

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

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## Report Documentation Page

*Form Approved*  
OMB No. 0704-0188

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1. REPORT DATE <b>02 APR 2012</b>	2. REPORT TYPE <b>Presentation</b>	3. DATES COVERED <b>02-03-2012 to 02-04-2012</b>			
4. TITLE AND SUBTITLE <b>Sandwich Plates With A Compressible Core Impacted by Blast Loading</b>		5a. CONTRACT NUMBER			
		5b. GRANT NUMBER			
		5c. PROGRAM ELEMENT NUMBER			
6. AUTHOR(S) <b>Terry Hause</b>		5d. PROJECT NUMBER			
		5e. TASK NUMBER			
		5f. WORK UNIT NUMBER			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000</b>		8. PERFORMING ORGANIZATION REPORT NUMBER <b>#22704</b>			
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) <b>U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000</b>		10. SPONSOR/MONITOR'S ACRONYM(S) <b>TARDEC</b>			
		11. SPONSOR/MONITOR'S REPORT NUMBER(S) <b>#22704</b>			
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>Presentation for SAE World Congress 2012.</b>					
14. ABSTRACT <b>1. The face sheets fulfill the Love-Kirchoff assumptions and are thin compared with the core. 2. The bonding between the face sheets and the core is assumed to be perfect. 3. The kinematic boundary conditions at the interfaces between the core and the facings are satisfied. 4. The core is assumed to be a weak orthotropic transversely compressible core carrying only the transverse strains and the normal strain. 5. The shock wave pressure is uniformly distributed on the front face of the sandwich plate.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT  <b>Public Release</b>	18. NUMBER OF PAGES  <b>26</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

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## Acknowledgements

The author would like to express thanks to the U.S. Army RDECOM TARDEC for their support and funding under the Independent Laboratory In-House Research Program (ILIR)

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## Outline

1. Motivation
2. Basic Assumptions and Preliminaries
3. Theoretical Developments
4. Blast Loading
5. Results
6. Concluding Remarks

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

## Motivation

- High bending stiffness and strength to weight ratio
- Excellent thermal and sound insulation
- Increased durability under a thermo-mechanical loading environment
- Tight thermal distortion tolerances
- Lightweight in structure

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## Basic Assumptions and Preliminaries

1. The face sheets fulfill the Love-Kirchoff assumptions and are thin compared with the core.
2. The bonding between the face sheets and the core is assumed to be perfect.
3. The kinematic boundary conditions at the interfaces between the core and the facings are satisfied.
4. The core is assumed to be a weak orthotropic transversely compressible core carrying only the transverse strains and the normal strain.
5. The shock wave pressure is uniformly distributed on the front face of the sandwich plate.

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

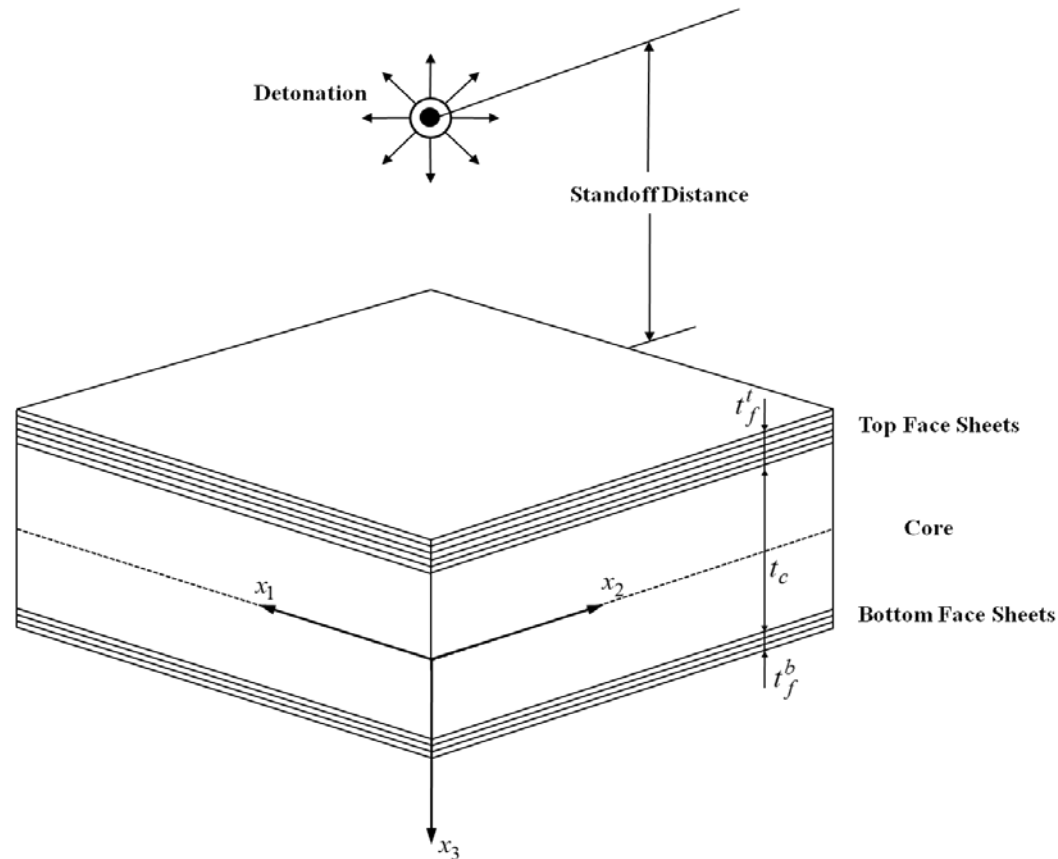


Fig 1. An asymmetric sandwich plate under blast loading

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## Theoretical Developments

### Displacement Field

#### Top Face

$$v_{\alpha}^t = u_{\alpha}^a + u_{\alpha}^d - \left( x_3 + \frac{t_c + t_f^t}{2} \right) u_{3,\alpha}^a - \left( x_3 + \frac{t_c + t_f^t}{2} \right) u_{3,\alpha}^d$$

$$v_3^t = u_3^a + u_3^d$$

#### Bottom Face

$$v_{\alpha}^b = u_{\alpha}^a - u_{\alpha}^d - \left( x_3 - \frac{t_c + t_f^b}{2} \right) u_{3,\alpha}^a + \left( x_3 - \frac{t_c + t_f^b}{2} \right) u_{3,\alpha}^d$$

$$v_3^b = u_3^a - u_3^d$$

#### Core

$$v_{\alpha}^c = u_{\alpha}^a - \left( \frac{t_f^t - t_f^b}{4} \right) u_{3,\alpha}^a - \left( \frac{t_f^t + t_f^b}{4} \right) u_{3,\alpha}^d - \frac{2x_3}{t_c} u_{\alpha}^d + \left( \frac{t_f^t + t_f^b}{2t_c} \right) x_3 u_{3,\alpha}^a + \left( \frac{t_f^t - t_f^b}{2t_c} \right) x_3 u_{3,\alpha}^d + \left( \frac{4x_3^2}{t_c^2} - 1 \right) \Phi_{\alpha}^c$$

$$v_3^c(x, y, z, t) = u_3^a - \frac{2x_3}{t_c} u_3^d$$

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Note:

the Greek indices have the range 1, 2, while the Latin indices have the range 1, 2, 3 and unless otherwise stated, Einstein's summation convention over the repeated indices is assumed. Also,  $\partial_i$  denotes partial differentiation with respect to the coordinates  $x_i$ , while superscripts  $t$  and  $b$  indicate the association with the top and bottom facings respectively.

Also,

$$u_i^a = \frac{1}{2}(u_i^t + u_i^b), \quad u_i^d = \frac{1}{2}(u_i^t - u_i^b)$$

represent the average and the half difference of the face sheet mid-surface displacements while, the core displacements,  $\Phi_\alpha^c$  warping functions of the core.

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## Non-Linear Strain-Displacement Relationships

The strain-displacement relationships given by the Lagrangian Strain-Displacement Relationships used in conjunction with the Von-Karman assumptions is given in indicial notation as

$$\gamma_{11} = v_{1,1} + \frac{1}{2}(v_{3,1})^2$$

$$\gamma_{22} = v_{2,2} + \frac{1}{2}(v_{3,2})^2$$

$$\gamma_{33} = v_{3,3} + \frac{1}{2}(v_{3,3})^2$$

$$\gamma_{23} = \frac{1}{2}(v_{2,3} + v_{3,2}) + \frac{1}{2}v_{3,2}v_{3,3}$$

$$\gamma_{13} = \frac{1}{2}(v_{1,3} + v_{3,1}) + \frac{1}{2}v_{3,1}v_{3,3}$$

$$\gamma_{12} = \frac{1}{2}(v_{1,2} + v_{2,1}) + \frac{1}{2}v_{3,1}v_{3,2}$$

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## Constitutive Equations

Both the top and bottom face sheets are considered to be constructed from unidirectional fiber reinforced anisotropic laminated composites, the axes of orthotropy not necessarily being coincident with the geometrical axes. The stress-strain relationships for each lamina of the facings becomes

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ & \bar{Q}_{22} & \bar{Q}_{26} \\ \text{Sym} & & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \\ 2\gamma_{12} \end{Bmatrix}$$

Where,  $\bar{Q}_{ij}$  for  $i, j = (1, 2, 6)$  are the *Transformed plane-stress reduced stiffness measures*.

The stress-strain relationships for the orthotropic core with the geometrical and material axes coincident are expressed as

$$\tau_{33}^c = E^c \gamma_{33}^c, \quad \tau_{13}^c = G_{13}^c \gamma_{13}^c, \quad \tau_{23}^c = G_{23}^c \gamma_{23}^c$$

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Equations of Motion-Hamilton's Variational Principle

$$\int_{t_0}^{t_1} (\delta U - \delta W - \delta T) dt = 0$$

$U$  = strain energy,

$W$  = represent the work done by external forces

$T$  = represent the kinetic energy

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$$\delta U = \int_A \left( \int_{-t_c/2-t_f}^{-t_c/2} \tau_{\alpha\beta}^t \delta\gamma_{\alpha\beta}^t dx_3 + \int_{-t_c/2}^{+t_c/2} \tau_{i3}^c \delta\gamma_{i3}^c dx_3 + \int_{t_c/2}^{t_c/2+t_f} \tau_{\alpha\beta}^b \delta\gamma_{\alpha\beta}^b dx_3 \right) dA$$

Where  $\tau_{ij}$  are the tensorial components of the second Piola-Kirchoff stress tensor, while  $A$  is attributed to the area of the sandwich plate.

$$\delta W = \int_A \left( \hat{q}_3^t(x_1, x_2, t) \delta v_3^t + \hat{q}_3^b(x_1, x_2, t) \delta v_3^b \right) dA - \int_A \left( 2C^t \dot{v}_3^t \delta v_3^t + 2C^c \dot{v}_3^c \delta v_3^c + 2C^b \dot{v}_3^b \delta v_3^b \right) dA$$

Where  $q^t(x_1, x_2, t)$  denotes the transverse pressure loading from a spherical air-blast and  $C$  is the structural damping coefficient per unit area of the plate.

$$\int_{t_0}^{t_1} \delta T dt = \int_{t_0}^{t_1} \int_A - \left( \int_{-t_c/2-t_f}^{-t_c/2} \rho_f^t \ddot{v}_3^t \delta v_3^t dx_3 + \int_{-t_c/2}^{t_c/2} \rho^c \ddot{v}_3^c \delta v_3^c dx_3 + \int_{t_c/2}^{t_c/2+t_f} \rho_f^b \ddot{v}_3^b \delta v_3^b dx_3 \right) dA dt$$

Where  $\rho^c$  and  $\rho_f^t, \rho_f^b$  are the mass densities of the core and the top and bottom face sheets, respectively, and  $\ddot{v}_3$  denotes the transverse acceleration.

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## Equations of Motion

$$\delta u_\alpha^a : N_{\alpha\beta,\beta}^a = 0$$

$$\delta u_\alpha^d : N_{\alpha\beta,\beta}^d + \frac{N_{\alpha 3}^c}{t_c} = 0$$

$$\delta \Phi_\alpha^c : M_{\alpha 3}^c = 0$$

$$\begin{aligned} \delta u_3^a : & u_{3,\alpha\beta}^a N_{\alpha\beta}^a + M_{\alpha\beta,\alpha\beta}^a + u_{3,\alpha\beta}^d N_{\alpha\beta}^d + \frac{1}{t_c} \left( \frac{2t_c + t_f^t + t_f^b}{4} - u_3^d \right) N_{\alpha 3,\alpha}^c - \frac{2}{t_c} u_{3,\alpha}^d N_{\alpha 3}^c \\ & - \left( \frac{t_f^t \rho^t + t_f^b \rho^b + t_c \rho^c}{2} \right) \ddot{u}_3^a - \left( \frac{t_f^t \rho^t - t_f^b \rho^b}{2} \right) \ddot{u}_3^d - \left( \frac{C^t + C^b}{2} + C^c \right) \dot{u}_3^a \\ & - \left( \frac{C^t - C^b}{2} \right) \dot{u}_3^d + \frac{\hat{q}_3^t + \hat{q}_3^b}{2} = 0 \end{aligned}$$

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$$\begin{aligned}
 \delta u_3^d : & \quad u_{3,\alpha\beta}^a N_{\alpha\beta}^d + M_{\alpha\beta,\alpha\beta}^d + u_{3,\alpha\beta}^d N_{\alpha\beta}^a + \left(1 - \frac{2}{t_c} u_3^d\right) N_{33}^c + \left(\frac{t_f^t - t_f^b}{4t_c}\right) N_{\alpha 3,\alpha}^c \\
 & - \frac{1}{2} \left(t_f^t \rho^t + t_f^b \rho^b + \frac{t_c \rho^c}{3}\right) \ddot{u}_3^d - \left(\frac{t_f^t \rho^t - t_f^b \rho^b}{2}\right) \ddot{u}_3^a - \left(\frac{C^t + C^b}{2}\right) \dot{u}_3^d - \left(\frac{C^t - C^b}{2}\right) \dot{u}_3^a \\
 & + \frac{\hat{q}_3^t - \hat{q}_3^b}{2} = 0
 \end{aligned}$$

Where, the global stress resultants and stress couples are defined as

$$\left(N_{\alpha\beta}^a, M_{\alpha\beta}^a\right) = \frac{1}{2} \left\{ \left(N_{\alpha\beta}^t + N_{\alpha\beta}^b\right), \left(M_{\alpha\beta}^t + M_{\alpha\beta}^b\right) \right\}$$

$$\left(N_{\alpha\beta}^d, M_{\alpha\beta}^d\right) = \frac{1}{2} \left\{ \left(N_{\alpha\beta}^t - N_{\alpha\beta}^b\right), \left(M_{\alpha\beta}^t - M_{\alpha\beta}^b\right) \right\}$$

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and the local stress resultants and stress couples are given as:

$$\{N_{\alpha\beta}^t, M_{\alpha\beta}^t\} = \int_{-t_c/2-t_f^t}^{-t_c/2} \tau_{\alpha\beta}^t \left\{ 1, \left( x_3 + \frac{t_c + t_f^t}{2} \right) \right\} dx_3$$

$$\{N_{\alpha\beta}^b, M_{\alpha\beta}^b\} = \int_{t_c/2}^{t_c/2+t_f^b} \tau_{\alpha\beta}^b \left\{ 1, \left( x_3 - \frac{t_c + t_f^b}{2} \right) \right\} dx_3$$

$$\{N_{i3}^c, M_{i3}^c\} = \int_{-t_c/2}^{-t_c/2} \tau_{i3}^c(1, x_3) dx_3, \quad (i = 1, 2, 3)$$

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## Boundary Conditions

For the case of *simply supported boundary conditions*, the boundary conditions become:  
Along the edges  $x_n = (0, L_n)$

$$N_{nn}^a = N_{nn}^d = N_{nt}^a = N_{nt}^d = M_{nn}^a = M_{nn}^d = u_3^a = u_3^d = 0$$

$n$  and  $t$  are the normal and tangential directions to the boundary. When  $n = 1, t = 2$   
and when  $n = 2, t = 1$

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Solving the governing equations, results in two nonlinear coupled second order ordinary differential equations in terms of the modal amplitudes. These are given as:

$$m_1 \ddot{w}_{mn}^a + C \dot{w}_{mn}^a + C_{10}^a w_{mn}^a + C_{11}^a w_{mn}^a w_{mn}^d + C_{12}^a w_{mn}^a (w_{mn}^d)^2 + C_{30}^a (w_{mn}^a)^3 = \frac{q_{mn}}{2}$$

$$m_2 \ddot{w}_{mn}^d + C \dot{w}_{mn}^d + C_{01}^d w_{mn}^d + C_{02}^d (w_{mn}^d)^2 + C_{03}^d (w_{mn}^d)^3 + C_{20}^d (w_{mn}^a)^2 + C_{21}^d (w_{mn}^a)^2 w_{mn}^d = \frac{q_{mn}}{2}$$

The coefficients  $C_{10} - C_{12}, C_{30}, C_{01} - C_{03}, C_{20}, C_{21}$  are expressions which depend on the material and geometrical properties of the structure.

These two governing differential equations are then solved using the 4th Order runge-Kutta Method.

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

## Blast Loading

For a free in-air spherical air burst, the pressure profile over time is given in figure 4 as

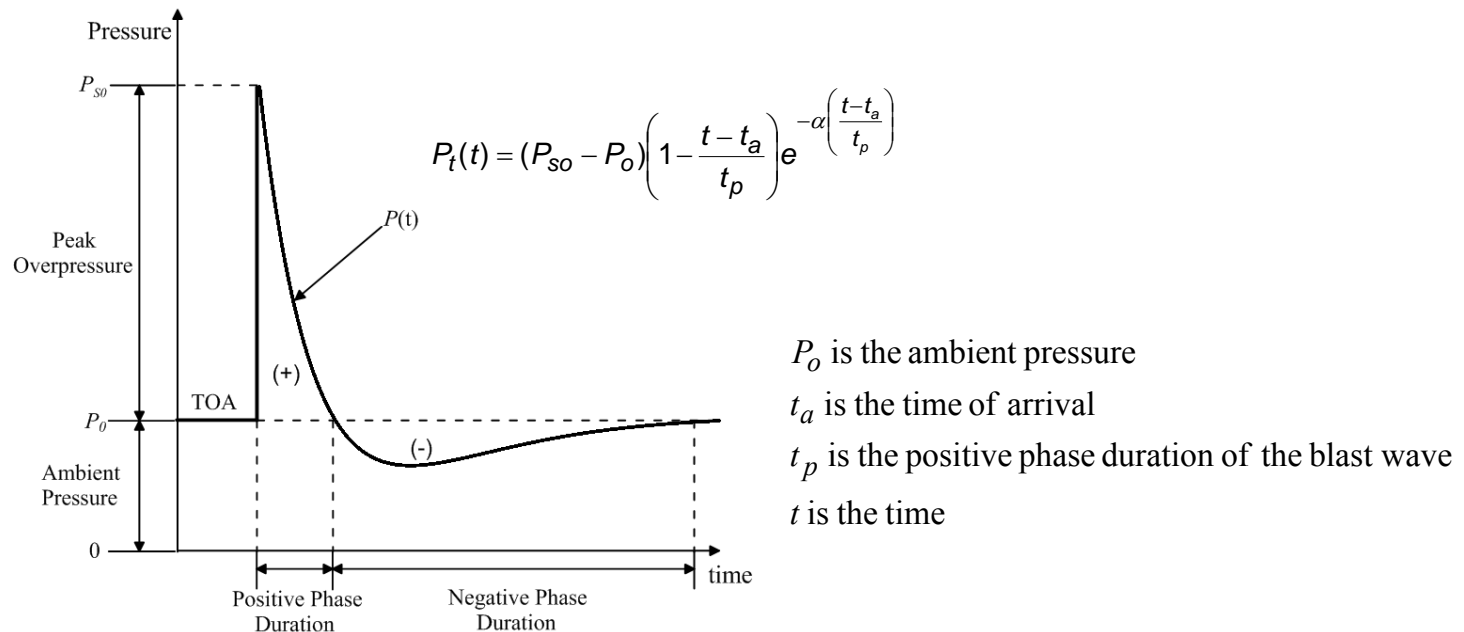


Fig 2. Incident Profile of a blast wave

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

## Results-Validation

To validate the present approach, the dynamic response of a simply supported plate impacted by a uniform pressure pulse was chosen from R.S. Alwar et Al.

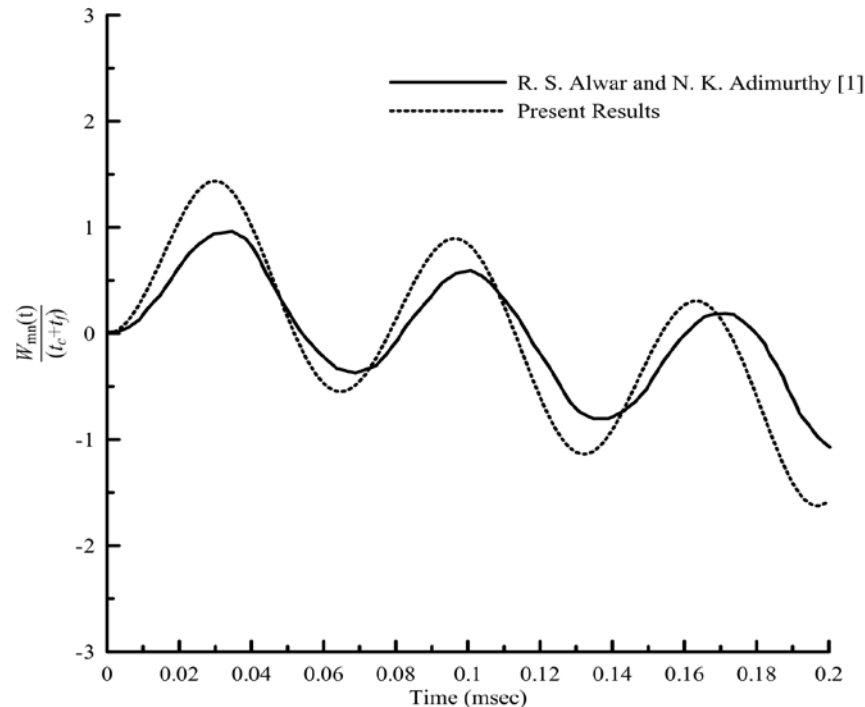


Fig 3. The non-dimensional global deflection-time response of a simply supported sandwich plate impacted by a uniform pressure pulse

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

## Results-Present

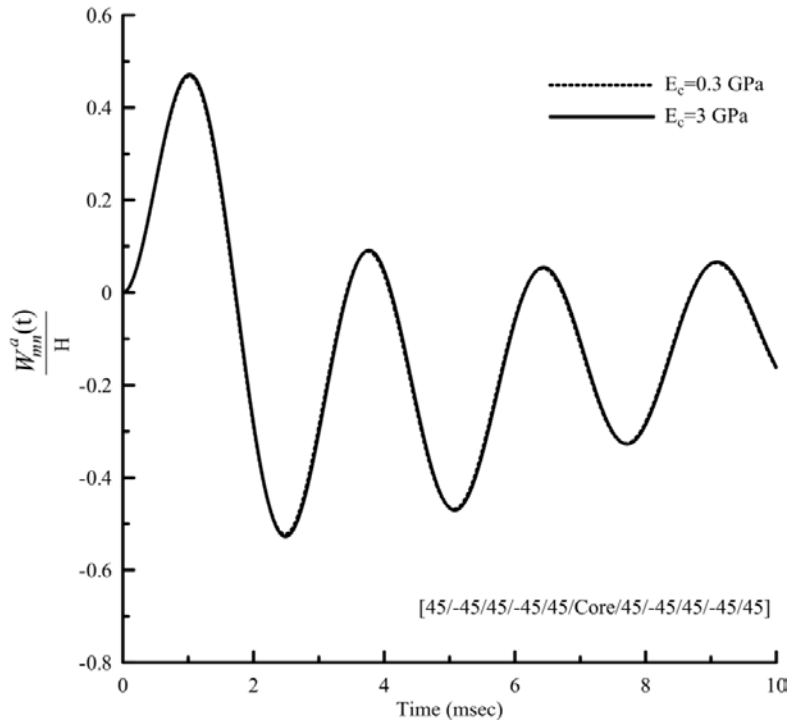


Fig. 4 The effect of the transverse modulus of the core on the global response of a sandwich plate with a fixed fiber orientation and stacking sequence.

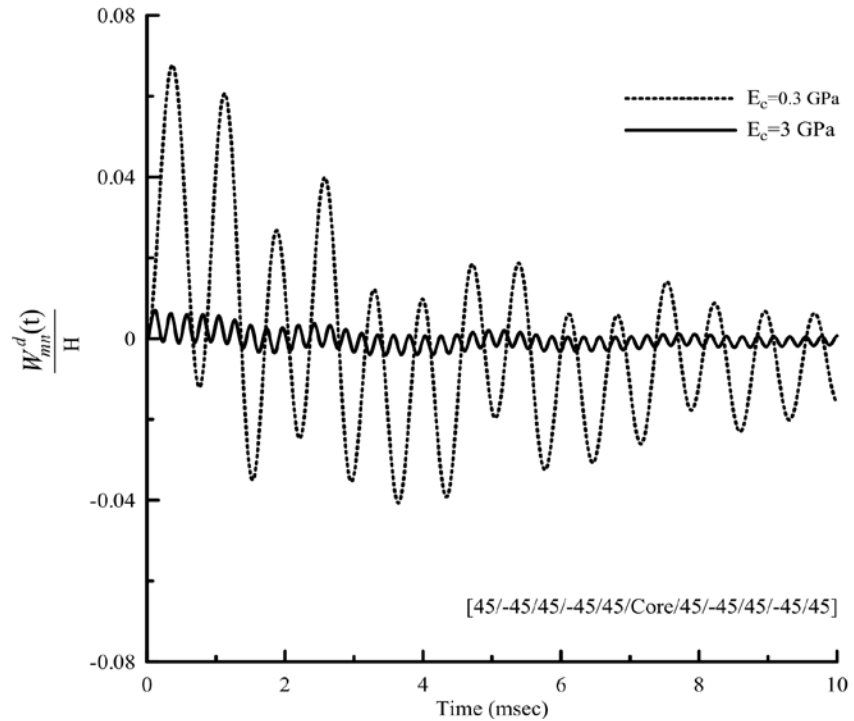


Figure 5 - The counterpart of Figure 4 for the wrinkling response of a sandwich plate.

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

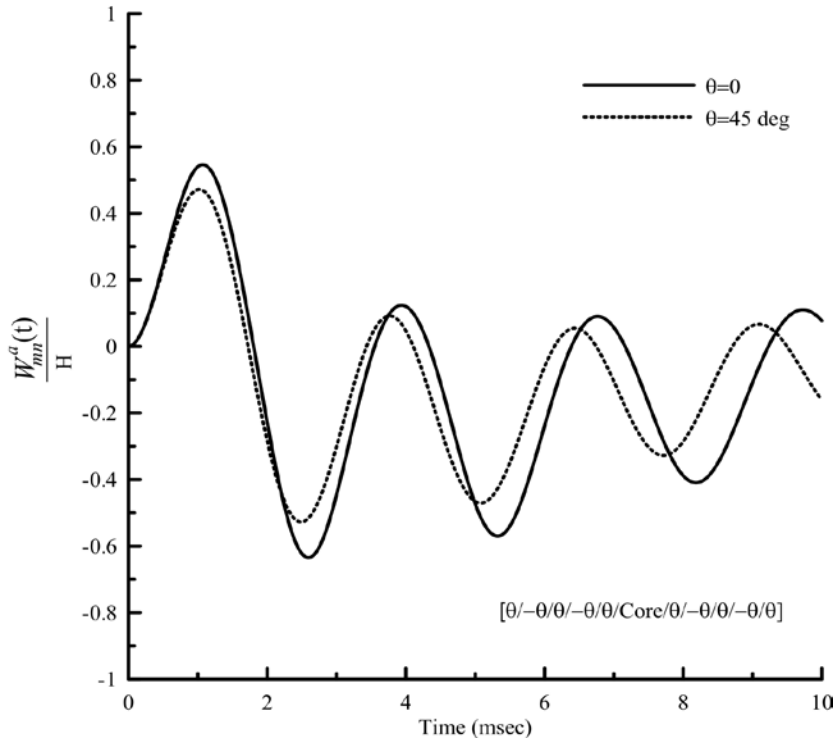


Fig 6. The effect of the fiber orientation and stacking sequence of the facings on the global response of a sandwich plate.

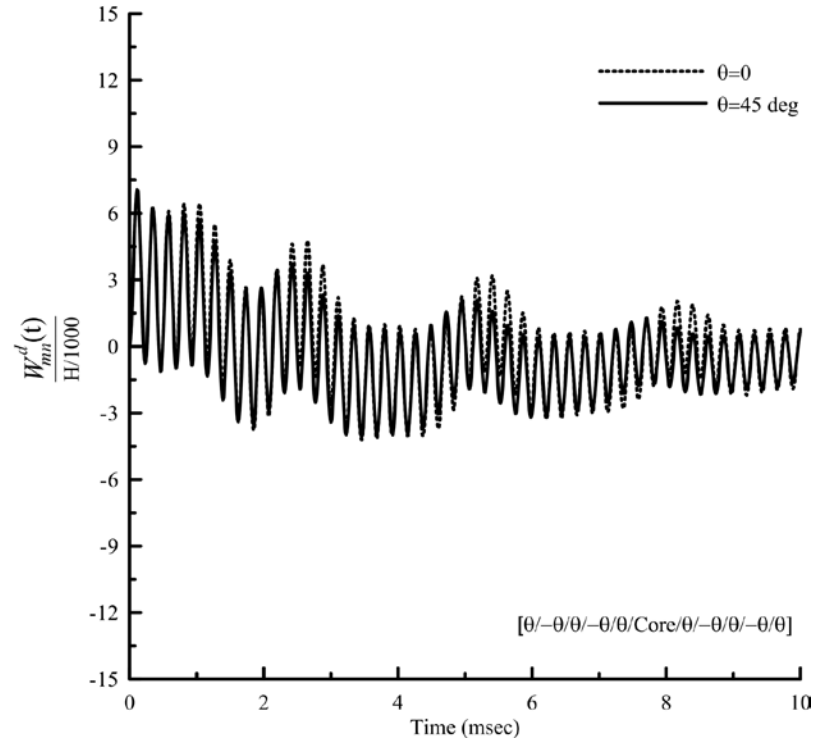


Figure 7. The counterpart of Figure 6 for the wrinkling response of a sandwich plate.

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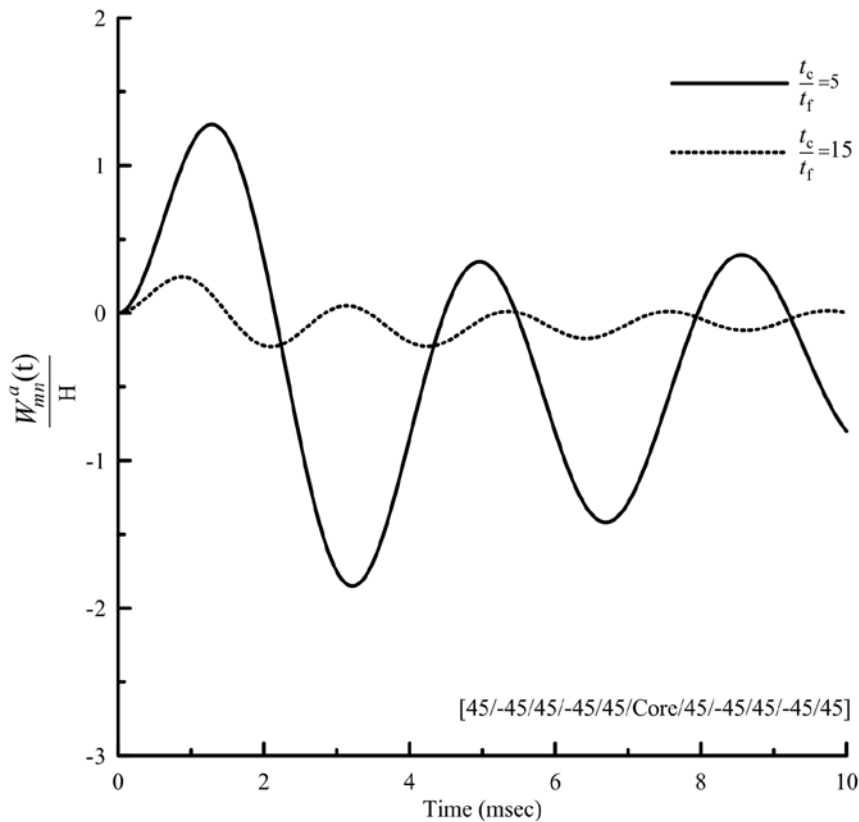


Fig 8. The effect of the core-facing thickness ratio on the global deflection-time history of a sandwich plate with a fixed fiber orientation and stacking sequence

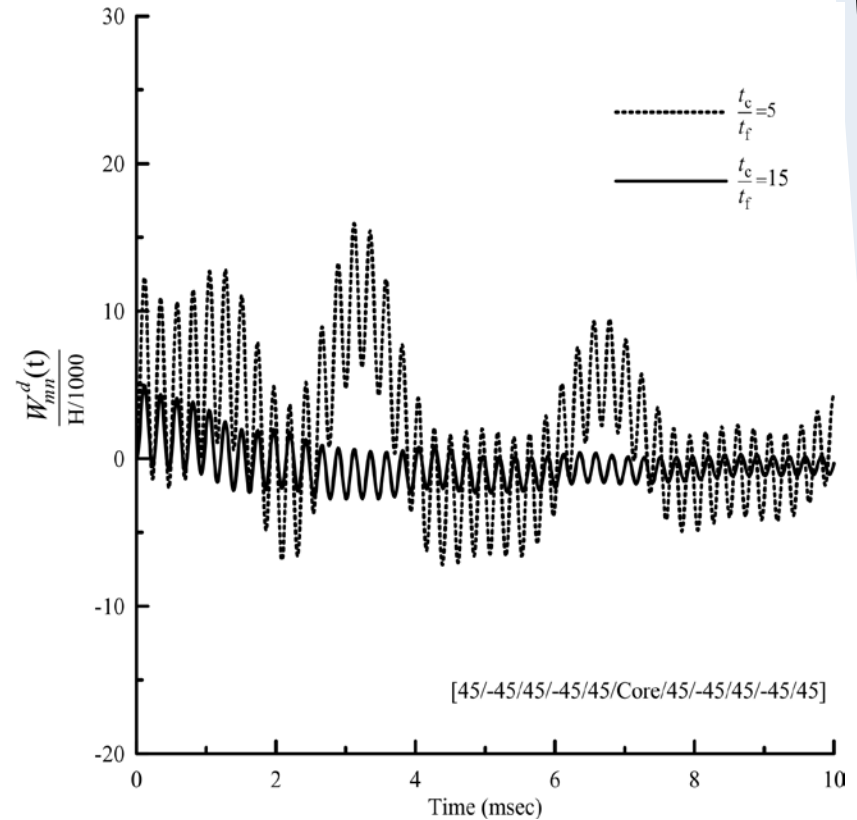


Fig 9. The counterpart of Figure 8 for the wrinkling response.

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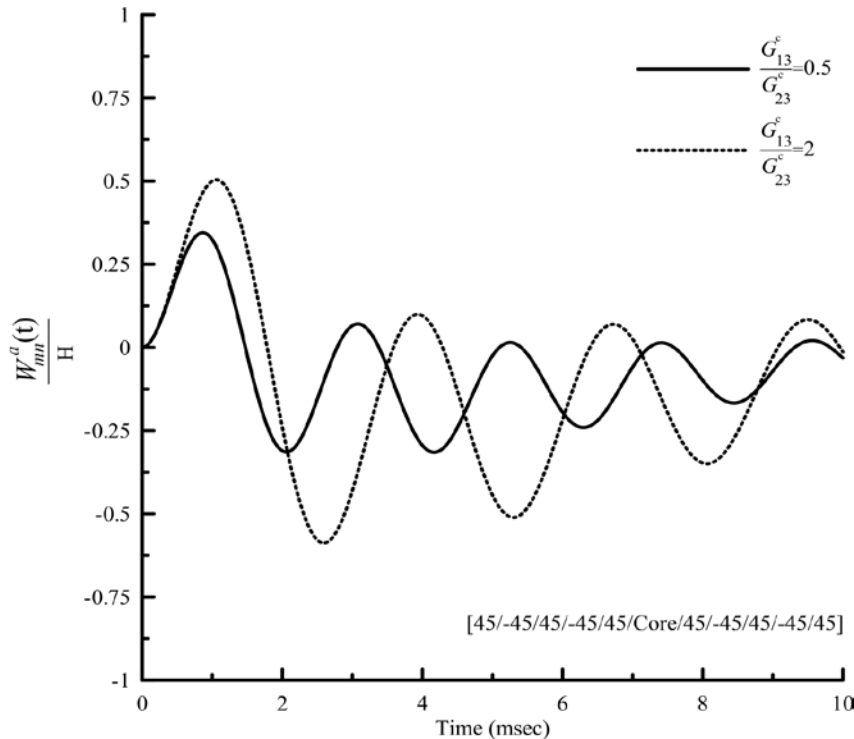


Figure 10. The effect of the core shear modulus ratio on the deflection-time history of angle-ply laminated sandwich plate.

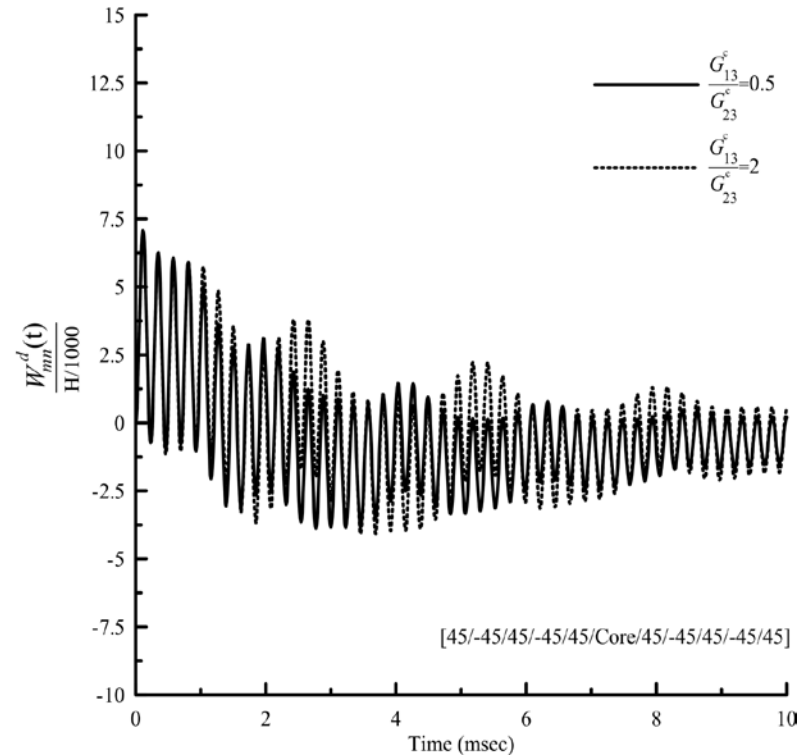


Fig 11. The counterpart of Figure 10 for the wrinkling response.

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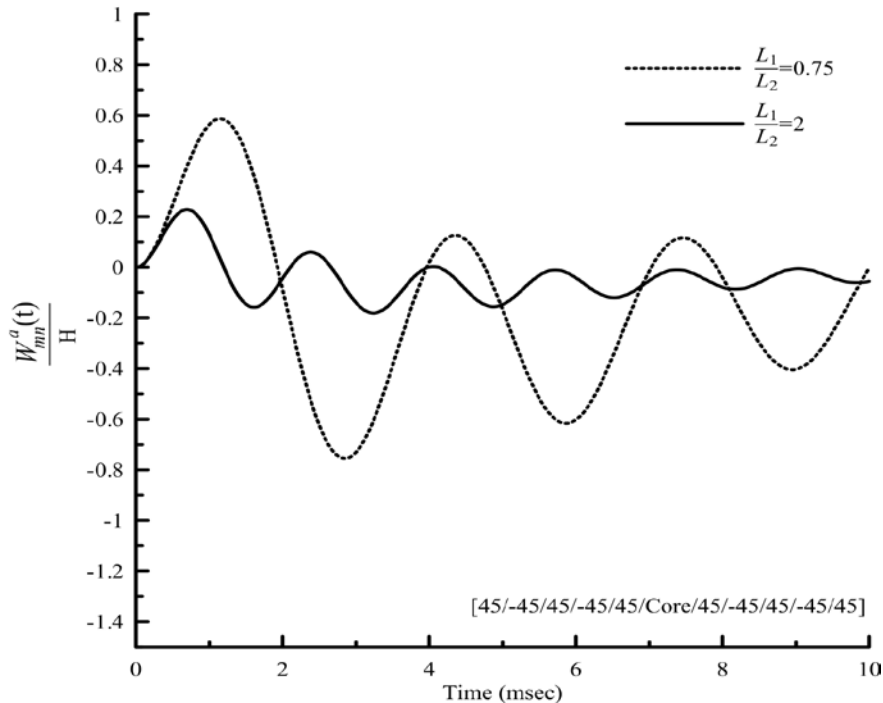


Fig 12. The effect of the aspect ratio on the global response of a sandwich plate.

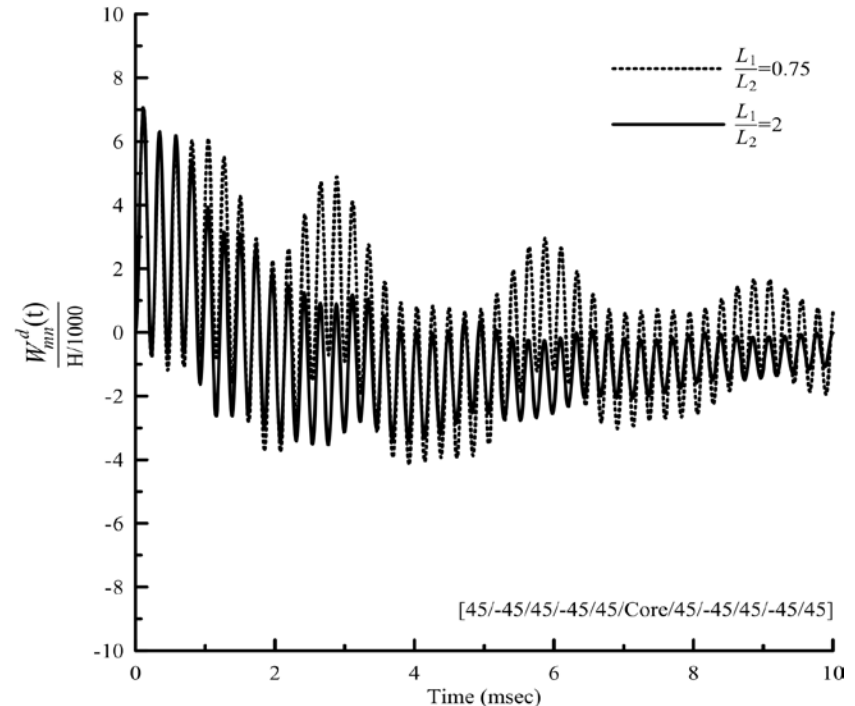


Fig 13. The counterpart of Figure 12 for the wrinkling response.

# Sandwich Plates With A Compressible Core Impacted by Blast Loading

## Concluding Remarks

The effect of a number of important geometrical and material parameters were analyzed with conclusions drawn.

Some of the important conclusions were :

- the wrinkling response seems to be diminished as the young's modulus of the core is increased. The same is the case for larger rates of decay.
- For thicker cores, both the global and wrinkling responses are less severe.
- It was also revealed that the compressibility of the core has only a marginal effect upon the global response of the sandwich plate.
- One should keep in mind that both the stress and strain profiles should be determined to determine possible failure of the structure.