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Recursive LMMSE Centralized Fusion with Recombination of Multi-Radar Measurements

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***Abstract*—For target tracking with radar measurements, recursive LMMSE (Linear Minimum Mean Squared Error) filtering outperforms the popular measurement conversion based Kalman filters, which have some serious drawbacks in terms of both estimation accuracy and credibility. The existing recursive LMMSE with measurements from a single radar is first extended to the multi-radar case. It is then shown that recombination plays an important role in performance improvement for recursive LMMSE centralized fusion using multiple radars. Here, "recombination" means shuffling all scalar measurements from the multiple radars, dimension by dimension. This differs from the case of centralized fusion with linear measurements from multiple sensors. Numerical simulation examples are provided to illustrate the use of recombination in recursive LMMSE centralized fusion for the nonlinear radar measurements.**

Keywords: Centralized fusion, recombination, recursive LMMSE filtering, nonlinear filtering, radar measurements, target tracking.

I. Introduction

In tracking applications, target dynamics are usually modeled in Cartesian coordinates, while the radar measurements may be directly available in polar or spherical coordinates. Due to the nonlinear relationship between the Cartesian coordinates and polar/spherical coordinates, tracking in Cartesian coordinates using measurements available in sensor coordinates is in essence a nonlinear filtering problem. The general nonlinear filtering problem, consisting of point estimation and density estimation, has been widely studied and a lot of results are available, e.g., extended Kalman filter (EKF), unscented filtering (UF) [1], the second-order Stirling interpolation based filter (DD2) [2], and Gaussian filter [3] for point estimation and particle filter [4] for density estimation. They can all be applied to target tracking in Cartesian coordinates with measurements in polar/spherical coordinates. However, since the nonlinear relationship between the Cartesian coordinates and polar/spherical coordinates is known explicitly, specifically designed nonlinear filters may perform better.

In a track-while-scan surveillance system, the measurements are usually range, bearing, elevation and range rate in polar or spherical coordinates [5]. Specifically designed nonlinear filtering algorithms for this type of problem abound. One popular way is to convert the measurement model in the polar/spherical coordinates to the Cartesian coordinates so that the converted measurement model takes a pseudo linear form with respect to the target state described in the Cartesian coordinates and the Kalman filter can be applied. Unfortunately, the converted measurement errors are state dependent and have bias. Numerous ways have been proposed to debias the converted measurements [5]. For instance, [6]–[8] proposed to use additive debiasing, and [9], [10] proposed to use multiplicative debiasing. [10], [11] extended debiased conversion to the case when range rate measurements are also available. In a scan-while-track surveillance system, such as a phased array radar, the measurements are usually range and two direction cosines [5]. Specifically designed nonlinear filtering algorithms for this type of problem are also available. For example, by using the first-order and second-order Taylor series expansions, [8], [12] developed Kalman filters for this type of problem, again based on debiased measurement model conversion.

As pointed out in [5], [13], the Kalman filter based on debiased conversion of the measurement model from the polar/sphericla coordinates to the Cartesian coordinates has the following serious drawbacks: a) the converted measurement errors are state dependent; b) their covariances are estimated conditioned on the measurement or state; c) the converted measurement error sequence is not white any more. However, the Kalman filter assumes the measurement noise is independent of the state, its covariance is unconditional, and it is white. These drawbacks cannot be overcome in the existing debiased conversion based Kalman filters. To be free of these drawbacks, [13] and [14] proposed recursive LMMSE filtering with respect to the converted measurements for track-while-scan system and scan-while-track system, respectively.

For centralized fusion with multiple sensors, a basic idea is to stack the measurements from all sensors first and then use the stacked measurements to replace measurements in the traditional estimation method. One may think that centralized fusion done this way has the best performance achievable—there is no room to improve performance of multi-sensor centralized fusion done this way. This is true for multi-sensor centralized fusion with linear measurements. However, this paper shows that this is not true for multi-radar

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14. ABSTRACT
For target tracking with radar measurements, recursive LMMSE (Linear Minimum Mean Squared Error) filtering outperforms the popular measurement conversion based Kalman filters, which have some serious drawbacks in terms of both estimation accuracy and credibility. The existing recursive LMMSE with measurements from a single radar is first extended to the multi-radar case. It is then shown that recombination plays an important role in performance improvement for recursive LMMSE centralized fusion using multiple radars. Here, ?recombination? means shuffling all scalar measurements from the multiple radars, dimension by dimension. This differs from the case of centralized fusion with linear measurements from multiple sensors. Numerical simulation examples are provided to illustrate the use of recombination in recursive LMMSE centralized fusion for the nonlinear radar measurements.

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centralized fusion. The recursive LMMSE for target tracking with measurements from a single radar in a track-while-scan system is first extended to the multi-radar case. Illustrative examples then verify that recombination[1] plays a key role in performance improvement using multiple radars.

This paper is organized as follows. Sec. II formulates the problem and summarizes recursive LMMSE filtering for a single radar. Sec. III presents recursive LMMSE centralized fusion with measurements from multiple radars. Sec. IV presents recursive LMMSE centralized fusion with recombined radar measurements. Sec. V provides illustrative examples to show the use of recombination in recursive LMMSE centralized fusion. Sec. VI gives conclusions.

II. Problem Formulation

For the convenience of discussion, consider only the following target dynamics described in a three-dimensional Cartesian coordinate system

$$X_k = F_{k-1}X_{k-1} + G_{k-1}W_{k-1}$$

where $X_k = [\ \mathsf{x}_k \quad \dot{\mathsf{x}}_k \quad \mathsf{y}_k \quad \dot{\mathsf{y}}_k \quad \mathsf{z}_k \quad \dot{\mathsf{z}}_k\ ]'$ is the target state vector at time k, which consists of position and velocity along each direction, $\langle W_k \rangle$ is a zero-mean white noise sequence with covariance Q_k, $E[X_0] = \bar{X}_0$, $\text{cov}(X_0) = P_0$, and $\text{cov}(X_0, W_k) = 0$.

Suppose that M radars co-located at the origin of the Cartesian coordinates are used to observe the target motion. In spherical coordinates, the measurements from the i-th radar at time k are range r_k^i, bearing (or azimuth) b_k^i and elevation e_k^i. They are observed as

$$\begin{aligned} r_k^i &= \mathsf{r}_k + \tilde{r}_k^i \\ b_k^i &= \mathsf{b}_k + \tilde{b}_k^i \\ e_k^i &= \mathsf{e}_k + \tilde{e}_k^i \end{aligned}$$

where

$$\begin{aligned} \mathsf{r}_k &= (\mathsf{x}_k^2 + \mathsf{y}_k^2 + \mathsf{z}_k^2)^{1/2} \\ \mathsf{b}_k &= \tan^{-1}(\mathsf{y}_k/\mathsf{x}_k) \\ \mathsf{e}_k &= \tan^{-1}(\mathsf{z}_k/(\mathsf{x}_k^2 + \mathsf{y}_k^2)^{1/2}) \end{aligned}$$

are the corresponding groud truth; $\langle \tilde{r}_k^i \rangle$, $\langle \tilde{b}_k^i \rangle$ and $\langle \tilde{e}_k^i \rangle$, which are independent of each other and of X_0 and $\langle W_k \rangle$, are all zero-mean white Gaussian measurement noise sequences with standard deviations σ_r^i, σ_b^i and σ_e^i, respectively. It is also assumed that these noises are independent across radars.

The radar measurements in the spherical coordinates can be converted into their counterparts in the Cartesian coordinates through

$$\begin{aligned} x_k^i &= r_k^i \cos e_k^i \cos b_k^i \\ y_k^i &= r_k^i \cos e_k^i \sin b_k^i \\ z_k^i &= r_k^i \sin e_k^i \end{aligned}$$

[1]In this paper, recombination means shuffling all scalar measurements from the multiple radars, dimension by dimension.

For the ease of comparison, the recursive LMMSE filtering for X_k is summarized below [13].

Define

$$\begin{aligned} Z_k^i &= [\ x_k^i \quad y_k^i \quad z_k^i\ ]' \\ \lambda_1^i &= E[\cos \tilde{b}_k^i] = e^{-(\sigma_b^i)^2/2} \\ \lambda_2^i &= E[\cos^2 \tilde{b}_k^i] = (1 + e^{-(\sigma_b^i)^2})/2 \\ \lambda_3^i &= E[\sin^2 \tilde{b}_k^i] = (1 - e^{-(\sigma_b^i)^2})/2 \\ \mu_1^i &= E[\cos \tilde{e}_k^i] = e^{-(\sigma_e^i)^2/2} \\ \mu_2^i &= E[\cos^2 \tilde{e}_k^i] = (1 + e^{-(\sigma_e^i)^2})/2 \\ \mu_3^i &= E[\cos^2 \tilde{e}_k^i] = (1 - e^{-(\sigma_e^i)^2})/2 \\ \Omega_i &= \text{diag}\{\lambda_1^i \mu_1^i, \lambda_1^i \mu_1^i, \mu_1^i\} \end{aligned}$$

and the recursive filtering operation at time k

$$[\hat{X}, \hat{P}] = \text{filtering}(\bar{X}, \bar{P}, \tilde{Z}, C, S)$$

with

$$\begin{aligned} \hat{X} &= F_{k-1}\bar{X} + CS^{-1}\tilde{Z} \\ \hat{P} &= F_{k-1}\bar{P}F_{k-1}' + G_{k-1}Q_{k-1}G_{k-1}' - CS^{-1}C' \end{aligned}$$

Given $\hat{X}_{k-1|k-1}^i = E^*[X_{k-1}|\{Z_t^i\}_{t=1}^{k-1}]$ and $P_{k-1|k-1}^i = \text{MSE}(\hat{X}_{k-1|k-1}^i|\{Z_t^i\}_{t=1}^{k-1})$, where $E^*[x|z]$ stands for LMMSE estimate of x using z and $\text{MSE}(\hat{x}|z) = E[(x-\hat{x})(x-\hat{x})'|z]$, the recursive LMMSE filtering for X_k at the i-th radar can be obtained as

$$\begin{aligned} &[\hat{X}_{k|k}^i, P_{k|k}^i] \\ &\quad = \text{filtering}(\hat{X}_{k-1|k-1}^i, P_{k-1|k-1}^i, \tilde{Z}_{k|k-1}^i, C_{k|k-1}^i, S_k^i) \end{aligned}$$

where

$$\begin{aligned} \tilde{Z}_{k|k-1}^i &= Z_k^i - \hat{Z}_{k|k-1}^i \\ \hat{Z}_{k|k-1}^i &= E^*[Z_k^i|\{Z_t^i\}_{t=1}^{k-1}] \\ &= \Omega_i[\ \hat{X}_{k|k-1}(1) \quad \hat{X}_{k|k-1}(3) \quad \hat{X}_{k|k-1}(5)\ ]' \\ C_{k|k-1}^i &= \text{cov}(\tilde{X}_{k|k-1}, \tilde{Z}_{k|k-1}^i) \\ &= \Omega_i[\ P_{k|k-1}(:,1) \quad P_{k|k-1}(:,3) \quad P_{k|k-1}(:,5)\ ] \\ S_k^i &= [S_k^i(m,n)]_{m,n=1}^3 \\ S_k^i(1,1) &\approx \lambda_2^i \mu_2^i P_{k|k-1}^i(1,1) + \lambda_3^i \mu_2^i P_{k|k-1}^i(3,3) \\ &\quad + \alpha_k^i(\lambda_2^i[\hat{X}_{k|k-1}^i(1)]^2 + \lambda_3^i[\hat{X}_{k|k-1}^i(3)]^2) + \beta_k^i(1) \\ S_k^i(2,2) &\approx \lambda_2^i \mu_2^i P_{k|k-1}^i(3,3) + \lambda_3^i \mu_2^i P_{k|k-1}^i(1,1) \\ &\quad + \alpha_k^i(\lambda_3^i[\hat{X}_{k|k-1}^i(1)]^2 + \lambda_2^i[\hat{X}_{k|k-1}^i(3)]^2) + \beta_k^i(2) \\ S_k^i(3,3) &\approx \mu_2^i P_{k|k-1}^i(5,5) + \mu_3^i[P_{k|k-1}^i(1,1) + P_{k|k-1}^i(3,3) \\ &\quad + \mu_2^i(\sigma_r^i)^2[\hat{X}_{k|k-1}^i(5)]^2/(\bar{\mathsf{r}}_k^i)^2 \\ &\quad + \mu_3^i(\sigma_r^i)^2(\bar{\mathsf{d}}_k^i)^2/(\bar{\mathsf{r}}_k^i)^2] + \beta_k^i(3) \\ S_k^i(1,2) &= S_k^i(2,1) \approx (\lambda_2^i - \lambda_3^i)\mu_2^i P_{k|k-1}^i(1,3) \\ &\quad + \alpha_k^i \hat{X}_{k|k-1}^i(1)\hat{X}_{k|k-1}^i(3) + \beta_k^i(4) \\ S_k^i(1,3) &= S_k^i(3,1) \approx \lambda_1^i(\mu_2^i - \mu_3^i)[P_{k|k-1}^i(1,5) + (\sigma_r^i)^2 \\ &\quad \cdot \hat{X}_{k|k-1}^i(1)\hat{X}_{k|k-1}^i(3)/(\bar{\mathsf{r}}_k^i)^2] + \beta_k^i(4)\hat{X}_{k|k-1}^i(1) \end{aligned}$$

$$
\begin{aligned}
S_k^i(2,3) &= S_k^i(3,2) \approx \lambda_1^i(\mu_2^i - \mu_3^i)[P_{k|k-1}^i(3,5) + (\sigma_r^i)^2 \\
&\quad \cdot \hat{X}_{k|k-1}^i(3)\hat{X}_{k|k-1}^i(5)/(\bar{\mathsf{r}}_k^i)^2] + \beta_k^i(4)\hat{X}_{k|k-1}^i(3) \\
\bar{\mathsf{r}}_k^i &= ((\bar{\mathsf{d}}_k^i)^2 + [\hat{X}_{k|k-1}^i(5)]^2)^{1/2} \\
\bar{\mathsf{d}}_k^i &= ([\hat{X}_{k|k-1}^i(1)]^2 + [\hat{X}_{k|k-1}^i(3)]^2)^{1/2} \\
\alpha_k^i &= \mu_2^i(\sigma_r^i)^2/(\bar{\mathsf{r}}_k^i)^2 + \mu_3^i[\hat{X}_{k|k-1}^i(5)]^2/(\bar{\mathsf{d}}_k^i)^2 \\
&\quad + \mu_3^i(\sigma_r^i)^2[\hat{X}_{k|k-1}^i(5)]^2/(\bar{\mathsf{d}}_k^i\bar{\mathsf{r}}_k^i)^2 \\
\beta_k^i &= [\beta_k^i(j)]_{j=1}^5 \\
\beta_k^i(1) &= [\lambda_2^i\mu_2^i - (\lambda_1^i\mu_1^i)^2][\hat{X}_{k|k-1}^i(1)]^2 \\
&\quad + \lambda_3^i\mu_2^i[\hat{X}_{k|k-1}^i(3)]^2 \\
\beta_k^i(2) &= [\lambda_2^i\mu_2^i - (\lambda_1^i\mu_1^i)^2][\hat{X}_{k|k-1}^i(3)]^2 \\
&\quad + \lambda_3^i\mu_2^i[\hat{X}_{k|k-1}^i(1)]^2 \\
\beta_k^i(3) &= [\mu_2^i - (\mu_1^i)^2][\hat{X}_{k|k-1}^i(5)]^2 + \mu_3^i(\bar{\mathsf{d}}_k^i)^2 \\
\beta_k^i(4) &= [\mu_2^i(\lambda_2^i - \lambda_3^i) - (\lambda_1^i\mu_1^i)^2]\hat{X}_{k|k-1}^i(1)\hat{X}_{k|k-1}^i(3) \\
\beta_k^i(5) &= [\lambda_1^i(\mu_2^i - \mu_3^i) - \lambda_1^i(\mu_1^i)^2]\hat{X}_{k|k-1}^i(5)
\end{aligned}
$$

We first extend the above recursive LMMSE filtering with measurments from a single radar to the multi-radar centralized fusion case. Then we are mainly interested to see whether there is still room to improve performance under the recursive LMMSE filteing framework.

III. Recursive LMMSE Centralized Fusion with Radar Measurements

A basic idea for centralized fusion is to stack the measurements from all sensors first and then use the stacked measurements as the measurements for estimation.

Let

$$
Z_k = [\ (Z_k^1)' \quad \cdots \quad (Z_k^M)'\ ]',\ Z^k = \{Z_t\}_{t=1}^k \qquad (1)
$$

Then the following theorem for multi-radar centrazlied fusion holds.

Theorem 1: Given $\hat{X}_{k-1|k-1} = E^*[X_{k-1}|Z^{k-1}]$ and $P_{k-1|k-1} = \text{MSE}(\hat{X}_{k-1|k-1}|Z^{k-1})$, the recursive LMMSE centralized fusion for X_k can be obtained as

$$
\begin{aligned}
&[\hat{X}_{k|k}, P_{k|k}] \\
&= \text{filtering}(\hat{X}_{k-1|k-1}, P_{k-1|k-1}, \tilde{Z}_{k|k-1}, C_{k|k-1}, S_k)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{Z}_{k|k-1} &= Z_k - \hat{Z}_{k|k-1} \\
\hat{Z}_{k|k-1} &= [\ (\hat{Z}_{k|k-1}^1)' \quad \cdots \quad (\hat{Z}_{k|k-1}^M)'\ ]' \\
C_{k|k-1} &= [\ C_{k|k-1}^1 \quad \cdots \quad C_{k|k-1}^M\ ] \\
S_k &= \begin{bmatrix} S_k^1 & S_k^{1,2} & \cdots & S_k^{1,M} \\ S_k^{2,1} & S_k^2 & \cdots & S_k^{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ S_k^{M,1} & S_k^{M,2} & \cdots & S_k^M \end{bmatrix} \\
S_k^{i,j} &= \Omega_i P_{k|k-1}([1,3,5],[1,3,5])\Omega_j,\ i \neq j,\ i,j = 1,\cdots,M
\end{aligned}
$$

$$
\begin{aligned}
&P_{k|k-1}([1,3,5],[1,3,5]) \\
&= \begin{bmatrix} P_{k|k-1}(1,1) & P_{k|k-1}(1,3) & P_{k|k-1}(1,5) \\ P_{k|k-1}(3,1) & P_{k|k-1}(3,3) & P_{k|k-1}(3,5) \\ P_{k|k-1}(5,1) & P_{k|k-1}(5,3) & P_{k|k-1}(5,5) \end{bmatrix}
\end{aligned}
$$

Proof:

The LMMSE estimator $E^*[X_k|Z^k]$ always has the following quasi-recursive form [15]

$$
\begin{aligned}
\hat{X}_{k|k} &= E^*[X_k|Z^k] = E^*[X_k|Z^{k-1}, Z_k] \\
&= \hat{X}_{k|k-1} + C_{k|k-1}S_k^{-1}(Z_k - \hat{Z}_{k|k-1}) \\
P_{k|k} &= \text{MSE}(\hat{X}_{k|k}|Z^k) = P_{k|k-1} - C_{k|k-1}S_k^{-1}C'_{k|k-1}
\end{aligned}
$$

From the stacked measurement (1), we have

$$
\begin{aligned}
\hat{Z}_{k|k-1} &= E^*\left[Z_k|Z^{k-1}\right] \\
&= [\ E^*[Z_k^1|Z^{k-1}]' \quad \cdots \quad E^*[Z_k^M|Z^{k-1}]'\ ]' \\
&= [\ (\hat{Z}_{k|k-1}^1)' \quad \cdots \quad (\hat{Z}_{k|k-1}^M)'\ ]'
\end{aligned}
$$

$$
\begin{aligned}
&C_{k|k-1} = \text{cov}(\tilde{X}_{k|k-1}, \tilde{Z}_{k|k-1}) \\
&= [\ \text{cov}(\tilde{X}_{k|k-1}, \tilde{Z}_{k|k-1}^1) \quad \cdots \quad \text{cov}(\tilde{X}_{k|k-1}, \tilde{Z}_{k|k-1}^M)\ ] \\
&= [\ C_{k|k-1}^1 \quad \cdots \quad C_{k|k-1}^M\ ]
\end{aligned}
$$

$$
S_k = \text{cov}(\tilde{Z}_{k|k-1}) = \begin{bmatrix} S_k^1 & S_k^{1,2} & \cdots & S_k^{1,M} \\ S_k^{2,1} & S_k^2 & \cdots & S_k^{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ S_k^{M,1} & S_k^{M,2} & \cdots & S_k^M \end{bmatrix}
$$

$$
\begin{aligned}
S_k^{i,j} &= \text{cov}(\tilde{Z}_{k|k-1}^i, \tilde{Z}_{k|k-1}^j) \\
&= [S_k^{i,j}(m,n)]_{m,n=1}^3,\ i \neq j,\ i,j = 1,\cdots,M
\end{aligned}
$$

From Lemma 2 of [13], it follows that

$$
\begin{aligned}
S_k^{i,j}(1,1) &= \text{cov}(\tilde{x}_{k|k-1}^i, \tilde{x}_{k|k-1}^j) \\
&= E[x_k^i x_k^j] - E[\hat{x}_{k|k-1}^i \hat{x}_{k|k-1}^j]
\end{aligned}
$$

where

$$
\begin{aligned}
&E[\hat{x}_{k|k-1}^i \hat{x}_{k|k-1}^j] \\
&= \lambda_1^i\mu_1^i\lambda_1^j\mu_1^j E[\hat{X}_{k|k-1}(1)\hat{X}_{k|k-1}(1)] \\
&= \lambda_1^i\mu_1^i\lambda_1^j\mu_1^j E[\hat{\mathsf{x}}_{k|k-1}\hat{\mathsf{x}}_{k|k-1}] \\
&E[x_k^i x_k^j] \\
&= E[((\mathsf{r}_k + \tilde{r}_k^i)\cos(\mathsf{e}_k + \tilde{e}_k^i)\cos(\mathsf{b}_k + \tilde{b}_k^i)) \\
&\cdot ((\mathsf{r}_k + \tilde{r}_k^j)\cos(\mathsf{e}_k + \tilde{e}_k^j)\cos(\mathsf{b}_k + \tilde{b}_k^j))] \\
&= E[(\mathsf{x}_k \cos\tilde{b}_k^i \cos\tilde{e}_k^i - \mathsf{y}_k \sin\tilde{b}_k^i \cos\tilde{e}_k^i \\
&+ \frac{\mathsf{x}_k}{\mathsf{r}_k}\tilde{r}_k^i \cos\tilde{b}_k^i \cos\tilde{e}_k^i - \frac{\mathsf{y}_k}{\mathsf{r}_k}\tilde{r}_k^i \sin\tilde{b}_k^i \cos\tilde{e}_k^i \\
&- \frac{\mathsf{x}_k\mathsf{z}_k}{\mathsf{d}_k}\cos\tilde{b}_k^i \sin\tilde{e}_k^i + \frac{\mathsf{y}_k\mathsf{z}_k}{\mathsf{d}_k}\sin\tilde{b}_k^i \sin\tilde{e}_k^i \\
&- \frac{\mathsf{x}_k\mathsf{z}_k}{\mathsf{d}_k\mathsf{r}_k}\tilde{r}_k^i \cos\tilde{b}_k^i \sin\tilde{e}_k^i + \frac{\mathsf{y}_k\mathsf{z}_k}{\mathsf{d}_k\mathsf{r}_k}\tilde{r}_k^i \sin\tilde{b}_k^i \sin\tilde{e}_k^i)
\end{aligned}
$$

$$
\begin{aligned}
&\cdot (\mathsf{x}_k \cos \tilde{b}_k^j \cos \tilde{e}_k^j - \mathsf{y}_k \sin \tilde{b}_k^j \cos \tilde{e}_k^j \\
&+ \frac{\mathsf{x}_k}{\mathsf{r}_k} \tilde{r}_k^j \cos \tilde{b}_k^j \cos \tilde{e}_k^j - \frac{\mathsf{y}_k}{\mathsf{r}_k} \tilde{r}_k^j \sin \tilde{b}_k^j \cos \tilde{e}_k^j \\
&- \frac{\mathsf{x}_k \mathsf{z}_k}{\mathsf{d}_k} \cos \tilde{b}_k^j \sin \tilde{e}_k^j + \frac{\mathsf{y}_k \mathsf{z}_k}{\mathsf{d}_k} \sin \tilde{b}_k^j \sin \tilde{e}_k^j \\
&- \frac{\mathsf{x}_k \mathsf{z}_k}{\mathsf{d}_k \mathsf{r}_k} \tilde{r}_k^j \cos \tilde{b}_k^j \sin \tilde{e}_k^j + \frac{\mathsf{y}_k \mathsf{z}_k}{\mathsf{d}_k \mathsf{r}_k} \tilde{r}_k^j \sin \tilde{b}_k^j \sin \tilde{e}_k^j)] \\
&= \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j E[\mathsf{x}_k \mathsf{x}_k] \\
\mathsf{d}_k &= (\mathsf{x}_k^2 + \mathsf{y}_k^2)^{1/2}
\end{aligned}
$$

Thus

$$
\begin{aligned}
S_k^{i,j}(1,1) &= \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j (E[\mathsf{x}_k \mathsf{x}_k] - E[\hat{\mathsf{x}}_{k|k-1} \hat{\mathsf{x}}_{k|k-1}]) \\
&= \mathrm{cov}(\tilde{\mathsf{x}}_{k|k-1}) = \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(1,1)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
S_k^{i,j}(1,2) &= \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(1,3) \\
S_k^{i,j}(1,3) &= \lambda_1^i \mu_1^i \mu_1^j P_{k|k-1}(1,5) \\
S_k^{i,j}(2,1) &= \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(3,1) \\
S_k^{i,j}(2,2) &= \lambda_1^i \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(3,3) \\
S_k^{i,j}(2,3) &= \lambda_1^i \mu_1^i \mu_1^j P_{k|k-1}(3,5) \\
S_k^{i,j}(3,1) &= \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(5,1) \\
S_k^{i,j}(3,2) &= \mu_1^i \lambda_1^j \mu_1^j P_{k|k-1}(5,3) \\
S_k^{i,j}(3,3) &= \mu_1^i \mu_1^j P_{k|k-1}(5,5)
\end{aligned}
$$

In summary,

$$
S_k^{i,j} = \Omega_i P_{k|k-1}([1,3,5],[1,3,5]) \Omega_j
$$

□

Remark: Compared with the single radar case in Sec. II, only the off-digonal blocks of S_k are new.

Remark: According to [13], under the recursive LMMSE filtering framework, $\hat{Z}_{k|k-1}^i$ and $C_{k|k-1}^i$ can be obtained exactly, but S_k^i needs approximation. However, for multi-radar centralized fuison, $S_k^{i,j}$ can also be obtained exactly. This makes the recursive LMMSE multi-radar centralized fusion much easier unexpectedly.

Remark: In practice, multiple radars may work asynchronously due to different sampling rates, different initial times, etc. For this case, we can align measurements made at different time instants to a common fusion time [16]. However, due to common process noise, measurement noise of each radar will be correlated with the process noise, and measurement noises across radars will be correlated too. Further work will be done to extend multi-radar recursive LMMSE centralized fusion above to the asynchronous case.

One may think that Theorem 1 presents the *optimal* multi-radar centralized LMMSE fusion. However, this is not the case. As shown below, although recombination of linear measurements do not improve performance of centralized fusion, it does for multi-radar centralized LMMSE fusion.

IV. Recursive LMMSE Centralized Fusion with Recombined Radar Measurements

Let us first examine the use of recombination for centralized fusion with linear measurements.

A. Linear Measurements

Suppose that we have M sensors of non-radar type. Each of them observes the target position in Cartesian coordinates directly as

$$
\begin{aligned}
x_k^i &= \mathsf{x}_k + \tilde{x}_k^i \\
y_k^i &= \mathsf{y}_k + \tilde{y}_k^i \\
z_k^i &= \mathsf{z}_k + \tilde{z}_k^i
\end{aligned}
$$

where $i = 1, 2, \cdots, M$, and $\langle \tilde{x}_k^i \rangle$, $\langle \tilde{y}_k^i \rangle$ and $\langle \tilde{z}_k^i \rangle$, which are independent of each other and of X_0 and $\langle W_k \rangle$, are all zero-mean white Gaussian measurement noise sequences with standard deviations σ_x^i, σ_y^i and σ_z^i, respectively. It is also assumed that these noises are independent across sensors.

The measurement from the i-th sensor can be rewritten in a compact form as

$$
Z_k^i = D_k X_k + V_k^i
$$

where

$$
\begin{aligned}
Z_k^i &= [\ x_k^i \quad y_k^i \quad z_k^i\ ]' \\
D_k &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
V_k^i &= [\ \tilde{x}_k^i \quad \tilde{y}_k^i \quad \tilde{z}_k^i\ ]' \\
E[V_k^i] &= 0_{3\times 1},\ R_k^i = \mathrm{cov}(V_k^i) = \mathrm{diag}\{(\sigma_x^i)^2, (\sigma_y^i)^2, (\sigma_z^i)^2\}
\end{aligned}
$$

Define

$$
\begin{aligned}
Z_k &= [\ (Z_k^1)' \quad \cdots \quad (Z_k^M)'\ ]' \\
Z^k &= \{Z_t\}_{t=1}^k \\
H_k &= [\ D_k' \quad \cdots \quad D_k'\ ]' \\
R_k &= \mathrm{diag}\{R_k^1, \cdots, R_k^M\}
\end{aligned}
$$

Given $\hat{X}_{k-1|k-1}^o = E^*[X_{k-1}|Z^{k-1}]$ and $P_{k-1|k-1}^o = \mathrm{MSE}(\hat{X}_{k-1|k-1}^o|Z^{k-1})$, it is well known that centralized fusion of X_k can be obtained as

$$
\begin{aligned}
&[\hat{X}_{k|k}^o, P_{k|k}^o] \\
&\quad = \mathrm{filtering}(\hat{X}_{k-1|k-1}^o, P_{k-1|k-1}^o, \tilde{Z}_{k|k-1}^o, C_{k|k-1}, S_k)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{Z}_{k|k-1}^o &= Z_k - H_k \hat{X}_{k|k-1}^o \\
C_{k|k-1} &= P_{k|k-1}^o H_k' \\
S_k &= H_k P_{k|k-1}^o H_k' + R_k
\end{aligned}
$$

For each of the x, y and z dimensions, order the standard deviations of all M sensors at the fusion center. Then we have

$$
\begin{aligned}
\sigma_x^{(1)} &\le \sigma_x^{(2)} \le \cdots \le \sigma_x^{(M)} \\
\sigma_y^{(1)} &\le \sigma_y^{(2)} \le \cdots \le \sigma_y^{(M)} \\
\sigma_z^{(1)} &\le \sigma_z^{(2)} \le \cdots \le \sigma_z^{(M)}
\end{aligned}
$$

where $\sigma_x^{(1)}$ is the best of $\{\sigma_x^i\}_{i=1}^M$, $\sigma_x^{(2)}$ the second best, $\sigma_x^{(M)}$ the M-th best, i.e., the worst. Likewise for y and z dimensions.

Now construct M virtual sensors at the fusion center, each observing the target position in all three dimensions. Denote $Z_k^{(i)} = [\ x_k^{(i)} \quad y_k^{(i)} \quad z_k^{(i)}\ ]'$ as the measurements of the i-th virtual sensor along all three dimensions. Suppose that $\sigma_x^{(i)}$ is the original $\sigma_x^{l_i}$, $\sigma_y^{(i)}$ the original $\sigma_y^{m_i}$, and $\sigma_z^{(i)}$ the original $\sigma_z^{n_i}$, $l_i, m_i, n_i = 1, 2, \cdots, M$. Then measurements of the i-th virtual sensor can be constructed as

$$x_k^{(i)} = x_k^{l_i},\ y_k^{(i)} = y_k^{m_i},\ z_k^{(i)} = z_k^{n_i}$$

Correspondingly,

$$Z_k^{(i)} = D_k X_k + V_k^{(i)}$$

where

$$\begin{aligned} V_k^{(i)} &= [\ \tilde{x}_k^{(i)} \quad \tilde{y}_k^{(i)} \quad \tilde{z}_k^{(i)}\ ]' = [\ \tilde{x}_k^{l_i} \quad \tilde{y}_k^{m_i} \quad \tilde{z}_k^{n_i}\ ]' \\ E[V_k^{(i)}] &= 0_{3\times 1} \\ R_k^{(i)} &= \text{cov}(V_k^{(i)}) = \text{diag}\{(\sigma_x^{l_i})^2, (\sigma_y^{m_i})^2, (\sigma_z^{n_i})^2\} \end{aligned}$$

Remark: The above procedure is called *recombination* in this paper. After recombination, virtual sensor 1 is the best of all M virtual sensors in terms of measurement accuracy in all three dimensions; virtual sensor 2 is the second best, and so on. Before recombination, it is hard to rank sensors if all three dimensions are considered.

Remark: To distinguish from the original sensors, we use "virtual sensors" to emphasize that they are the result of recombination and exist only conceptually at the fusion center.

Define $\bar{Z}_k$, $\bar{Z}^k$, $\bar{R}_k$ as the counterparts of Z_k, Z^k, R_k for the stacked measurements from the ranked virtual sensors. Given $\hat{X}_{k-1|k-1}^r = E^*[X_{k-1}|\bar{Z}^{k-1}]$ and $P_{k-1|k-1}^r = \text{MSE}(\hat{X}_{k-1|k-1}^r|\bar{Z}^{k-1})$, centralized fusion of X_k with the recombined measurements can be obtained as

$$\begin{aligned} &[\hat{X}_{k|k}^r, P_{k|k}^r] \\ &= \text{filtering}(\hat{X}_{k-1|k-1}^r, P_{k-1|k-1}^r, \tilde{Z}_{k|k-1}^r, \bar{C}_{k|k-1}, \bar{S}_k) \end{aligned}$$

where

$$\begin{aligned} \tilde{Z}_{k|k-1}^r &= \bar{Z}_k - H_k \hat{X}_{k|k-1}^r \\ \bar{C}_{k|k-1} &= P_{k|k-1}^r H_k' \\ \bar{S}_k &= H_k P_{k|k-1}^r H_k' + \bar{R}_k \end{aligned}$$

Then a natual question arises. If $\hat{X}_{k-1|k-1}^o = \hat{X}_{k-1|k-1}^r$ and $P_{k-1|k-1}^o = P_{k-1|k-1}^r$, are $\hat{X}_{k|k}^o = \hat{X}_{k|k}^r$ and $P_{k|k}^o = P_{k|k}^r$? That is, if centralized fusion with the original measurements (denoted by superscript "o") and centralized fusion with the recombined measurements (denoted by superscript "r") have the same estimation results at time $k-1$, will they be so at time k? The answer is yes for the linear measurement case. This can be easily shown as follows.

Let $p = 3(i-1)$, $\bar{l}_i = 3(l_i - 1)$, $\bar{m}_i = 3(m_i - 1)$, $\bar{n}_i = 3(n_i - 1)$. Construct a $3M \times 3M$ square matrix T_k such that all its entries are zeros except that the $(p+1, \bar{l}_i + 1)$th, $(p+2, \bar{m}_i + 2)$th, and $(p+3, \bar{n}_i + 3)$th entries are all 1. Then it can be easily seen that

$$\bar{Z}_k = T_k Z_k$$

T_k is nothing but a $3M \times 3M$ elementary row transformation matrix, so it is invertible. Thus by the property of LMMSE estimation, centralized fusion with the original measurements and centralized fusion with the recombined measurements will be the same if they have the same initialization.

It is clear now that recombination does not improve performance of centralized fusion with linear measurements. This also explains why we do not have the concept of virtual sensors above.

B. Radar Measurements

Similarly as above, we can also recombine multi-radar measurements at the fusion center. For each measurement component, i.e., range, bearing and elevation, order the standard deviations of all radars at the fusion center.

Now construct M virtual radars at the fusion center, each observing range, bearing and elevation of the target. Denote $Z_k^{(i)} = [\ x_k^{(i)} \quad y_k^{(i)} \quad z_k^{(i)}\ ]'$ as the converted measurements of the i-th virtual radar in all three dimensions. Suppose that $\sigma_r^{(i)}$ is the original $\sigma_r^{l_i}$, $\sigma_b^{(i)}$ the original $\sigma_b^{m_i}$, and $\sigma_e^{(i)}$ the original $\sigma_e^{n_i}$, $l_i, m_i, n_i = 1, 2, \cdots, M$. Then converted measurements of the i-th virtual radar can be constructed as

$$\begin{aligned} x_k^{(i)} &= r_k^{l_i} \cos e_k^{n_i} \cos b_k^{m_i} \\ y_k^{(i)} &= r_k^{l_i} \cos e_k^{n_i} \sin b_k^{m_i} \\ z_k^{(i)} &= r_k^{l_i} \sin e_k^{n_i} \end{aligned}$$

Define

$$\begin{aligned} \bar{Z}_k &= [\ (Z_k^{(1)})' \quad \cdots \quad (Z_k^{(M)})'\ ]' \\ \bar{Z}^k &= \{\bar{Z}_t\}_{t=1}^k \end{aligned}$$

Given $\hat{X}_{k-1|k-1}^r = E^*[X_{k-1}|\bar{Z}^{k-1}]$ and $P_{k-1|k-1}^r = \text{MSE}(\hat{X}_{k-1|k-1}^r|\bar{Z}^{k-1})$, from Theorem 1, centralized fusion of X_k with the recombined measurements can be obtained as

$$\begin{aligned} &[\hat{X}_{k|k}^r, P_{k|k}^r] \\ &= \text{filtering}(\hat{X}_{k-1|k-1}^r, P_{k-1|k-1}^r, \tilde{Z}_{k|k-1}^r, \bar{C}_{k|k-1}, \bar{S}_k) \end{aligned}$$

where

$$\begin{aligned} \tilde{Z}_{k|k-1}^r &= \bar{Z}_k - \hat{\bar{Z}}_{k|k-1} \\ \hat{\bar{Z}}_{k|k-1} &= [\ (\hat{Z}_{k|k-1}^{(1)})' \quad \cdots \quad (\hat{Z}_{k|k-1}^{(M)})'\ ]' \\ \bar{C}_{k|k-1} &= [\ C_{k|k-1}^{(1)} \quad \cdots \quad C_{k|k-1}^{(M)}\ ] \\ \bar{S}_k &= \begin{bmatrix} S_k^{(1)} & S_k^{(1),(2)} & \cdots & S_k^{(1),(M)} \\ S_k^{(2),(1)} & S_k^{(2)} & \cdots & S_k^{(2),(M)} \\ \vdots & \vdots & \ddots & \vdots \\ S_k^{(M),(1)} & S_k^{(M),(2)} & \cdots & S_k^{(M)} \end{bmatrix} \end{aligned}$$

$$S_k^{(i),(j)} = \Omega_{(i)} P^r_{k|k-1}([1,3,5],[1,3,5])\Omega_{(j)}$$
$$i \neq j,\ i,j = 1,\cdots,M$$
$$\Omega_{(i)} = \text{diag}\{\lambda_1^{m_i}\mu_1^{n_i}, \lambda_1^{m_i}\mu_1^{n_i}, \mu_1^{n_i}\}$$

The fact that recombination does help improve performance in the case of radar measurements is verified by the numerical examples in the next section.

V. Examples and Discussions

Next, we compare performance of the recursive LMMSE multi-radar centralized fusion with the original measurments (CF-o) and with recombined measurements (CF-r) to see whether recombination helps.

Consider the following three-dimensional discrete-time constant velocity motion model [17] of a target

$$X_k = F_{k-1}X_{k-1} + G_{k-1}W_{k-1}$$

where

$$X_k = [\ \mathsf{x}_k \quad \dot{\mathsf{x}}_k \quad \mathsf{y}_k \quad \dot{\mathsf{y}}_k \quad \mathsf{z}_k \quad \dot{\mathsf{z}}_k\ ]',\ X_0 \sim \mathcal{N}(\bar{X}_0, P_0)$$
$$F_k = \text{diag}\{\mathcal{F},\mathcal{F},\mathcal{F}\},\ G_k = \text{diag}\{\mathcal{G},\mathcal{G},\mathcal{G}\}$$
$$\mathcal{F} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \mathcal{G} = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}, T = 2\text{ s}$$
$$W_k \sim \mathcal{N}([\ 0 \quad 0 \quad 0\ ]', \text{diag}\{(q_k^x)^2, (q_k^y)^2, (q_k^z)^2\})$$

Three radars co-located at the origin of the Cartesian coordinates were used to observe the target motion. For the i-th radar, the standard deviations of the range, bearing and elevation measurement errors are σ_r^i, σ_b^i and σ_e^i, respectively, $i = 1,2,3$.

The estimation accuracy measures used are root mean-squared (RMS) position and velocity errors. The filter credibility measures used are the noncredibility index (NCI) [18] and inclination indicator (II) [18], which were shown to be better than the average normalized estimation error squared (ANEES) widely used in the tracking literature.

All results below were averaged over 500 Monte Carlo runs. All filters were initialized with

$$\hat{X}_{0|0} = \bar{X}_0,\ P_{0|0} = P_0$$

and

$$\bar{X}_0 = [\ -50\text{km} \quad 500\text{m/s} \quad 200\text{km} \quad 0\text{m/s} \quad 100\text{km} \quad 0\text{m/s}\ ]'$$
$$P_0 = \text{diag}\{10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2, 10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2, 10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2\}$$
$$q_k^x = q_k^x = q_k^x = 0.01\text{m/s}^2$$

Two typical cases were considered to illustrate the use of recombination. In the first case, the difference in measurement accuracy of different sensors for the same component is huge. In the second case, the difference is not significant.

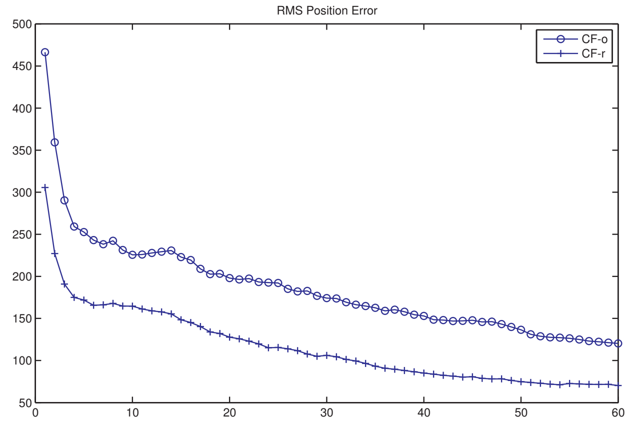

Figure 1. RMS Position Error (case 1)

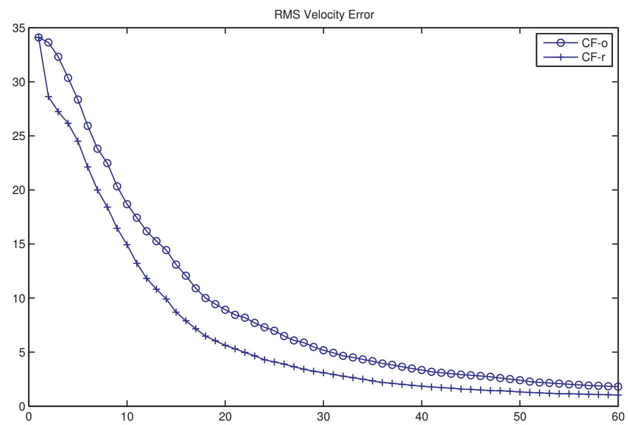

Figure 2. RMS Velocity Error (case 1)

Figs. 1 through 4 show comparison results for **Case 1**, in which

$$\sigma_r^1 = 10\text{m},\ \sigma_b^1 = 10\text{mrad},\ \sigma_e^1 = 100\text{mrad};$$
$$\sigma_r^2 = 100\text{m},\ \sigma_b^2 = 1\text{mrad},\ \sigma_e^2 = 10\text{mrad};$$
$$\sigma_r^3 = 1000\text{m},\ \sigma_b^3 = 100\text{mrad},\ \sigma_e^3 = 1\text{mrad};$$

From the simulation results, it can be seen that in terms of estimation accuracy, centralized fusion with recombined measurements beats the one with the original measurements significantly, especially in position (by about 100m). For filter credibility, two fusion methods are almost the same and they are both credible.

Figs. 5 through 8 show comparison results for **Case 2**, in which

$$\sigma_r^1 = 10\text{m},\ \sigma_b^1 = 20\text{mrad},\ \sigma_e^1 = 50\text{mrad};$$
$$\sigma_r^2 = 100\text{m},\ \sigma_b^2 = 10\text{mrad},\ \sigma_e^2 = 20\text{mrad};$$
$$\sigma_r^3 = 200\text{m},\ \sigma_b^3 = 50\text{mrad},\ \sigma_e^3 = 10\text{mrad};$$

From the simulation results, it can be seen that performance difference is still significant, especially during the steady state. While both methods are considered credible, the one with the original measurements is somewhat more credible.

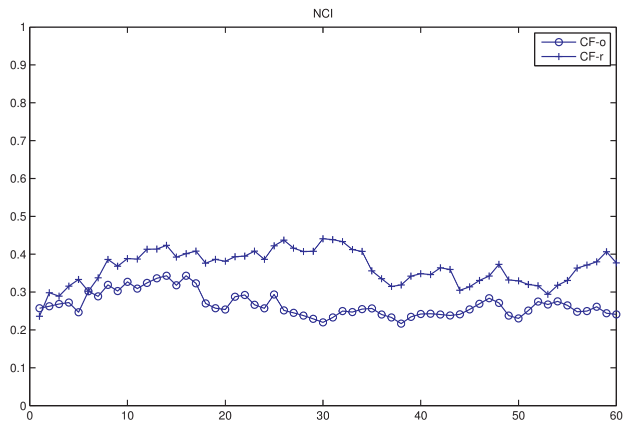

Figure 3. NCI (case 1)

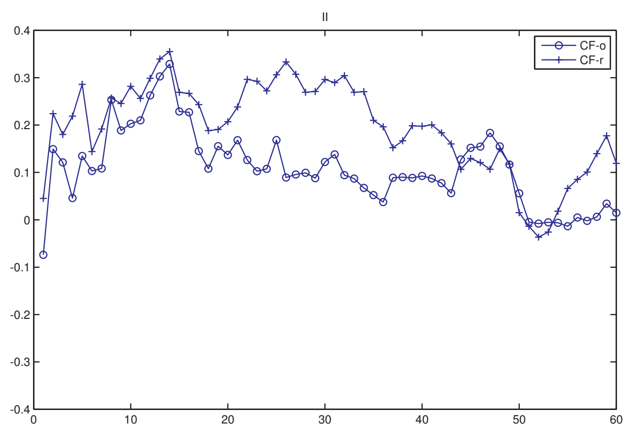

Figure 4. II (case 1)

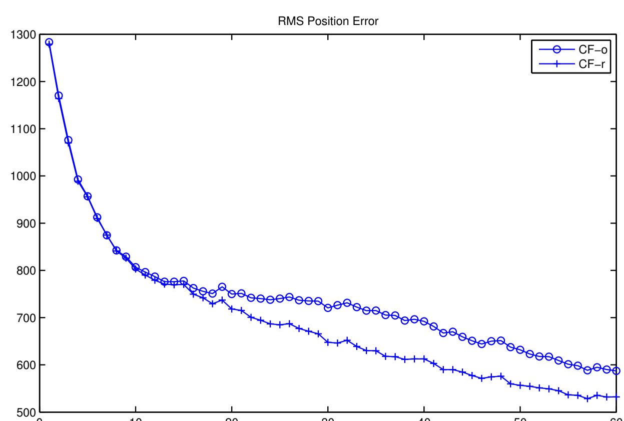

Figure 5. RMS Position Error (case 2)

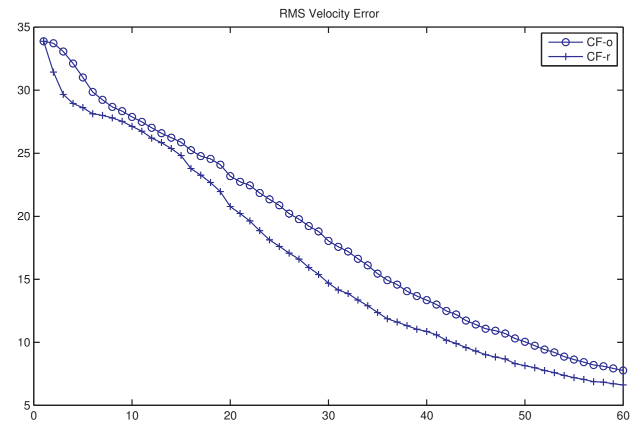

Figure 6. RMS Velocity Error (case 2)

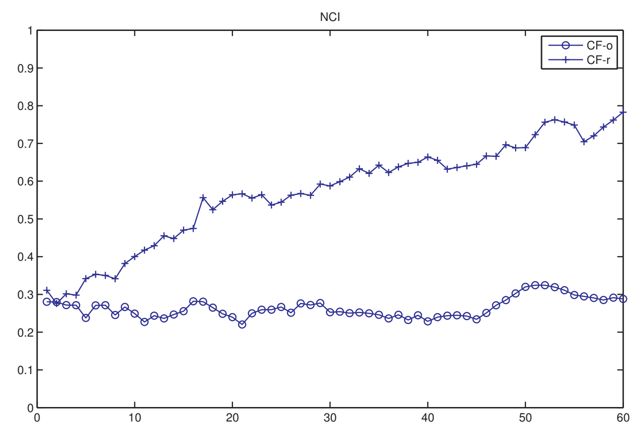

Figure 7. NCI (case 2)

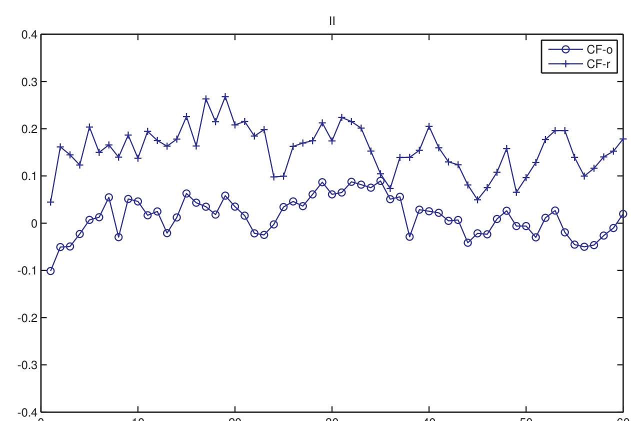

Figure 8. II (case 2)

Remark: LMMSE estimation is the best in the linear class, but "in which linear class?" is a crucial question. Centralized fusion with the original measurements is linear in the original measurements, while centralized fusion with recombined measurements is linear in the recombined measurements. Due to the nonlinear conversion from spherical coordinates to Cartesian coordinates, they are two different linear classes. So from the above numerical examples, it can be seen that the linear class of the recombined measurements is better than the one of the original measurements in terms of estimation accuracy and filter credibility. When measurements are linear, recombination does not change the linear class. This explains why recombination helps improve performance of multi-radar recursive LMMSE centralized fusion.

VI. Conclusions

By following the classical measurements stacking method, recursive LMMSE filtering for a single radar in a track-while-scan system has been extened to the case of centralized fusion with multiple radars. It is then shown that centralized fusion done this way does not provide the best LMMSE performance achievable. Interestingly, centralized fusion by following the measurement stacking method can be improved simply by ranking scalar measurements dimension by dimension and recombining these measurements. This differs from the case of centralized fusion with linear measurements.

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