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IMPROVED CORRELATION FOR BLOWOUT OF BLUFF-BODY STABILIZED FLAMES (PREPRINT)

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Improved Correlation for Blowout of Bluff-Body Stabilized Flames

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The lean extinction limit is one measure of the stability of combustion systems. Over the past 60 years, many papers have been written on the subject of extinction of bluff-body flame holders. Early in the study of this subject, numerous experiments were conducted over a range of flame holders, pressures, temperatures, and fuels. The authors typically attempted to derive empirical correlations for the lean limit as a function of global conditions that appeared to have arbitrary exponents. In general, these authors concluded that the extinction appeared to be some function of Damköhler number. More recently, with the advent of high-speed diagnostics and computers, new observations concerning the extinction process have been made, with the most general conclusion being that the extinction process is a wake phenomenon, where the flame is highly strained and dominated by large vortices. In the present paper a new correlation for lean extinction is derived using a linear least-squares fit and more than 800 data points from historical and current experiments. Fits of various dimensionless parameters are made, but the best fit is that of a Damköhler number with ignition delay as the chemical time scale, verifying many previous conclusions. Finally, it is concluded that flame-holder size--not shape--is the driving parameter that represents the flame-holder geometry.

I. Introduction

Flame stabilization by a bluff body is commonly employed in combustion systems, including industrial boilers, gas turbine engines, and scramjet engines.¹⁻³ Bluff bodies are used in these applications where high-speed flows would not normally stabilize a flame and where control of the location, spread, and efficiency are desirable. Stable flames facilitate robust operation of aero gas turbine engines. For power-generation turbines and industrial boilers, low pollutant emissions are essential. To achieve low NO_x emissions, these systems often operate near the lean limit. Understanding blowout in these systems is key to long-term operation and profitability of the device while meeting emissions standards.

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A. Traditional Extinction Research

For the past 60 years, bluff-body stabilized flames have been studied in great detail. In 1950, DeZubay conducted seminal experiments on disk-shaped flame holders⁴ over a range of pressures and inlet velocities relevant to the propulsion systems of the time. DeZubay studied a range of Reynolds numbers from 90,000 to 680,000. In Fig. 1 the extinction equivalence ratio is plotted as function of a correlation parameter derived by DeZubay.

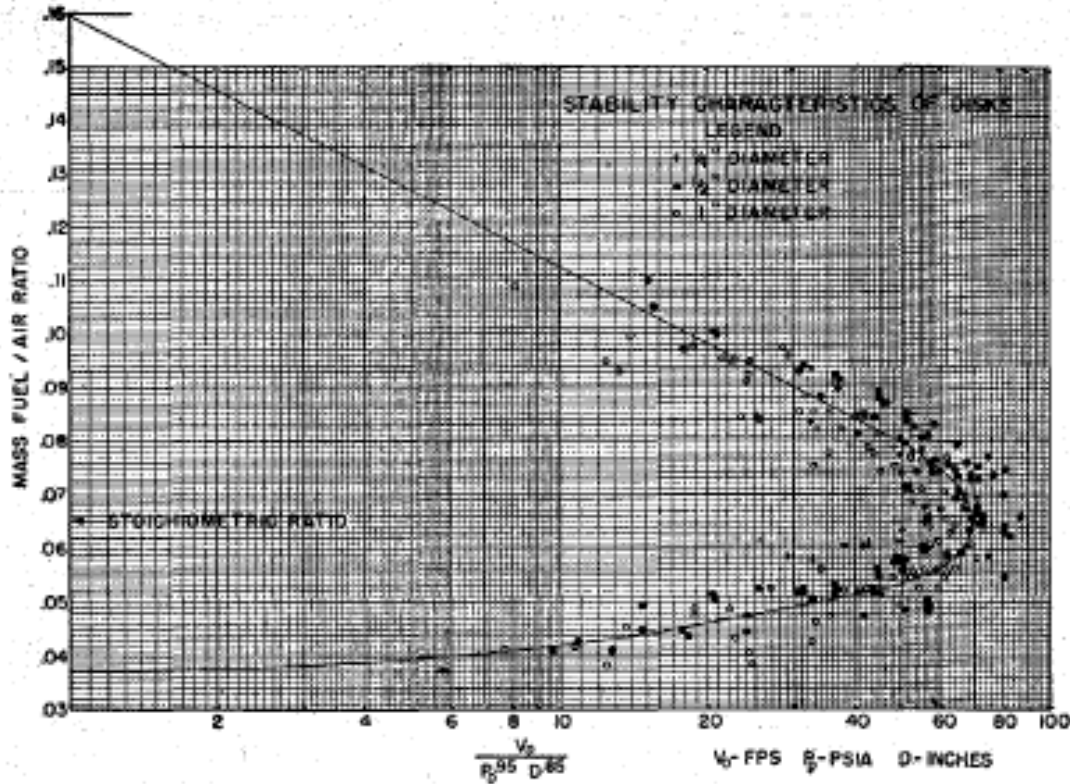


Figure 1. Blowout correlation of DeZubay.

Equation 1 is the formula for the original DeZubay correlation. In the correlation, the values for pressure (in psi) and velocity (in ft per s) are the “global” or inlet values. The length scale (in in.) is the diameter of the disk. Exponents near unity indicate a strong correlation with the parameter. The curve fit proposed by DeZubay fits only the lean side of the curve and fails for “DeZubay numbers” greater than 50. The method that DeZubay used to estimate the exponents for the fit is unclear from his paper.

$$DeZ = \frac{U}{P^{0.95} D^{0.85}} \quad (1)$$

The inlet temperature throughout DeZubay’s experiment was ambient; therefore, his original correlation does not include the inlet temperature of the rig. A modified form of the DeZubay correlation parameter that includes temperature dependence is, thus, more widely used than the original parameter:

$$DeZ_T = \frac{U * 10^4}{P^{0.95} D^{0.85} T^{1.2}} \quad (2)$$

Here T is the inlet temperature in Rankine. Although unpublished, this form of the DeZubay correlation parameter appears to fit the temperature-dependent data well for a DeZubay number below 200. Following DeZubay’s paper were several other papers of note, including those of Nakanishi et al. (1953), Coward and Jones (1952), Henzel and

Bryant (1954), Prince et al. (1956), and King (1957).⁵⁻⁹ These papers provide data over a wide range of pressures, temperatures, velocities, and geometries. Some of these data were used in the current work to derive an improved lean-blowout correlation.

Zukoski was the next author to document significant findings, which he published in his thesis in 1954 and in a paper with Marble in 1955.^{10,11} Zukoski and Marble discuss the similarities of extinction events and put forward hypotheses on the physical significance of these similarities. Their intent was to identify those parameters that influenced extinction and those that were not sensitive. They found that the relevant parameters could be used to derive a simple formula for extinction and concluded that parameters describing the fluid mechanics and chemical reactions were separate competing parameters. They discuss the dynamics of stable flames and define a “critical ignition delay time” that must be less than the residence time of the hot wake. From the competition between the fluid mechanic and chemical reaction time scales, they define a Damköhler-like correlation parameter, where the chemistry time scale is defined as an ignition delay:

$$Da_{ign} = \frac{D}{U\tau_{ign}}, \quad (3)$$

Ozawa (1971) authored an excellent review of the early work in lean extinction.¹² He drew some interesting conclusions and cited pressure, temperature, velocity, geometry, fuel type, and heat loss as key parameters for bluff-body stability/blowout. However, Ozawa’s paper is ITAR controlled, and more detailed findings cannot be reported here.

Ballal and Lefebvre (1979) conducted a more contemporary investigation of bluff-body flames.¹³ They studied the influence of many global parameters, including temperature, pressure, velocity, turbulence level, flame-holder size, and flame-holder blockage ratio, on what they termed “weak extinction.” From their research they also proposed a correlation for various global parameters; Eq. (4) is that correlation.

$$\phi_{LBO} = \left[\frac{2.25(1 + 0.4U(1 + 0.1Tu))}{p^{0.25}T\left(e^{\frac{T}{150}}\right)D(1 - B)} \right]^{0.16} \quad (4)$$

In the correlation, U is the velocity, Tu the turbulence intensity (%), T the temperature, p the pressure, and B the dimensionless blockage. All units are MKS. Ballal and Lefebvre concluded that a primary influence on the weak extinction limits was inlet temperature, with lesser influences being air velocity and turbulence level. Despite experimental results which indicated that pressure does not strongly affect the weak extinction limit, the pressure exponent in the correlation is 0.25. However, Ballal and Lefebvre concluded that the weak extinction limit was independent of pressure.

Also in 1979, Plee and Mellor conducted an analysis of characteristic time scales and their influence on the lean extinction of bluff bodies.¹⁴ In their research they defined time scales associated with turbulent mixing, homogeneous chemical kinetics, liquid-droplet evaporation, and fuel injection. They then studied the effect of these time scales on the lean extinction of three different flame holders, one being similar to that of Ballal and Lefebvre. They found a linear correlation with pressure, inlet temperature, inlet velocity, flame-holder geometry, fuel type, and injector size. Like Zukoski and Marble, they concluded that extinction was a competition between a fluid mechanic and a chemical time scale.

In 1988 Stwalley and Lefebvre conducted research on “irregular”-shaped flame holders.¹⁵ In this research, slots and notches were cut in the flame holders to affect vortex shedding in their wake. This research is similar to that recently conducted by Kim et al.^{16,17} Stwalley and Lefebvre found that unless the flame-holder treatment had a significant influence on the wake of the bluff body, no appreciable impact on the extinction was observed. Kim et al. drew similar conclusions for uncarborated configurations where the overall width of the flame holder was not changed. The primary conclusion of Stwalley and Lefebvre was that flame-holder size was the most important parameter in determining aerodynamic blockage and that the shape of the flame holder played “a very minor role.”

Most recently in collaborative research with the Air Force Research Laboratory, Knaus et al. and Roach et al.^{18, 19} explored the impact of local conditions in the wake on flame-holder extinction. In this research the isothermal Fluent LES solution was used to predict local hydrodynamics. The assumption was that the isothermal and

combusting hydrodynamics were similar at the lean extinction. Local chemical time scales were predicted using the extinction strain rate from a single-step global chemistry calculation with CHEMKIN. The tool predicted local Damköhler number as a ratio of these quantities. The chemistry simulation was progressed in decreasing equivalence ratio, and the local Damköhler number was evaluated over the entire domain. In the simulation when a sufficient portion of the computational region was above a certain threshold, the user determined that the bluff body was blown out. The value of predicting local Damköhler number as opposed to simply calculating the Damköhler number using a global correlation was unclear from this research.

B. Contemporary Extinction Research

Over the last decade many papers have discussed the dynamics of bluff-body flames and their impact on vortex shedding in the wake of the flame holder. With the advent of fast computers and high-speed cameras, the physical understanding of these flames has improved significantly. Bluff-body stabilized flames can be characterized in two ways. For conditions where the flame is stable, at high equivalence ratios, the flame is stabilized in the shear layer that is formed on the upstream surface of the flame holder. These shear layers provide substantial turbulent mixing of reactants and products and, under certain conditions, hold a flame. The bluff body also provides a wake or recirculation zone in which reactants and hot products can mix at relatively low velocity and long residence times as compared to the shear layer of the bluff body.

The dynamics of the near-blowout flame have been disputed for more than 50 years. As early as 1951, Nichols and Field described large-scale pulsations in the bluff-body wake when the equivalence ratio was near the extinction limit.²⁰ In 1991, Hertzberg et al. made high-speed velocity measurements of vortex shedding in the wake and found “discrete peaks” in the spectrum of the data near blow off.²¹

More recently Mehta and Soteriou (2003) and Erickson et al. (2006) commented on vortex shedding as it relates to bluff-body flame blow-out.^{22, 23} In their work they conducted detailed modeling of bluff-body stabilized flames. In their 2003 study they modeled a bluff-body flame at a Reynolds number of 20,000 and found that the flame was dominated not by large Karman-Street vortices but by much smaller vortices. They concluded that the baroclinic effect of the temperature rise across the flame suppresses the Karman-Street-type vortex shedding that is typically observed behind these bluff bodies under non-combusting conditions. In 2006 they conducted another model study in which the temperature rise across the flame was varied. They concluded that at lower temperature ratios across the flame, $T_{\text{burned}}/T_{\text{unburned}} < \sim 2$, the flame was dominated not by small turbulent vortices but by large Karman-Street-type vortices. These same structures were also captured by Porumbel and Menon (2006) and Fureby (2006) in their combusting Large Eddy Simulation (LES) investigations.^{24, 25}

Ongoing research is being conducted by Tim Lieuwen at the Georgia Institute of Technology.^{3, 26, 27} In 2003 he reported the results of research on precursors to extinction. Acoustic characterization of near-blowout bluff-body flames was conducted, and appreciable increases in acoustic events prior to blowout were found. In 2007 experiments were conducted at low Reynolds numbers, and two stages of blowout were reported. The first is the onset of “holes” in the flame where the flame is highly stretched as the equivalence ratio decreases. As the equivalence ratio continues to decrease, large-scale structures are observed to be emanating from the wake and the flame front in the shear layer begins to “flap,” resembling isothermal wake shedding. The authors hypothesize that the large-scale motion brings relatively cold reactants into the wake region, which then fails to light, and the flame finally extinguishes. They compare these observations to those of Zukoski and conclude, unlike Zukoski, that the extinction event appears to be a local phenomenon.

They then attempt to develop a local or wake Damköhler number. They compile blowout data available in the literature and recast the data in terms of a Damköhler number. Lip conditions and the boundary-layer momentum thickness of the shear layer are used in the developed parameter. Momentum thickness is derived from the data and is represented by:

$$\frac{\theta}{D} = \frac{35}{\sqrt{\text{Re}}} \quad (5)$$

The chemical time scale in the parameter is calculated using a Perfectly Stirred Reactor (PSR) model. The minimum residence time required for the PSR to realize complete combustion is used as the kinetic time scale.

In our research,^{28,29} we agree with the findings previously discussed. We also assert that a substantial change in the wake occurs as the extinction limit is approached. We conclude that the vortex dynamics-- and not the geometry--is the dominant mechanism for bluff-body flame extinction. This conclusion is supported by the lean blowout data, by the high-speed images, and by reference data from NACA.

El-Asrag et al. (2008) also examined the idea of the Damköhler number being the proper parameter for extinction³⁰ and modeled three flames with the same “characteristic Damköhler number.” They showed that the wake flame controls stability, and their model exhibits the asymmetric von Karman vortex shedding observed in other studies. They hypothesize that the flame holder has two stability mechanisms. First, the wake provides sufficient residence time and chemical heat release to ignite new reactants. Second, the shear layer generated around the wake provides a turbulent mechanism to mix the fresh air with the combustion products from the recirculation zone.

Researchers at the University of Connecticut are also studying the extinction of bluff-body flames.^{31, 32} They have drawn conclusions similar to those reported previously. They add to the discussion their observation that the local extinction appears to move closer to the wake as global extinction is approached and the reaction zone shortens. They cite blowout of the shear layer and wake stabilization as key precursors to extinction.

Driscoll and Rasmussen (2005) studied non-premixed bluff-body flames.³³ They developed a Damköhler-like scaling parameter for their experiment and six previous experiments. The research included experiments on non-premixed flames that were stabilized in wall cavities, bluff bodies, and struts with direct injection into the wake zone. They also identified a Damköhler number that is different from those proposed by Zukoski and Ozawa. Non-premixed conditions introduce a different flame competition. They report that extinction for non-premixed flames is controlled by the imbalance between the flame-propagation speed and the gas velocity. The important region for this comparison is the point where the non-premixed fuel and air is stoichiometric. From this (I feel that a word is missing here) they develop their parameter.

C. Objective of Current Research

Although in previous studies correlations have been developed for lean blowout, a definitive correlation has not been established. In DeZubay’s original work the inlet temperature of the rig did not vary; thus, a modification of his parameter was required. In addition, DeZubay’s correlation results in a large spread in the blowout prediction when applied to the data collected for the current study, as shown in Fig. 2.

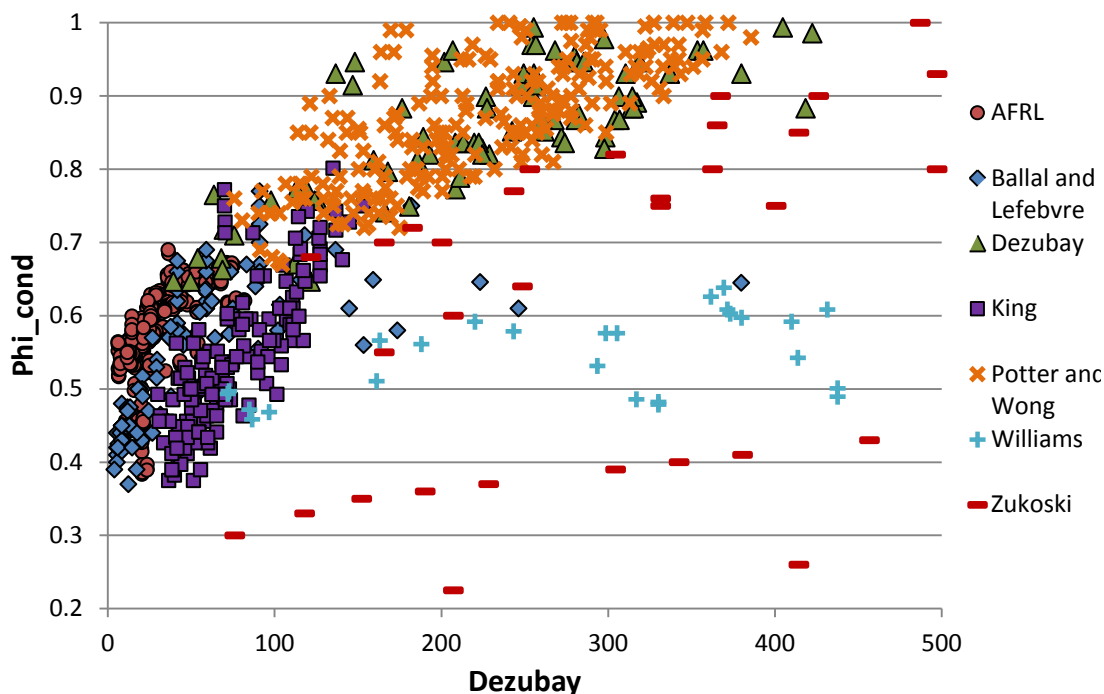


Figure 2. DeZubay correlation with data collected for current study.

The Ballal and Lefebvre paper¹³ contains conflicting conclusions on the effect of pressure on lean blowout; some other researchers, such as King, studied the effects of individual parameters on blowout but did not develop a correlation for predicting blowout. In addition, no attempt is made in previous studies to non-dimensionalize the parameters.

The objective of the current research effort is twofold. The first goal is to provide detailed experimental data on bluff-body stabilized flames. From these data and existing experimental data, an improved extinction parameter can be derived and correlated. Discussions of the physics of the extinction will be used to evaluate several dimensionless quantities, which will then be correlated with the extinction data to determine whether they have a dominant or secondary effect on the extinction. The second goal is to enhance the phenomenological understanding of the bluff-body flame stabilization. A main assertion of the current work is that the changing flame speeds and ignition delay times near blowout may be the important factors that dictate extinction and that the extinction is a competition between chemistry and fluid dynamics. Vortex shedding typically changes from Kelvin-Helmholtz as the equivalence ratio decreases and may be an effect--not a cause--of extinction.

II. Experimental Setup

A. Experimental Rig

Experiments were carried out in an atmospheric-pressure combustion rig at the Air Force Research Laboratory at Wright-Patterson AFB. The facility can provide air-inlet temperatures up to 800R with electric heaters. The rig can also flow vitiated air at temperatures up to 1800R. Mach numbers of 0.4 and Reynolds numbers of 150,000 can be reached with the available air flow. Multiple fuels can be used in the facility, including propane and JP-8; for the experiment discussed in this paper, only gaseous propane was used. The propane is premixed upstream of the flame holder.

Figure 3 is a schematic of the test rig, with a v-gutter bluff body installed. The air flow is right to left. The cross section of the rig is 6 in. high by 5 in. wide. The flame-holder trailing edge is 23 in. from the inlet of the test section, and the flame holder spans the full height of the rig. A perforated plate 3 in. downstream of the inlet provides flow straightening and sets turbulence levels.

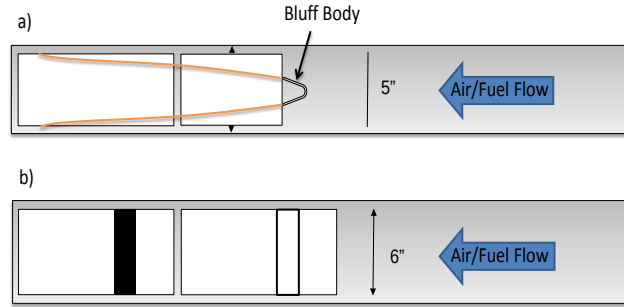


Figure 3. Schematic of bluff-body rig, showing optical access: a) top view b) side view.

B. Lean Blowouts and Correlation Setup

Lean-blowout measurements were made on four types of bluff bodies: various sizes of v-gutters, a cylinder, a plate, and a square cylinder. Figure 4 depicts the cross sections of these flame holders. Blowout was achieved by reaching the desired inlet conditions and decreasing the fuel flow until the flame had visibly blown out. A data point was then recorded. High-speed images and acoustic measurements were also recorded near blowout.

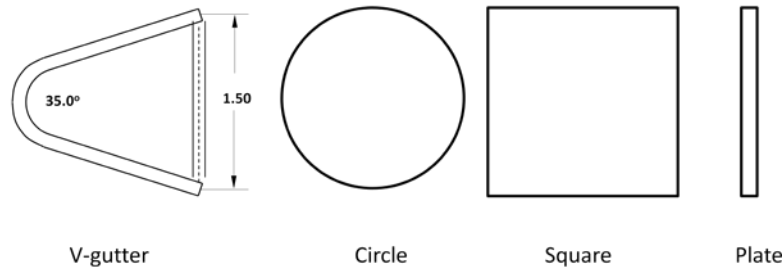


Figure 4. Various bluff-body flame holders studied.

The data points from these experiments were combined with those obtained from the publications of other authors, including Ballal and Lefebvre,¹³ DeZubay,⁴ King,⁹ Potter and Wong,³⁴ Williams,³⁵ Zukoski,¹⁰ and Chaudhuri et al.³⁶ Figure 5 is a summary of the experimental parameters from each study. An input file of more than 850 individual points was created that included the relevant parameters from each study such as inlet conditions and flame-holder characteristics. The least-squares curve fit function in the Matlab optimization toolbox, which

employs a trust-region reflective algorithm, was used to find an optimal correlation. The desired equation form was selected before running the correlation code.

Author(s)	Lip Velocity (ft/s)	Flame-holder Diameter (in)	Inlet Temp. (R)	Inlet Pressure (psia)	Reynolds Number	Fuel Type	Flame-holder Shape(s)
Huelskamp, et al.	21-171	0.375-1.5	523-1034	Atmospheric	4800-86,000	Propane	Open and closed v-gutter, cylinder, plate, square cylinder
Ballal and Lefebvre	34-497	0.8-4.92	540-1035	2.9-14.5	12,000-506,000	Propane	45° hollow cone
DeZubay	86-706	0.25-1	550	3-15	4200-345,000	Hopane	Disk
King	403-743	1.5	1260-1860	5.2-12.5	14,000-87,000	JP-4	V-gutter ring
Potter and Wong	79-765	0.375-1	540-550	3.7-13.8	7,300-260,000	Propane	Cylinder
Williams	48-390	0.016-0.498	540	14.7	400-86,000	City Gas	Cylinder
Zukoski	81-1206	0.01-0.25	610-860	14.7	200-57,000	Hydrogen, Methane, Gasoline	Cylinder
UConn	66-128	0.375	513-531	14.7	13,000-25,000	Propane	V-gutter

Figure 5. Summary of experimental parameters from data used in this work

C. Imaging Setup

Bluff-body flame chemiluminescence was captured using a high-speed Phantom v7.1 camera (Fig. 6), which has a monochrome CMOS detector that is capable of capturing 4800 frames-per-second (fps) for a full frame. In the current experimental setup, images were collected at 10,000 fps, with a maximum resolution of 496 x 344 pixels encompassing 165 mm x 102 mm. Chemiluminescence was collected through an 85-mm Nikkor lens with an f-stop of 1.4 and exposure times of $\sim 95 \mu\text{s}$. A polished stainless-steel mirror with an Al:MgF coating was placed at the top of the experimental rig to allow imaging of the axial plane of the flame. This axial plane represents an optical slice of the turbulent shear layers along which chemiluminescence caused by the flame fronts can represent the nature of the flame-shedding mechanism.

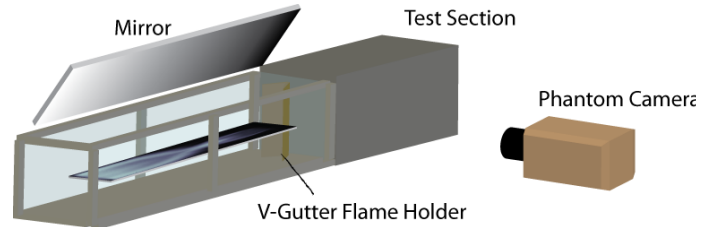


Figure 6. Side view of imaging setup using Phantom high-speed camera and aluminum mirror.

III. Correlation Results and Discussion

A. Correlation with U/D, P, and T

The correlation code was first run with the same parameters as those used in DeZubay's correlation: velocity, length scale, and inlet pressure and temperature. Figure 7 shows the correlation results with all of the available data. The R-squared value is 0.07, and the plot shows a number of outliers.

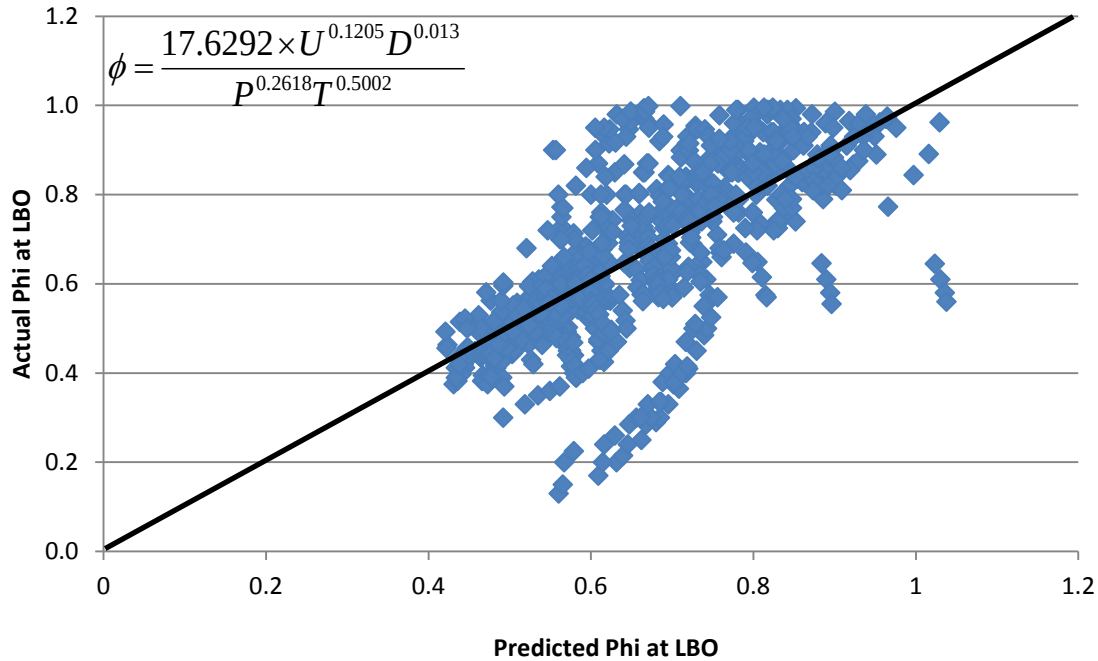


Figure 7. Correlation with all data, $R^2=0.07$

Some references (Zukoski) cite a Reynolds number of 10,000 as a transition point in the lean-blowout behavior, and many of the outliers in the first correlation were points taken at low Reynolds numbers. Figure 8 shows the correlation results with all points taken at Reynolds numbers of more than 10,000. The filtered dataset improved the R-squared value from 0.07 to 0.62.

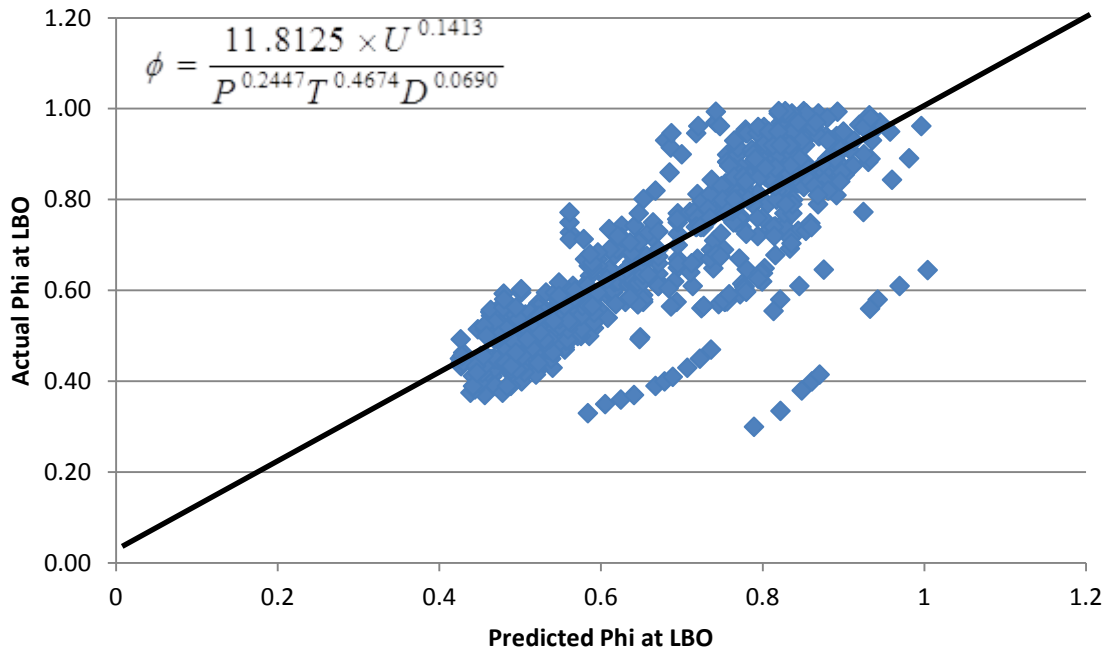


Figure 8. Correlation with data taken at Reynolds number of more than 10,000, $R^2=0.62$.

The dataset was then filtered again so that it contained only data points that were taken on flame holders with a diameter of 0.375 in. or greater. Some of the sources had tested flame holders that were much smaller, on the order

of 0.01in., but since these flame holders would not serve a practical purpose in modern combustion systems, these data were discarded. In addition, the smallest flame holder used for the experiments in the current work was 0.375 in. in diameter. Discarding the smallest diameter flame-holder data further improved the R-squared value to 0.85, with the results shown in Fig. 9.

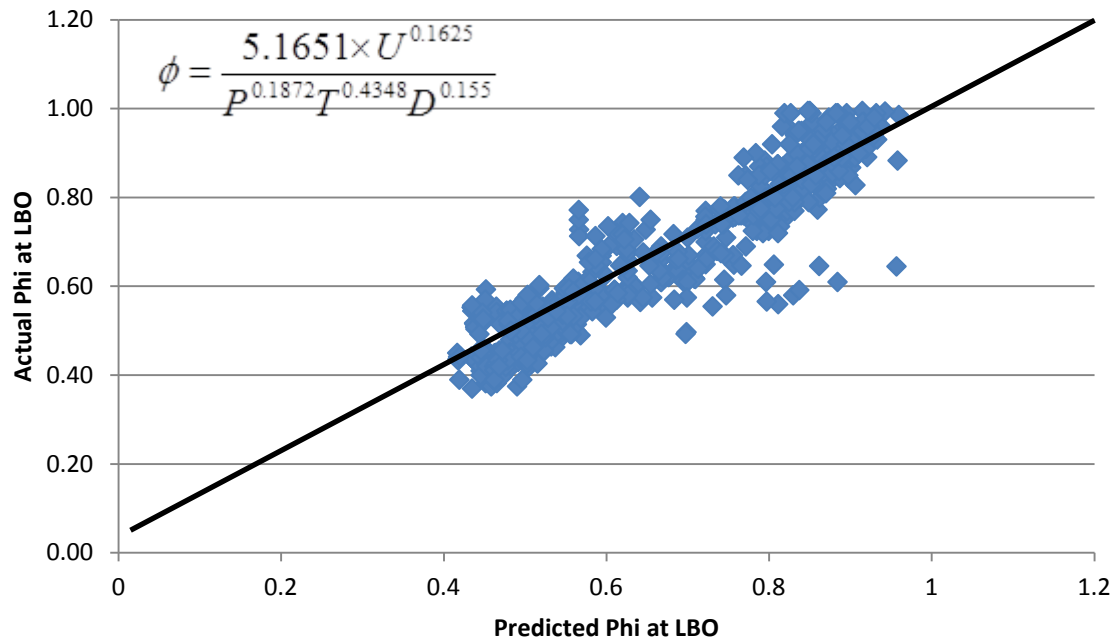


Figure 9. Data taken at Reynolds number of more than 10,000 and diameter of more than 0.375in,

The correlation was then run on a dataset that was filtered by diameter only, with the requirement remaining that the flame holder must have a width of 0.375 in. but allowing low-Reynolds-number data points (Fig. 10).

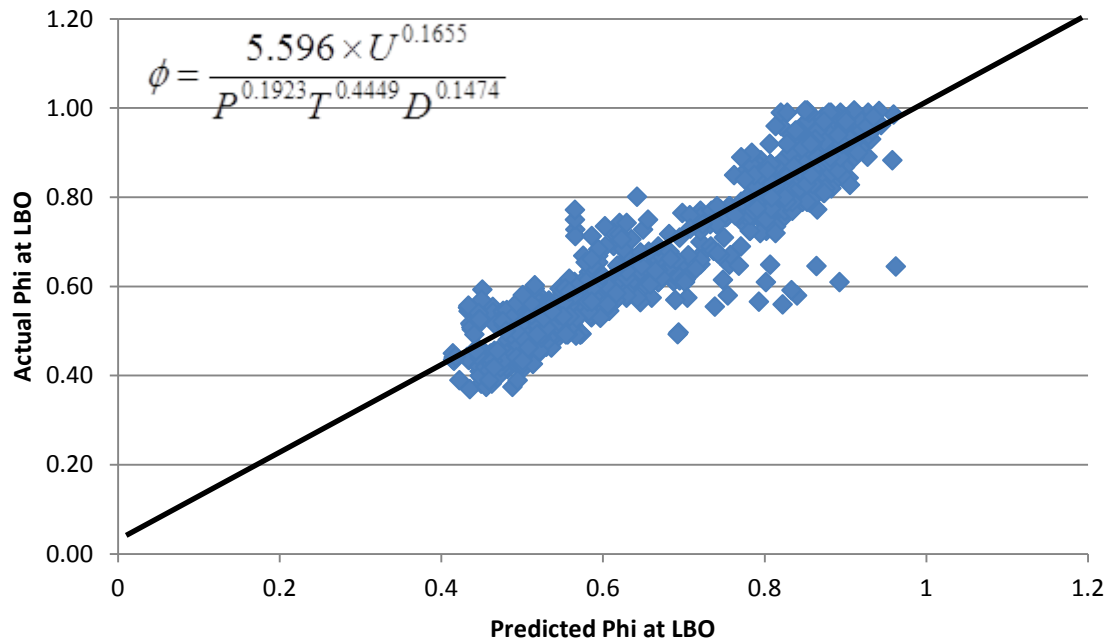


Figure 10. Correlation with data at diameter of more than 0.375 in., $R^2=0.85$.

This did not cause the correlation to deteriorate and provided a larger dataset. The low-Reynolds-number points were included in subsequent correlations. Some additional outliers were discarded because of an inability to corroborate the results of the source paper.

The exponents on the velocity and diameter parameters were very similar and were grouped together to form a single parameter, U/D . The resulting equation, with an R-squared value of 0.88, was:

$$\phi_{predicted} = \frac{5.0115 \times \left(\frac{U}{D}\right)^{0.1493}}{P^{0.2192} T^{0.4108}} \quad (6)$$

The results, separated by source, are shown in Fig. 11.

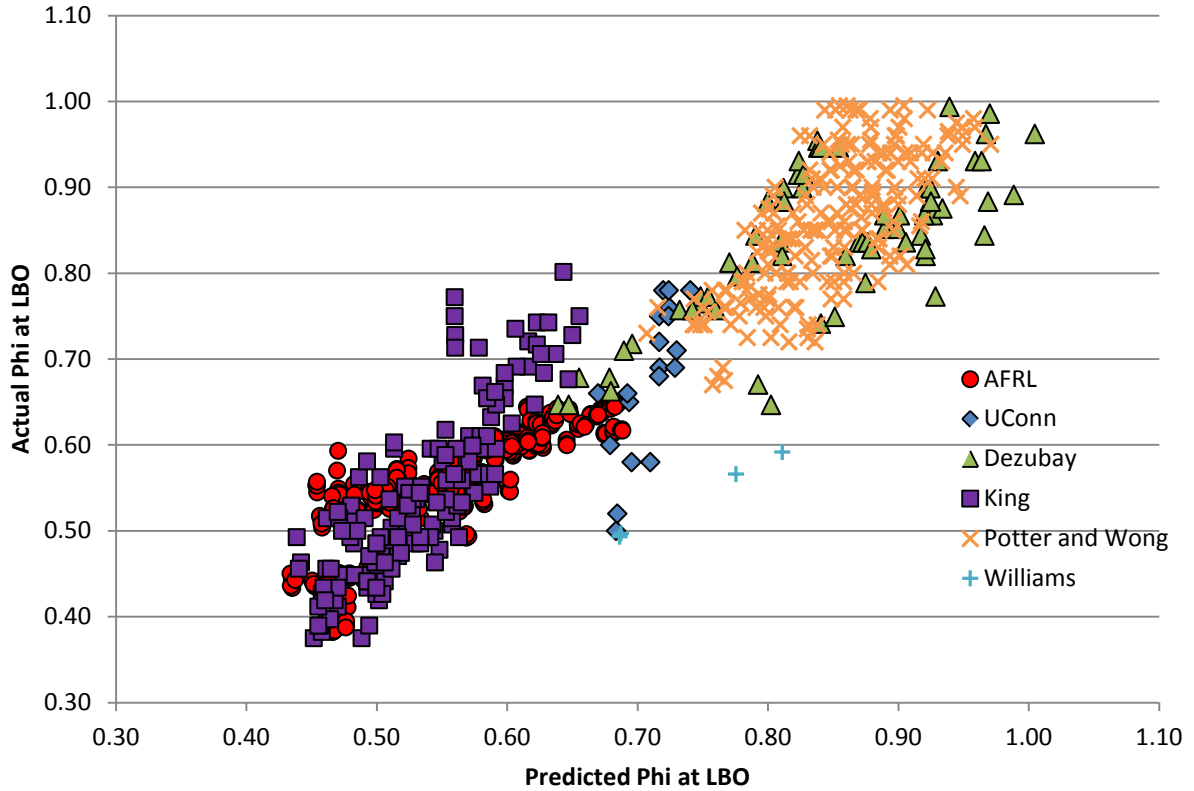


Figure 11. Correlation results for $D > 0.375$, with U/D as parameter, $R^2 = 0.88$.

B. Damköhler Number

The ratio of U/D was recognized as a fluid dynamic time scale. This led the study in the direction of a global Damköhler number. As discussed in the Introduction section, many of the early authors also found a correlation with Damköhler number, even though they did not have specific curve fits to a kinetics time scale available for comparison. For a bluff body to maintain a flame, the hot combustion products must be able to ignite the fresh reactants. If the ignition delay time is longer than the associated fluid dynamic time scale, the reactants will not be able to ignite, causing blowout. In addition, adequate flame speed is required to propagate the flame downstream of the flame holder. Equation 7 is the equation for laminar flame speed for propane at an equivalence ratio of 0.60 as a function of pressure and temperature and was derived using the chemical kinetics model of Gokulakrishnan et al. 2009. (I don't see a reference for this in the Reference Section)

$$U_l \approx 1e^{-6} P^{-0.22} T^{2.54} \quad (7)$$

A data set for the ignition delay time of heptane was derived using an n-heptane detailed mechanism, including vitiated flow. From that set of data, the ignition delay was curve fit for pressure and temperature in the region for temperatures above 1000K. The plot of these data is in Fig. 12. The curve fit past the NTC region, as shown in Eq. (8), is:

$$IDT \approx P^{-0.9404} e^{17235/T} \quad (8)$$

where pressure is in Pa and temperature in Kelvin.

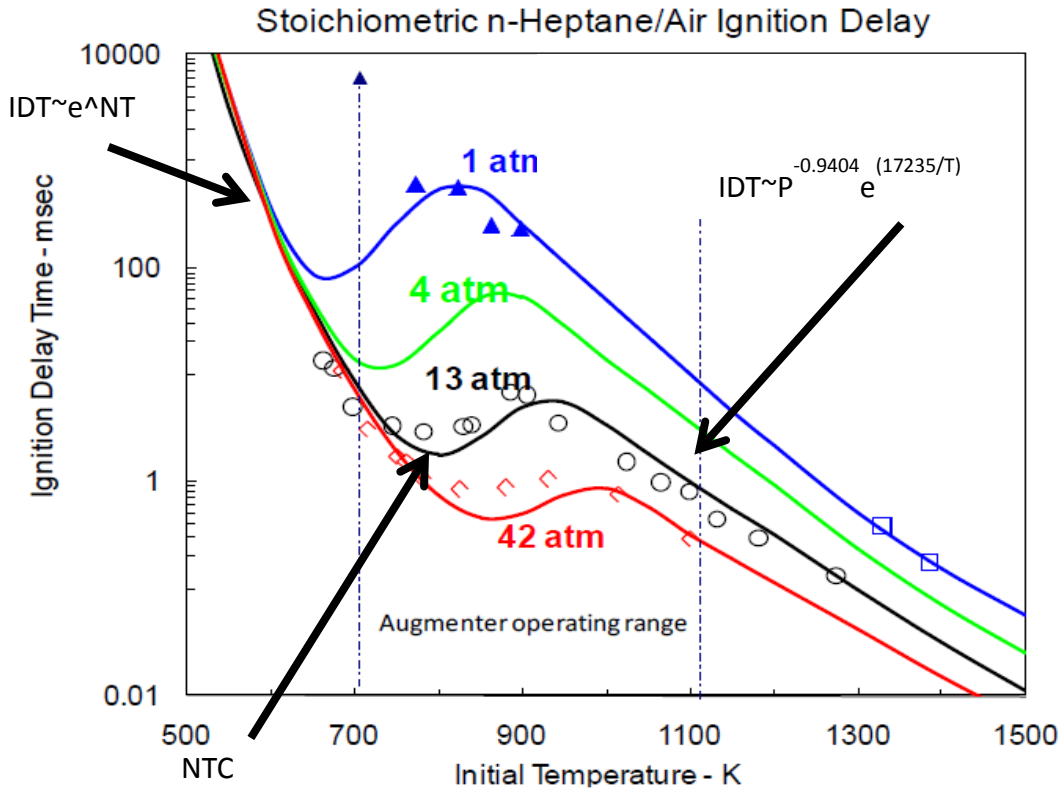


Figure 12. Ignition delay time of heptane.

The curve fit before the NTC region also has an exponential relationship with temperature, although this region loses much of its dependence on pressure. Because of the exponential nature of the ignition delay time, the form of the correlation equation was changed to provide an exponential form for temperature. This had little effect on the R-squared value, which remained near 0.88. The results are shown in Fig. 13. This correlation is capable of predicting the equivalence ratio at lean blowout with substantial accuracy and can be explained physically as a global Damköhler number, with U/D providing a fluid dynamic time scale and the pressure and temperature determining the kinetic reaction rate. Data from the King study appear to follow a slightly different trend, but this may be explained by differences in fuel type. The remainder of the data was obtained on propane or a similar fuel, while the King data were obtained on JP-4.

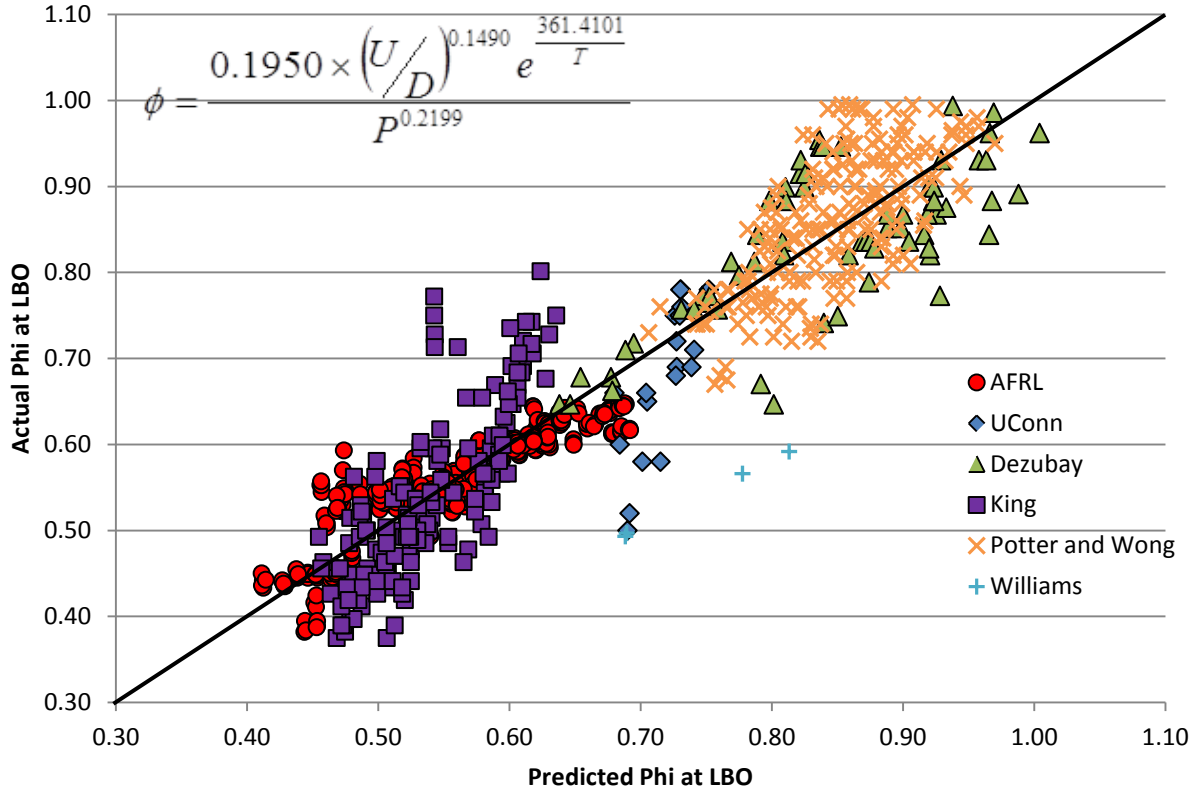


Figure 13. Correlation fit of data using an exponential for inlet temperature, $R^2=0.88$.

C. Reynolds Number

Once a strong correlation to Damköhler number was established, the focus of the work shifted to determining the degree to which other dimensionless parameters affect lean blowout. Because of its importance in characterizing flows, the Reynolds number was chosen for further study. The form used in the correlation for Reynolds number was:

$$\phi_{predicted} = A(Re)^x \quad (9)$$

The correlation resulted in an A value of 0.3439 and an x value of 0.0617. The R-squared value was -20.448. A negative R-squared value indicates that the correlation provided a worse prediction of the equivalence ratio at lean blowout than if the mean equivalence ratio had been used to predict each blowout. Figure 14 shows the actual equivalence ratio at lean blowout versus that predicted by the Reynolds number correlation.

D. Prandtl Number

The Prandtl number is used in heat transfer to compare how quickly the momentum (or velocity) of a fluid diffuses relative to its heat. The Prandtl number is defined as:

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{k} \quad (10)$$

The form used to correlate the Prandtl number was the same as that used to correlate the Reynolds number. The correlation resulted in an A value of 0.4026 and an x value of -1.395. The R-squared value was -203.715. Figure 15 shows the actual equivalence ratio at lean blowout versus the predicted equivalence ratio. As suggested by the negative R-squared value, the Prandtl number does not correlate with the equivalence ratio at lean blowout.

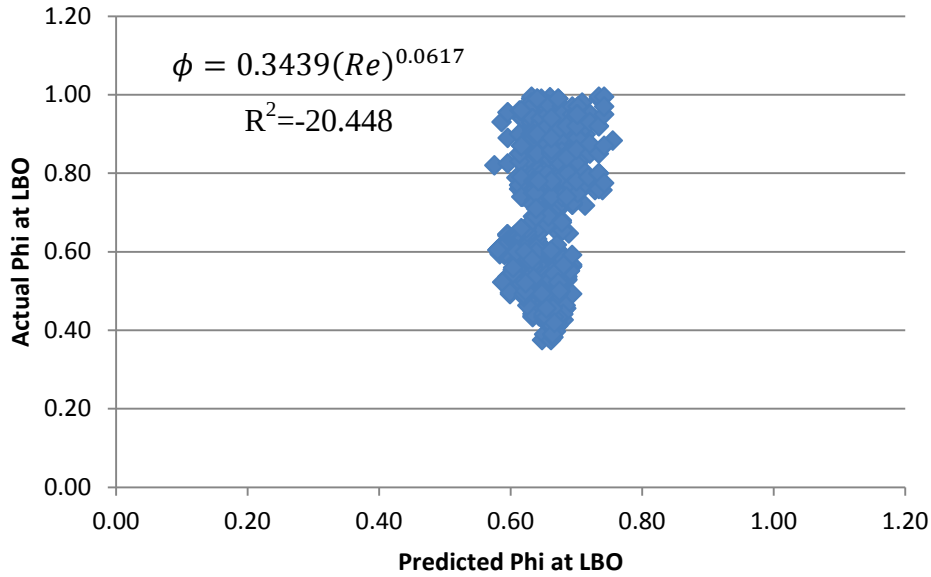


Figure 14: Correlation results with Reynolds number.

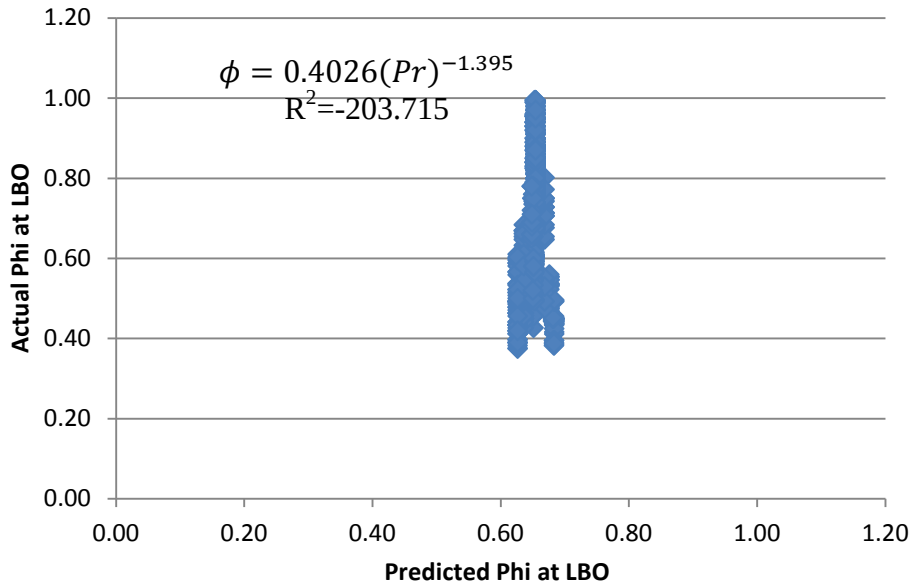


Figure 15: Correlation results with Prandtl number.

E. Strouhal Number

In many of the contemporary papers cited in the Introduction section, the conclusion was drawn that near lean blowout, the flame begins to exhibit behavior similar to that of von Karman vortex shedding. The authors conclude that the shedding contributes to lean blowout due to excessive flame stretching. If this behavior does contribute to lean blowout, it could be expected that the blowout would correlate with the characteristic frequency, or Strouhal number, of the flame holder, which is a dimensionless parameter used to describe the vortex shedding behind a bluff body. The Strouhal number is defined in Eq. (11).

$$St = \frac{fL}{U} \quad (11)$$

The Strouhal number varies with bluff-body shape and is defined by the above relationship, where f is the shedding frequency measured behind the bluff body, L a characteristic length scale (in this case, the diameter of the flame holder), and U the velocity of the flow (in this study, taken to be the lip velocity). The Strouhal numbers of the flame holders used in this study were obtained by taking the autospectrum of the velocity signal from a hot wire. For more information on the experimental setup for these measurements, see Appendix A. The results were verified through a comparison to the results of Blevins (1985),³⁷ Roshko (1954),^{38,39} Younger et al. (1951),⁴⁰ Campioli et al. (2005),⁴¹ and Brun et al. (2008).⁴² Figure 16 shows the results of this work plotted along with those reported by Brun et al. (2005) (I don't see a Brun in 2005 in the Reference Section. Do you mean 2008-Ref. 42?) for a square cylinder. The data from the present study closely match those obtained by Brun, et al.

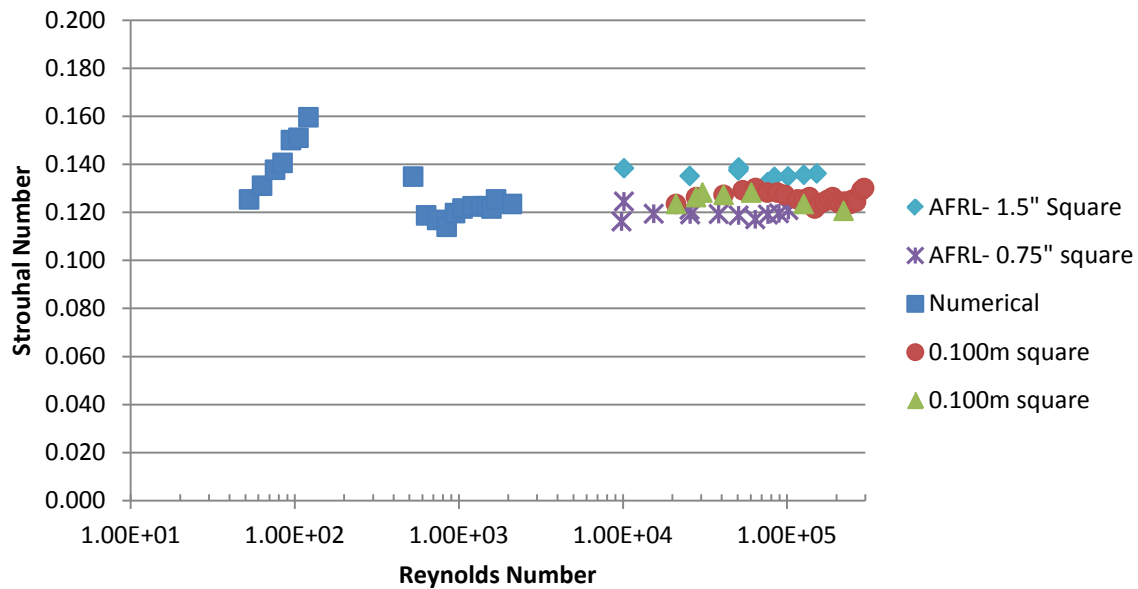


Figure 16. Comparison of Strouhal numbers for square-cylinder bluff body.

Hot wire measurements at Reynolds numbers from 10,000 to 150,000 were also made on open v-gutters, a closed v-gutter, a cylinder, and a flat plate. The characteristic length scales of these bluff bodies ranged from 0.375 to 1.5 in. Strouhal numbers for each bluff body over the range of Reynolds numbers are shown in Fig. 17. For a plot of all historical and experimental Strouhal number results, see Appendix B.

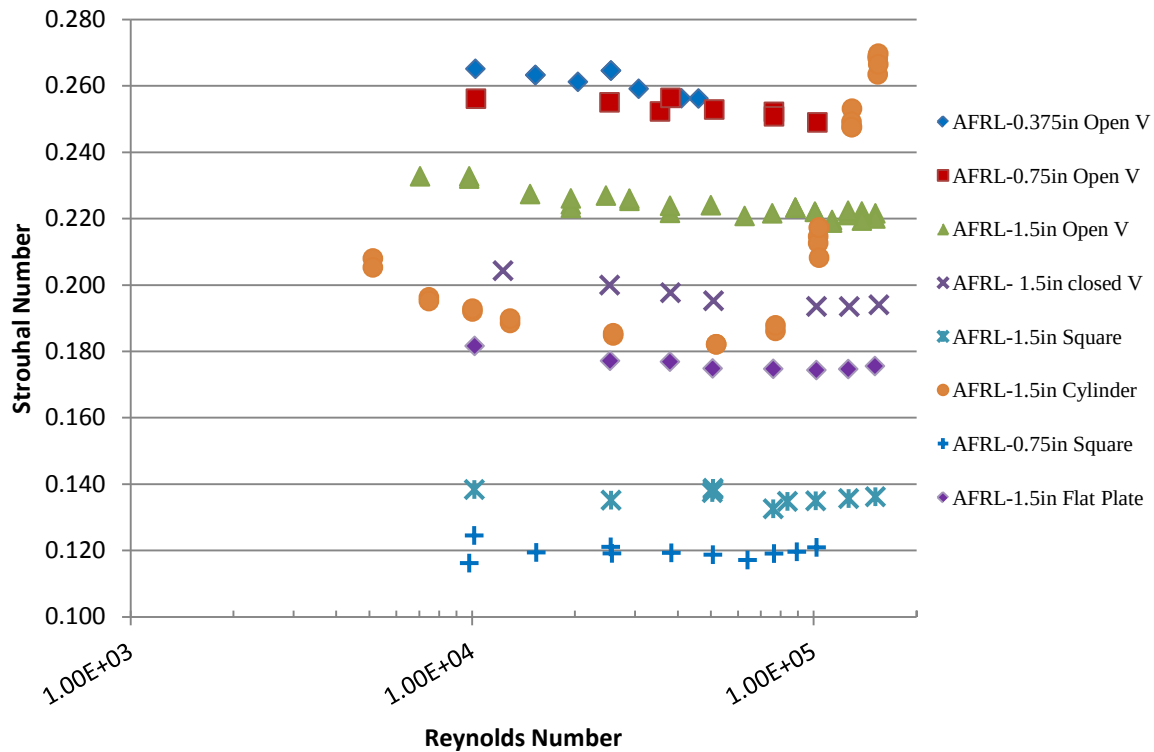


Figure 17. Strouhal number results from AFRL data.

The Reynolds number dependence of the Strouhal number is particularly apparent in the results for the 1.5-in. cylinder, while most other shapes produced a fairly constant Strouhal number. Measurements were also made on rectangular bluff bodies, but these results are not reported here because of difficulties in identifying a dominant frequency above the growing turbulence as the Reynolds number was increased. After the Strouhal number for each bluff body was established, these results were added to the lean-blowout data, and the correlation code was run. The form employed in the correlation was the same as that used for both the Reynolds number and the Prandtl number:

$$\phi_{predicted} = A(St)^x \quad (12)$$

The correlation resulted in an A value of 0.1095 and an x value of -1.1515. The R-squared value was -1.0494. As illustrated in Fig. 18 and through the negative R-squared value from the correlation, the Strouhal number does not appear to have a direct effect on lean blowout. In addition, including the Strouhal number in the Damköhler number correlation does not improve the R-squared value significantly, and the exponent of the Strouhal number is very small, -0.07.

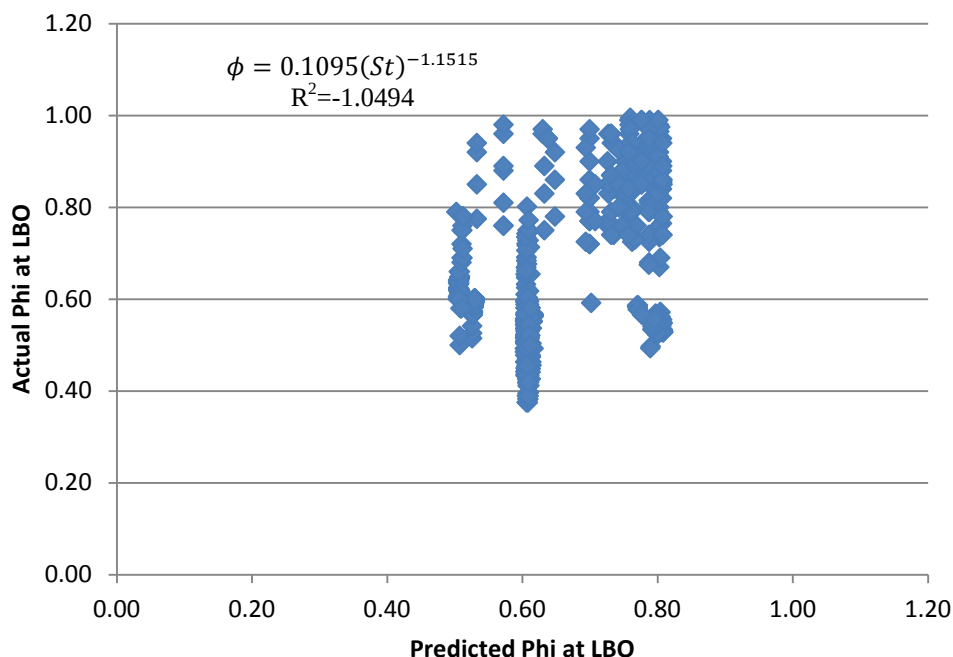


Figure 18. Correlation results with Strouhal number.

IV. Imaging Results

High-speed images were recorded near blowout for five flame-holder geometries: an open v-gutter, a cylinder, a square cylinder, and a plate, all with a diameter of 1.5 in., and an open v-gutter with a diameter of 0.75 in. Figure 19 contains a still image of all five flame holders near blowout. The flames behind the cylinder and the 0.75-in. open v-gutter appear to display an asymmetric mode similar to Von Karman shedding. The square cylinder also shows this tendency. However, the flames behind the 1.5-in. v-gutter and the flat plate do not exhibit this sinuous structure near blowout. This suggests that an asymmetric flame structure is not a necessary precursor to lean blowout. Additional work by Kostka et al. (2011) using Proper Orthogonal Decomposition (POD) on high-speed imaging shows that near lean blowout, the flame is not dominated by ordered, asymmetric shedding, but simply becomes more disorganized.⁴³ Lean-blowout measurements were also made for each flame-holder geometry with a 1.5-in. diameter at the same inlet conditions, and each flame holder blew out at an equivalence ratio $\sim 0.54 \pm 5\%$. This, together with the lack of correlation of Strouhal number with lean blowout, indicates that the shape of the flame holder does not have a strong effect on blowout. Blowout is simply governed by the geometric size of the flame holder. This conclusion also was drawn in many of the papers discussed in the Introduction section.

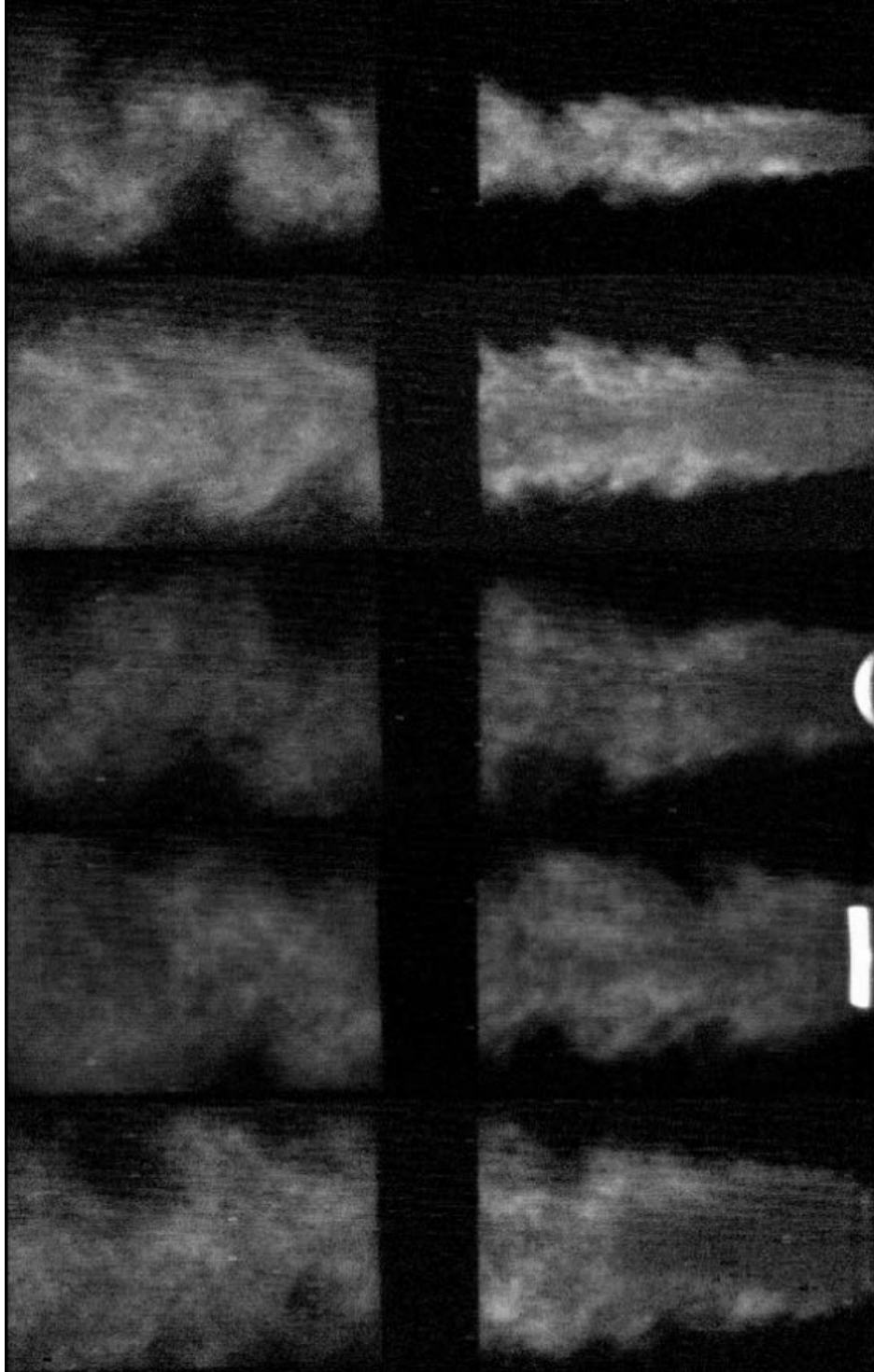


Figure 19. High-speed images of flames near blowout. From top to bottom, bluff-body geometries are: 0.75-in. open v-gutter, 1.5-in. open v-gutter, 1.5-in. cylinder, 1.5-in. square cylinder, and 1.5-in. flat plate.

V. Conclusions and Future Work

The correlation

$$\phi_{predicted} = \frac{0.1950 \times \left(\frac{U}{D}\right)^{0.1493} e^{\frac{361.4101}{T}}}{P^{0.2199}} \quad (13)$$

is essentially a global Damköhler number. Like the conclusions drawn by the authors of earlier papers, the lean extinction for the more than 800 data points that were correlated in the present work appears to correlate with a wake time scale; U/D represents the fluid-dynamic time scale behind the bluff body, while the pressure and temperature determine the chemical time scale with the same functionality as the ignition delay time of the inlet reactants. The ability of the correlation to predict lean blowout suggests that blowout is largely determined by Damköhler number rather than other dimensionless parameters presented throughout this work.

In the present work Reynolds, Prandtl, Strouhal, and Schmidt numbers did not correlate with the extinction data. For a Reynolds number between 4000 and 344,000, a lean extinction dependence on Reynolds number is not apparent. The Prandtl number also does not appear to have any correlation with lean blowout, which would indicate that momentum--not thermal diffusion--at the flame dominates the extinction physics. This also eliminates the Schmidt number as a dimensionless number that is important to blowout because of its dependence on Prandtl number.

We assume that the Strouhal number is representative of the shape of the flame holder. The Strouhal number is the dimensionless frequency of the vortex that is shed behind the flame holder, and each flame holder has a specific, unique Strouhal number. Given this, the extinction data did not correlate with the Strouhal number. The high-speed images appear to support this conclusion since flame holders with the same width and at the same inlet conditions displayed very different wakes and yet they had the same extinction limit, within experimental error. An investigation of the shedding frequency of the flame vortex behind the wake during combustion will be required to lend further support to this conclusion.

Some spread is still present in the data of the final correlation, Eq. (13). The chemical time scale used is based on a fit of ignition delay time, which includes an exponential term with temperature. The constant in the exponent will be a function of the fuel. In the large set of experimental data, several different fuels were used. Further research will involve conducting experiments with different fuels to determine the fuel effect and publishing constants for the curve fit, Eq. (13), so that it can be adjusted for fuel.

Appendix A: Hot-Wire-Anemometry Measurements

To measure the magnitude of the fluctuating velocity, a TSI Model 1210 hot-wire sensor was used at the end of the experimental rig. The hot wire was used in conjunction with an IFA 300 anemometer. The TSI 1210 has a single cylindrical platinum fiber that is maintained at a constant temperature by the IFA 300 anemometer. For a heated cylinder in cross-flow, the heat transfer from the cylinder is a function of the Reynolds number of the flow over the cylinder. The Nusselt number [Eq. (A-1)] is a dimensionless number that represents the average temperature gradient at the surface of the heated cylinder. In Eq. (A-1), h is the convection coefficient, k the thermal conductivity of the air, and L the cylinder diameter.

$$Nu = \frac{hL}{k} \quad (A-1)$$

Hilpert's(?) derived an empirical correlation (Incorporera and DeWitt, 1985)(I don't see this reference in the Reference Section.) for constant-temperature cylinders in cross-flow, Eq. (A-2). The correlation relates the Nusselt number to the Reynolds and Prandtl numbers. For a(?)

$$Nu_D = C Re_D^m Pr^{1/3} \quad (A-2)$$

For a constant-temperature wire, the heat transfer is a function of the current that is applied to the film by the anemometer.

$$I_w^2 R_w = h A_w (T_w - T_\infty) \quad (A-3)$$

where the voltage of the wire is:

$$V_w = I_w R_w \quad (\text{A-4})$$

Solving for h:

$$h = \frac{V_w^2}{R_w \pi D_w L_w (T_w - T_\infty)} \quad (\text{A-5})$$

From Eq. (A-1), the Nusselt number is:

$$Nu_w = \frac{V_w^2}{R_w \pi D_w L_w k_w (T_w - T_\infty)} \quad (\text{A-6})$$

The anemometer controls the probe voltage through a bridge circuit. For a bridge circuit, the voltage is related to current by

$$V_b = I_w (R_w + R_c) \quad (\text{A-7})$$

$$V_b = \left(\frac{V_w}{R_w} \right) (R_w + R_c) \quad (\text{A-8})$$

$$V_w = \frac{V_b R_w}{R_w + R_c} \quad (\text{A-9})$$

Substituting, the Nusselt number becomes

$$Nu_D = \frac{V_b^2 R_f^2}{(R_f + R_c)^2 \pi L_f k_f (T_f - T_\infty)} \quad (\text{A-10})$$

In Eq. (A-10) the Nusselt number is a function of the square of the bridge voltage of the anemometer.

When Eqs. (A-2) and (A-10) are combined, the Reynolds number of the wire is observed to be(?) a function of the square of the bridge voltage of the anemometer, Eq. (A-11). Each hot wire was calibrated using a Pitot probe. The mean velocity magnitude was measured with the Pitot probe for the three experimental rigs over a range of blower settings. At the same time, the hot wire probe was placed in the flow, and the bridge voltage was recorded. The hot wire Reynolds number and the square of the bridge voltage were calculated for each condition. These data were then plotted for each hot wire over the range of velocities tested. Figure A-1 shows the Reynolds number calibration for the hot wire probes that were available for the experiment.

$$\text{Re}^m = f(V_b^2) \quad (\text{A-11})$$

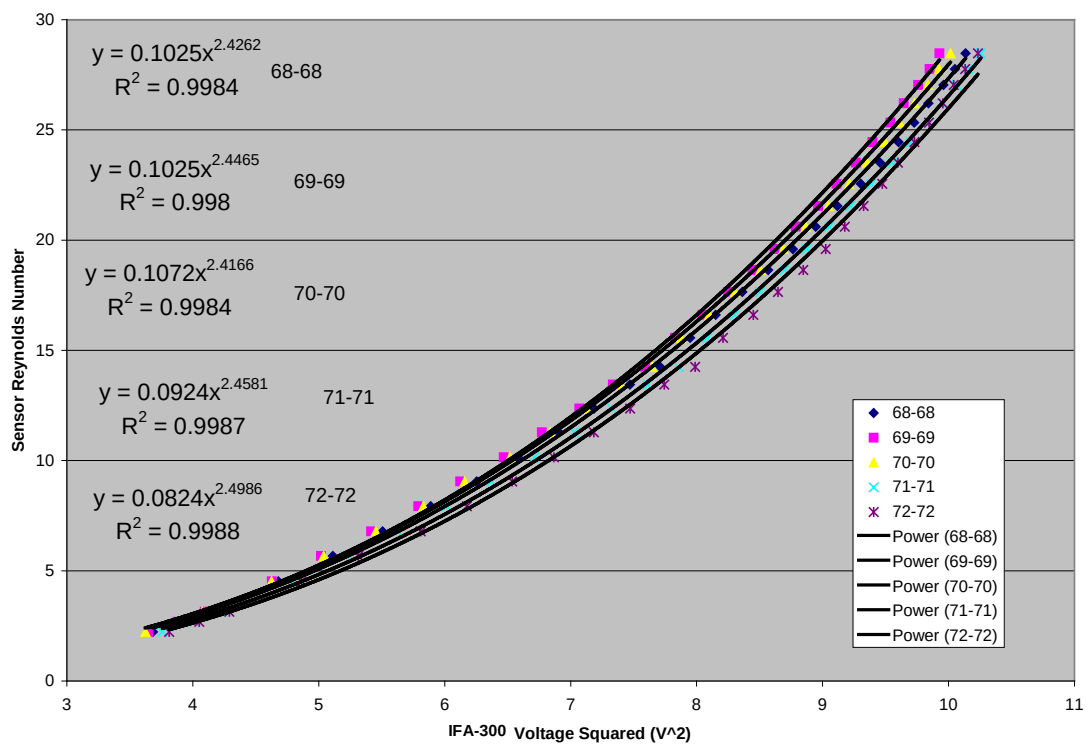


Figure A-1 Calibration data for TSI 1210 hot wire probes.

Appendix B: Strouhal Number Results

Figure B-1 shows the results of Strouhal number measurements made in the current work plotted with those of past researchers.

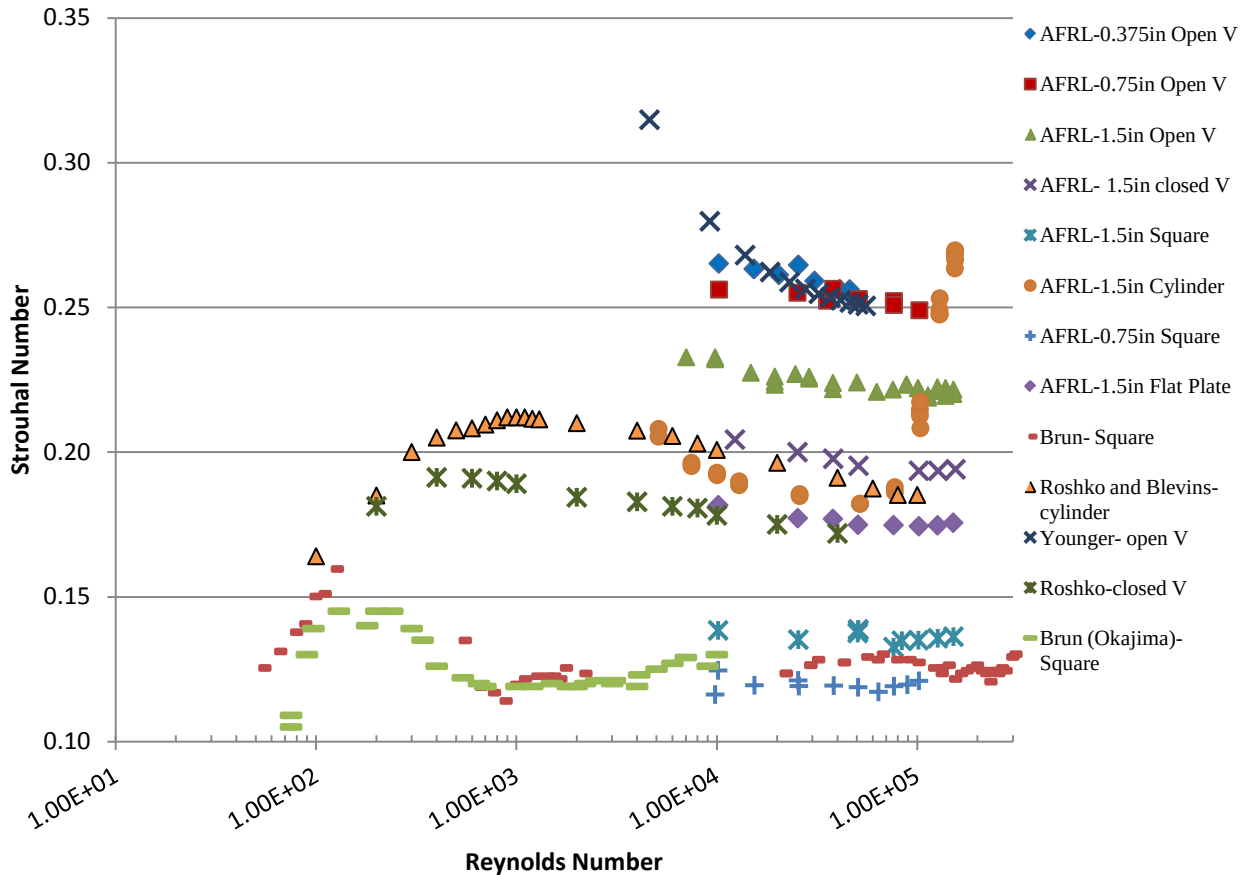


Figure B-1. Strouhal number results from AFRL and historical data.

Acknowledgments

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