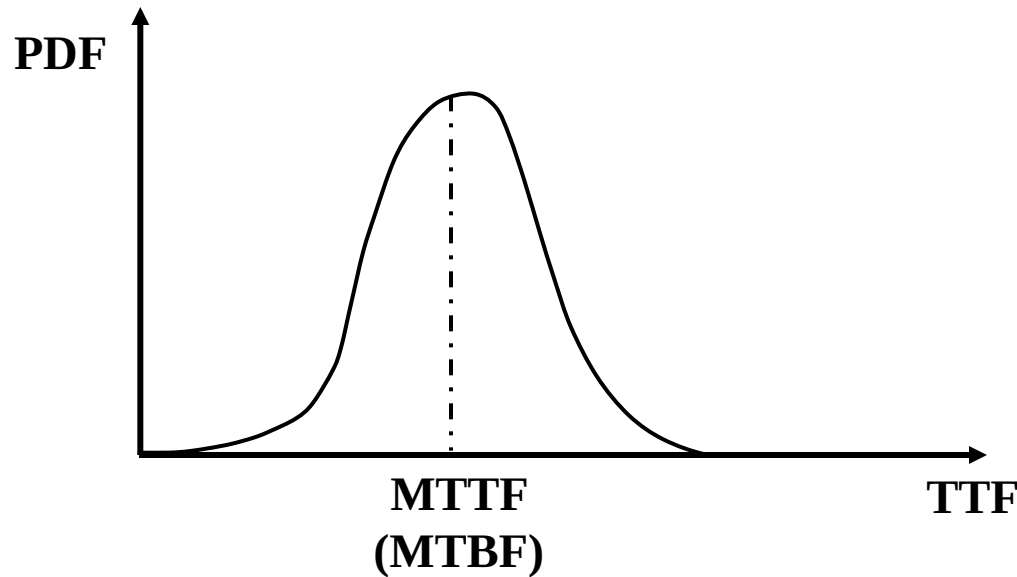
Determine average # of failures $k_{\Delta t}(t)$ among N systems in each Δt . Then :

$$\lambda(t) = \frac{k_{\Delta t}(t)}{N\Delta t}$$

Calculation of Hazard Rate $\lambda(t)$

For **constant** hazard rate systems :

$$\lambda = \frac{1}{MTBF}$$



For most engineering systems, the hazard rate **IS NOT** constant. To estimate it, we need the PDF of TTF for each component.

Reliability Calculation

All we need for calculating the reliability of a system (non-repairable** or **repairable**) is the system PDF of time to failure (TTF)**

We use :

- **Data to estimate the PDF of TTF **for each component****
- **Monte Carlo simulation to estimate the PDF of TTF for the **system****

Estimation of the PDF (or CDF) of the TTF (TBF) using Limited, Censored Data

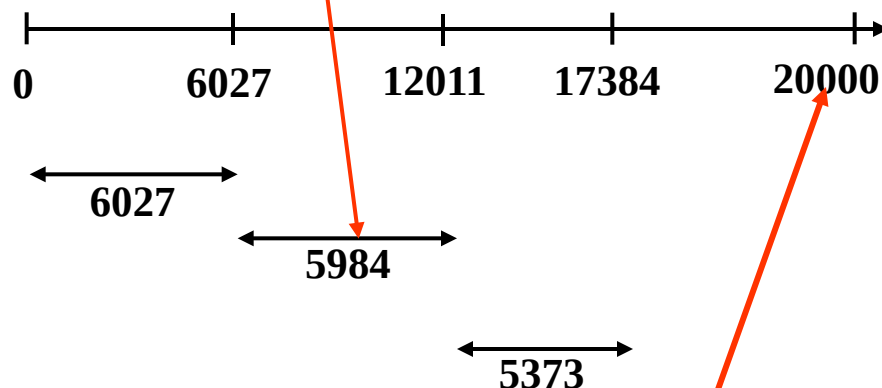
➤ Censored MLE Approach

Limited Data

Group L1

Original data		Updated data	
Vehicle#	mileage	Vehicle#	mileage
10	741	1	10247
4	5273	2	9044
<u>7</u>	<u>6027</u>	2	8977
5	7398	3	13984
6	7495	3	4064
2	9044	4	5273
1	10247	4	9747
8	12008	5	7398
<u>7</u>	<u>12011</u>	5	7611
9	12014	6	7495
10	12074	6	7516
3	13984	7	6027
5	15009	7	5984
6	15011	7	5373
4	15020	8	12008
<u>7</u>	<u>17384</u>	9	12014
2	18021	10	741
3	18048	10	11333

**Time Between Failures
(TBF)**



Censoring Mileage

Censored MLE Approach

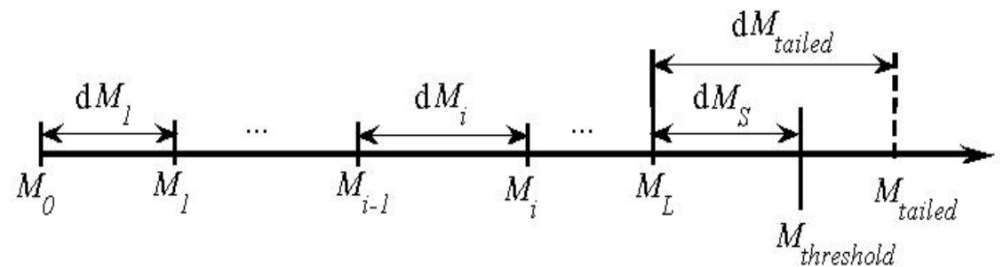
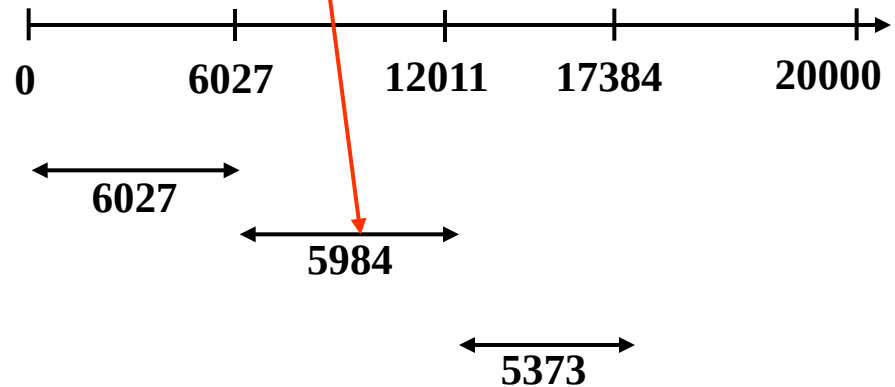
- Using available limited data (TBFs and censoring mileage), **“estimate” PDF of TBF** using a **censored MLE** approach.
- **Tail** sample the PDF of previous step to “enhance” the original limited data.
- Using “enhanced” data from previous step, **“better estimate”** the PDF of TBF using an **uncensored MLE** approach.
- Using the PDF of previous step, a **Bootstrap** approach estimates **statistics of TBF** (e.g. distribution of MTBF, distribution of TBF standard deviation, etc.)

Notation

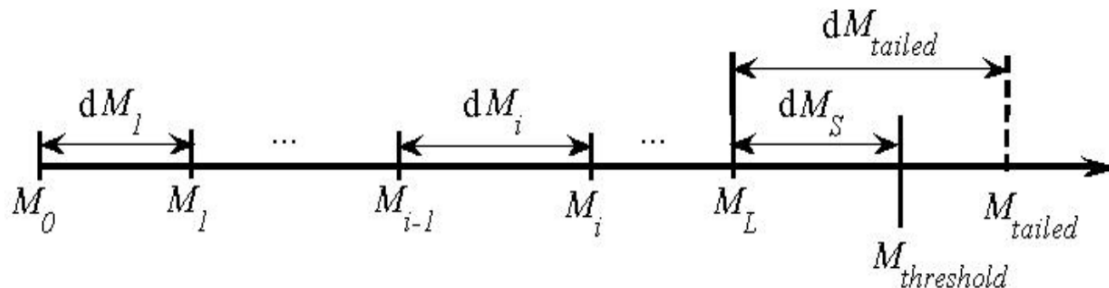
Group L1

Original data		Updated data	
Vehicle#	mileage	Vehicle#	mileage
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Time Between Failures (TBF)

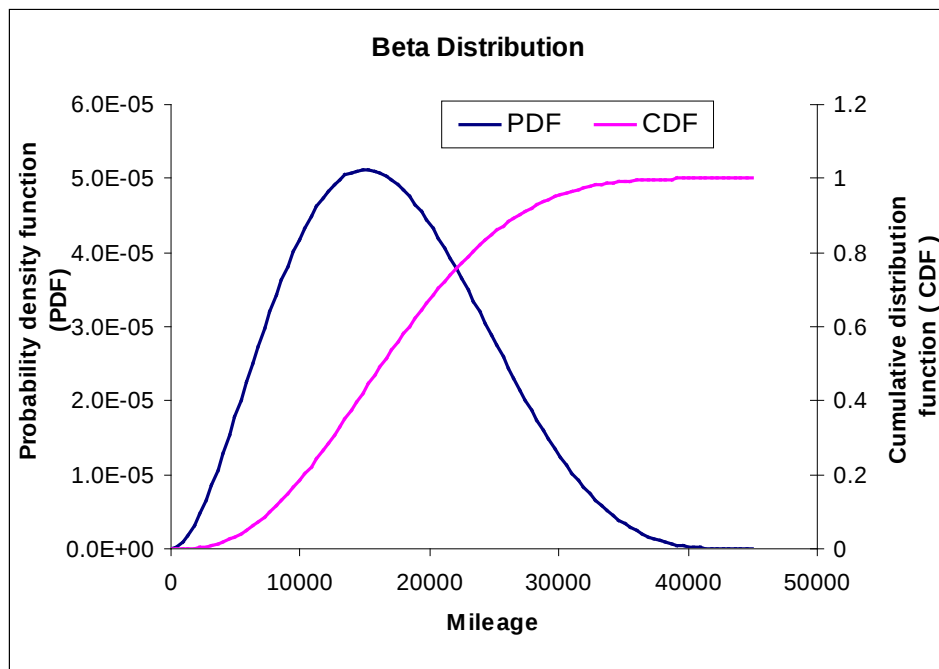


Observation / Assumption



$$dM_i = X_i \sim \beta(A, B, p, q), \quad (A \leq X_i \leq B, \text{ and } p > 0, q > 0)$$

$$f(x, A, B, p, q) = \beta(p, q)^{-1} (x - A)^{p-1} (B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \leq x \leq B, \text{ and } p > 0, q > 0)$$



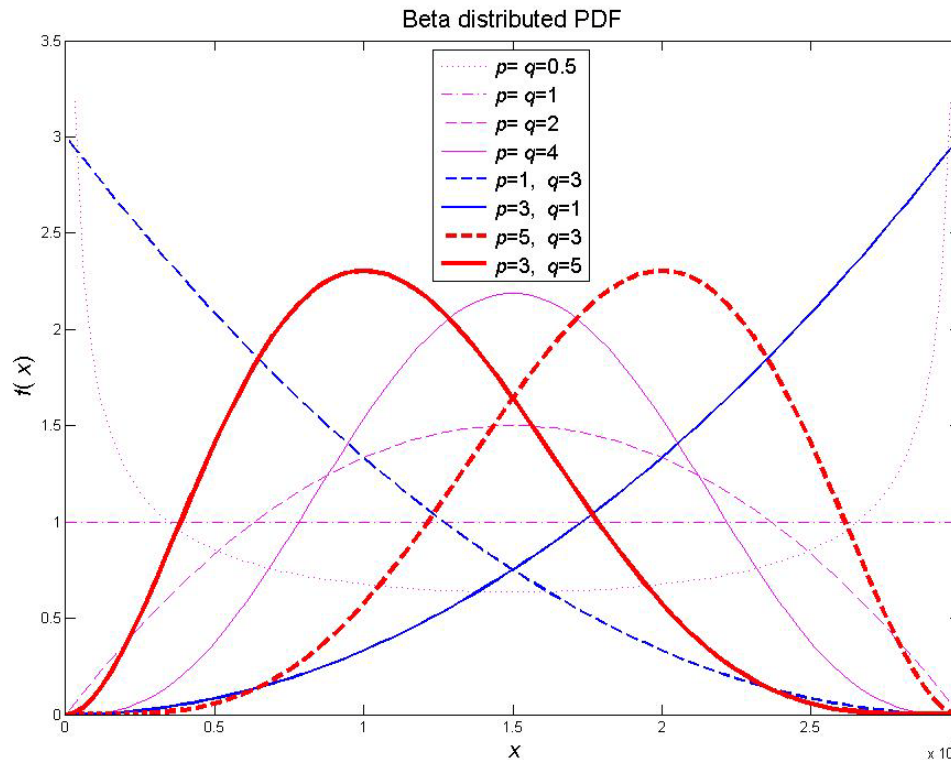
$$A = 0$$

$$B = 45,000 \text{ miles}$$

$$p = 3, q = 5$$

Observation / Assumption

- **Beta distribution family** is used to model TBF.



$A=0, B = 30000$

$$f(x, A, B, p, q) = \beta(p, q)^{-1} (x - A)^{p-1} (B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \leq x \leq B, \text{ and } p > 0, q > 0)$$

MLE Approach

Determines parameters (A, B, p, q) of “most likely” **Beta distribution** using available data. It provides Likelihood function in Bayesian estimation.

Censored MLE

$$\underset{A, B, p, q}{Max} \prod_{i=1}^{N_F} f(x_i, A, B, p, q) \prod_{j=1}^{N_s} [1 - F(x_j, A, B, p, q)]$$

Diagram annotations:

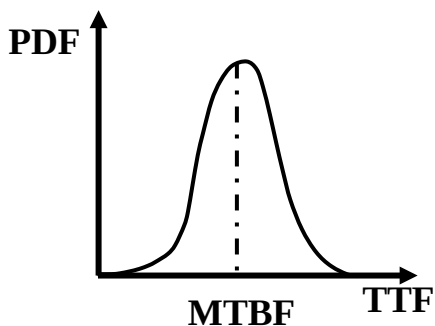
- Red arrow from “# of recorded failures” to N_F
- Red arrow from “# of survivals” to N_s
- Red arrow from “Beta PDF” to $f(x_i, A, B, p, q)$
- Red arrow from “Beta CDF” to $F(x_j, A, B, p, q)$

Uncensored MLE

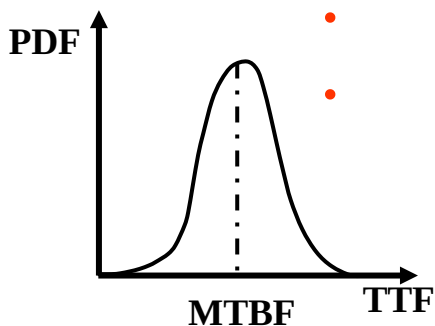
$$\underset{A, B, p, q}{Max} \prod_{i=1}^N f(x_i, A, B, p, q)$$

System Reliability and Reliability Allocation

System (Vehicle) Reliability



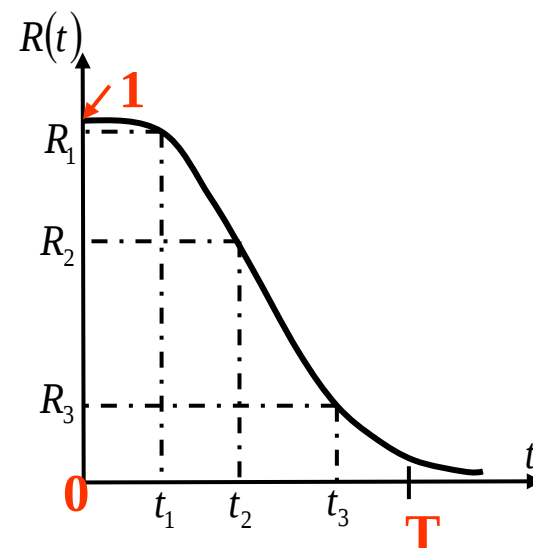
Comp. 1



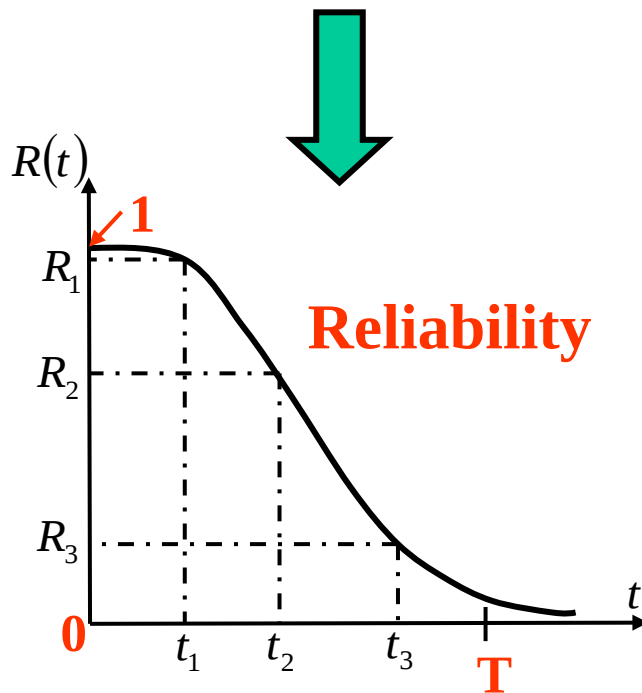
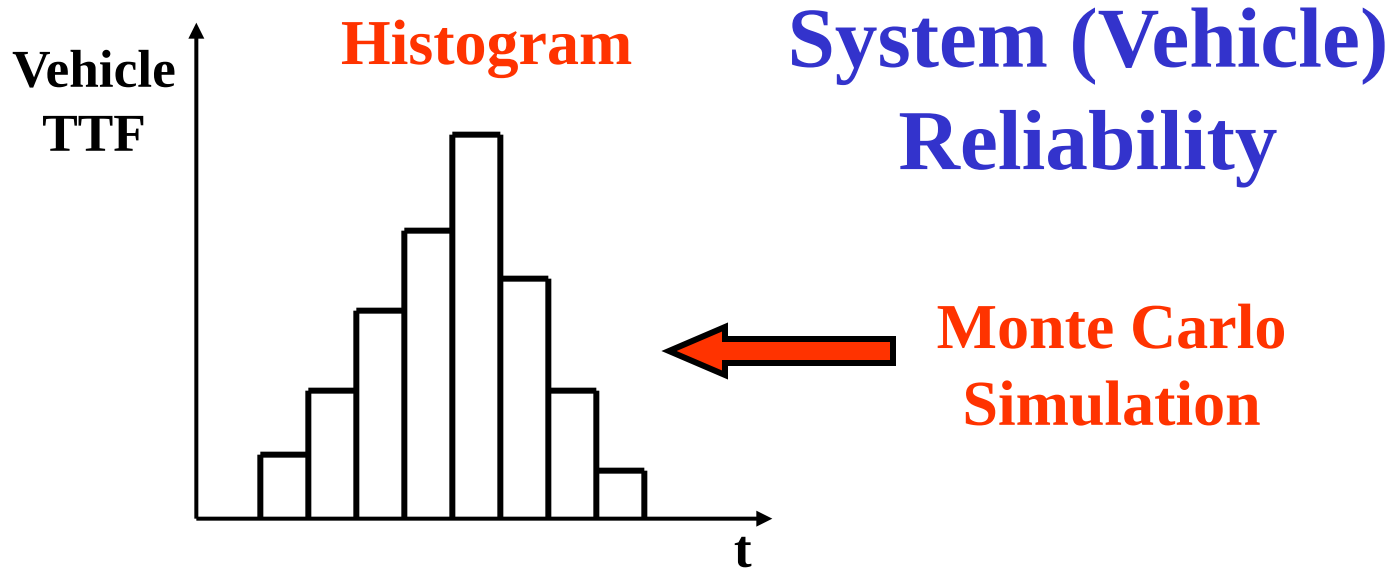
Comp. n

Vehicle

Vehicle $R(t)$



**Second Software
Demonstration**

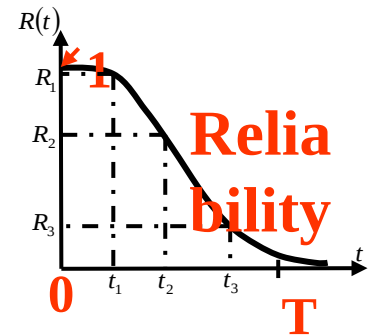


For Vehicle :

$$MTBF = \int_0^{\infty} R(t)dt$$

Reliability Allocation

Specify system (vehicle) reliability



Optimization

Determine **required** reliability of EACH component

This optimization problem **DOES NOT**
have a **unique** solution

Reliability Allocation

One way to get a unique solution is to trade-off reliability and associated cost

$$\begin{array}{ll} \min_{\underline{R}_{comp}} & \text{Cost} \\ \text{s. t.} & \text{System Reliability} = R^t \end{array}$$

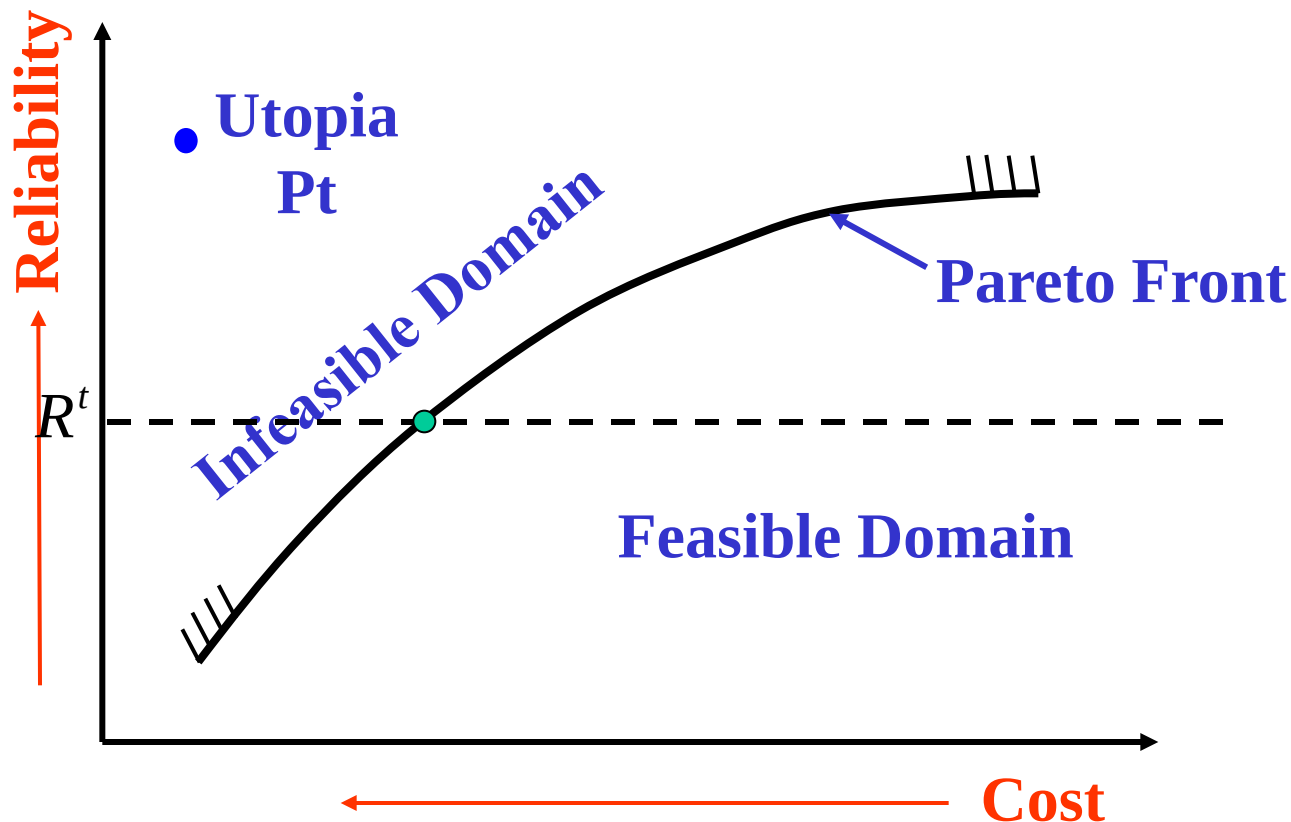
Target system reliability



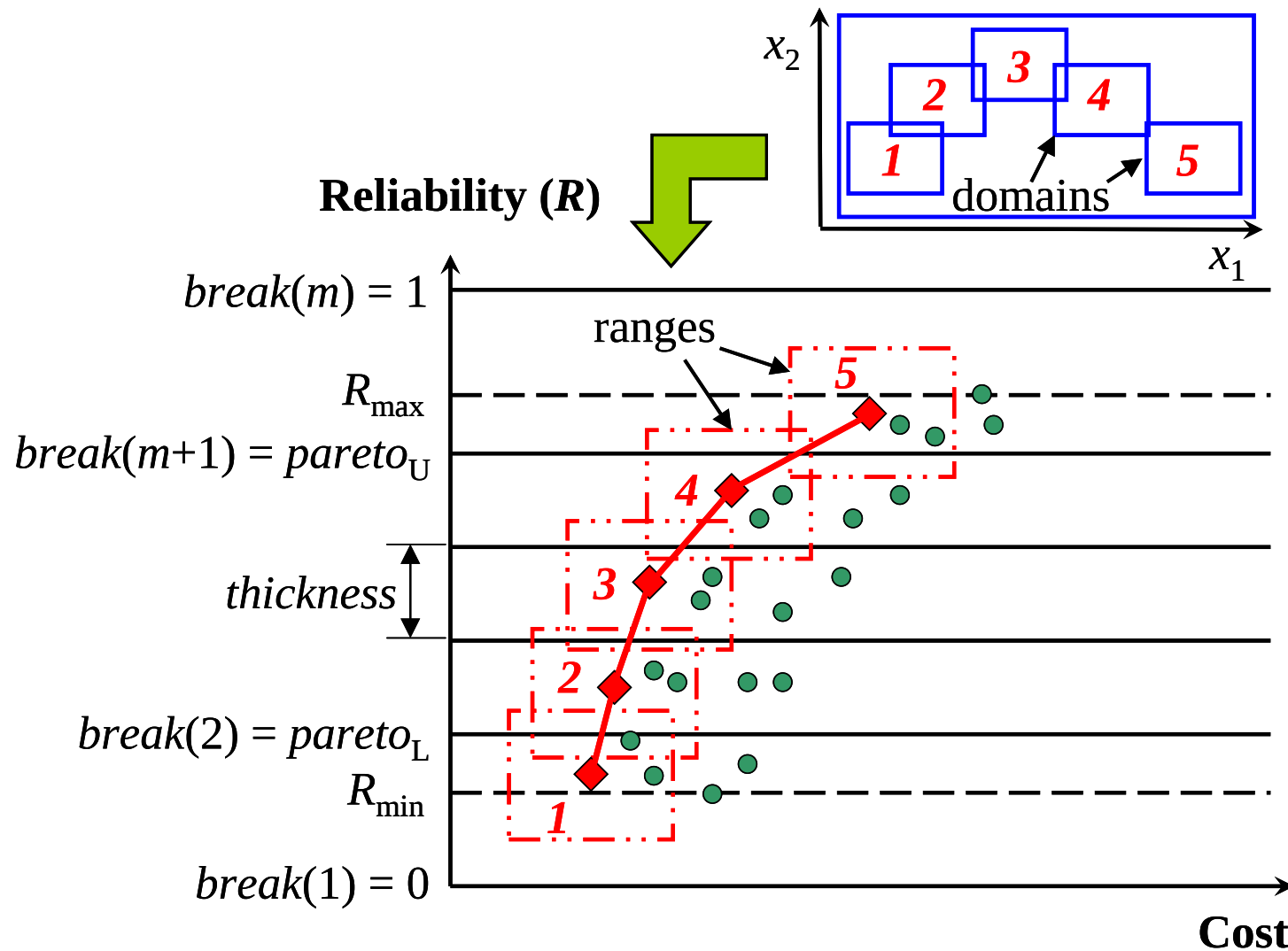
By varying R^t , we get the so called “Pareto Frontier.”

Reliability vs Risk of Failure (Cost)

We want to **maximize Reliability** and simultaneously **minimize Risk of failure (cost)**



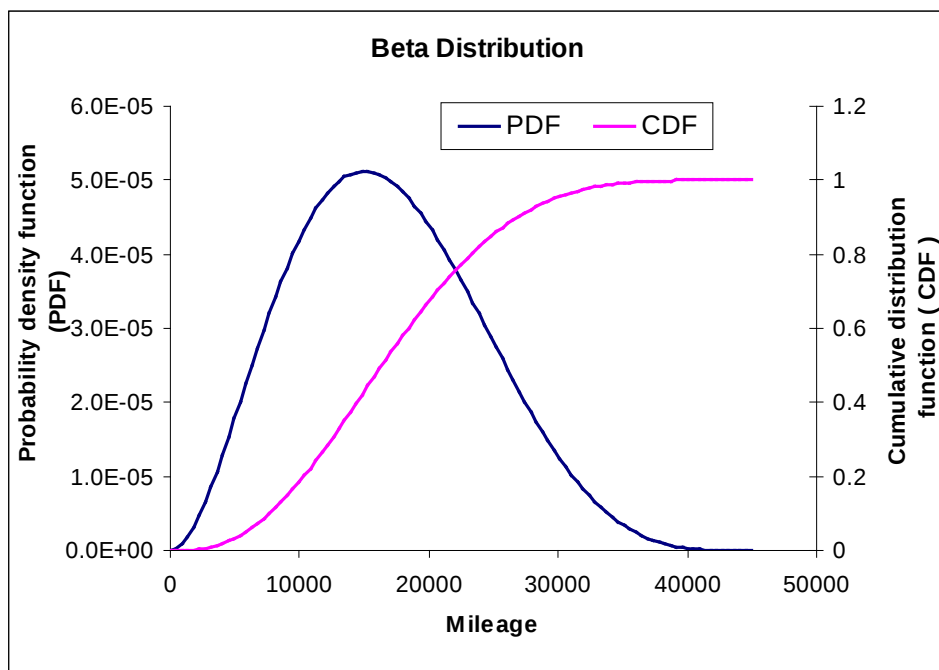
Reliability – Cost Pareto Front Calculation



$$\bar{\sigma} = \frac{\sigma}{B-A}$$

Definition of Design Variables

$$f(x, A, B, p, q) = \beta(p, q)^{-1} (x - A)^{p-1} (B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \leq x \leq B, \text{ and } p > 0, q > 0)$$



$$\mu = MTBF$$

Assume constant COV

Then:

$$\bar{\mu} = \frac{\mu - A}{B - A} \quad \bar{\sigma} = \frac{\sigma}{B - A}$$

$$p = \bar{\mu} \left(\frac{\bar{\mu}(1 - \bar{\mu})}{\bar{\sigma}^2} - 1 \right),$$

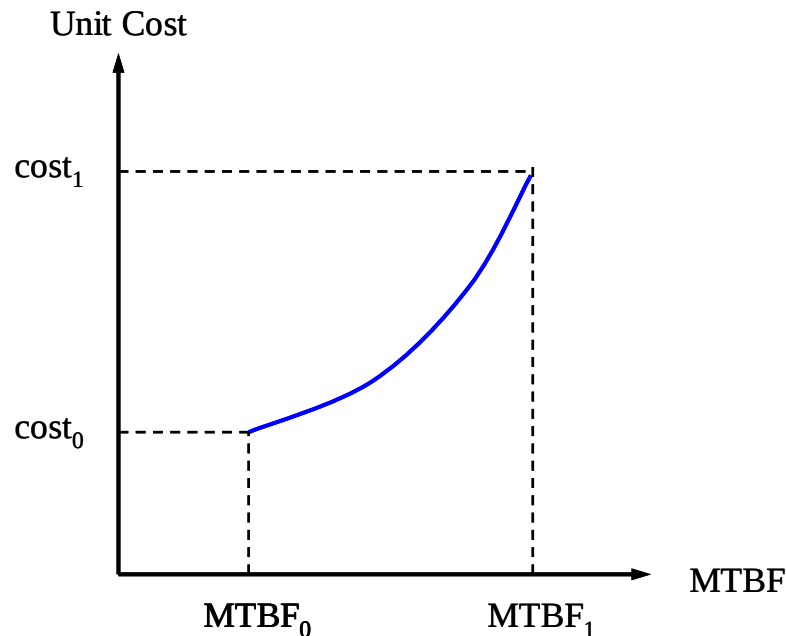
$$q = (1 - \bar{\mu}) \left(\frac{\bar{\mu}(1 - \bar{\mu})}{\bar{\sigma}^2} - 1 \right)$$

Reliability-Cost Relation

$$\text{cost} = \text{cost}_0 e^{k(\text{MTBF}/\text{MTBF}_0 - 1)} \quad : \text{For each component}$$

$$\text{Cost} = \sum_{i_c=1}^{N_c} \left[\text{cost}_0 e^{k(\text{MTBF}/\text{MTBF}_0 - 1)} (1 + \text{failure counts}) \right]_{i_c}$$

For system with N_c components

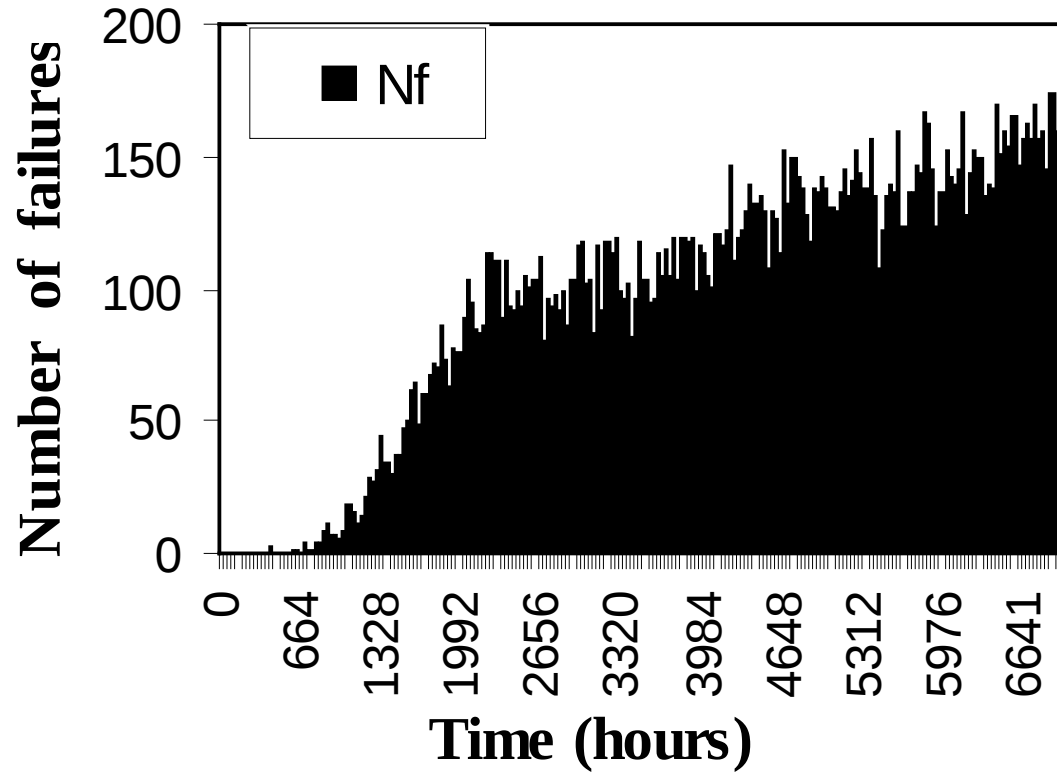


Example

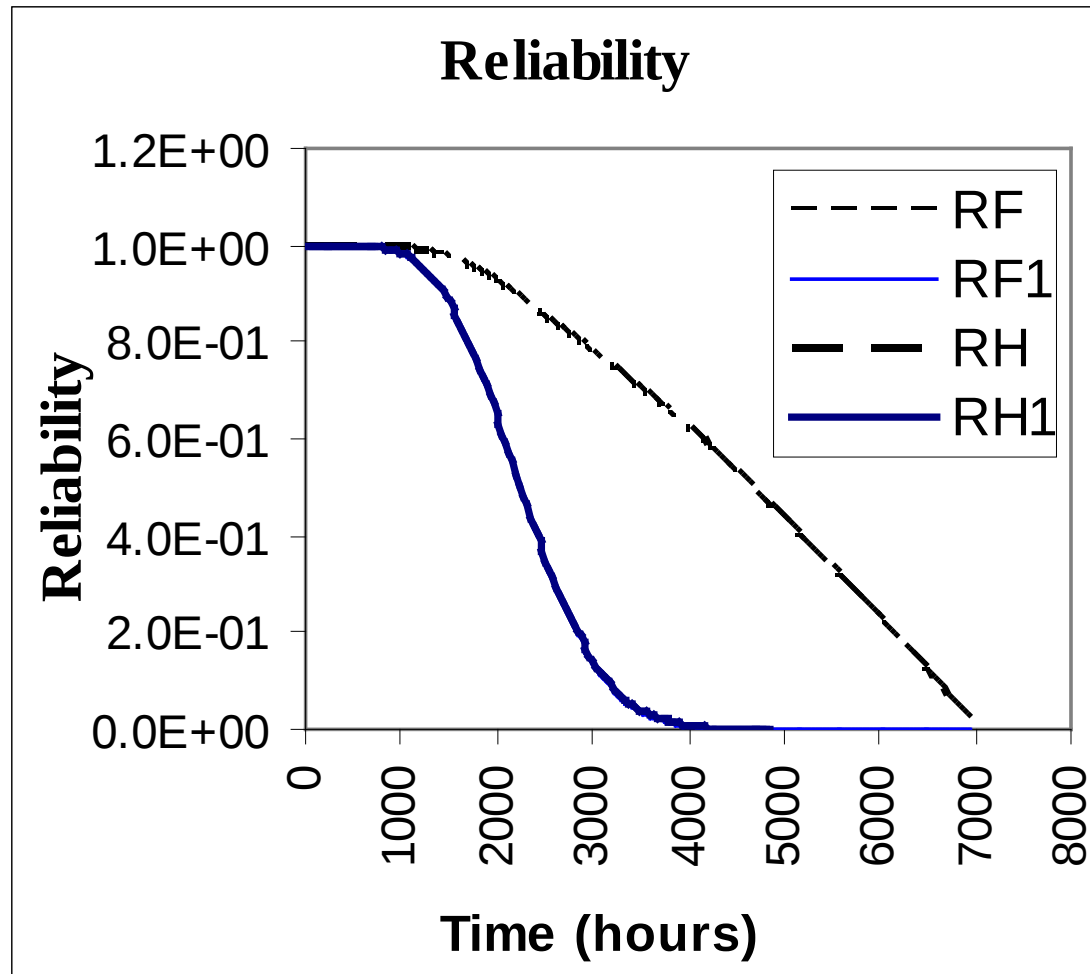
Input Information

<i>Component Number Comp No.</i>	<i>Baseline MTBF in hours (MTBF₀)</i>	<i>Coefficient of Variation</i>	<i>B_{factor}</i>	<i>Baseline cost (Cost₀)</i>	<i>k</i>
1	4076	0.3	3	\$27,500.00	1
2	15000	0.3	3	\$7,000.00	1
3	26510	0.3	3	\$3,000.00	1
4	40000	0.3	3	\$5,000.00	1
5	18000	0.3	3	\$5,000.00	1
6	8000	0.3	3	\$500.00	1
7	31809	0.3	3	\$22,500.00	1
8	9520	0.3	3	\$30,000.00	1
9	9713	0.3	3	\$12,500.00	1
10	2330	0.3	3	\$20,000.00	1
11	40000	0.3	3	\$27,500.00	1
12	8614	0.3	3	\$1,000.00	1
13	45000	0.3	3	\$30,000.00	1
14	20000	0.3	3	\$3,000.00	1
15	25000	0.3	3	\$15,000.00	1

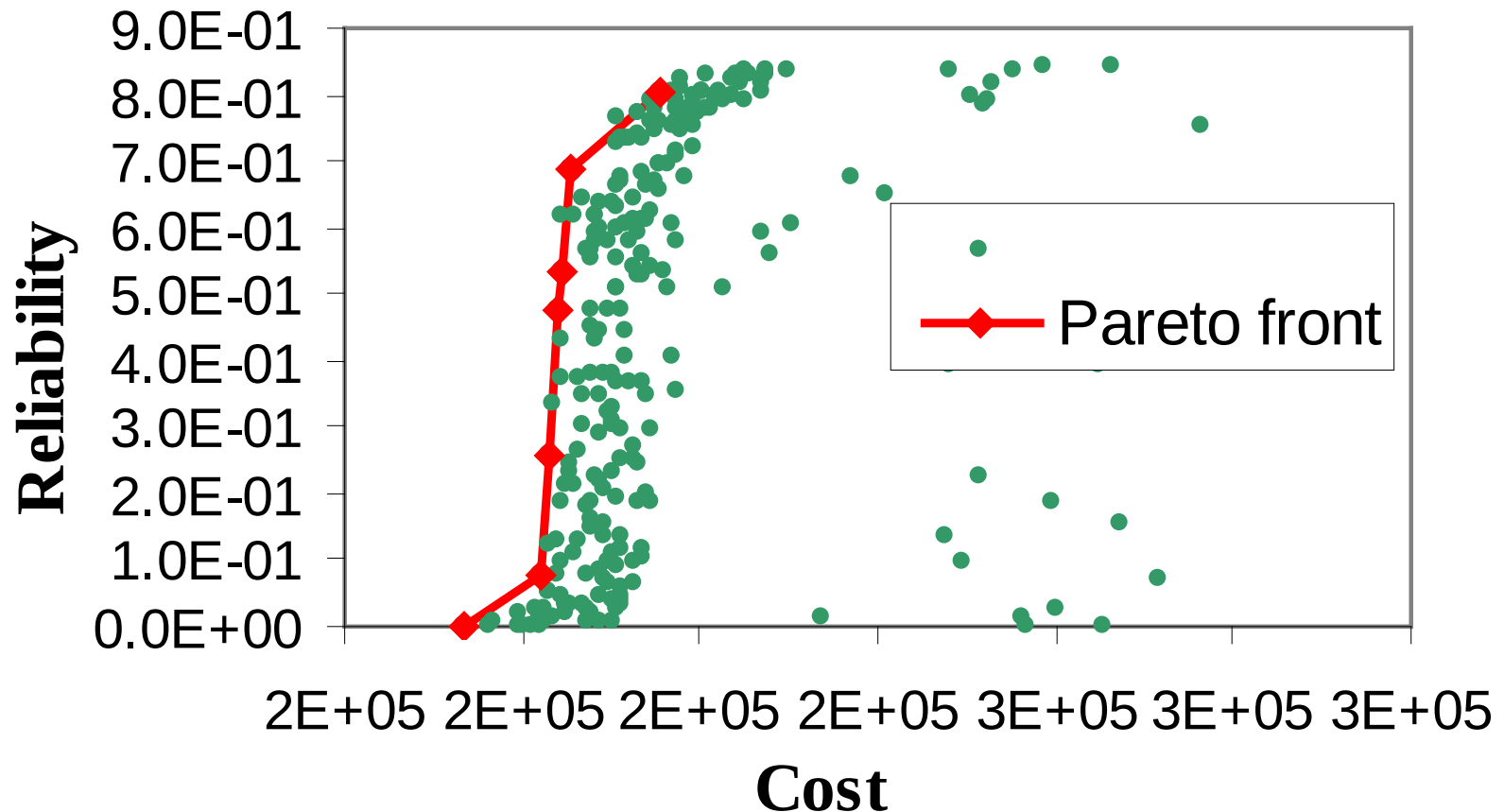
Histogram of System failures



Reliability Comparison between Repairable And Non-Repairable System



System Reliability-Cost Pareto Front



Q & A