



**A Decision Analysis Perspective on Multiple
Response Robust Optimization**

THESIS

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OPTIMIZATION

THESIS

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Abstract

Decisions in which multiple objectives must be optimized simultaneously occur frequently in government, military, and industrial settings. One method a decision maker (e.g., a design engineer) may use to assist in multiple response optimization situations is the application of a desirability function. The decision maker specifies the desirability function parameters so as to express his or her own preferences with respect to the objectives under consideration. An informed specification of the parameters is essential so that the desirability function accurately describes the decision maker's value trade-offs and risk preference. Misapplication of the desirability function may result in the selection of an optimal policy that is inconsistent with the stated preferences. This thesis examines the desirability function from a decision analysis perspective. In particular, utility transversality provides the basis for an analysis of the implicit value trade-off and risk attitude assumptions attendant to the desirability function.

A limitation of the desirability function is its failure to explicitly account for response variability. A robust solution accounts for not only the expected response, but the variance as well. Assessing a utility function over desirability as a means to describe the decision maker's risk preference produces a robust operating solution that is consistent with those preferences. This thesis examines robustness as it applies to the desirability function, using a decision analysis perspective. In particular, a robust manufacturing solution is identified for a wire-bonding process, seen often in the quality and reliability engineering design literature. An exponential utility function over desirability is applied to regression equations developed from a Box-Behnken design. Monte Carlo simulation enables specification of the robust solution.

Using decision analysis methods, this methodology is applied to a practical problem currently facing the Air Force Research Laboratory (AFRL). Contributing to AFRL's Robust Decision Making Strategic Technology Team program, this thesis examines robustness in the context of national policy-making in country stability situation. Different levels of diplomatic, informational, military, and economic (DIME) instruments of national policy are investigated to examine how they affect the political, military, economic, social, infrastructure, and information (PMESII) systems of a nation. AFRL's National Operational Environment Model (NOEM) serves as the basis for the analysis of a scenario involving the Democratic Republic of Congo. A D-optimal design of experiments enables identification of a robust national policy. Employment of a multiattribute utility function that satisfies the axioms of expected utility theory ensures that the policy is consistent with the decision maker's stated preferences.

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Jonathan S. Findley

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A DECISION ANALYSIS PERSPECTIVE ON MULTIPLE RESPONSE ROBUST OPTIMIZATION

I. Introduction

Often, a decision must be made that optimizes a single objective, such as a company desiring to maximize profit. Other times, decisions must be made to simultaneously optimize a set of multiple objectives. Such decisions can occur within any organization.

Examples of multiple objective decisions in government include selecting the proper regional road network that balances the environmental, social and economic impacts of the region's municipalities within a fixed budget [7]. Another example is selecting the best location for a new airport. The competing objectives in this case are minimizing construction costs, while also minimizing transit time for travelers, maximizing safety and minimizing noise pollution [21].

Examples in military settings include designing a military aircraft where maximum range and maximum speed are two competing objectives with maximum payload capacity [25]. Another example is developing a stabilization, security, transition, and reconstruction operation (SSTRO) where the nation's positive political, military, economic, and social indicators are desired [5].

Examples in industry include gum extraction from plant seeds where maximum extraction yield, viscosity, hue, and emulsion stability needs to be balanced with minimum protein content [28]. Another example is machining parts where the balance of metal removal rate and surface roughness needs to be optimized [35]. One's personal life also holds examples of such decisions such as buying a truck where one desires maximum power with maximum fuel economy.

Often, objectives conflict so that selecting an alternative resulting in the simultaneous optimum for each objective is infeasible. Consider the truck buyer desiring increased power in a new truck with increased fuel economy. Nearly always, the truck with the most power will not also have the highest fuel economy. This “ideal” truck does not exist. In these cases, a compromise solution to balance these competing objectives must be found. This correct balance is based in large part on the application and objectives in question as well as the preferences of the decision makers in charge. Although many organizations have a group of people making these types of decisions, this thesis assumes a single decision maker with no loss of generality. Several areas of study exist to assist the decision maker with such decisions.

Decision analysis is a combination of mathematical and logical methods used to assist a decision maker in choosing the appropriate decision in an uncertain world. Using the axioms of utility theory, mathematical models can be created that best describe the preferences of the decision maker [18]. These preferences include the decision maker’s trade-offs between different value measures and the decision maker’s probabilistic preference between different outcomes [18]. Decision analysis can apply to both single attribute situations as well as multiple attribute situations.

Some decision analysis practitioners prefer to treat multiple attribute applications in a way that examines each attribute separately in terms of the decision maker’s preferences for that attribute. Marginal utility functions are developed for each attribute; these separate functions are then combined into a single multiattribute utility function based on certain assumptions made about preferential independence, utility independence and additive independence amongst the attributes [23].

Other decision analysis practitioners prefer to formulate a value function directly considering the deterministic trade-offs between the multiple attributes. A utility function is then assessed for this value measure [1]. Matheson and Abbas propose

the idea of utility transversality in a utility function assigned over a value function [31]. This concept relates the risk aversion functions of the individual attributes to the value trade-off functions between these attributes. Abbas also relates a decision maker’s risk aversion over value to the decision maker’s multiattribute risk aversion [2].

Often, a decision maker is interested in adjusting a set of controllable inputs to optimize a set of outputs. Response Surface Methodology (RSM) is a set of statistical and mathematical methods used to develop or improve processes [32]. Used widely in industry, most RSM applications optimize a response variable which is a function of one or more input variables. These functions can be known exactly such as through a chemical or engineering process. Other times, the underlying function is not known and is estimated using various statistical methods [32].

In many cases, more than one response variable is important to a process. One example is a machining process with machining parameters as input variables and two response variables: removal rate and surface roughness. The optimum decision might be the proper combination of machining parameters that simultaneously maximizes removal rate and minimizes surface roughness [35].

Numerous multiple response optimization models exist in current RSM literature [15, 9, 13, 24, 45]. Harrington introduces one such model [15]. Harrington’s desirability function transforms each response to an individual desirability level between zero and one. The optimum strategy is then the one which produces the highest geometric mean of the individual desirability levels [15].

Derringer and Suich modified the form of the individual desirability function to be more flexible [13]. They change the individual desirability function to provide the decision maker more control over how quickly desirability moves from zero to one as the response moves from its worst value to its target value.

In employing the desirability function, care must be taken by the analyst to choose parameters consistent with the decision maker's own preferences. The desirability function exhibits implicit and explicit assumptions regarding risk attitude, value trade-offs, and attribute independence. Potential problems may result from using a multiple response optimization model without fully understanding these assumptions. Kros and Mastrangelo introduce the idea of applying utility theory to examine the assumptions and preferences underlying desirability functions [29]. They attempt to compare and contrast assumptions regarding risk preferences, trade-offs, and relationships between the multiple attributes inherent in Derringer and Suich's desirability function [29].

This thesis examines the desirability function from a decision analysis perspective. This analysis provides knowledge about how best to employ the desirability function in a manner consistent with the decision maker's value trade-offs and risk attitude.

In the current climate of budgetary constraints across industrial and governmental organizations, proper analysis of a decision situation is vital to maximizing the limited resources available. When confronted with a situation where multiple objectives need to be considered, the analyst needs to understand the assumptions inherent to the model chosen so that it is consistent with the decision maker's preferences and the proper solution for the organization can be found.

The rest of this thesis is organized as follows. Chapter 2 presents a review of relevant literature. Chapter 3 develops the methodology used to examine the desirability function and the methodology used to find a robust optimum solution from a decision analysis perspective. Chapter 4 presents the analysis of a robust optimization solution in a wire bonding process experiment in the semiconductor industry using various assumptions about risk attitude and value trade-offs. Chapter 5 presents the analysis of

a robust optimization solution in a nation-building (i.e., SSTRO) example. Chapter 6 offers significant findings, recommendations, and suggestions for future work.

II. Review of Related Literature

2.1 Organization

This chapter reviews the related literature applicable to the thesis. Section 2.2 gives an overview of decision analysis including risk preferences and modeling uncertainty in cases of single attributes and multiple attributes.

Section 2.3 discusses Response Surface Methodology and how the desirability function is used to find an optimum setting in a multiple response situation. The idea of a robust optimum point in a noisy environment is also discussed.

Section 2.4 discusses stability operations and their place in the United States security plan are then discussed. The Air Force Research Lab models such operations with their National Operational Environment Model.

2.2 Decision Analysis

In a system of unknowns such as a business venture where future profits and product demand growth can only be estimated, decision analysis is an excellent tool to balance the factors, both certain and uncertain, that apply to a given decision situation. Ronald Howard describes decision analysis as a cycle encompassing deterministic, probabilistic, and informational phases to settle on a logically best decision [18]. The deterministic phase establishes certain relationships between the variables within the problem. The probabilistic phase introduces the uncertainties and risk preference of the decision maker. The informational phase determines the value of gathering more information. If more information is to be gathered, the cycle is repeated until no new information is deemed necessary by the decision maker [18].

The uncertainty involved in a business investment venture is not something easily measured. Howard describes two types of probabilities: objective and subjective. An

objective probability is one that is measured after several instances of the uncertain event occur. For example, finding after 1,000 coin flips that the coin came up heads approximately 500 times could lead one to objectively assign a 50% probability to that coin coming up heads on the next coin flip. Alternatively, a subjective probability is one which is assigned based on individual knowledge about the nature of the uncertain event. For example, assigning a 50% probability to a coin one has never seen flipped before based on the fact the coin appears to be 'fair' is subjective [18]. In most cases, a particular business venture cannot be tried several times to see how likely it is to be successful. Only a subjective probability can be assigned to its success based on analytical knowledge of the nature of this and similar ventures.

2.2.1 Utility Theory.

Howard describes utility theory as encompassing five axioms concerning the idea of lotteries [18]. A lottery is a set of outcomes in which exactly one occurs. The first axiom requires transitivity in preference. If three alternatives, A, B, and C, are available and if an individual prefers A to B and prefers B to C, this individual must prefer A to C.

The second axiom calls for a probabilistic preference. Consider one who prefers A to B to C. Then, a preference probability, p , must exist where this person is indifferent to accepting B for certain or accepting a lottery which produces A with probability p or produces C with probability $(1 - p)$. When a particular value of p is found, B becomes the certain equivalent (CE) of the lottery between A and C. Figure 1 shows how this lottery and its CE are usually depicted in decision analysis literature.

The third axiom involves substitution. The CE of a lottery can be exchanged or substituted for the lottery itself in any situation without any changes of preference for the decision maker.

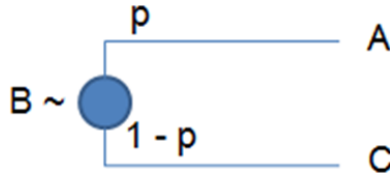


Figure 1. B is the certain equivalent of the lottery between A and C

The fourth axiom states the acceptance of probability as the means to describe uncertainty and also expresses indifference in the ordering of lotteries in the decision at hand. It states that multiple levels of lotteries may be replaced by a single lottery with the prizes and associated probabilities calculated by the laws of probability. In essence, this axiom states that the lottery itself holds no intrinsic value to the decision maker.

The fifth axiom states if a decision maker is faced with two lotteries, each with outcomes of A or B, and the decision maker prefers A to B, the decision maker must prefer the lottery that yields A with the higher probability.

2.2.2 Risk Preference.

Any practical decision opportunity which involves uncertainty must take into account the decision maker's preference towards risk. Howard points out that few people would be willing to give up next year's salary for a 50% chance of doubling it versus receiving no salary for the year although this represents a fair proposition [18]. A person indifferent to the preceding deal is considered to be risk neutral. Howard suggests that most people and organizations are risk-averse. A risk-averse individual's CE for a given lottery is less than the expected value of the outcome of that lottery [18].

2.2.3 Utility Function.

One way to calculate a decision maker's risk preference is to find the preference probabilities for all feasible alternatives within the current decision space. Depending on the situation and the number of alternatives, this can become quite cumbersome. The utility function encodes the decision maker's risk preference. This function associates an outcome from a lottery or other uncertain situation with a utility value. Moreover, the decision maker's utility of a lottery is the expected value of the u-values of the lottery's outcomes. From the example in Figure 1, the decision maker's utility for the lottery would be $u_1 = p * U(A) + (1 - p) * U(C)$. If the decision maker prefers one lottery to another, the preferred lottery's utility will be higher than the other.

Howard also points out that although these u-values can model the decision maker's preferences among various alternatives or lotteries, the actual magnitude of a utility means nothing on its own [18]. Comparing utilities cannot be used to show how strongly one alternative is preferred to another. It can only be used for ranking purposes [18].

Arrow and Pratt [6, 37] introduce the risk aversion function,

$$\gamma(y) = -\frac{u''(y)}{u'(y)}. \quad (2.1)$$

They show the negative ratio of a utility function's second derivative over its first derivative correctly measures the local risk aversion at any given point along a utility function. This measure assumes that the utility function in question is monotonically increasing and is twice differentiable. If $\gamma(y) = 0$, the utility function is describing risk neutral behavior. If $\gamma(y) > 0$, it is describing risk averse behavior and if $\gamma(y) < 0$, it is describing risk seeking behavior [37].

2.2.4 Modeling Uncertainty.

Decision analysts employ several methods to model uncertainty. Two common methods are using a discretized approximation of the continuous distribution or using a Monte Carlo simulation over the distribution [11]. A discretized approximation simplifies the calculations involved with using a continuous function. When the continuous probability function is unknown, it can be estimated by using known data to estimate the parameters of a Beta distribution.

The Beta distribution is a flexible continuous probability distribution given over a set bounded range [3]. Equation 2.2 shows the density function.

$$Beta(\alpha, \beta, a, b, x) = \frac{(x - a)^{\alpha-1}(b - x)^{\beta-1}}{\int_a^b (x - a)^{\alpha-1}(b - x)^{\beta-1} dx} \quad (2.2)$$

Where a and b are the lower and upper bounds of the domain, and α and β are the two parameters of the Beta distribution [3].

Abbas et al. [3] describe assessing the data into fractiles and then using Matlab's *fminsearch* function to estimate the two parameters $(\hat{\alpha}, \hat{\beta})$ by minimizing the squared errors from the value, x , from the inverse of its related probability p . Equation 2.3 shows the relevant expression,

$$\min_{\hat{\alpha}, \hat{\beta}} \sum_{i=1}^n (X_i - \hat{X}_i)^2, \quad (2.3)$$

where $\hat{X}_i = BetaInverse(p_i, \hat{\alpha}, \hat{\beta}, a, b)$, X_i is the midpoint of the i -th fractile, n is the number of fractiles used, and $\hat{\alpha}, \hat{\beta} > 0$. The parameters a and b are chosen by the user and p_i is the probability associated with the i -th fractile [3].

Monte Carlo simulation is a method to model the uncertainty in a given system. Assume an uncertainty can be modeled by a known distribution; a computer then

produces a large number of random numbers from that distribution. The utilities of the associated outcomes are then used to find the expected utility of the lottery [11].

Often multiple, correlated random variables must be modeled in the decision model. Sklar [44] states given a joint cumulative distribution function (CDF) $F(x_1, \dots, x_n)$ with marginal CDFs $F_1(x_1), \dots, F_n(x_n)$, the joint CDF can be written as a function of its marginals,

$$F(x_1, \dots, x_n) = C[F_1(x_1), \dots, F_n(x_n)]. \quad (2.4)$$

The function, C , is called a copula. If each marginal CDF, F_i , is continuous, then C is a unique function for the given joint CDF [44].

Clemen and Reilly [10] describe how to create the joint multivariate random sample for a Monte Carlo simulation using the multivariate normal copula: 1) generate a vector of random numbers $(y_1, \dots, y_n)$ from a multivariate normal random number generator using the desired correlation matrix, $\mathbf{R}$, 2) the standard normal distribution function, $\Phi(y_i)$ is calculated for each of the n variables, 3) the inverse marginal distribution functions for each variable is then used to calculate the vector of required variates, $(F_1^{-1}[\Phi(y_1)], \dots, F_n^{-1}[\Phi(y_n)])$ for the Monte Carlo simulation.

2.2.5 Multiattribute Utility Theory.

Two common methods for formulating multiattribute utility functions include creating a multiattribute value function using deterministic trade-offs between the attributes and then assessing a single attribute utility function over that value or by assessing conditional utility functions over each of the individual attributes and then combining these into a single multiattribute utility function by using various independence assumptions [1].

Some of the independence assumptions made while forming a multiattribute utility function include preferential independence, utility independence, and additive independence. To describe these independence assumptions, consider a multiattribute decision situation with up to three attributes, y_1, y_2, y_3 where each attribute falls within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, 2, 3$. Assume the attributes can be ordered such that (y_1^0, y_2^0, y_3^0) is the least preferred prospect and (y_1^*, y_2^*, y_3^*) is the most preferred prospect.

The attributes, y_1 and y_2 , are said to be preferentially independent of y_3 if a given deterministic prospect $(y_1^{(2)}, y_2^{(2)}, y_3)$ is preferred to or indifferent to $(y_1^{(1)}, y_2^{(1)}, y_3)$ regardless of the value of y_3 [22]. Equation 2.5 displays this concept.

$$(y_1^{(2)}, y_2^{(2)}, y_3) \succeq (y_1^{(1)}, y_2^{(1)}, y_3), \forall y_3 \quad (2.5)$$

In a two attribute decision space with attributes, y_1 and y_2 , y_1 is said to be utility independent of y_2 if the conditional preferences of y_1 given a certain value of y_2 do not depend on the value of y_2 [22]. In other words, the conditional utility function over y_1 given a fixed value of y_2 is a positive linear transformation of the conditional utility function over y_1 given y_2 is fixed at any other value [22]. If y_1 is utility independent of y_2 , then the two-attribute utility function must be of the form in Equation 2.6,

$$u(y_1, y_2) = g(y_2) + h(y_2)u(y_1, y_2^{(1)}), \quad (2.6)$$

where $g(y_2)$ and $h(y_2)$ are both functions that depend on y_2 only and $u(y_1, y_2^{(1)})$ is the conditional utility function over y_1 given $y_2 = y_2^{(1)}$ [23].

Two attributes, y_1 and y_2 , are said to be additive independent if the comparison between any two prospects depends only on the marginal preference structure of the two attributes. Consider two levels of each attribute, $y_1^{(1)}, y_1^{(2)}, y_2^{(1)}, y_2^{(2)}$. If y_1 and y_2

are additive independent, then the lottery with equal chances of the two prospects, $(y_1^{(1)}, y_2^{(1)})$ and $(y_1^{(2)}, y_2^{(2)})$ is equivalent to the lottery with the two prospects, $(y_1^{(1)}, y_2^{(2)})$ and $(y_1^{(2)}, y_2^{(1)})$, with equal chances [23].

If two attributes are additive independent, the two-attribute utility function is additive and can be written as in Equation 2.7.

$$u(y_1, y_2) = k_{y_1} u_{y_1}(y_1) + k_{y_2} u_{y_2}(y_2) \quad (2.7)$$

The utility function is normalized such that $u(y_1^0, y_2^0) = 0$ and $u(y_1^*, y_2^*) = 1$. The marginal utility function, $u_{y_i}(y_i)$, is normalized such that $u_{y_i}(y_i^0) = 0$ and $u_{y_i}(y_i^*) = 1$ ($i = 1, 2$). The constants, k_{y_1} and k_{y_2} are calculated such that $k_{y_1} = u(y_1^*, y_2^0)$ and $k_{y_2} = u(y_1^0, y_2^*)$ [23].

Richard [39] defines multivariate risk aversion as the condition in which the decision maker prefers a lottery with an even chance of the prospects, $(y_1^{(1)}, y_2^{(2)})$ or $(y_1^{(2)}, y_2^{(1)})$, where $y_i^{(1)} < y_i^{(2)}$, $i = 1, 2$ to a lottery with even chances for $(y_1^{(1)}, y_2^{(1)})$ or $(y_1^{(2)}, y_2^{(2)})$. Indifference between these two lotteries is referred to as multivariate risk neutrality. A preference of the second lottery to the first is multivariate risk seeking behavior. Richard shows that the sign of the mixed partial derivative of the utility function indicates the multivariate risk preference expressed by the utility function [39]. Table 1 gives the results.

Table 1. Sign of utility function's mixed partial derivative compared to multivariate risk preference

Sign of $\frac{\partial^2}{\partial y_1 \partial y_2} U(y_1, y_2)$	Multivariate Risk Preference
-	averse
0	neutral
+	seeking

2.2.6 Utility Transversality.

Matheson and Abbas introduce the concept of utility transversality in the multi-attribute case where a utility function is assigned over a multiattribute value function [31]. Equation 2.8 shows the general formulation of such a utility function,

$$U(y_1, \dots, y_n) = U_V(V(y_1, \dots, y_n)), \quad (2.8)$$

where $y_1, \dots, y_n$ are the attributes under consideration, $V(y_1, \dots, y_n)$ is the deterministic value function over these attributes, and $U_V(V)$ is the utility function with respect to value [31].

Assuming that the utility and value functions are both monotonically increasing and twice differentiable, the risk aversion function with respect to a single attribute, $y_i, i = 1, \dots, n$, is given in Equation 2.9,

$$\gamma_{y_i}^U = -\frac{U''_{y_i}}{U'_{y_i}}, \quad (2.9)$$

where $U''_{y_i} = \partial^2 U(y_1, \dots, y_n) / \partial y_i^2$ and $U'_{y_i} = \partial U(y_1, \dots, y_n) / \partial y_i$ [31].

Considering the two dimensional case where $U(y_1, y_2) = U_V(V(y_1, y_2))$ and following the chain rule of differentiation, Matheson and Abbas [31] show that the risk aversion with respect to a single variable is

$$\gamma_{y_1}^U = \gamma_V^U V'_{y_1} + \gamma_{y_1}^V. \quad (2.10)$$

The risk aversion with respect to value is γ_V^U . The partial derivative of value with respect to y_1 is $V'_{y_1} = \partial V(y_1, y_2)/\partial y_1$. Matheson and Abbas define $\gamma_{y_1}^V$ as the value function's contribution to the risk aversion with respect to y_1 as

$$\gamma_{y_1}^V = -\frac{V''_{y_1}}{V'_{y_1}}, \quad (2.11)$$

where $V''_{y_1} = \partial^2 V(y_1, y_2)/\partial y_1^2$ [31]. This is analogous to the definition of risk aversion shown in Equation 2.1.

The risk aversion with respect to a single attribute is defined completely by the risk aversion with respect to value and the form of the deterministic value function itself. Since the value function is deterministic and can be assessed by specifying the deterministic tradeoffs between attributes, then assessing the utility function over value determines the risk aversion function for all attributes [31].

Matheson and Abbas define the utility transversality relation as the relationship between the risk aversion functions of the various attributes and is shown in Equation 2.12 [31].

$$\gamma_{y_1}^U = (\gamma_{y_2}^U - \gamma_{y_2}^V)t + \gamma_{y_1}^V \quad (2.12)$$

The value, t , is the deterministic tradeoff function between the attributes y_1 and y_2 along an isopreference contour, calculated as in Equation 2.13 [31].

$$t(y_1, y_2) = \frac{V'_{y_1}}{V'_{y_2}} = -\frac{dy_2}{dy_1}|_{\text{isopreference contour}} \quad (2.13)$$

2.2.7 Attribute Dominance.

Abbas and Howard introduced the concept of attribute dominant utility functions [4]. This special class of multiattribute utility functions satisfies four conditions.

Consider a two-attribute decision situation where a given prospect can be written as (y_1, y_2) and the two attributes, y_1 and y_2 , fall within the respective ranges, $y_1 \in [y_1^0, y_1^*]$ and $y_2 \in [y_2^0, y_2^*]$. Assume the attributes can be ordered such that (y_1^0, y_2^0) is the least preferred prospect and (y_1^*, y_2^*) is the most preferred. Assume the attributes are mutually preferentially independent and the prospects are arranged such that for any $y_1^{(1)}, y_1^{(2)}, y_2^{(1)}, y_2^{(2)}$, the following conditions hold [4].

$$\begin{aligned} y_1^{(2)} \geq y_1^{(1)} &\Rightarrow (y_1^{(2)}, y_2) \succeq (y_1^{(1)}, y_2) \forall y_2 \in [y_2^0, y_2^*] \text{ and} \\ y_2^{(2)} \geq y_2^{(1)} &\Rightarrow (y_1, y_2^{(2)}) \succeq (y_1, y_2^{(1)}) \forall y_1 \in [y_1^0, y_1^*] \end{aligned} \quad (2.14)$$

Assume a multiattribute utility function $U_{y_1 y_2}(y_1, y_2)$ exists with range as given in Equation 2.15 [4].

$$0 \leq U_{y_1 y_2}(y_1, y_2) \leq 1, \forall y_1 \in [y_1^0, y_1^*], y_2 \in [y_2^0, y_2^*] \quad (2.15)$$

Based on this formulation, the utility of the prospects, (y_1^0, y_2^0) and (y_1^*, y_2^*) , are as shown in Equation 2.16 [4].

$$U_{y_1 y_2}(y_1^0, y_2^0) = 0, U_{y_1 y_2}(y_1^*, y_2^*) = 1 \quad (2.16)$$

With the preceding assumptions, an attribute dominance utility function is one where a prospect, (y_1, y_2) , is a least preferred prospect if at least one of its attributes, y_1 or y_2 , is at its minimum value. Equation 2.17 describes this condition [4].

$$U_{y_1 y_2}(y_1^0, y_2^0) = U_{y_1 y_2}(y_1^0, y_2) = U_{y_1 y_2}(y_1, y_2^0) = 0, \\ \forall y_1 \in [y_1^0, y_1^*], y_2 \in [y_2^0, y_2^*] \quad (2.17)$$

2.3 Response Surface Methodology

Response Surface Methodology (RSM) is a set of mathematical and statistical tools used to optimize systems [33]. RSM provides techniques to estimate a model of the system of interest by running an experiment varying the input variables of interest and measuring the response of interest [32]. A function is estimated from the results of such an experiment as shown in Equation 2.18

$$y = \beta \mathbf{x} + \epsilon \quad (2.18)$$

where y is the response in question, $\mathbf{x}$ is the vector of input variables, β is a vector of coefficients estimated from the experiment and ϵ is a random variable representing the random noise within the system [32].

Equation 2.18 can be extended to multiple responses using a vector of responses, $\mathbf{y}$, a matrix of coefficients, $\mathbf{B}$, and a vector of random noise variables, $\boldsymbol{\epsilon}$. Equation 2.19 shows such an equation.

$$\mathbf{y} = \mathbf{B}\mathbf{x} + \boldsymbol{\epsilon} \quad (2.19)$$

Classic RSM and regression techniques are built around systems occurring in nature or in man-made environments. Special considerations must be given when analyzing computer simulation models. Common random numbers are commonly used when comparing different simulation scenarios since they can sharpen the compari-

son between scenarios. When attempting to estimate the simulation model with a regression polynomial, non-overlapping pseudo-random number (PRN) streams must be used at the different design points so the outputs are independently identically distributed as RSM methods assume [27].

One other difference in a computer simulation experiment is that it can be accomplished sequentially without needing to consider blocking the the analysis. In a classic experiment, the experimenter would generally randomize the order of the design points to remove any ordering bias. If an augmented design were to be added to this experiment, the experimenter would have to analyze the data with that sequence in mind since the augmented design points were not randomized with the original set. A computer simulation experiment can be sequenced similarly without any additional concern to the change in analysis due to the augmented design. The order of the experiment does not affect the computer output if a non-overlapping random stream of PRNs is used to initiate any design points [27].

2.3.1 Desirability Function.

Harrington introduces the desirability function in order to find an optimal solution when optimizing multiple, often competing, objectives simultaneously [15]. The desirability function scales each objective measure, y_i , to a scale between 0 and 1, denoted by d_i . The overall desirability of a given solution is the geometric mean of the individual desirabilities as shown in Equation 2.20 [15]. The optimal solution is the vector of y_i 's that produces the largest overall desirability, D .

$$D = (d_1 d_2 \cdots d_n)^{\frac{1}{n}} \tag{2.20}$$

Derringer and Suich modify the function for the individual objective desirabilities so that they are more flexible [13]. Equation 2.21 shows the function for an individual objective's desirability where a maximum value of the response, y_i , is desired.

$$d_i = \begin{cases} 0, & y_i \leq L_i \\ \left[\frac{y_i - L_i}{T_i - L_i} \right]^{r_i}, & L_i < y_i < T_i \\ 1, & y_i \geq T_i \end{cases} \quad (2.21)$$

The parameters, L_i , T_i , and r_i , are provided by the decision maker. The parameter L_i indicates the minimum acceptable value of y_i . Any value of y_i below L_i is unacceptable. This qualitative characteristic is expressed in the desirability function by forcing $d_i = 0$ which then forces the overall desirability to $D = 0$ as well. The parameter T_i is an optimal value of y_i or the point at which any more y_i would give no additional value. The parameter r_i controls how quickly the individual objective desirability increases from 0 to 1 as y_i increases from L_i to T_i . Figure 2 depicts how different values of r_i affect the desirability function.

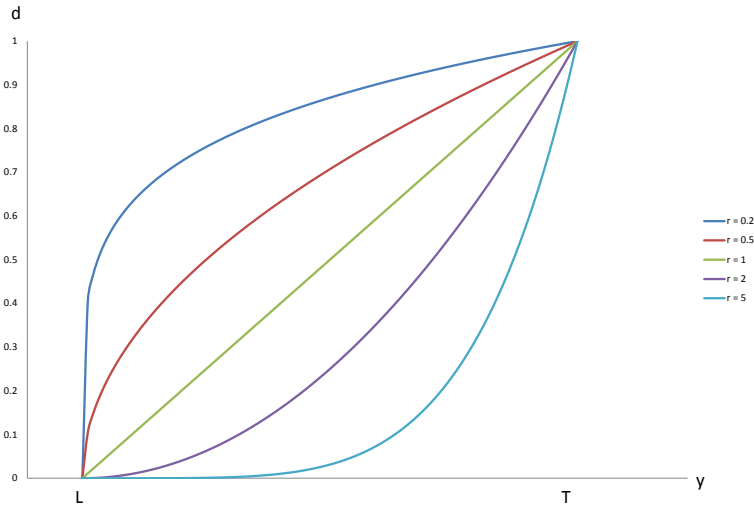


Figure 2. Maximizing desirability functions, d_i , for various levels of r_i [13]

Equation 2.22 shows the function for an individual objective's desirability where a minimum value of the response, y_i , is desired.

$$d_i = \begin{cases} 1, & y_i \leq T_i \\ \left[\frac{y_i - U_i}{T_i - U_i} \right]^{r_i}, & T_i < y_i < U_i \\ 0, & y_i \geq U_i \end{cases} \quad (2.22)$$

The parameters, U_i , T_i , and r_i , are similarly provided by the decision maker. The parameter U_i is the maximum value of y_i . Any value above this is unacceptable to the decision maker and will drive the overall desirability to $D = 0$. The parameter r_i is similar to its use in the maximizing function. The parameter T_i is the optimal value of y_i or the point at which any level of y_i below this would warrant little extra value [13].

At times, a specific target value is desired. In this case, a two-sided transformation is applicable as shown in Equation 2.23.

$$d_i = \begin{cases} 0, & y_i \leq L_i \\ \left[\frac{y_i - L_i}{T_i - L_i} \right]^{s_i}, & L_i < y_i \leq T_i \\ \left[\frac{y_i - U_i}{T_i - U_i} \right]^{t_i}, & T_i < y_i < U_i \\ 0, & y_i \geq U_i \end{cases} \quad (2.23)$$

The parameters, L_i and s_i , are synonymous to L_i and r_i in equation 2.21. The parameters, U_i and t_i , are synonymous to U_i and r_i in equation 2.22. The parameter T_i is the optimal value of the objective, y_i . When $y_i = T_i$, the individual desirability, $d_i = 1$ [13]. Figure 3 depicts two-sided transformations for various levels of s_i and t_i .

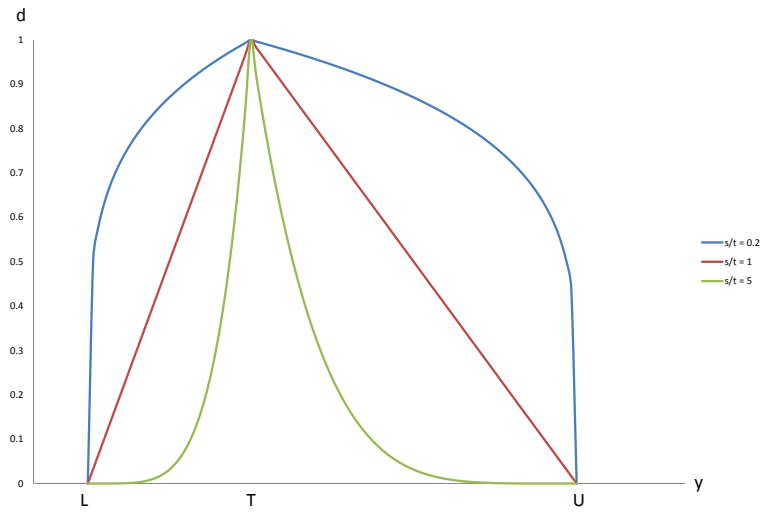


Figure 3. Two-sided desirability functions, d_i , for various levels of s_i and t_i [13]

2.3.2 Robust Optimization.

The optimal solution found in a multiple response situation using the desirability function does not take uncertainty into consideration and does not consider the variability of the responses [36].

Figure 4 illustrates the concept of a robust solution. The x -axis shows an input variable affecting a maximum response along the y -axis. At $x = x_2$, point B is clearly the global maximum within this range. However, as x is allowed to vary from x_2 by r , the response drops Δ_B , from y_2 to y'_2 . Conversely, if the local optimum point A is considered and x is allowed to vary from x_1 by r , the response drops Δ_A , from y_1 to y'_1 . In this situation, where $\Delta_A < \Delta_B$, point A can be considered more robust than point B [16].

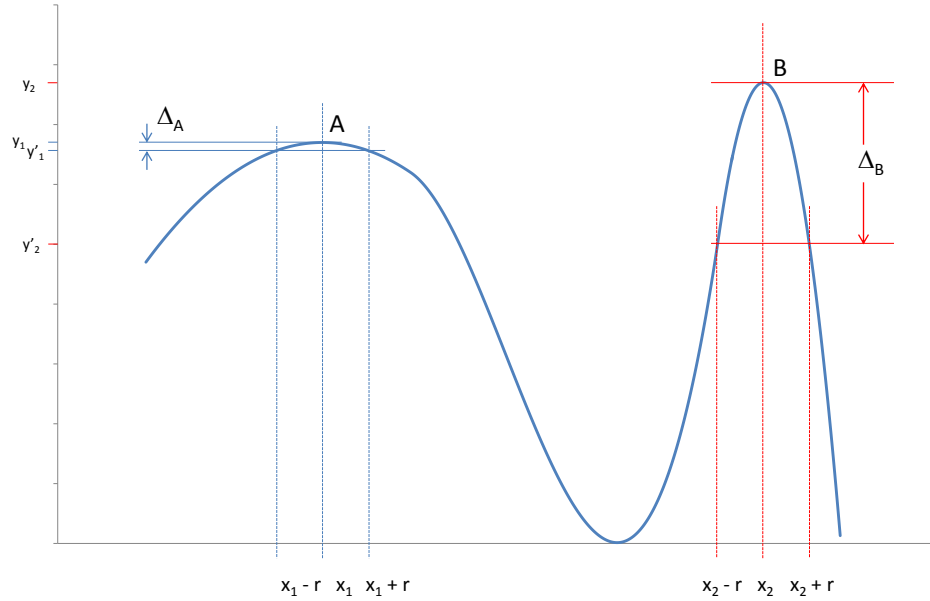


Figure 4. Illustration of Robust Solution [16]

From a decision analysis perspective, the preference between point A and point B is dependent upon the decision maker's risk preference. A sufficiently risk averse

decision maker would prefer point A. A decision maker tending toward risk neutral or risk seeking behavior would prefer point B instead.

Taguchi introduces robust parameter design [46, 47] for situations where uncontrollable or nuisance factors exist and a solution that is insensitive to the variability of these nuisance factors is desired.

Vining and Myers present a constrained optimization method using predictive regression models for both the process mean and variance [51]. The robust solution is found by either constraining the mean while minimizing the variance or constraining the variance while optimizing the mean [51].

While these two methods focus on a single response of interest, Peterson, Miró-Quesado and del Castillo present a robust approach to a multiple response problem using Monte Carlo simulation [36]. After calculating the response equations to the seemingly unrelated regression (SUR) model, they calculate the probability a given set of inputs would give an acceptable set of output values using results from a Monte Carlo simulation.

2.4 Stability, Security, Transition, and Reconstruction Operations

The National Security Strategy (NSS) [34] outlines the overall security strategy for the nation. President Obama states the greatest threat to the American people is the possibility of violent extremists obtaining weapons of mass destruction. Engagement with other countries, especially emerging nations, is key to the NSS [34]. President Obama notes the need for an interagency approach to this end. Assisting developing countries join the world economy, manage their security needs, and assist their governments to lead their nations with an eye to human rights is imperative. The military's role includes excelling at counterterrorism and stability operations while maintaining readiness to address the full range of military operations [34].

Stability operations are essential to keeping a safe and secure environment for U.S. interests abroad. Department of Defense Instruction (DoDI) 3000.05 [48] states stability operations are a core U.S. military mission and should be conducted across the range of military operations. Further, U.S. military operations must strike “appropriate balance between offensive and defensive operations and stability operations in all phases” [49].

Military stability operations are planned based on effects based operations (EBO) as outlined by the United States Joint Forces Command [50]. A nation or region in question is characterized as a system of systems in terms of its political, military, economic, social, infrastructure, and information (PMESII) systems. The effects of diplomatic, informational, military and economic (DIME) instruments of national policy is then modeled to estimate the PMESII effects within the nation or region in question [50].

The current NSS is interested in developing diplomatic relations, security measures, and economic markets in African nations [34]. The U.S. is particularly concerned about assisting the Democratic Republic of Congo (DRC) in its efforts to create a secure, stable government. Concern still exists about human rights violations in the DRC after its recent presidential election [12].

2.5 National Operational Environment Model

The National Operational Environment Model (NOEM) is a simulation model representing a nation-state or region used to test a variety of courses of actions [42]. NOEM subscribes to the DIME and PMESII paradigm while modeling nation security and stability [43]. The main contributions to NOEM come from Richardson [40], Robbins [41], and Fensterer [14]. Richardson [40] applies system dynamics modeling

techniques to a Stability and Reconstruction effort at a national level. This allows a statistical analysis of various high-level policy choices.

Robbins [41] improves upon this idea by developing a more detailed model, the Stabilization and Reconstruction Operations Model (SROM), which allows for more comprehensive analysis of the nation-building efforts. SROM models DIME and PMESII interactions at the sub-national, regional level which allows testing of a wide variety of policy options on a national or regional basis.

Fensterer [14] uses a Value Focused Thinking strategy to capture the important values of nation state stability. Using Department of Defense (DoD) guidance, Fensterer suggests five fundamental values of stability: Economy, Governance, Rule of Law, Security, and Social Well-Being.

Based on this research, SROM was re-engineered at the Air Force Research Laboratory to become NOEM [42]. NOEM contains several modules to model the complex and critical DIME and PMESII interactions within a region. Although most modules contain deterministic models, an Agent-Based model represents the population and provides a stochastic element to the overall model [42].

III. Methodology

3.1 Research Methodology

This chapter develops the methodology used to examine the desirability function and the methodology used to find a robust optimum solution from a decision analysis perspective. Section 3.2 an analysis of the desirability function from a decision analysis perspective. Section 3.2.1 analyzes the desirability function as a value function. Sections 3.2.2 and 3.2.3 analyze a utility function assessed over the desirability function. Section 3.3 develops the methodology taken to find a robust decision point within a multiple response system.

3.2 Desirability Function Analysis

This section presents an analysis of Derringer and Suich's desirability function [13] from a decision analysis perspective. Two different cases are examined. In the first case, assume the desirability function is a value function. In the second case, assume the desirability function is a utility function. The analysis focuses on the two-dimensional case with responses, y_1 and y_2 . The analysis is generalized to an n -dimensional case, which is presented in Appendix A.

The desirability function for the 2-dimensional case is

$$D = (d_1 d_2)^{\frac{1}{2}}. \quad (3.1)$$

Consider a system with n inputs, $(x_1, \dots, x_n)$. Each input, x_i falls within the experimental region defined by $x_i \in [x_i^0, x_i^*]$, $i = 1, \dots, n$. These inputs cause changes in two responses, y_1 and y_2 which each fall within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, 2$. This analysis focuses on the case where a maximum response is desired as displayed

in Equation 2.21 with no loss of generality since the other forms of the desirability function can be transformed into this case. Only the non-trivial piece of Equation 2.21 where $L_i < y_i < T_i, i = 1, 2$ is considered. Equation 3.2 displays this non-trivial piece.

$$d_i = \left[\frac{y_i - L_i}{T_i - L_i} \right]^{r_i}, \quad L_i < y_i < T_i \quad (3.2)$$

Where $[L_i, T_i] \subseteq [y_i^0, y_i^*], i = 1, 2$. This forces $0 < D < 1$.

3.2.1 Desirability function as value function.

Consider the case in which the desirability function is used as a value function. The desirability function can be used as a value function if the decision maker assumes the expected responses from the regression equations are deterministic. The deterministic trade-off between the two attributes can then be examined using the tradeoff function as defined in Chapter II.

$$t(y_1, y_2) = \frac{D'_{y_1}}{D'_{y_2}} = - \frac{dy}{dx} \Big|_{\text{isopreference contour}} \quad (3.3)$$

The deterministic tradeoffs between the two attributes can then be stated.

$$t(y_1, y_2) = \frac{r_1}{r_2} \left[\frac{y_2 - L_2}{y_1 - L_1} \right] \quad (3.4)$$

Equation 3.4 indicates how many units of y_2 the decision maker is willing to give up in order to increase y_1 by one unit, at the point (y_1, y_2) . In this case, the tradeoff is a ratio of the two exponents r_1 and r_2 multiplied by the ratio of the differences between the two current values of y_i and their respective minimum acceptable values, L_i . Table 2 shows how the tradeoff changes as each response or parameter is increased while all others are held constant. Interestingly, the two target values, T_1 and T_2 ,

cancel out of the function when the ratio of the two partial derivatives of the overall desirability function is calculated.

Table 2. Tradeoff varies with respect to different variables increasing

Variable increasing (all else constant)	Effect on $t(y_1, y_2)$
y_1	decreases
y_2	increases
r_1	increases
r_2	decreases
L_1	increases
L_2	decreases

The desirability function tends to move the responses away from their minimum values. When a response is pushed near its minimum value, its individual desirability, d_i , tends to zero.

For a constant value of y_2 , $t(y_1, y_2)$ decreases as y_1 increases. As y_1 increases from its minimum, d_1 increases from zero which increases the overall desirability. The further y_1 is from its minimum, the less y_2 a decision maker is willing to give up for further increases in y_1 .

For a constant value of y_1 , $t(y_1, y_2)$ increases as y_2 increases. The same argument applies as above. The further y_2 is from its minimum, the more y_2 a decision maker is willing to give up for further increases in y_1 .

Consider the following example where y_2 is 10 units above its minimum value, y_1 is two units over its minimum, and $r_1 = r_2 = 1$. The decision maker is willing to reduce y_2 by five units to increase y_1 by one unit. See Equation 3.5.

$$t(L_1 + 2, L_2 + 10) = \frac{1}{1} \left[\frac{10}{2} \right] = 5 \quad (3.5)$$

Now let y_2 be one unit above its minimum value not 10 units, as above and keep all other values the same. The decision maker is only willing to reduce y_2 by 0.5 units

to increase y_1 by one unit. Equation 3.6 depicts this result. After y_2 is reduced from 10 to 1, the decision maker does not have as much excess y_2 to pay in order to increase y_1 . The decreased level of y_2 results in a lower tradeoff between the two attributes.

$$t(L_1 + 2, L_2 + 1) = \frac{1}{1} \left[\frac{1}{2} \right] = 0.5 \quad (3.6)$$

Increasing r_1 increases the trade-off. As r_1 increases, d_1 decreases for a given value of y_1 . A higher r_1 implies the higher values of y_1 are worth a premium; the decision maker is willing to decrease more of y_2 in order to increase y_1 . A smaller r_1 allows a much lower y_1 to achieve higher desirability.

Increasing r_2 decreases the tradeoff. As r_2 increases, d_2 decreases for a given value of y_2 . A higher r_2 implies the higher values of y_2 are worth a premium; the decision maker is willing to decrease y_2 less in order to increase y_1 .

Increasing L_1 increases the tradeoff. The minimum level of y_1 is being increased, forcing d_1 closer to zero for a fixed y_1 . As y_1 approaches its minimum value (by raising that minimum aspiration level), the decision maker is willing to decrease y_2 more to increase y_1 .

Increasing L_2 decreases the tradeoff. The minimum level of y_2 is being increased, forcing d_2 closer to zero for a fixed y_2 . As y_2 approaches its minimum value (by raising that minimum aspiration level), the decision maker is willing to decrease y_2 less to increase y_1 .

This tradeoff analysis conflicts with the analysis Kros and Mastrangelo [29] apply to the desirability function. Their analysis incorrectly takes the partial derivative of the desirability function with respect to d_i and concludes that the desirability function implies a constant trade-off of one [29]. Their analysis considers the trade-off with respect to the two competing desirabilities. However, the trade-off between

two attributes needs to be calculated with respect to the attributes themselves [31], not their desirability. In this case, that is with respect to the responses, y_1 and y_2 .

3.2.2 Assessing a utility function over the desirability function.

When uncertainty is present in a decision situation, a utility function is required for proper assessment. As described in Chapter II, one method of formulating a multiattribute utility function is to assign a single attribute utility function over a multiattribute value function. Assume a 2-dimensional deterministic desirability function (Equation 3.1). Consider the case of constant risk aversion over desirability as a value function. Assume the following utility function

$$U = 1 - e^{-\gamma D}, \quad (3.7)$$

where D is the value of the desirability function and γ is the constant risk aversion with respect to this desirability. Assume the decision maker is risk averse with respect to desirability (i.e., $\gamma > 0$). Howard [19] notes through practical experience that most decision makers are risk averse in their attitude for risk taking.

Following the concepts developed by Matheson and Abbas [31], the risk aversion function for each attribute is expressed. The utility function's risk aversion with respect to the first response, y_1 , is examined first. The risk aversion with respect to the second response follows.

The risk aversion of the utility function with respect to the first attribute, y_1 , is

$$\gamma_{y_1}^U = \gamma_D^U D'_{y_1} + \gamma_{y_1}^D. \quad (3.8)$$

Consider each term in Equation 3.8. The risk aversion for the utility function with respect to the desirability function is simply

$$\gamma_D^U = \gamma. \quad (3.9)$$

The partial derivative of D with respect to y_1 is

$$D'_{y_1} = \frac{r_1}{2(T_1 - L_1)} \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2} - 1} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}}. \quad (3.10)$$

The contribution of the desirability function to the risk aversion of the attribute y_1 is

$$\gamma_{y_1}^D = \frac{1 - \frac{r_1}{2}}{y_1 - L_1}. \quad (3.11)$$

Substituting these expressions into Equation 3.8 provides the risk aversion expressed by this utility function with respect to y_1 .

$$\gamma_{y_1}^U = \frac{\gamma r_1}{2(y_1 - L_1)} \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} + \frac{1 - \frac{r_1}{2}}{y_1 - L_1} \quad (3.12)$$

From Equation 3.12, it is clear that the risk aversion with respect to y_1 is dependent on the ranges of both responses, the values of both responses, the exponent assigned to each response, and the constant risk aversion coefficient in the utility function over desirability. Based on the assumptions made in this case, every segment of this expression is positive with the exception of the numerator of the desirability function's contribution to risk aversion, $1 - \frac{r_1}{2}$. The sign of this contribution depends on the exponent r_1 and the sign switches at $r_1 = 2$. Examining the cases of r_1 less than, equal to, and greater than two provides further insight.

For $r_1 < 2$, each component of Equation 3.12 is positive; therefore, the utility function indicates risk aversion with respect to y_1 . That being said, this aversion to

risk varies with respect to changes in the inputs to the risk aversion function. Table 3 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

Table 3. Risk aversion varies with respect to different variables increasing when $r_1 < 2$

Variable increasing (all else constant)	Effect on $\gamma_{y_1}^U$
γ	increases
y_1	decreases
y_2	increases
r_1	varies
r_2	decreases
L_1	increases
L_2	decreases
T_1	decreases
T_2	decreases

As γ , the risk aversion coefficient with respect to desirability, increases, showing increased risk aversion over desirability, the risk aversion with respect to an individual attribute increases.

As y_1 increases, the utility function displays a decreased risk aversion to y_1 . The closer y_1 is to its target value, the decision maker can better afford taking a risk in y_1 .

The variables associated with the other response, y_2 , r_2 and L_2 , affect risk preference due to the interaction within the desirability function. As mentioned in Chapter II and explicitly indicated by $\gamma_{y_1}^U$, the risk aversion with respect to a single attribute is defined in part by the form of the value function. In this case, the desirability function is a value function. The risk attitude reaction to increases in y_2 and r_2 match exactly with the tradeoff reactions in Table 2.

How $\gamma_{y_1}^U$ varies with respect to r_1 is more complicated. Equation 3.13 shows the partial derivative of the risk aversion, $\gamma_{y_1}^U$, with respect to r_1 .

$$\frac{\partial}{\partial r_1} \gamma_{y_1}^U = \frac{1}{4(y_1 - L_1)} \left(\gamma \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} \left[2 + r_1 \ln \left[\frac{y_1 - L_1}{T_1 - L_1} \right] \right] - 2 \right) \quad (3.13)$$

This partial derivative is not strictly positive or negative. The sign of this partial derivative depends on the expression within the parentheses. If $d_1 \leq e^{-2}$, then $\frac{\partial}{\partial r_1} \gamma_{y_1}^U < 0$. However, the converse is not true.

Figure 5 shows isopreference curves for various values of γ , with d_1 on the horizontal axis and d_2 on the vertical axis. If (d_1, d_2) lies on an isopreference curve, $\frac{\partial}{\partial r_1} \gamma_{y_1}^U = 0$. If (d_1, d_2) lies southwest of this isopreference curve, $\frac{\partial}{\partial r_1} \gamma_{y_1}^U < 0$ and if (d_1, d_2) lies northeast of this curve, $\frac{\partial}{\partial r_1} \gamma_{y_1}^U > 0$.

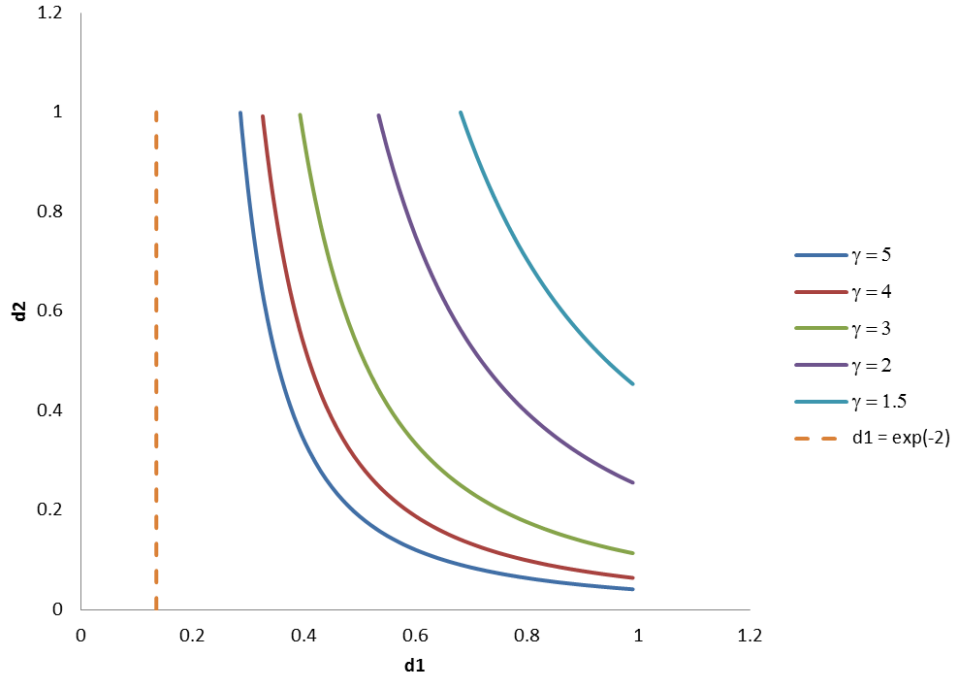


Figure 5. Isopreference curves of γ with respect to (d_1, d_2) that make $\frac{\partial}{\partial r_1} \gamma_{y_1}^U = 0$

The decision maker must be aware of unintentional consequences of adjusting r_1 . Figure 5 shows how increasing r_1 increases risk aversion with respect to y_1 for a particular area of the desirability while simultaneously decreasing risk aversion with respect to y_1 for its complimentary area. As γ decreases, the decision maker is moving towards risk neutrality. This action also decreases the area where $\frac{\partial}{\partial r_1}\gamma_{y_1}^U > 0$ and increases the area where $\frac{\partial}{\partial r_1}\gamma_{y_1}^U < 0$

As L_1 increases, risk aversion increases. The minimum level of y_1 is being increased, forcing d_1 closer to zero for a fixed y_1 . As y_1 approaches its minimum value (by raising that minimum aspiration level) the utility function expresses more risk aversion with respect to y_1 .

As T_1 increases, risk aversion decreases. Since a fixed value of y_1 is being placed further from its target, the utility function expresses less risk aversion. Increasing the size of the interval $[L_1, T_1]$ decreases risk aversion with respect to y_1 while decreasing the size of the interval $[L_1, T_1]$ increases risk aversion with respect to y_1 .

As T_2 increases, risk aversion decreases. Since a fixed value of y_2 is being placed further from its target, the utility function expresses less risk aversion.

For $r_1 = 2$, parts of Equation 3.12 cancel out, leaving only positive expressions. It follows that the utility function expresses risk aversion with respect to y_1 . This aversion to risk varies with respect to changes in the inputs to the risk aversion function. Table 4 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

The only difference between Table 4 and Table 3 (denoted by the asterisk) is that the response y_1 has no effect on the risk aversion expressed with respect to y_1 . This is due to the fact that the exponent $r_1/2 = 1$ and y_1 then cancel out of $\gamma_{y_1}^U$. When $r_1 = 2$, the desirability function is said to be risk-contribution neutral with respect to y_1 since $\gamma_{y_1}^D = 0$ [31].

Table 4. Risk aversion varies with respect to different variables increasing when $r_1 = 2$

Variable increasing (all else constant)	Effect on $\gamma_{y_1}^U$
γ	increases
y_1	no effect*
y_2	increases
r_2	decreases
L_1	increases
L_2	decreases
T_1	decreases
T_2	decreases

For $r_1 > 2$, the desirability function's contribution to risk aversion, $\gamma_{y_1}^D$, is negative. The utility function can express risk seeking, risk neutral, or risk aversion with respect to y_1 . Table 5 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

Table 5. Risk aversion varies with respect to different variables increasing when $r_1 > 2$

Variable increasing (all else constant)	Effect on $\gamma_{y_1}^U$
γ	increases
y_1	increases*
y_2	increases
r_1	varies
r_2	decreases
L_1	varies*
L_2	decreases
T_1	decreases
T_2	decreases

When comparing the results shown in Table 5 with the results shown in Table 3, it is seen that the utility function displays increasing risk aversion with respect to y_1 as the response, y_1 , increases. Increasing risk aversion is not how most decision makers prefer their risk aversion to be modeled.

Another change in this case is how L_1 affects risk aversion with respect to y_1 . Equations 3.14 and 3.15 display the partial derivative of $\gamma_{y_1}^U$ with respect to L_1 .

$$\frac{\partial}{\partial L_1} \gamma_{y_1}^U = \frac{1}{2(y_1 - L_1)^2} \left[\gamma r_1 \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} \left(1 - \frac{r_1(T_1 - y_1)}{2(T_1 - L_1)} \right) + 2 - r_1 \right] \quad (3.14)$$

$$= \frac{1}{2(y_1 - L_1)^2} \left[\gamma r_1 D \left(1 - \frac{r_1(T_1 - y_1)}{2(T_1 - L_1)} \right) + 2 - r_1 \right] \quad (3.15)$$

This partial derivative is no longer monotonic. Table 6 displays the sign of $\frac{\partial}{\partial L_1} \gamma_{y_1}^U$ based on the value of r_1 . When $r_1 \in [2, \frac{2(T_1 - L_1)}{T_1 - y_1}]$ the sign of $\frac{\partial}{\partial L_1} \gamma_{y_1}^U$ can be either positive or negative depending on the relationship of the parameters within the brackets of Equation 3.15.

Table 6. Sign of $\frac{\partial}{\partial L_1} \gamma_{y_1}^U$ based on r_1

Value of r_1	Sign of $\frac{\partial}{\partial L_1} \gamma_{y_1}^U$
$r_1 < 2$	+
$2 < r_1 < \frac{2(T_1 - L_1)}{T_1 - y_1}$	+ / 0 / -
$r_1 > \frac{2(T_1 - L_1)}{T_1 - y_1}$	-

The decision maker must be aware of the unintended consequences adjusting the bounds on responses has on risk preference.

3.2.3 Conjugate Desirability Function.

The exponential utility function over desirability example produced a case where the utility function over value was from a different family of utility functions than the marginal utility functions over the attributes. In many cases, a decision maker may want to express the same risk preference for all the attributes as is expressed for the given value function. In considering this case, Matheson and Abbas present the idea of a conjugate value function [31].

A value function is said to be conjugate to a utility function over value if the marginal utility function over an attribute is of the same family as the utility function over value [31].

Consider a logarithmic utility function over desirability as shown in Equation 3.16.

$$U = \ln D \quad (3.16)$$

The risk aversion with respect to desirability is

$$\gamma_D^U = \frac{1}{D}. \quad (3.17)$$

The partial derivative of D with respect to y_1 , D'_{y_1} , and the contribution of the desirability function to the risk aversion of y_1 , $\gamma_{y_1}^D$ are shown in Equations 3.10 and 3.11 respectively. The risk aversion with respect to y_1 is

$$\gamma_{y_1}^U = \frac{1}{y_1 - L_1}. \quad (3.18)$$

Matheson and Abbas state “the product value function is conjugate to the logarithmic utility function” [31]. It follows that the desirability function is conjugate to the logarithmic utility function.

3.2.4 Desirability function as a utility function.

If the desirability function itself is assumed to be a utility function, it follows Abbas and Howard’s definition of an attribute dominance utility function [4] since its value is zero if any of the attributes is at or below its minimum acceptable value. When

considering the desirability function in this manner, it is denoted as D^d . Equation 3.19 shows the two-dimensional case when $L_i < y_i < T_i, i = 1, 2$.

$$D_{y_1 y_2}^d(y_1, y_2) = \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} \quad (3.19)$$

Equation 3.20 gives the marginal utility function for the first attribute.

$$D_{y_1}^d = D_{y_1 y_2}^d(y_1, T_2) = \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \quad (3.20)$$

Equation 3.21 gives the conditional utility function for the second attribute given a certain level of the first attribute.

$$D_{y_2|y_1}^d(y_2|y_1) = \frac{D_{y_1 y_2}^d(y_1, y_2)}{D_{y_1}^d} = \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} \quad (3.21)$$

Since Equation 3.21 does not depend on the value of y_1 , the conditional utility function is equal to the marginal utility function and it is seen that the desirability function displays utility independence of the two attributes. The desirability function can then be expressed as in Equation 3.22.

$$D_{y_1 y_2}^d(y_1, y_2) = D_{y_1}^d(y_1) D_{y_2}^d(y_2) \quad (3.22)$$

With this assumption of utility independence, the analyst can simply assess the marginal utility functions of each attribute separately and then multiply them to create the overall desirability function. Equation 3.22 is a special case of Equation 2.6 which depicts the general case of a utility function expressing utility independence.

The attribute y_1 is utility independent of y_2 if the conditional preferences of y_1 given a certain value of y_2 do not depend on the value of y_2 [22]. If the attributes are not utility independent, the analyst should not use the desirability function as a utility function.

Equation 3.23 gives the risk aversion of the marginal utility function of the first attribute.

$$\gamma_{y_1} = \frac{1 - \frac{r_1}{2}}{y_1 - L_1} \quad (3.23)$$

Kros and Mastrangelo present erroneous conclusions regarding the risk aversion of the desirability function [29]. For $r_i = 2$, the desirability function reflects risk neutrality with respect to y_i . For $r_i < 2$ the desirability function reflects decreasingly risk averse behavior. For $r_i > 2$, the desirability function reflects decreasingly risk seeking behavior. This breakpoint in modeling risk preference is dependent on the number of objectives. When generalized to n -dimensions, the desirability function reflects risk neutrality when $r_i = n$. The desirability function reflects risk averse and risk seeking behavior when $r_i < n$ and $r_i > n$ respectively.

The mixed partial derivative of the desirability function is shown in Equation 3.24.

$$\frac{\partial^2 D_{y_1 y_2}^d(y_1, y_2)}{\partial y_1 \partial y_2} = \frac{r_1 r_2}{4(y_1 - L_1)(y_2 - L_2)} \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2} \right]^{\frac{r_2}{2}} \quad (3.24)$$

This mixed partial derivative is always positive over the range $L_i < y_i < T_i, i = 1, 2$. Therefore, the desirability function displays multivariate risk seeking behavior [39]. This type of behavior makes sense in an attribute dominance utility function, since the overall utility is zero if any one of the attributes is at its minimum level.

The fact that the desirability function sets the overall desirability to zero if any of its attributes is at or below its minimum values, regardless of the values of the remaining attributes merits discussion. Consider the following three attribute relation.

$$D(L_1 + \epsilon, L_2 + \epsilon, L_3 + \epsilon) > D(L_1 - \epsilon, T_2, T_3) = 0, \quad (3.25)$$

where $\epsilon > 0$ is small. Consider the right hand side of Equation 3.25. Even with the second two attributes at their target values, since the first attribute is slightly below its minimum value, its desirability is zero. Consider the left hand side of Equation 3.25. With all three attributes slightly above their minimum values, the desirability is a small positive number and is therefore greater than the right hand side. Care must be taken to ensure a valid minimum value is decided upon before applying the desirability function. An elevated minimum value could reject a potentially acceptable decision point. Moreover, changing the minimum values of the responses also affects the trade-off and risk aversion functions. In decision analysis, a value (or utility) function is stipulated over any valid domain of interest in the attempt to avoid such errors (i.e., $L_i = y_i^0$ and $T_i = y_i^*$, $\forall i$).

3.3 A Decision Analysis Perspective on Robust Optimization

A set of regression equations is developed from an experimental design over the decision space. Once the decision maker selects the proper user-defined parameters, the maximum desirability is calculated and its location found within the feasible region.

Since the desirability function is not differentiable throughout its range, a non-gradient optimization algorithm must be used. The Hooke and Jeeves Direct Search algorithm [17, 20, 8] is implemented in Excel using Visual Basic for Applications (VBA). Since the starting point is critical when using such an algorithm over such un-smooth surfaces, multiple starting points are chosen throughout the feasible region. This is accomplished by taking a random sampling of the feasible region or by dividing the feasible region into a grid of equally spaced vertices along each dimension of the decision space. The code iterates through each starting point for the direct search algorithm. The code outputs the location and desirability of all local maxima. To

confirm the results of the direct search algorithm, the local maxima is also calculated using the differential evolution algorithm within Matlab [38]. Once the deterministic local maxima are calculated, the Direct Search algorithm is re-run calculating the desirability function in a Monte Carlo simulation.

Clemen and Reilly discuss using regression equations for decision analysis. These equations estimate the conditional expected value of a response given a set of input variables. The difference between the observed value and the predicted value of the response is the residual. This set of residuals can be used to estimate the continuous distribution function (CDF) of the random error term, ϵ , as described in Equation 2.18 in Chapter II [11]. This CDF is approximated with a Beta distribution using the residuals as the fractile midpoints in Matlab's *fminsearch* function as described in Abbas et al [3] and reviewed in Section 2.2.4.

The correlation of the residuals, $\mathbf{R}$, is calculated and then the random samples for the Monte Carlo simulation is calculated using the multivariate normal copula [10].

Matlab's *mvnrnd* function generates a matrix of random vectors from a multivariate standard normal distribution with correlation $\mathbf{R}$. Equation 3.26 displays how the vector of random samples $(\epsilon_1, \dots, \epsilon_n)$ is calculated,

$$(\epsilon_1, \dots, \epsilon_n) = (F_1^{-1}[\Phi(y_1)], \dots, F_n^{-1}[\Phi(y_n)]), \quad (3.26)$$

where Φ is the standard normal CDF and F_i^{-1} , $i = 1, \dots, n$ is the i -th inverse Beta marginal CDF estimated from the residuals.

Using each local maximum as the starting point, the solution to

$$\max_{\mathbf{y}^{(1)}} \sum_{i=1}^n \left(\frac{1}{n} D(\mathbf{y}^{(1)} + \boldsymbol{\epsilon}_i) \right) \quad (3.27)$$

is calculated where n is the number of random samples used in the Monte Carlo simulation, $\boldsymbol{\epsilon}_i$ is i -th vector random samples and $\boldsymbol{y}^{(1)}$ is the vector of responses found with the highest expected desirability. The global maximum is the robust solution point for the system. Sensitivity analysis indicates the robustness of the solution.

IV. Wire Bonding Experiment Findings

4.1 Multi-response optimization of semiconductor manufacturing process

Del Castillo et al. [9] introduce an experiment to illustrate a multi-response optimization problem using the desirability function. The experiment is executed to optimize a wire bonding process in the semiconductor industry. The experiment has three input factors and six responses. Table 7 contains the three factors and their low and high values used in the experiment. Table 8 contains the response descriptions and their acceptable ranges and target values.

Table 7. Factors and levels for example experiment [9]

Factor	Factor Name	Levels	
		Low	High
A	Flow Rate	40	120
B	Flow Temp	200	450
C	Block Temp	150	350

Table 8. Response descriptions and ranges for multiple response example [9]

Response - description	Min	Max	Target
y_1 = maximum temperature at Position A	185	195	190
y_2 = beginning bond temperature at position A	170	195	185
y_3 = finish bond temperature at position A	170	195	185
y_4 = maximum temperature at position B	185	195	190
y_5 = beginning bond temperature at position B	170	195	185
y_6 = finish bond temperature at position B	170	195	185

Excel Analysis ToolPak is used to run an ordinary least squares (OLS) multiple linear regression on the data del Castillo et al. provide from the experiment to estimate the response equations [9].

$$\hat{y}_1 = -18.404 + 0.567x_2 + 0.530x_3 - 0.002x_2x_3 \quad (4.1)$$

$$\hat{y}_2 = 37.463 + 0.150x_1 + 0.173x_2 + 0.141x_3 \quad (4.2)$$

$$\hat{y}_3 = 33.413 + 0.166x_1 + 0.128x_2 + 0.204x_3 \quad (4.3)$$

$$\hat{y}_4 = 45.616 + 0.767x_1 + 0.065x_2 + 0.079x_3 - 0.008x_1^2 + 0.002x_1x_2 \quad (4.4)$$

$$\hat{y}_5 = 51.443 + 0.230x_1 + 0.046x_2 + 0.165x_3 - 0.004x_1^2 + 0.001x_1x_2 \quad (4.5)$$

$$\hat{y}_6 = 36.066 + 0.239x_1 + 0.049x_2 + 0.274x_3 - 0.003x_1^2 + 0.001x_1x_2 \quad (4.6)$$

4.2 Deterministic optimum setting

The two-sided desirability function from Equation 2.23 is used to find the optimum operating environment within the experimental region described in Table 7. Using the Min, Max, and Target values from Table 8 as the values of L_i , U_i , T_i , $i = 1, \dots, 6$ and Equations 4.1 through 4.6, the Hooke-Jeeves (HJ) algorithm finds the maximum desirability and its operating solution for various risk preferences. Table 9 contains the exponent parameters used to express various risk attitudes.

Table 9. Exponent parameters used to express various risk attitudes

	Risk Attitude		
	Averse	Neutral	Seeking
s_i, t_i	3	6	9

In each of the risk attitude cases described in Table 9, the same two local optimum solutions are found. Adjusting the exponents s_i and t_i ($i = 1, \dots, 6$) such that $s_i = t_i$ ($i = 1, \dots, 6$) will not change the optimum solution because the trade-off functions, $t(y_i, y_j)$, $i \neq j$ does not change.

Table 10 displays the two local optimum solutions and their predicted responses. The desirability values shown are from the risk neutral case where $s_i = t_i = 6$ ($i = 1, \dots, 6$). The first column contains the global optimum solution.

Table 10. Local deterministic operating solutions

Local Optimum Points		
$\mathbf{x_1}$	120.000	82.192
$\mathbf{x_2}$	436.168	450.000
$\mathbf{x_3}$	320.297	330.453
$\hat{\mathbf{y}}_1$	186.391	185.939
$\hat{\mathbf{y}}_2$	176.162	174.318
$\hat{\mathbf{y}}_3$	174.378	171.955
$\hat{\mathbf{y}}_4$	193.277	192.861
$\hat{\mathbf{y}}_5$	173.139	172.910
$\hat{\mathbf{y}}_6$	184.364	185.000
$\mathbf{D}$	0.0023	0.0006

These two local optimum solutions represent the two regions, A and B, where $D > 0$ in the experimental region. Figure 6 represents Region A by displaying desirability as Block Temperature and Flow Rate vary with Flow Temperature fixed at $x_2 = 436.17$. Figure 7 represents Region B by displaying desirability as Block Temperature and Flow Rate vary with Flow Temperature fixed at $x_2 = 450$.

The figures displaying Regions A and B with Flow Temperature and Block Temperature held constant are presented in Appendix C. From all three dimensions of x , Region A appears larger than Region B. Region A also contains the global optimum solution.

Both del Castillo et al [9] and Kros and Mastrangelo [29] display results regarding a global maximum for this experimental region using $s_i = t_i = 1$, $i = 1, \dots, 6$. Table 11 compares the maximum solution displayed in both of these articles.

Del Castillo et al.'s solution is displayed in the first column of Table 11. Kros and Mastrangelo's solution is in the second column; it matches del Castillo et al.'s solution closely. The third column is the solution found using the regression equations

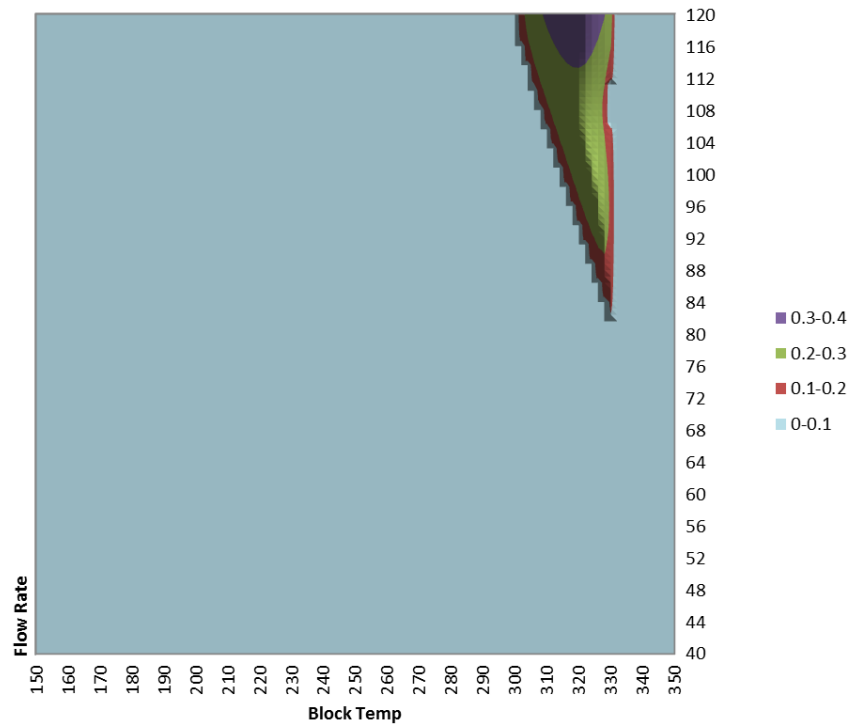


Figure 6. Value of D as Block Temp and Flow Rate vary (Flow Temp = 436.17)

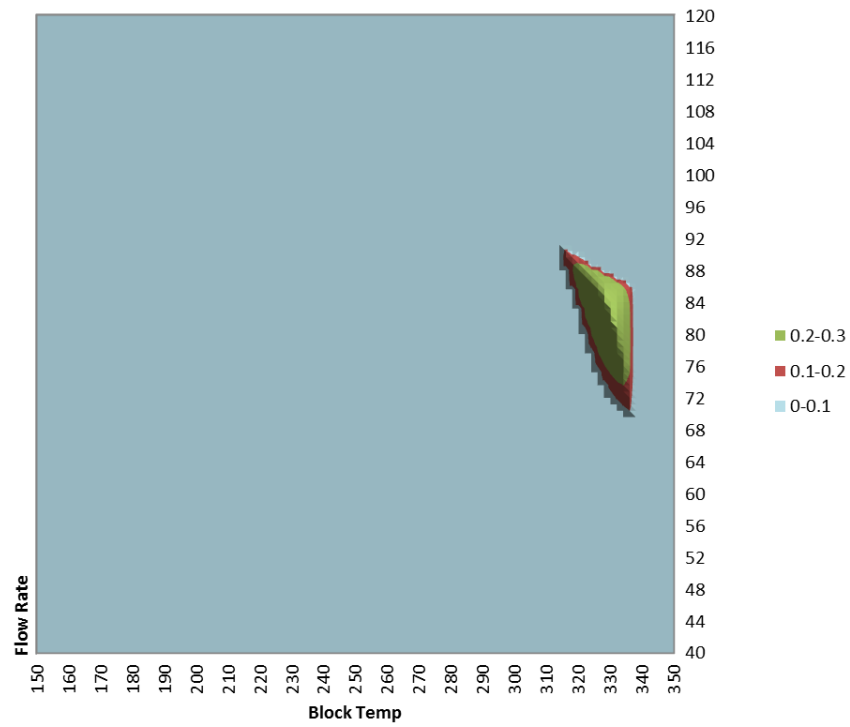


Figure 7. Value of D as Block Temp and Flow Rate vary (Flow Temp = 450)

Table 11. Comparing global maximum found for the del Castillo example[9, 29]

	DC	KM	Self
x_1	84.312	84.14	84.113
x_2	450	450	450
x_3	329.73	329.66	329.902
$\hat{y}_1$	186.1	186.06	186.02
$\hat{y}_2$	174.5	174.49	173.92
$\hat{y}_3$	172.1	172.02	172.06
$\hat{y}_4$	192.6	192.63	192.62
$\hat{y}_5$	173.1	173.04	173.07
$\hat{y}_6$	185	184.95	185.00
D	0.3076	0.3058	0.2989

stated by del Castillo et al. [9] in the HJ algorithm. Interestingly, this set of equations produces a global maximum in Region B instead of region A as when using Equations 4.1 through 4.6.

4.3 Monte Carlo Simulation

The residuals based on the above model are used to estimate the marginal distribution functions for the six responses. Appendix B contains the residuals for this regression model. The lower bound A for each response's Beta distribution is chosen by rounding its lowest residual down to the next integer value. The upper bound B for each response is chosen by rounding its highest residual up to the next integer value. Using these bounds, the α and β parameters are estimated using Matlab's *fminsearch* function. Table 12 contains the estimated parameters of the six marginal Beta distribution functions.

Table 12. Estimated parameters for the marginal Beta distribution functions

	Residual y_1	Residual y_2	Residual y_3	Residual y_4	Residual y_5	Residual y_6
α	0.9980	0.9047	1.3739	0.8615	1.1511	1.1428
β	1.1914	0.9218	1.3411	0.9916	1.5732	1.2991
A	-15	-11	-16	-9	-5	-5
B	31	21	30	18	11	10

The correlation matrix $\mathbf{R}$ is calculated from the residuals as shown in Table 13.

Table 13. Correlation matrix of the residuals

1	0.7672	0.6728	0.4184	0.5533	0.3700
0.7672	1	0.8351	0.6260	0.8313	0.6369
0.6728	0.8351	1	0.5084	0.6988	0.5849
0.4184	0.6260	0.5084	1	0.8838	0.9232
0.5533	0.8313	0.6988	0.8838	1	0.9122
0.3700	0.6369	0.5849	0.9232	0.9122	1

Using Matlab's *mvnrnd* function and the normal multivariate copula method, a set of 10,000 sample residuals are constructed for the Monte Carlo simulation. To confirm this random sample is from the intended distribution, the Beta distribution parameter estimates and correlation matrix of the sample is calculated for comparison with the residual data estimates. Table 14 contains the Beta parameters fit to the sample.

Table 14. Beta distribution parameters fit to the Monte Carlo samples

	Residual y_1	Residual y_2	Residual y_3	Residual y_4	Residual y_5	Residual y_6
α	0.9688	0.8878	1.3584	0.8569	1.1485	1.1336
β	1.1618	0.9125	1.337	0.9789	1.571	1.2833

Table 15 contains the correlation matrix of the random sample. Both the parameters from the six marginal distributions and the correlation matrix from the random sample resemble those calculated from the residuals.

Table 15. Correlation matrix for the Monte Carlo samples

1	0.7545	0.6592	0.4121	0.5452	0.3660
0.7545	1	0.8211	0.6092	0.8169	0.6224
0.6592	0.8211	1	0.4859	0.6791	0.5665
0.4121	0.6092	0.4859	1	0.8712	0.9127
0.5452	0.8169	0.6791	0.8712	1	0.9056
0.3660	0.6224	0.5665	0.9127	0.9056	1

The Monte Carlo simulation is run using the random sample. First, the desirability function is treated as a utility function. The HJ algorithm is started at both of the local maxima displayed in Table 10. Risk aversion is modeled as described in Table 9. Table 16 displays the robust optimal solutions when $s_i = t_i = 3, 6, 9$ ($i = 1 \dots, 6$).

Table 16. Robust optimal solutions varying by risk aversion

s_i, t_i	3	6	9
$\mathbf{x}_1$	120	120	120
$\mathbf{x}_2$	407.190	429.415	431.174
$\mathbf{x}_3$	349.934	339.771	337.703
$\mathbf{E}(\mathbf{D})$	0.0212	0.0037	0.0009

The robust optimal solution moves slightly as the exponents vary, but it remains near Region A. The value of $E(D)$ decreases as the exponent increases due to the decreasing effect the exponent has on d_i . It appears, in this case, adjusting the exponents in the desirability function does not affect which area of the operating region contains the robust optimal solution when the desirability function is used as a utility function. This is most likely due to Region A containing the highest peak desirability and also being the larger region where $D > 0$.

The robust optimal solution is also calculated using the desirability function as a value function and assessing an exponential utility function over it.

$$U = 1 - e^{-\gamma D} \quad (4.7)$$

The exponents, s_i, t_i ($i = 1, \dots, 6$) vary as described in Table 9 in addition to varying the risk aversion coefficient, $\gamma = 0.2, 1, 5$. Table 17 contains the set of robust optimal solutions found in this case.

The robust optimal solution moves slightly as the exponents and γ vary, but it remains near Region A. The value of $E(U)$ decreases as the exponent increases due to the decreasing effect the exponent has on d_i . The value of $E(U)$ increases as γ

Table 17. Robust optimal solutions varying γ , s_i , t_i

γ	s_i, t_i	3	6	9
0.2	$\mathbf{x}_1$	120	120	120
	$\mathbf{x}_2$	407.176	429.413	431.175
	$\mathbf{x}_3$	349.990	339.778	337.703
	$\mathbf{E}(\mathbf{U})$	0.0042	0.0007	0.0002
1	$\mathbf{x}_1$	120	120	120
	$\mathbf{x}_2$	407.192	429.348	431.329
	$\mathbf{x}_3$	350	339.854	337.769
	$\mathbf{E}(\mathbf{U})$	0.0196	0.0035	0.0009
5	$\mathbf{x}_1$	120	120	120
	$\mathbf{x}_2$	407.227	429.018	431.416
	$\mathbf{x}_3$	350	340.449	337.873
	$\mathbf{E}(\mathbf{U})$	0.0739	0.0156	0.0043

increases due to the increasing effect it has on $\mathbf{U}$. It appears, in this case, adjusting the exponents or γ does not affect which area of the operating region contains the robust optimal solution when an exponential utility function is assessed over the desirability function. This is most likely due to Region A containing the highest peak desirability and also being the larger region where $D > 0$.

The desirabilities calculated by adding the 10,000 sample residuals to the various optimum solutions found provide insight into the robustness of the solutions. Consider the case when $s_i = t_i = 3$ ($i = 1, \dots, 6$). Table 18 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

Table 18. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 3$

	Deterministic	$\gamma = 0.2$	$\gamma = 1$	$\gamma = 5$
$\mathbf{E}(\mathbf{D})$	0.0109	0.0212	0.0212	0.0212
Max $\mathbf{D}$	0.5129	0.4112	0.4109	0.4104

The deterministic solution produces a higher maximum desirability when adding the set of 10,000 random residuals to it. However it produces a lower average de-

sirability. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 8 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

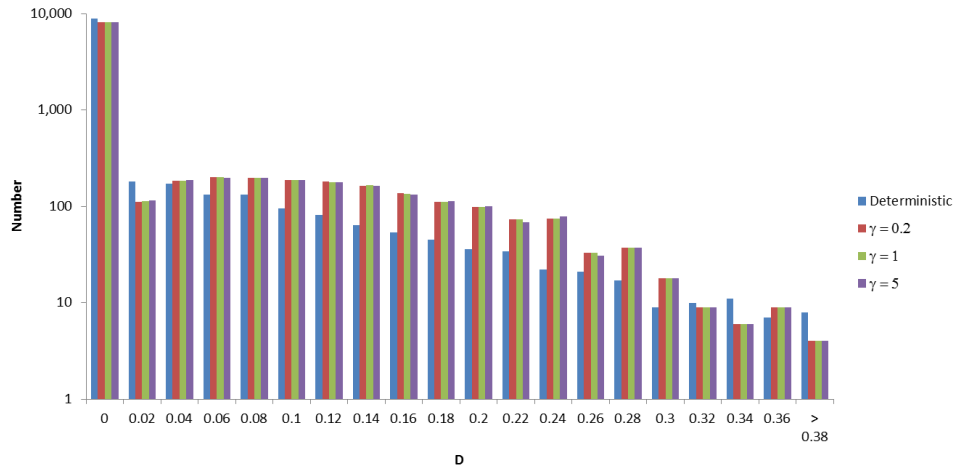


Figure 8. Compare the distribution of D between different values of γ and the deterministic optimum when $s_i = t_i = 3$)

The horizontal axis shows desirability. The vertical axis shows the number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, a risk averse or risk neutral decision maker would prefer one of the robust solutions. A sufficiently risk seeking decision maker would prefer the deterministic solution.

Consider the exponential utility function

$$U = e^{-\gamma D} \quad (4.8)$$

where $\gamma < 0$. This utility function describes risk seeking behavior with respect to desirability and $\gamma_D^U = \gamma$. When $\gamma < -16.5$, the deterministic solution is preferred to the three robust solutions. This extreme risk seeking attitude is rarely modeled in practice.

Consider the case when $s_i = t_i = 6$ ($i = 1, \dots, 6$). Table 19 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

Table 19. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 6$

	Deterministic	$\gamma = 0.2$	$\gamma = 1$	$\gamma = 5$
$E(D)$	0.0018	0.0037	0.0037	0.0037
Max D	0.2631	0.2873	0.2884	0.2893

The deterministic solution produces a lower maximum desirability and lower average desirability when adding the set of 10,000 random residuals to it. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 9 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

The horizontal axis shows desirability. The vertical axis shows number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, since the robust solutions had the highest maximum desirability and

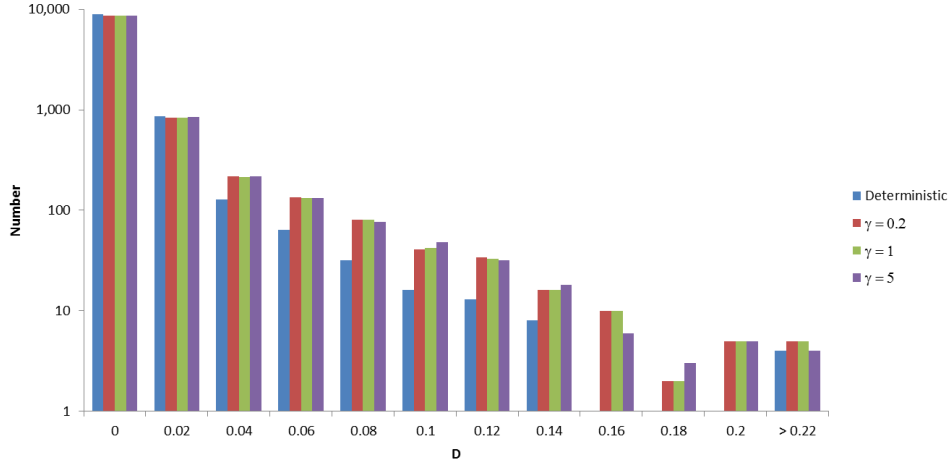


Figure 9. Compare the distribution of D between different values of γ and the deterministic optimum when $s_i = t_i = 6$)

highest average desirability, a decision maker would prefer a robust solution over the deterministic solution regardless of risk preference.

Consider the case when $s_i = t_i = 9$ ($i = 1, \dots, 6$). Table 20 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

Table 20. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 9$

	Deterministic	$\gamma = 0.2$	$\gamma = 1$	$\gamma = 5$
$E(D)$	0.0004	0.0009	0.0009	0.0009
Max D	0.1350	0.1488	0.1462	0.1434

The deterministic solution produces a lower maximum desirability and lower average desirability when adding the set of 10,000 random residuals to it. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 10 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

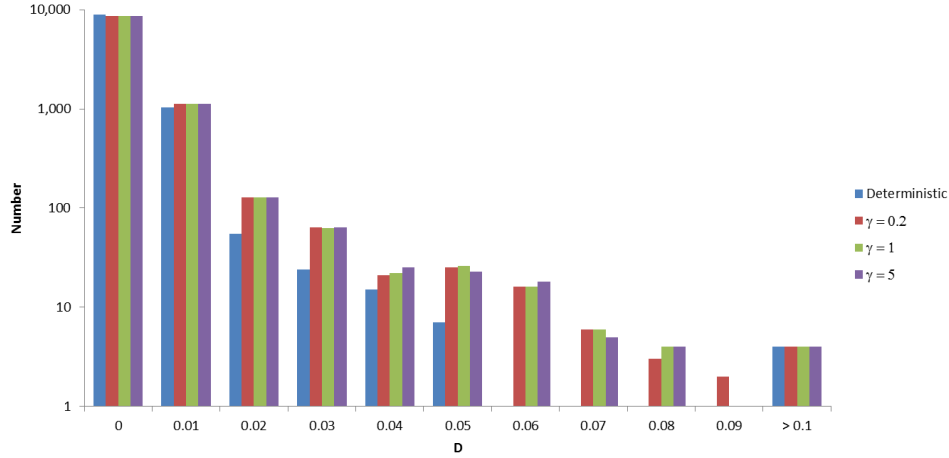


Figure 10. Compare the distribution of D between different values of γ and the deterministic optimum when $s_i = t_i = 9$)

The horizontal axis shows desirability. The vertical axis shows number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, since the robust solutions had the highest maximum desirability and highest average desirability, a decision maker would prefer a robust solution over the deterministic solution regardless of risk preference.

V. Robust Stability Operations Policy Findings

5.1 National Operational Environment Model Experiment

To illustrate the decision analysis approach to robust optimization, a stability operations policy optimization problem is investigated within the Air Force Research Laboratory's (AFRL) National Operational Environment Model (NOEM).

An experiment is designed to investigate how various diplomatic, informational, military and economic (DIME) instruments of national power affect the Democratic Republic of Congo's (DRC) political, military, economic, social, infrastructure, and information (PMESII) systems.

This experiment investigates the effect 14 DIME factors have on two PMESII responses. Table 21 lists the factors with brief descriptions. Table 22 lists the units and minimum and maximum values for the experimental factors.

National debt and total number of activists are the two indicators of the DRC's PMESII systems. Minimizing both responses is preferred and is indicative of a more stable government.

Each design point is run for three simulated years (1095 days) with two replications. A list of random seed generators is generated in Excel and each design point is assigned a seed from this list. NOEM outputs debt and activist data for every simulated day. This experiment considers the arithmetic mean of the last 30 days of the simulation's debt and activist output as the two responses for debt and activists respectively.

Table 21. NOEM DRC experiment factors and descriptions

Factor	Description
Diplomatic	
Stimulus maximum	Maximum amount of money allocated daily to government stimulus
Stimulus Spending %	Percentage of government funding earmarked for stimulus
Government Corruption	Proportion of government income not available for use
Military	
Initial Police Forces	Police forces at the start of the simulation
Police Forces Goal	Long term goal for police forces
Jail Term	Mean jail term for arrested activist
Mean Adjudication Processing Time	Mean time spend in adjudication process
Adjudication Rate	Average rate of adjudication process
Economic	
Tax Rate	Income and production tax rate
Interest Rate	Government debt interest rate
Long Term Government Employee Share	Long term proportion of workers employed by the government
Government Wages	Mean annual wage paid to government employees
Infrastructure Spending %	Percentage of government funding earmarked for infrastructure
Services Spending %	Percentage of government funding earmarked for providing services

Table 22. NOEM DRC experiment factors, units, minimum values, maximum values

Factor	Units	Min	Max
Diplomatic			
Stimulus maximum	\$	0	6,000,000
Stimulus Spending %	%	0	100
Government Corruption	n/a	0.05	0.15
Military			
Initial Police Forces	personnel	52,500	137,500
Police Forces Goal	personnel	112,500	187,500
Jail Term	days	3	100
Mean Adjudication Processing Time	day	0.1	0.9
Adjudication Rate	per day	0.01	0.13
Economic			
Tax Rate	n/a	0.03	0.5
Interest Rate	n/a	0.0225	0.0675
Long Term Government Employee Share	n/a	0.001	0.05
Government Wages	\$	1400	4200
Infrastructure Spending %	%	0	100
Services Spending %	%	0	100

The experiment executes a D-optimal design. D-optimality focuses on good model coefficient estimation. It does this by choosing design points so that the determinant of the moment matrix $\mathbf{M}$ is maximized [32].

$$|\mathbf{M}| = \frac{|\mathbf{X}'\mathbf{X}|}{N^p} \quad (5.1)$$

where $\mathbf{X}$ is the design matrix, N is the number of experiment runs and p is the number of parameters [32]. An experimental design consisting of 139 design points with two replicates each is chosen to create a full quadratic model with cubic terms.

A deterministic value function, V , is formulated to describe the preferred relationship between the two responses.

$$V = 18.164 - 5.75 \times 10^{-10}y_1 - 1.01 \times 10^{-4}y_2, \quad (5.2)$$

where y_1 is debt in dollars, and y_2 is the number of activists.

$$t(y_2, y_1) = \frac{V'_{y_2}}{V'_{y_1}} = 176,439.50 \quad (5.3)$$

The value function V is describing a tradeoff where the decision maker would be willing to increase the debt by \$176,439.50 in order to reduce activists by one.

This particular form of a value function assumes a constant trade-off between the two responses throughout the entire response space. Consider two examples, one with \$3 billion of debt and 500 activists, and a second with \$3 billion of debt and 50,000 activists. It is unlikely a decision maker would have the same tradeoff between debt and number of activists in these two cases. Given \$3 billion of debt, a decision maker would more likely have a higher tradeoff when faced with 50,000 activists as opposed to when faced with 500 activists. Moreover, given a particular number of activists, a decision maker would more likely have a decreasing tradeoff as debt increases.

Consider an exponential utility function assessed over V

$$U = \begin{cases} 1 + e^{-\gamma V}, & \gamma < 0 \\ V, & \gamma = 0 \\ 1 - e^{-\gamma V}, & \gamma > 0 \end{cases} \quad (5.4)$$

After assessing a utility function over value, the risk aversion with respect to the two attributes can be calculated. The objective in this experiment is to maximize V which minimizes y_1 and y_2 . Since this is a decreasing value function, the risk aversion functions with respect to these attributes change some of the signs in Equations 2.11 and 2.10.

$$\gamma_{y_i}^V = -\frac{V''_{y_i}}{V'_{y_i}}, \quad i = 1, 2 \quad (5.5)$$

$$\gamma_{y_i}^U = -\gamma_V^U V'_{y_i} + \gamma_{y_i}^V, \quad i = 1, 2 \quad (5.6)$$

This value function is additive, so its contribution to risk aversion is zero.

$$\gamma_{y_i}^V = 0, \quad i = 1, 2 \quad (5.7)$$

The risk aversion of the utility function with respect to the two attributes are

$$\gamma_{y_1}^U = 5.75 \times 10^{-10} \gamma \quad \text{and} \quad (5.8)$$

$$\gamma_{y_2}^U = 1.01 \times 10^{-4} \gamma. \quad (5.9)$$

In cases where constant risk aversion is indicated by the utility function, one must be cognizant of saturation effect. The utility function, $U = 1 - e^{-\gamma V}$, equals one as V approaches positive utility. When V is greater than $\approx 1/\gamma$, the utility function is nearly a horizontal line. This saturation effect improperly models risk preference at these higher levels of V . This limitation of the exponential utility function must be considered when selecting reasonable values of γ .

5.2 Experiment Results

After the experiment is run, the data is analyzed in Design-Expert. Ordinary least squares (OLS) regression is applied and two regression equations to predict y_1 and y_2 are developed.

The model for y_1 did not pass the lack-of-fit test. However the R-Squared, adjusted R-Squared and predicted R-Squared measures are all high so this model is accepted. After a natural log transform is applied to the y_2 response this model does pass the lack-of-fit test. Its R-Squared, adjusted R-Squared and predicted R-Squared are also high. These results are displayed in Table 23. The coefficients for these two functions appear in Appendix D.

Table 23. Statistics for the two regression equations

	y_1	$\ln y_2$
R-Squared	0.9882	0.9920
Adj R-Squared	0.9773	0.9847
Pred R-Squared	0.9631	0.9686

5.3 Deterministic optimum setting

The Hooke-Jeeves (HJ) algorithm finds the maximum value and corresponding operating solution. One thousand random starting points within the experimental region are generated using the uniform psuedo-random number generator in Microsoft Excel Visual Basic for Applications (VBA). This procedure produces 657 local maxima.

The set of local maxima is reduced using a K means clustering algorithm with Matlab's *kmeans* function. This algorithm classifies the 657 points into K groups by minimizing the sum of squares of Euclidean distances between the points and their corresponding cluster centroid [30].

To find the proper number of clusters K , K is chosen to vary between 2 and 50 and the total sum of squared distances in each case is graphed against K to find a point where the trade-off between the sum and K is balanced. Figure 11 displays this graph.

Based on this chart, 13 clusters are chosen which gives a total sum of squared distances of 788.05. The maximum value point from each cluster is chosen to represent

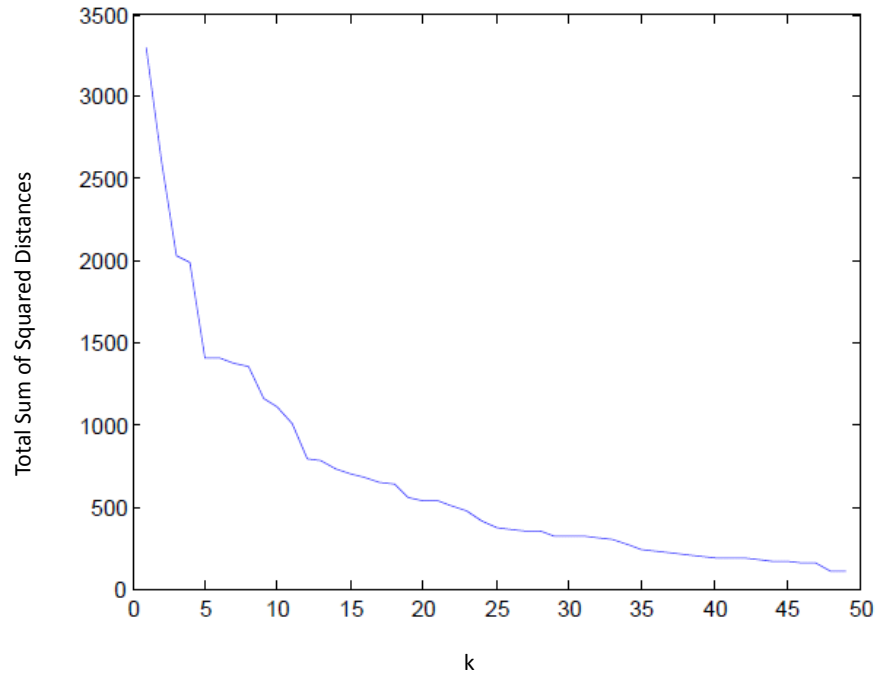


Figure 11. Total Sum of Squared Distances in all K clusters by K

its cluster as a starting point for the Monte Carlo simulation. These points are displayed in Table 24.

Table 24. Local deterministic optimum policies

Adjud Proc Time	Infra Spend	Govt Employ	Interest Rate	Police Goal	Tax Rate	Adjud Rate	Stim Pct	Govt Wage	Service Spend	Corrupt Pct	Police Init	Jail Term	Stim	V
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	
0.9	100	0.05	0.0675	165611	0.480	0.130	100	4188.61	0	0.141	52500	87	0	15,826
0.9	100	0.001	0.0225	164944	0.480	0.110	100	4200.00	0	0.130	52500	100	0	14,501
0.1	4.22	0.001	0.0225	166091	0.386	0.010	0	1400.00	0	0.050	137500	79	6000000	14,208
0.9	1.54	0.005	0.0225	166388	0.480	0.089	100	4200.00	0	0.125	137500	98	0	14,131
0.9	10.11	0.001	0.0225	167133	0.461	0.116	0	3985.57	0	0.050	137500	88	0	14,067
0.9	100	0.004	0.0225	162504	0.480	0.110	100	4200.00	0	0.130	137500	96	0	13,957
0.9	4.87	0.001	0.0225	169242	0.458	0.108	0	1400.00	0	0.050	137500	88	0	13,833
0.9	100	0.05	0.0675	169510	0.465	0.130	100	1400.00	0	0.138	52500	88	0	13,215
0.9	6.41	0.001	0.0225	167085	0.422	0.090	0	1400.00	0	0.050	137500	83	6000000	12,623
0.9	14.33	0.001	0.0225	166446	0.461	0.118	0	3970.42	0	0.050	52500	92	0	11,950
0.9	100	0.001	0.0225	157331	0.470	0.110	0	4200.00	0	0.118	137500	90	0	11,490
0.9	100	0.001	0.0225	153712	0.439	0.093	0	4101.30	0	0.123	137500	82	6000000	10,514
0.9	100	0.05	0.0675	155220	0.383	0.130	0	1400.00	80.11	0.129	52500	65	6000000	5,090

5.4 Monte Carlo Simulation

The residuals from the regression equations are used to estimate the marginal distribution functions for the two responses. Appendix D contains the residuals for this regression model. The lower bound A for each response's Beta distribution is chosen by rounding its lowest residual down to the next integer value. The upper bound B for each response is chosen by rounding its highest residual up to the next integer value. Using these bounds, the α and β parameters are estimated using Matlab's *fminsearch* function. Table 25 contains the estimated parameters of the two marginal Beta distribution functions.

Table 25. Estimated parameters for the marginal Beta distribution functions

	Residual y_1	Residual $\ln y_2$
α	17.7591	255.1359
β	7.9396	255.7977
A	-4.422E+09	-3
B	6.389E+09	6

The correlation matrix $\mathbf{R}$ is calculated from the residuals as shown in Table 26.

Table 26. Correlation matrix of the residuals

	1	-0.1912
	-0.1912	1

Using Matlab's *mvnrnd* function and the normal multivariate copula method, a set of 10,000 sample residuals are constructed for the Monte Carlo simulation. To confirm this random sample is from the intended distribution, the Beta distribution parameter estimates and correlation matrix of the sample is calculated for comparison with the residual data estimates. Table 27 contains the Beta parameters fit to the sample.

Table 27. Beta distribution parameters fit to the Monte Carlo samples

	Residual	Residual
	y_1	$\ln y_2$
α	18.0711	260.9499
β	8.0698	261.6797

Table 28 contains the correlation matrix of the random sample. Both the parameters from the two marginal distributions and the correlation matrix from the random sample appear to resemble those calculated from the residuals.

Table 28. Correlation matrix for the Monte Carlo samples

1	-0.1833
-0.1833	1

The Monte Carlo simulation is run using the random sample. In this model, the regression equation for y_2 contains a natural log transformation on the response. The random sample is not simply added to the responses. The Monte Carlo simulation allows one to compute the expected utility for a particular vector $\mathbf{x}$. Any point, $\mathbf{x}^*$ gives an expected utility,

$$\sum_{i=1}^n \left(\frac{1}{n} U_V \left(V \left(f_1(\mathbf{x}^*) + \epsilon_{1i}, e^{\ln(f_2(\mathbf{x}^*)) + \epsilon_{2i}} \right) \right) \right) \quad (5.10)$$

where $f_i(\mathbf{x}^*)$ is the expected response y_i given an input of $\mathbf{x}^*$ ($i = 1, 2$), n is the number of random samples used in the Monte Carlo simulation, ϵ_{1i} and ϵ_{2i} are the i -th random samples from the distributions of the residuals for y_1 and y_2 respectively, V is the multiattribute value function, and U_V is the utility function assessed over value. The optimization problem is to then choose a point $\mathbf{x}^*$ that maximizes expected utility.

The HJ algorithm is started at each local maxima displayed in Table 24. Risk preferences are modeled by varying γ in the appropriate utility function in Equation

5.4. Table 29 contains the levels of γ chosen and the corresponding risk preference with respect to y_1 and y_2 .

Table 29. Levels of γ sampled in Monte Carlo simulation

γ	$\gamma_{y_1}^U$	$\gamma_{y_2}^U$
-0.01	-5.751E-12	-1.015E-06
0	0	0
0.01	5.751E-12	1.015E-06
0.02	1.150E-11	2.030E-06

Table 30 displays the robust solutions found when $\gamma = -0.01, 0, 0.01, 0.02$ alongside the deterministic solution. The robust solutions when $\gamma = 0, 0.01, 0.02$ are equal. They differ slightly from the deterministic solution and the robust solution when risk seeking behavior is modeled ($\gamma = -0.01$).

Table 30. Compare deterministic solution with robust solutions for $\gamma = -0.01, 0, 0.01, 0.02$

	Deterministic Solution	Robust Solutions $\gamma =$	
		-0.01	0, 0.01, 0.02
$\mathbf{x}_1$	0.9	0.9	0.9
$\mathbf{x}_2$	100	100	100
$\mathbf{x}_3$	0.05	0.05	0.05
$\mathbf{x}_4$	0.0675	0.0675	0.0675
$\mathbf{x}_5$	165481	165473	165473
$\mathbf{x}_6$	0.47811	0.47814	0.47806
$\mathbf{x}_7$	0.13	0.13	0.13
$\mathbf{x}_8$	100	100	100
$\mathbf{x}_9$	4188.76	4188.84	4188.84
$\mathbf{x}_{10}$	0	0	0
$\mathbf{x}_{11}$	0.14081	0.14081	0.14081
$\mathbf{x}_{12}$	52500	52500	52500
$\mathbf{x}_{13}$	87	87	87
$\mathbf{x}_{14}$	0	0	0
$\mathbf{E}(\mathbf{V})$	15.8259	15.8259	15.8259

This case illustrates a limitation in this method of finding a robust optimum solution. The methodology assumes constant variance in the response noise over the experimental region. The Monte Carlo simulation inputs noise to the responses,

$y_1, \dots, y_n$, and calculates a robust solution by examining how this noise affects the multiattribute value function. To illustrate this, consider the example robust solution in Chapter 2, redisplayed in Figure 12.

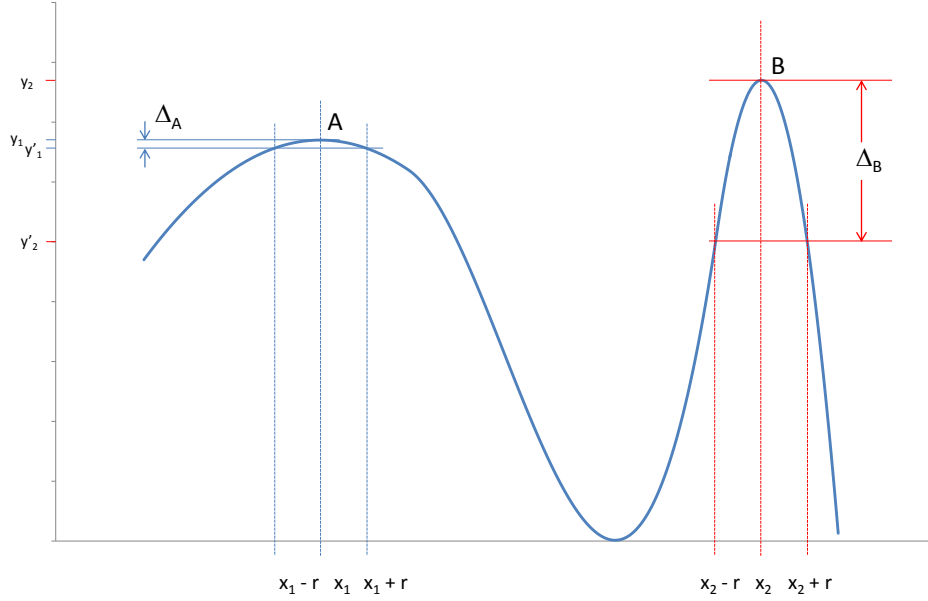


Figure 12. Illustration of Robust Solution [16]

Here, the horizontal axis represents the responses $(y_1, \dots, y_n)$ and the vertical axis represents $V(y_1, \dots, y_n)$. By properly specifying the decision maker's risk preference in the utility function $U_V(V(y_1, \dots, y_n))$, the proper robust solution can be found.

This method is dependent on the form of the value function in use. The value function illustrated in Figure 12 has curvature present. The value function used in this situation is an additive value function. It exhibits no curvature. Therefore this method will choose a robust solution at or near the optimum.

These three points are input into NOEM with 10 replicates run for each point to test the prediction of these optimum solutions. Table 31 contains the expected values and 95% confidence intervals for these three points.

Table 31. Average V and 95% confidence intervals from 10 replicates in NOEM for the three optimum points

	Deterministic	Robust Solution $\gamma =$	
		-0.01	$0, 0.01, 0.02$
E(V)	0.6577	0.7156	0.1472
95% Conf Int	[0.5837, 0.7316]	[0.6073, 0.8238]	[-0.1963, 0.4907]

The expected values ($E(V)$) displayed in Table 30 do not fall within the 95% confidence intervals found by running these points in NOEM. This is most likely due to no design point from the experiment lies near these calculated optima. A space-filling design such as a Latin hypercube design may present better results due to the experiment design points being more uniformly spread throughout the experimental region. Augmenting this D-optimal design with additional runs may also have presented a better model of the response surface. Moreover, a Kriging model of the system would most likely have described the surface better than a regression model. A Kriging model makes no assumption about the form of the underlying system response. Moreover, applications of Kriging models exist that model either variance homogeneity or variance heterogeneity [26].

VI. Summary, Conclusions and Recommendations

Decisions considering a set of multiple objectives occur everyday in government, military, and industrial settings. After understanding the decision maker's preferences regarding the trade-offs between the objectives and risk attitude, a utility function can be constructed to provide insight to the decision maker regarding such a decision.

When the decision situation concerns a system containing multiple inputs and outputs, response surface methodology provides a means to model the system with a set of equations. This set of equations can be used to inform the determination of value and utility functions that best describe the decision situation.

Many systems contain noise that effect the system outputs. In these cases, the best solution may not be simply the solution with the optimum expected output. The best solution may be in an operating location where the output varies less in the presence of noise. Depending on the form of the value function, this robust solution can be modeled and found by properly describing the risk attitude of the decision maker in a Monte Carlo simulation.

The desirability function has emerged as a popular method of scalarizing a multiple response optimization problem. It is a powerful function when the parameters are chosen properly to describe the decision maker's preferences.

In the case of the maximum is better desirability function, the lower bounds and target levels of the responses must be carefully chosen. A lower bound that is too high may eliminate desirable solutions. A target level that is too low may value multiple responses equally when one is more desirable.

When the desirability function is treated as a value function, the deterministic trade-offs described between responses are dependent upon the responses and their corresponding exponents and lower bounds. Trade-offs are not constant as Kros and

Mastrangelo calculated [29]. Once the lower bounds and targets are set, the exponents can be adjusted to properly describe the decision maker's trade-off preferences.

When a utility function is assessed over desirability, the risk preferences described depend on the form of the utility function. In the case of an exponential utility function, the risk preference with respect to the responses depends on the responses and their respective lower bounds, target levels, and desirability function exponents. Adjusting the lower bound or exponent has a non-monotonic effect on risk preference and must be adjusted carefully.

When the desirability function is treated as a utility function, there exists an implicit assumption of utility independence between all responses. The risk preference with respect to each response depends on the response, its exponent, and lower bound. The desirability function can describe risk averse, risk neutral, and risk seeking behavior depending on whether the exponent is less than, equal to, or greater than the number of responses. This is in contrast to Kros and Mastrangelo stating the risk preference described by the desirability function depends on whether the exponent is less than, equal to, or greater than one [29].

A decision analysis method of calculating a robust optimum solution using a utility function assigned over a value function in a Monte Carlo simulation is applied to a wire-bonding process experiment from the quality and reliability engineering design literature and to a national policy-making scenario within NOEM.

The usefulness of this method depends on the form of the value function. Since random noise is assumed to be constant throughout the experimental region, certain value functions (e.g., an additive value function) are affected uniformly by this random noise. When the value function exhibits curvature (e.g., the desirability function), the effect noise has on the variance in the value function can be measured and a

robust optimum solution can be found that is affected by changing the risk preference described by the utility function.

Additional research in this area includes analyzing other multiple response optimization functions. Any multiple response optimization function makes implicit and explicit assumptions regarding the decision maker's preferences. These assumptions should be properly analyzed so that the decision maker can make informed decisions.

The methodology presented for finding a robust optimum solution presented in this thesis provides insight in only certain forms of the value function. It does not provide additional insight about robustness when an additive value function is used. It can provide insight into a situation when a desirability function is used as the value function. Applying this method across a larger set of value functions should provide a fuller understanding about the types of functions for which this method provides insight about robustness.

The methodology presented here assumes random noise with constant variance throughout the experimental region. Further research can include modeling heterogeneous variance and applying a dynamic random sample that changes relative to the variance modeled at the current point in the experimental region within a Monte Carlo simulation.

The Kriging method can also estimate the response surface assuming either homogeneous or heterogeneous variance within the system. Further research includes how best to construct a Monte Carlo simulation that models the variance assumed in a Kriging model of the system.

In the current culture of constrained budgets, decision makers are faced more than ever with making decisions that encompass multiple, competing objectives. A method that properly describes the decision maker's preferences regarding risk and the trade-offs between the objectives at hand while also taking uncertainty into account will

serve to provide useful insight into the decision at hand. With that insight, the decision maker can proceed with confidence that he or she has made the best decision given current information.

Appendix A. Desirability function analysis in n -dimensions

Consider the desirability function in n -dimensions.

$$D = (d_1 \cdots d_n)^{\frac{1}{n}} \quad (\text{A.1})$$

Consider a system with m inputs, $(x_1, \dots, x_m)$. Each input, x_i falls within the experimental region defined by $x_i \in [x_i^0, x_i^*]$, $i = 1, \dots, m$. These inputs cause changes in two responses, y_1 and y_2 which each fall within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, \dots, n$. This analysis focuses on the case where a maximum response is desired as displayed in Equation 2.21 with no loss of generality since the other forms of the desirability function can be transformed into this case. Only the non-trivial piece of Equation 2.21 where $L_i < y_i < T_i$, $i = 1, \dots, n$ is considered. Equation A.2 displays this non-trivial piece.

$$d_i = \left[\frac{y_i - L_i}{T_i - L_i} \right]^{r_i}, L_i < y_i < T_i \quad (\text{A.2})$$

Where $[L_i, T_i] \subseteq [y_i^0, y_i^*]$, $i = 1, \dots, n$. This forces $0 < D < 1$.

The partial derivative of D with respect to y_i , ($1 \leq i \leq n$) is

$$D'_{y_i} = \frac{r_i}{n(T_i - L_i)} \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{n}} \cdots \left[\frac{y_i - L_i}{T_i - L_i} \right]^{\frac{r_i}{n} - 1} \cdots \left[\frac{y_n - L_n}{T_n - L_n} \right]^{\frac{r_n}{n}} \quad (\text{A.3})$$

The tradeoff function between y_i and y_j ($i \neq j$) is defined as

$$t(y_i, y_j) = \frac{D'_{y_i}}{D'_{y_j}} = - \frac{dy}{dx} \Big|_{\text{isopreference contour}} \quad (\text{A.4})$$

The deterministic tradeoffs between the two attributes can then be stated.

$$t(y_i, y_j) = \frac{r_i}{r_j} \left[\frac{y_j - L_j}{y_i - L_i} \right] \quad (\text{A.5})$$

Consider assessing an exponential utility function over D .

$$U = 1 - e^{-\gamma D} \quad (\text{A.6})$$

The risk aversion of the utility function with respect to y_i ($1 \leq i \leq n$) is

$$\gamma_{y_i}^U = \gamma_D^U D'_{y_i} + \gamma_{y_i}^D. \quad (\text{A.7})$$

The risk aversion for the utility function with respect to the desirability function is simply

$$\gamma_D^U = \gamma. \quad (\text{A.8})$$

The contribution of the desirability function to the risk aversion of the attribute y_i is

$$\gamma_{y_i}^D = \frac{1 - \frac{r_i}{n}}{y_i - L_i}. \quad (\text{A.9})$$

The risk aversion expressed by this utility function with respect to y_i is

$$\gamma_{y_i}^U = \frac{\gamma r_i}{n(y_i - L_i)} \left[\frac{y_1 - L_1}{T_1 - L_1} \right]^{\frac{r_1}{n}} \cdots \left[\frac{y_i - L_i}{T_i - L_i} \right]^{\frac{r_i}{n}} \cdots \left[\frac{y_n - L_n}{T_n - L_n} \right]^{\frac{r_n}{n}} + \frac{1 - \frac{r_i}{n}}{y_i - L_i}. \quad (\text{A.10})$$

Appendix B. Data from del Castillo example

Table 32. Data table from wire bonding experiment [9]

	x_1	x_2	x_3	y_1	y_2	y_3	y_4	y_5	y_6
	Flow	Flow	Block	Max	Begin	Finish	Max	Begin	Finish
n	Rate	Temp	Temp	Temp A	Bond A	Bond A	Temp B	Bond B	Bond B
1	40	200	250	139	103	110	110	113	126
2	120	200	250	140	125	126	117	114	131
3	40	450	250	184	151	133	147	140	147
4	120	450	250	210	176	169	199	169	171
5	40	325	150	182	130	122	134	118	115
6	120	325	150	170	130	122	134	118	115
7	40	325	350	175	151	153	143	146	164
8	120	325	350	180	152	154	152	150	171
9	80	200	150	132	108	103	111	101	101
10	80	450	150	206	143	138	176	141	135
11	80	200	350	183	141	157	131	139	160
12	80	450	350	181	180	184	192	175	190
13	80	325	250	172	135	133	155	138	145
14	80	325	250	190	149	145	161	141	149
15	80	325	250	180	141	139	158	140	148

Table 33. Residuals from del Castillo example experiment

	Residual	Residual	Residual	Residual	Residual	Residual
n	y_1	y_2	y_3	y_4	y_5	y_6
1	-12.558	-10.375	-6.575	-4.125	-3.5	-1.125
2	-11.558	-0.375	-3.825	8.375	3	4.375
3	-14.308	-5.625	-15.575	-5.875	-2	-1.875
4	11.692	7.375	7.175	6.625	4.5	3.625
5	10.692	9.125	9.8	8.375	5.25	4.375
6	-1.308	-2.875	-3.45	-8.625	-3.25	-4.625
7	-3.558	1.875	0.05	1.625	0.25	-1.375
8	1.442	-9.125	-12.2	-6.375	-4.25	-3.375
9	3.067	2.75	0.175	-5.357	-2.036	-2.857
10	-7.683	-5.5	3.175	-1.607	-1.536	-0.107
11	8.817	7.5	13.425	-1.107	2.964	1.393
12	-1.933	3.25	8.425	-1.357	-0.536	0.143
13	-2.933	-6	-6.2	0.143	-1.286	-1.857
14	15.067	8	5.8	6.143	1.714	2.143
15	5.067	0	-0.2	3.143	0.714	1.143

Appendix C. Feasible Region of del Castillo experiment

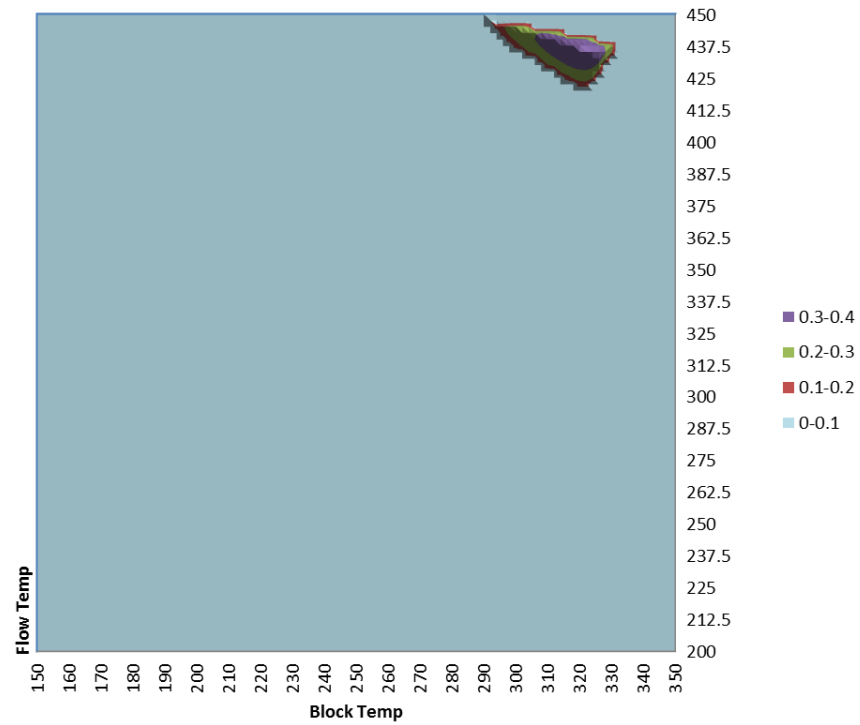


Figure 13. Value of D as Block Temp and Flow Temp vary (Flow Rate = 120)

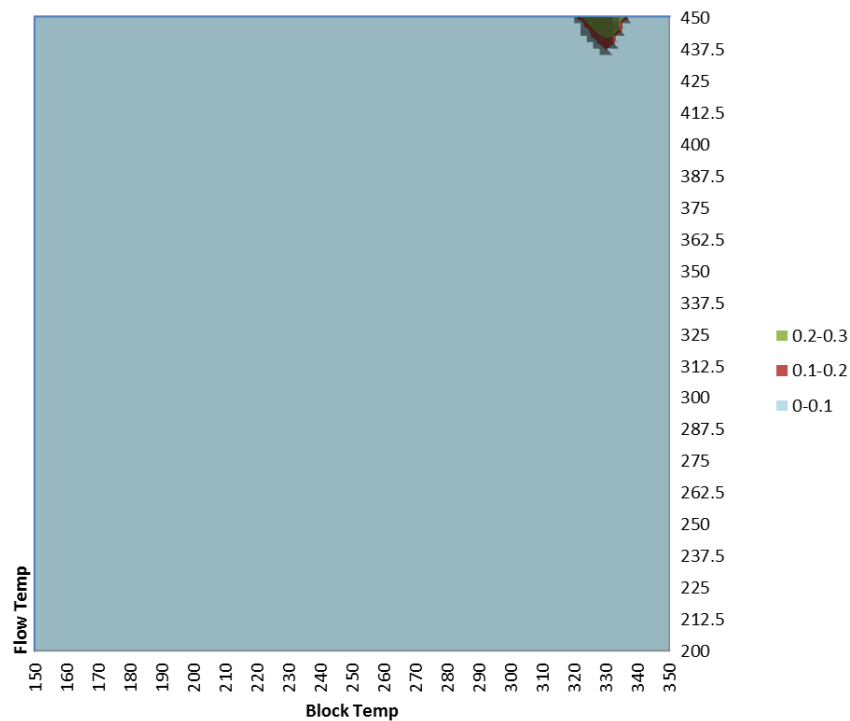


Figure 14. Value of D as Block Temp and Flow Temp vary (Flow Rate = 82.19)

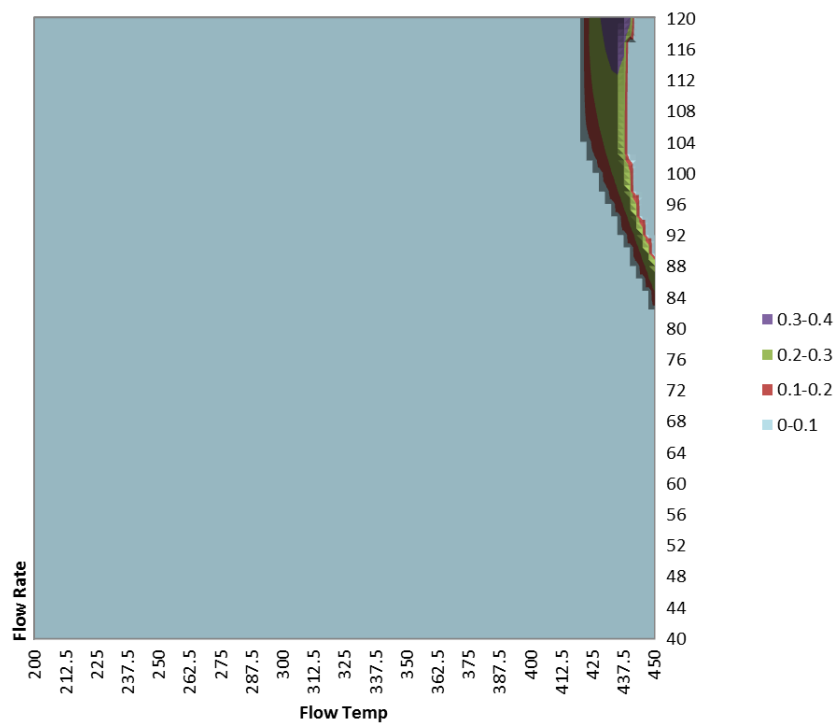


Figure 15. Value of D as Flow Temp and Flow Rate vary (Block Temp = 320.30)

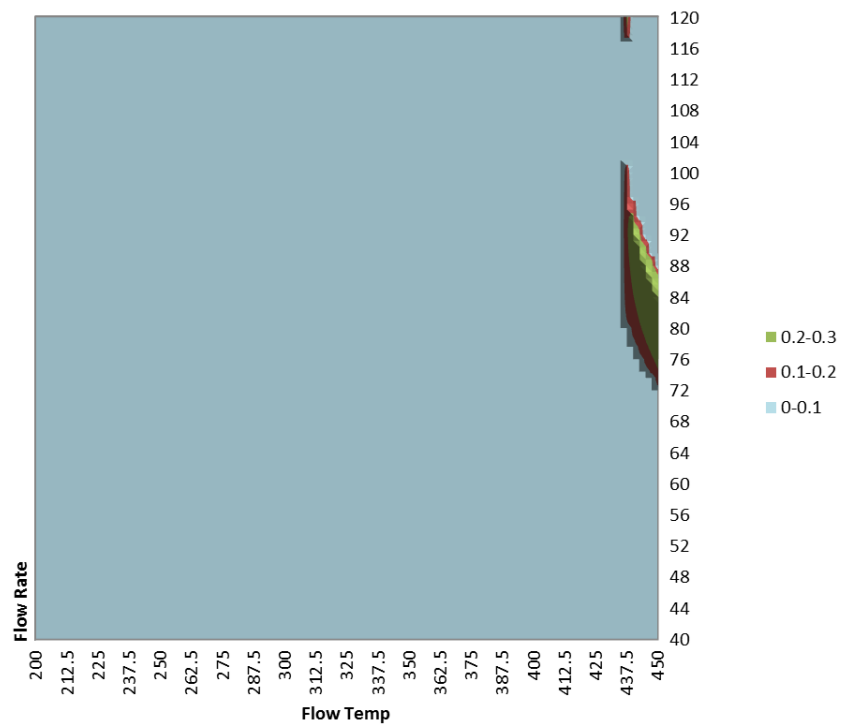


Figure 16. Value of D as Flow Temp and Flow Rate vary (Block Temp = 330.45)

Appendix D. Regression equation coefficients and residuals for NOEM experiment

Table 34. Coefficients for predicted Debt ($\hat{y}_1$)

	Coefficient
Factor	Estimate
Intercept	-1.2E+11
<i>A</i> -Adjud Proc Time	-1E+10
<i>B</i> -Infra Spend	-9.5E+07
<i>C</i> -Govt Employ	-5.2E+10
<i>D</i> -Interest Rate	-5.2E+11
<i>E</i> -Police Goal	2223638
<i>F</i> -Tax Rate	1.88E+10
<i>G</i> -Adjud Rate	2.41E+11
<i>H</i> -Stimulus Pct	3.01E+08
<i>J</i> -Govt Wage	34007472
<i>K</i> -Service Spend	52747611
<i>L</i> -Corrupt Pct	5.39E+11
<i>M</i> -Police Init	485203.9
<i>N</i> -Jail Term	1.7E+08
<i>O</i> -Stimulus	2870.185
<i>AB</i>	-1.4E+07
<i>AC</i>	-5.7E+10
<i>AD</i>	1.21E+10
<i>AE</i>	34766.79
<i>AF</i>	-5.8E+09

Continued on next page.

Table 34. (Debt model coefficients continued)

Factor	Coefficient Estimate
<i>AG</i>	-4.4E+10
<i>AH</i>	-3.5E+07
<i>AJ</i>	292440.8
<i>AK</i>	30246284
<i>AL</i>	1.04E+09
<i>AM</i>	9606.421
<i>AN</i>	-2E+07
<i>AO</i>	276.1704
<i>BC</i>	-5.2E+08
<i>BD</i>	-7.7E+08
<i>BE</i>	300.0216
<i>BF</i>	35535699
<i>BG</i>	-6.3E+07
<i>BH</i>	-205478
<i>BJ</i>	-12612
<i>BK</i>	-124684
<i>BL</i>	-1.3E+08
<i>BM</i>	277.5306
<i>BN</i>	91005.42
<i>BO</i>	-1.04642
<i>CD</i>	-1.2E+12
<i>CE</i>	578785.7

Continued on next page.

Table 34. (Debt model coefficients continued)

Factor	Coefficient Estimate
<i>CF</i>	1.22E+11
<i>CG</i>	6.9E+10
<i>CH</i>	-5.7E+08
<i>CJ</i>	20315991
<i>CK</i>	-9.3E+08
<i>CL</i>	-1.1E+12
<i>CM</i>	894714.9
<i>CN</i>	9.66E+08
<i>CO</i>	14794.99
<i>DE</i>	-581941
<i>DF</i>	3.29E+10
<i>DG</i>	-7.8E+11
<i>DH</i>	-8E+08
<i>DJ</i>	15254317
<i>DK</i>	-7.7E+08
<i>DL</i>	-3.3E+11
<i>DM</i>	520396.9
<i>DN</i>	1.44E+08
<i>DO</i>	5747.516
<i>EF</i>	-61603
<i>EG</i>	-286008
<i>EH</i>	-255.663

Continued on next page.

Table 34. (Debt model coefficients continued)

Factor	Coefficient Estimate
<i>EJ</i>	9.591571
<i>EK</i>	267.4753
<i>EL</i>	-74454
<i>EM</i>	-0.08278
<i>EN</i>	-231.426
<i>EO</i>	0.003944
<i>FG</i>	-5.3E+10
<i>FH</i>	-3.5E+07
<i>FJ</i>	-495916
<i>FK</i>	1.17E+08
<i>FL</i>	-7.5E+10
<i>FM</i>	10285.56
<i>FN</i>	22431989
<i>FO</i>	1334.28
<i>GH</i>	1.35E+08
<i>GJ</i>	-7219620
<i>GK</i>	-1.8E+08
<i>GL</i>	7.95E+10
<i>GM</i>	176310
<i>GN</i>	-5222336
<i>GO</i>	2426.442
<i>HJ</i>	-14446.1

Continued on next page.

Table 34. (Debt model coefficients continued)

Factor	Coefficient Estimate
<i>HK</i>	-177820
<i>HL</i>	-6.3E+08
<i>HM</i>	287.7633
<i>HN</i>	-365415
<i>HO</i>	13.19157
<i>JK</i>	2268.709
<i>JL</i>	-6658941
<i>JM</i>	-2.26307
<i>JN</i>	-1152.77
<i>JO</i>	0.126152
<i>KL</i>	-2E+08
<i>KM</i>	190.0535
<i>KN</i>	65779.41
<i>KO</i>	-5.14335
<i>LM</i>	-129731
<i>LN</i>	-6E+07
<i>LO</i>	-2756.72
<i>MN</i>	285.4206
<i>MO</i>	-0.00252
<i>NO</i>	3.376809
A^2	3.2E+10
B^2	3978377

Continued on next page.

Table 34. (Debt model coefficients continued)

Factor	Coefficient
	Estimate
C^2	2.2E+11
D^2	2.02E+13
E^2	-16.731
F^2	-1.4E+11
G^2	-2.5E+12
H^2	-285702
J^2	-12046.9
K^2	-828766
L^2	-5.4E+12
M^2	-5.3217
N^2	-4390076
O^2	-0.00124
A^3	-2.6E+10
B^3	-27719.5
C^3	3.87E+12
D^3	-1.6E+14
E^3	4.06E-05
F^3	2.04E+11
G^3	1.2E+13
H^3	-6288.01
J^3	1.29533
K^3	6608.289

Continued on next page.

Table 34. (Debt model coefficients continued)

	Coefficient
Factor	Estimate
L^3	1.95E+13
M^3	1.51E-05
N^3	26475.31
O^3	8.38E-11

Table 35. Coefficients for predicted activists ($\ln \hat{y}_2$)

	Coefficient
Factor	Estimate
Intercept	39.655
A -Adjud Proc Time	1.762
B -Infra Spend	0.054
C -Govt Employ	-141.256
D -Interest Rate	-97.080
E -Police Goal	-0.001
F -Tax Rate	3.105
G -Adjud Rate	-73.038
H -Stimulus Pct	-0.020
J -Govt Wage	-0.002
K -Service Spend	-0.009
L -Corrupt Pct	320.777
M -Police Init	9.50E-06

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient
	Estimate
<i>N</i> -Jail Term	-0.091
<i>O</i> -Stimulus	-6.10E-07
<i>AB</i>	-0.002
<i>AC</i>	12.913
<i>AD</i>	-1.894
<i>AE</i>	-7.71E-06
<i>AF</i>	0.151
<i>AG</i>	-4.845
<i>AH</i>	0.002
<i>AJ</i>	8.75E-05
<i>AK</i>	0.007
<i>AL</i>	1.518
<i>AM</i>	5.64E-06
<i>AN</i>	0.003
<i>AO</i>	-2.59E-08
<i>BC</i>	0.039
<i>BD</i>	0.035
<i>BE</i>	7.29E-09
<i>BF</i>	-0.005
<i>BG</i>	0.029
<i>BH</i>	-3.30E-06
<i>BJ</i>	1.39E-07

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient Estimate
<i>BK</i>	1.39E-06
<i>BL</i>	0.013
<i>BM</i>	-4.98E-08
<i>BN</i>	-6.05E-06
<i>BO</i>	-1.71E-10
<i>CD</i>	-168.524
<i>CE</i>	-1.44E-04
<i>CF</i>	-16.719
<i>CG</i>	46.315
<i>CH</i>	0.065
<i>CJ</i>	-0.002
<i>CK</i>	0.106
<i>CL</i>	192.671
<i>CM</i>	6.72E-05
<i>CN</i>	0.054
<i>CO</i>	-8.55E-07
<i>DE</i>	9.77E-05
<i>DF</i>	-34.473
<i>DG</i>	-81.507
<i>DH</i>	-0.030
<i>DJ</i>	0.008
<i>DK</i>	-0.067

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient
	Estimate
<i>DL</i>	-60.734
<i>DM</i>	-4.83E-05
<i>DN</i>	-0.001
<i>DO</i>	-9.20E-07
<i>EF</i>	1.98E-05
<i>EG</i>	2.05E-05
<i>EH</i>	-5.40E-08
<i>EJ</i>	-7.05E-11
<i>EK</i>	-5.09E-08
<i>EL</i>	5.38E-05
<i>EM</i>	7.64E-11
<i>EN</i>	-6.25E-08
<i>EO</i>	5.96E-13
<i>FG</i>	-0.475
<i>FH</i>	0.006
<i>FJ</i>	-0.001
<i>FK</i>	0.002
<i>FL</i>	-0.111
<i>FM</i>	7.26E-06
<i>FN</i>	-0.008
<i>FO</i>	-6.60E-09
<i>GH</i>	-0.026

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient
	Estimate
<i>GJ</i>	-0.001
<i>GK</i>	0.013
<i>GL</i>	0.190
<i>GM</i>	2.37E-05
<i>GN</i>	0.024
<i>GO</i>	4.19E-08
<i>HJ</i>	-1.26E-06
<i>HK</i>	1.38E-05
<i>HL</i>	0.046
<i>HM</i>	4.55E-08
<i>HN</i>	5.80E-05
<i>HO</i>	5.04E-10
<i>JK</i>	-1.34E-06
<i>JL</i>	-3.17E-04
<i>JM</i>	6.59E-10
<i>JN</i>	9.41E-07
<i>JO</i>	8.55E-13
<i>KL</i>	0.076
<i>KM</i>	-4.62E-08
<i>KN</i>	6.84E-05
<i>KO</i>	1.08E-10
<i>LM</i>	5.68E-05

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient
	Estimate
LN	0.058
LO	7.64E-07
MN	-1.84E-08
MO	-8.70E-13
NO	7.54E-10
A^2	-7.008
B^2	-0.001
C^2	7228.526
D^2	2707.111
E^2	4.72E-09
F^2	-10.416
G^2	1169.703
H^2	8.40E-05
J^2	7.16E-07
K^2	5.42E-05
L^2	-2686.560
M^2	-1.95E-10
N^2	0.001
O^2	1.84E-13
A^3	5.351
B^3	6.73E-06
C^3	-94855.894

Continued on next page.

Table 35. (Activist model coefficients continued)

Factor	Coefficient
	Estimate
D^3	-21722.951
E^3	-9.94E-15
F^3	11.252
G^3	-4959.153
H^3	5.56E-07
J^3	-7.44E-11
K^3	-2.09E-07
L^3	7234.906
M^3	1.87E-16
N^3	-5.68E-06
O^3	-1.70E-20

Table 36. Residuals from NOEM experiment

n	Residual	Residual
	y_1	$\ln y_2$
1	6.05E+08	0.01470
2	46501523	0.04140
3	-1.4E+08	-2.26903
4	-5.2E+08	0.02669
5	17163091	0.00104
6	2.93E+08	0.00221

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
7	6.07E+08	-0.03120
8	-2.1E+08	0.01059
9	-6.3E+07	-0.00359
10	-5.9E+08	-0.00723
11	-8.6E+08	-0.03596
12	5.91E+08	0.12739
13	3.27E+08	-0.00834
14	-4.7E+08	0.05884
15	-8.1E+08	0.04631
16	78070523	-0.00677
17	4.12E+08	0.02850
18	1.1E+08	0.00031
19	6.28E+08	-0.09888
20	3.9E+08	-0.03841
21	7.14E+08	-0.22464
22	-3.1E+08	-0.11321
23	-3E+08	0.01299
24	3.59E+08	-0.02980
25	-5.1E+08	0.05644
26	-8.7E+08	0.06156
27	5.54E+08	0.14357
28	-1.7E+08	0.01179

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
29	1.57E+08	0.00506
30	-6.1E+08	0.19792
31	60858431	-0.07986
32	-6.9E+08	-0.09111
33	4.65E+08	-0.16477
34	2.89E+08	-0.00784
35	2.45E+08	-0.02627
36	1.06E+08	0.00367
37	4.94E+08	-0.03780
38	-1.6E+07	-0.09847
39	-2.6E+08	0.11083
40	-1.9E+08	0.16481
41	-8.9E+07	-0.01404
42	-3.4E+08	0.02320
43	-5.5E+08	-0.03663
44	-2.5E+08	0.09646
45	14818895	-0.05011
46	-2.2E+08	-0.02335
47	8.54E+08	0.09151
48	1.62E+08	-0.00864
49	-1.4E+08	0.00599
50	1.95E+08	-0.03013

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	ln y_2
51	-6E+08	-0.06399
52	7.77E+08	-0.06574
53	-3.6E+08	0.00710
54	-7.7E+08	0.05698
55	-68764.7	0.00169
56	-6.6E+08	0.04321
57	1.07E+08	-0.03385
58	-2.7E+08	0.01978
59	1.89E+09	-0.14355
60	-4.5E+08	0.03494
61	-5.3E+08	0.00734
62	2.85E+08	-0.04153
63	6.09E+08	-0.03008
64	-4.2E+08	0.01458
65	1E+09	-0.14547
66	-4.4E+09	0.23652
67	-3.9E+08	0.02111
68	1.42E+08	-0.04746
69	7.52E+08	-0.01527
70	4.48E+08	-0.14286
71	5.77E+08	-0.07107
72	2424187	0.00200

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	ln y_2
73	-3.2E+07	0.00180
74	2.4E+08	-0.01881
75	-2.6E+07	-0.01237
76	1.91E+08	0.24609
77	1.26E+09	-0.07518
78	-5E+08	0.04178
79	-1.3E+08	0.05549
80	7.76E+08	-0.03767
81	2.86E+08	0.08801
82	8.21E+08	-0.07016
83	-2.2E+08	-0.00163
84	3.11E+08	-0.04712
85	2.03E+08	-0.00505
86	-3.6E+08	0.01480
87	4.66E+08	-0.09495
88	-3.8E+08	0.09892
89	5.65E+08	-0.12924
90	-2.3E+08	0.01413
91	-7.2E+07	0.00131
92	1.1E+09	-0.06148
93	-7.7E+08	0.04685
94	1.87E+08	-0.01776

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	ln y_2
95	24635434	0.00560
96	-5.5E+08	0.03593
97	-2.3E+08	0.02691
98	-4.1E+08	0.01791
99	-5.2E+08	0.03291
100	1.09E+09	-0.06534
101	-2.7E+07	0.17692
102	-6.9E+08	0.13842
103	-1.1E+08	0.10242
104	-1.3E+07	-0.09554
105	74725502	-0.01007
106	-9E+08	0.05072
107	-8.6E+07	0.00804
108	7.68E+08	-0.04576
109	77373775	-0.00131
110	-6.2E+08	0.08858
111	4.61E+08	-0.02004
112	2.87E+08	-0.03326
113	5.75E+08	-0.23979
114	-4.3E+08	0.01492
115	20782539	0.09571
116	20906907	-0.07693

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
117	6.44E+08	-0.02522
118	3.28E+08	-0.01519
119	1.39E+08	0.11427
120	-2.3E+08	-0.06090
121	1.02E+09	0.04245
122	-7E+08	0.14105
123	-1.6E+08	-0.00421
124	21789422	0.00127
125	-6.1E+08	0.03328
126	-2.2E+08	-0.00190
127	88528846	-0.00964
128	82073456	-0.02887
129	-5.4E+08	0.00532
130	-7.4E+08	0.04756
131	88697026	-0.01250
132	-7.8E+08	0.07467
133	-9.4E+07	-0.18327
134	-5.2E+08	-0.00225
135	5.25E+08	0.00848
136	3.65E+08	0.00283
137	3.14E+08	-0.04048
138	6.33E+08	-0.04005

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Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
139	2.58E+08	-0.00268
140	6.45E+08	-0.09468
141	4.89E+08	-0.06711
142	-3.1E+08	2.29606
143	-5.2E+08	0.02387
144	11918414	-0.00055
145	2.85E+08	-0.04482
146	6.06E+08	-0.03489
147	-2E+08	0.01573
148	-7.4E+07	0.01549
149	-5.9E+08	0.07362
150	63301594	0.08141
151	8.05E+08	-0.21154
152	3.28E+08	-0.00924
153	-5.3E+08	-0.02777
154	-8.1E+08	0.04623
155	84505897	-0.00937
156	4.07E+08	-0.07991
157	1.06E+08	-0.01242
158	6.27E+08	0.04279
159	3.99E+08	0.01167
160	6.79E+08	0.15372

Continued on next page.

Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
161	-3.5E+08	0.13814
162	-3E+08	0.02363
163	3.6E+08	-0.01006
164	-5.2E+08	0.01254
165	-8.8E+08	0.04320
166	5.77E+08	-0.19703
167	-2.2E+08	0.01890
168	1.68E+08	-0.03330
169	-5.9E+08	-0.12573
170	64175139	0.07030
171	-6.9E+08	0.17001
172	4.65E+08	0.11906
173	3.02E+08	-0.02906
174	2.46E+08	0.00041
175	1.09E+08	-0.01239
176	6.07E+08	-0.02233
177	-2.3E+07	-0.11000
178	-2.6E+08	0.12963
179	-1.9E+08	-0.15507
180	-8.9E+07	0.01462
181	-3.5E+08	0.00679
182	-5.5E+08	0.10022

Continued on next page.

Table 36. (Residuals continued)

n	Residual	Residual
	y_1	$\ln y_2$
183	-2.6E+08	-0.05576
184	14569111	0.05139
185	-2.2E+08	0.04875
186	1.03E+09	-0.20512
187	1.98E+08	-0.01911
188	-1.3E+08	0.00501
189	1.86E+08	0.00298
190	-1E+09	0.16006
191	6.57E+08	-0.02194
192	-1.6E+08	0.01964
193	-7.7E+08	0.03657
194	-1561419	0.00145
195	-4.4E+08	0.02101
196	1.2E+08	0.01994
197	-3E+08	0.01753
198	1.97E+09	-0.13089
199	-4.5E+08	0.01859
200	-6E+08	0.06112
201	1.21E+08	0.03122
202	5.93E+08	-0.02883
203	-4.9E+08	0.04262
204	9.95E+08	0.04702

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Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
205	-4.4E+09	0.23771
206	-3.6E+08	0.02866
207	1.85E+08	0.04997
208	7.1E+08	-0.06078
209	4.4E+08	0.08851
210	5.66E+08	0.00929
211	-1.7E+07	0.00199
212	-7.9E+07	0.00215
213	2.58E+08	-0.01779
214	-3E+07	-0.00057
215	1.86E+08	-0.27432
216	1.28E+09	-0.07517
217	-5.4E+08	0.02574
218	37521805	-0.05756
219	7.48E+08	-0.03814
220	2.94E+08	-0.10614
221	8.27E+08	-0.02067
222	-2.2E+08	0.02649
223	3.09E+08	-0.00408
224	1.92E+08	-0.00951
225	-2.7E+08	0.01268
226	4.97E+08	0.04459

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Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
227	-3.2E+08	-0.04864
228	59462511	0.09071
229	-6.5E+07	-0.01050
230	-1.4E+08	0.01054
231	1.14E+09	-0.06252
232	-8.5E+08	0.04684
233	1.94E+08	0.00299
234	-1.2E+08	-0.00682
235	-5.5E+08	0.01613
236	-2.8E+08	-0.00165
237	-5.5E+08	0.04358
238	-5.1E+08	0.03270
239	1.09E+09	-0.06537
240	5.74E+08	-0.20988
241	-6.9E+08	-0.06992
242	-5.7E+07	-0.08715
243	-6.1E+08	0.13442
244	1.34E+08	-0.00347
245	-9E+08	0.05345
246	-8.4E+07	0.00772
247	7.62E+08	-0.04690
248	891461.7	-0.00482

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Table 36. (Residuals continued)

	Residual	Residual
n	y_1	$\ln y_2$
249	-6.5E+08	-0.01392
250	3.75E+08	-0.02906
251	2.66E+08	0.00046
252	5.86E+08	0.17459
253	-4.3E+08	0.03501
254	20776848	0.05542
255	20998168	-0.07753
256	6.51E+08	-0.03974
257	3.28E+08	-0.00574
258	1.77E+08	-0.12397
259	-2.4E+08	0.08025
260	1.06E+09	-0.15035
261	-6.4E+08	-0.05990
262	-2.5E+08	0.02545
263	16457326	-0.00119
264	-5.5E+08	0.02066
265	-2.3E+08	0.04429
266	-6.4E+07	-0.00435
267	1.44E+08	0.01031
268	-5.4E+08	0.06502
269	-7.4E+08	0.02541
270	82312932	0.00280

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Table 36. (Residuals continued)

	Residual	Residual
n	y_1	ln y_2
271	-7.8E+08	0.01035
272	-2.1E+08	0.19584
273	-5.2E+08	0.06252
274	5.24E+08	-0.07190
275	3.56E+08	-0.03884
276	3.34E+08	-0.00970
277	6.4E+08	-0.03549
278	2.06E+08	-0.01601

Appendix E. Quad Chart

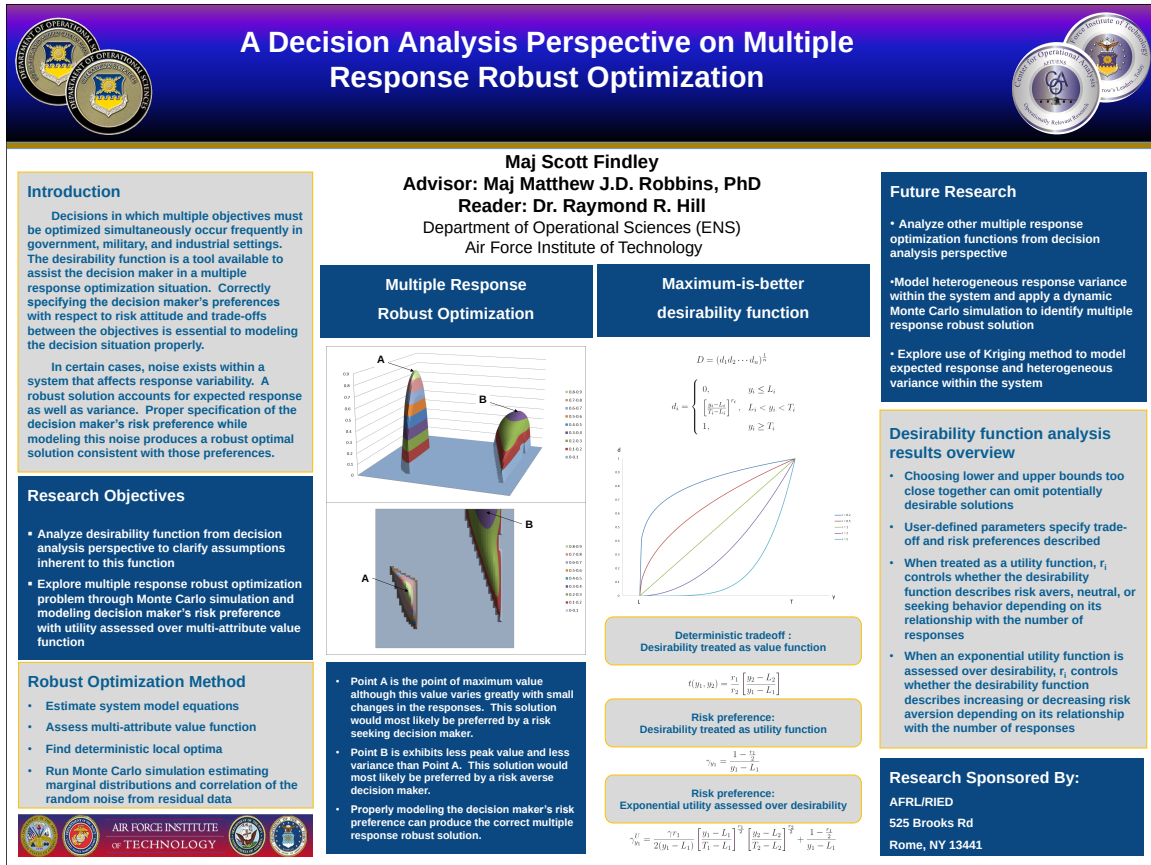


Figure 17. Quad Chart

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14. ABSTRACT Decisions in which multiple objectives must be optimized simultaneously occur frequently in government, military, and industrial settings. One method a decision maker may use to assist in such decisions is the application of a desirability function. An informed specification of the desirability function's parameters is essential to accurately describe the decision maker's value trade-offs and risk preference. This thesis uses utility transversality to analyze the implicit trade-off and risk attitude assumptions attendant to the desirability function. The desirability function does not explicitly account for response variability. A robust solution considers not only the expected response into account but also its variance. Assessing a utility function over desirability as a means to describe the decision maker's risk attitude produces a robust operating solution consistent with those preferences. This thesis examines robustness as it applies to the desirability function in a manufacturing experiment example. Different levels of diplomatic, informational, military and economic (DIME) instruments of national policy are investigated to examine their effect on the political, military, economic, social, infrastructure, and information (PMESII) systems of a nation, AFRL's National Operational Environment Model (NOEM) serves as a basis for identifying a robust national policy in a scenario involving the Democratic Republic of Congo.					
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