

Capacity of Hybrid Networks

Çağatay Çapar, Dennis Goeckel, Don Towsley, Richard Gibbens, and Ananthram Swami

Abstract—The rate at which information can be exchanged between nodes in a hybrid network is investigated. The network includes n ad hoc nodes and b base stations, where the ad hoc nodes can share information through the wired infrastructure in addition to wireless ad hoc communication. The per-node throughput scaling achievable in this hybrid network as n (and b) grows was studied in previous work for both one-dimensional and two-dimensional networks. This work completes this previous work in three ways. First, the wired network is modeled more realistically where the base stations are only connected to their nearest neighbors with finite bandwidth links as opposed to a fully connected network with infinite bandwidth. Second, through cut-set methods, we show upper bounds on the throughput which matches the lower bounds for a wide range of values of b . Finally, by establishing a new result on the maximum of a sequence of Poisson random variables (which is of independent interest), we improve previous lower bounds in the extreme case where the number of base stations scale almost on the same order with the number of ad hoc nodes, and also show a matching upper bound in that case for one-dimensional networks.

I. INTRODUCTION

Consider a wireless ad hoc network where n nodes are placed in a two-dimensional region, and are randomly matched into n source-destination pairs. In [1], Gupta and Kumar showed that the rate of information that can be shared by each pair scales as $1/\sqrt{n}$, which shows that the capacity of wireless networks does not scale well. Successive works to [1] try to improve this scaling by considering cases not assumed in the original network or communication model, such as node mobility [2], a sophisticated physical layer scheme [3], etc.

One perhaps more straightforward way to increase the capacity of an ad hoc network is to add infrastructure, i.e., an overlay wired network which helps carry information between the wireless nodes. In that case the network includes b base stations in addition to n wireless nodes, where the base stations are connected through a wired network. This type of a network is commonly referred to as a “hybrid network” [4], [5], and

the capacity that can be achieved by ad hoc nodes in a hybrid network is the problem of interest in this paper.

The scaling of capacity in hybrid networks has been studied in a number of works under different network and communication models starting with [4], [6]. Some of these works point to the same observation that whether or not infrastructure improves capacity scaling depends on how the number of base stations b scales as compared to n . In particular, the works of [4], [7], [8] show that the per-node throughput scaling of $1/\sqrt{n}$ remains the same if b grows at most with $\sqrt{n}$. Only after that point, does the capacity start to increase. On the other hand, [6], [9] studies the special case where $b = \Theta(n)$, and explore the possibility of providing each pair $\Theta(1)$ throughput. In particular, [9] shows that this is indeed possible for a fraction of node pairs arbitrarily close to one. In other related work, the multicast capacity in a hybrid network is studied in [10], while [11] extends this work to the case where the wireless nodes are mobile. Finally, [12] studies the case where the base stations are equipped with multiple antennas, and fading is assumed for the wireless channel.

Another work which studies capacity scaling in hybrid networks is [13]. Compared to other work in this area, two major differences in [13] are the interference model assumed, and the fact that one-dimensional hybrid networks are studied in addition to two-dimensional networks. Most of the work in hybrid networks use the simpler Protocol Model [1] for treating interference between wireless transmissions, where it is assumed that wireless transmissions cause no interference outside a certain radius from the transmitter and the same fixed rate for each active transmission can be achieved as long as this interference radius condition is satisfied. [13] uses a more realistic model which takes into account all of the interference power coming from the rest of the network and uses a model where the rate achieved is a function of the received signal-to-interference-and-noise-ratio (SINR). In addition, their results show that, in contrast to two-dimensional networks, the added infrastructure provides increased capacity even for a small number of base stations in one-dimensional hybrid networks, where pure ad hoc capacity is $1/n$. In particular, they show that nodes in a one-dimensional hybrid network can achieve a per-node throughput that scales as b/n . This linear increase is shown to hold for $b \log b \leq n$. In fact the same result also applies to two-dimensional networks; however, note that only when b grows faster than $\sqrt{n}$, does b/n provide a better throughput than pure ad hoc scaling of $1/\sqrt{n}$ for the 2-D case.

In this paper, we assume the same realistic interference model in [13], and study both 1-D and 2-D hybrid networks. Our results improve and complement the results in [13] in three ways. First, we assume a more realistic wired network,

C. Çapar, D. Goeckel, and D. Towsley are with the University of Massachusetts Amherst, Amherst, MA, 01003 USA. Email: {ccapar,goeckel}@ecs.umass.edu, towsley@cs.umass.edu. R. Gibbens is with the Computer Laboratory, University of Cambridge, Cambridge, UK. Email: richard.gibbens@cl.cam.ac.uk. A. Swami is with the Army Research Laboratory, Adelphi, MD, 20783 USA. Email: aswami@arl.army.mil.

This research was sponsored by the U.S. Army Research Laboratory and the U.K. Ministry of Defence and was accomplished under Agreement Number W911NF-06-3-0001. The views and conclusions contained in this document are those of the author(s) and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Army Research Laboratory, the U.S. Government, the U.K. Ministry of Defence or the U.K. Government. The U.S. and U.K. Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

Part of the results in this work was presented in a short paper in ACITA 2011.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 20 SEP 2012		2. REPORT TYPE		3. DATES COVERED 00-00-2012 to 00-00-2012	
4. TITLE AND SUBTITLE Capacity of Hybrid Networks				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Massachusetts,Amherst,MA,01003				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES Presented at the Annual Conference of International Technology Alliance (ACITA) 2012 held 17-21 Sep in Southampton, UK. U.S. Government or Federal Rights License					
14. ABSTRACT The rate at which information can be exchanged between nodes in a hybrid network is investigated. The network includes n ad hoc nodes and b base stations, where the ad hoc nodes can share information through the wired infrastructure in addition to wireless ad hoc communication. The per-node throughput scaling achievable in this hybrid network as n (and b) grows was studied in previous work for both one-dimensional and two-dimensional networks. This work completes this previous work in three ways. First, the wired network is modeled more realistically where the base stations are only connected to their nearest neighbors with finite bandwidth links as opposed to a fully connected network with infinite bandwidth. Second, through cutset methods, we show upper bounds on the throughput which matches the lower bounds for a wide range of values of b. Finally by establishing a new result on the maximum of a sequence of Poisson random variables (which is of independent interest), we improve previous lower bounds in the extreme case where the number of base stations scale almost on the same order with the number of ad hoc nodes, and also show a matching upper bound in that case for one-dimensional networks.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 8	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

where base stations are only connected to their nearest neighbors with wired links of finite bandwidth $f(n)$ which grows with n . This shows how the bandwidth limitations in the wired network can also constrain the throughput as opposed to the case in [13] where the wired network is assumed to have infinite capacity. Second, note that the results in [13] represent achievable results, i.e., they are shown by a construction providing the stated throughput values. Hence, it is of interest to consider whether these results can be further improved. Our second contribution is to show upper bounds on capacity by exploiting cut-set results adopted from recent work [14]. Our upper bounds show that the lower bounds shown in [13] are indeed tight for the range $b \log b \leq n$. Finally, we study the case where $b \log b \geq n$. By establishing a new result on the maximum of a sequence of Poisson random variables (with slowly growing mean values), we show that the achievable results for this range can be further improved, and in the one-dimensional case, match the upper bound we prove separately for this range of values.

The rest of the paper is organized as follows. Section II presents the network and communication model and the main results, which consists of lower and upper bounds. Section III provides a sketch of how these bounds are obtained. The cases of 1-D and 2-D networks are studied in Sections IV and V, respectively, and Section VI is the conclusion.

II. MODEL AND THE MAIN RESULTS

A. Network Model

The hybrid network we model consists of static ad hoc nodes and base stations. We consider both one-dimensional and two-dimensional networks. The 1-D network is inside the interval $[0, n]$, and the 2-D network is inside the square $[0, \sqrt{n}] \times [0, \sqrt{n}]$ (see Figs. 1, 2). Ad hoc nodes are distributed randomly according to a homogeneous Poisson point process with density $\lambda = 1$, so there are n nodes in the network on average in both 1-D and 2-D cases. Base stations are placed regularly in the network with a total of $b(n)$ base stations, where $b(n) \rightarrow \infty$, as n grows. For the 1-D network, the base stations are on a wired linear network with each base station connected to its nearest neighbors. In the 2-D case, the base stations are on the vertices of a square grid network. The capacity of each wired link is $f(n) < \infty$, where $f(n) \rightarrow \infty$, as n grows. Ad hoc nodes are matched into source-destination pairs uniformly at random such that each node is the destination for exactly one source node, and is the source for exactly one destination node. Base stations are neither the source nor the destination of any flow of information, and simply help carry the traffic between ad hoc node pairs.

B. Channel Model

The communication over the wireless channel is modeled such that, when node A transmits with transmit power P , the received power at node B is

$$P_{\text{rev},B} = P/d_{AB}^\alpha,$$

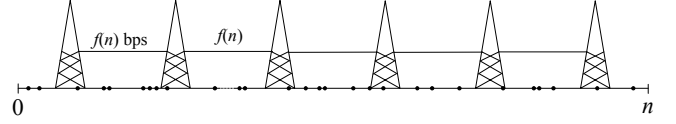


Fig. 1. The one-dimensional hybrid network consists of randomly placed ad hoc nodes (represented by dots) and regularly placed base stations in the interval $[0, n]$. The wired links between base station pairs can each carry $f(n)$ bits per second.

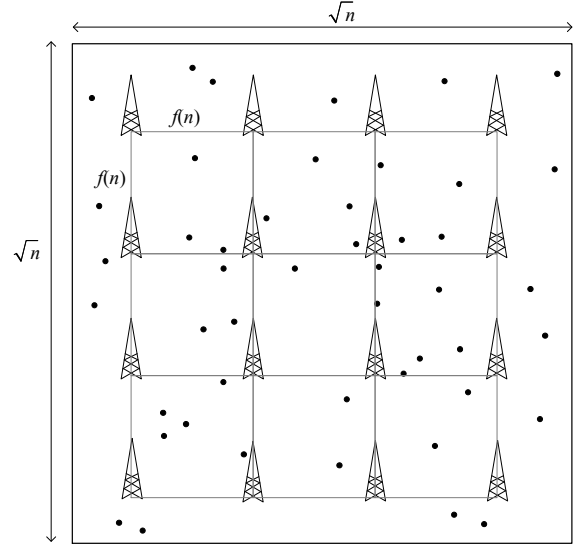


Fig. 2. The two-dimensional hybrid network consists of randomly placed ad hoc nodes (represented by dots) and regularly placed base stations on a square grid network inside the square $[0, \sqrt{n}] \times [0, \sqrt{n}]$. The wired links between base station pairs can each carry $f(n)$ bits per second.

where d_{AB} is the distance between nodes A, B , and $\alpha > 1$ in 1-D, $\alpha > 2$ in 2-D, is the path loss exponent. The received signal-to-interference-plus-noise ratio (SINR) at B is then

$$\text{SINR}_B = \frac{P_{\text{rev},B}}{N_0 + I_B}, \quad (1)$$

where N_0 is the power in the additive white Gaussian noise (AWGN) at the receiver, and I_B is the interference received at node B due to other transmissions in the network following the same path loss model. The same model is used for all wireless communications including communications between a node and a base station. The transmit power at the base stations can be higher than the ad hoc nodes; however, as will be seen in the analysis, the only requirement is that the base station can connect to the nearest ad hoc nodes.

The rate of information that can be sent from node A to B over the wireless link is a function of SINR_B . In this paper, we assume an SINR threshold model, where the transmission from A to B is successful if $\text{SINR}_B > \gamma$ for some threshold $\gamma > 0$. In that case the rate of information that can be sent from A to B is $W = \log_2(1 + \gamma)$ bits per second, i.e., a constant rate independent of n is achieved. This SINR model is more realistic than the Protocol Model employed in previous work

on hybrid networks [4], [10]. The rate of information on any wired link is $f(n)$ bps.

C. Main Results

Based on the above network and channel models, our main results in this paper are given in the following two theorems for the 1-D and 2-D hybrid networks, respectively.¹

Theorem 1: Consider a one-dimensional hybrid network inside the interval $[0, n]$, where ad hoc nodes are placed according to a Poisson point process with density $\lambda = 1$, and $b(n)$ base stations are regularly placed. The number of base stations $b(n) \rightarrow \infty$, as $n \rightarrow \infty$ with $b(n) = O(n)$. The links between base stations have a capacity of $f(n)$ bits per second. The per-node throughput shared by ad hoc nodes $T(n)$ in this network is upper-bounded as

$$T(n) = \begin{cases} O(\min\{\frac{b}{n}, \frac{f}{n}\}), & b \log b = O(n), \\ O\left(\min\left\{\frac{\log(\frac{\log b}{n/b})}{\log b}, \frac{f}{n}\right\}\right), & \text{otherwise,} \end{cases}$$

with high probability (w.h.p.), i.e., with probability one as $n \rightarrow \infty$. Furthermore, this upper bound is achievable w.h.p.

Theorem 2: Consider a two-dimensional hybrid network inside the square $[0, \sqrt{n}] \times [0, \sqrt{n}]$, where ad hoc nodes are placed according to a Poisson point process with density $\lambda = 1$, and $b(n)$ base stations are placed on the vertices of a square grid wired network with capacity of $f(n)$ bits per second on each link. The average per-node throughput $T(n)$ in this network is upper-bounded as

$$T(n) = \begin{cases} O(\frac{1}{\sqrt{n}}), & b = O(\sqrt{n}) \\ & \text{or } \sqrt{b}f = O(\sqrt{n}), \\ O(\min\{\frac{b}{n}, \frac{\sqrt{b}f}{n}\}), & b = w(\sqrt{n}), b \log b = O(n) \\ & \text{and } \sqrt{b}f = w(\sqrt{n}), \\ O\left(\min\left\{\sqrt{\frac{n}{b}} \frac{\log(\frac{\log b}{n/b})}{\log b}, \frac{\sqrt{b}f}{n}\right\}\right), & b \log b = w(n) \\ & \text{and } \sqrt{b}f = w(\sqrt{n}). \end{cases}$$

w.h.p. The upper bounds for the first two cases are achievable. For the last case (i.e., for $b \log b = w(n)$ and $\sqrt{b}f = w(\sqrt{n})$) we have the following lower bound:

$$T(n) = \Omega\left(\min\left\{\frac{\log(\frac{\log b}{n/b})}{\log b}, \frac{\sqrt{b}f}{n}\right\}\right) \text{ w.h.p.}$$

Remarks: Although the conditions and the expressions for the throughput values look complicated (especially for the 2-D case), the overall picture can be easily summarized. First of all, note that in almost all cases, the throughput is the minimum

¹The following order notation is used. $f(n) = O(g(n))$ if there exists a constant k such that $f(n) \leq kg(n)$ for n sufficiently large (for all $n > n_0$ for some n_0). $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$, and $g(n) = O(f(n))$. $f(n) \sim g(n)$ if $f/g \rightarrow 1$, $f = o(g)$ if $f/g \rightarrow 0$, which is equivalent to $g = w(f)$. Finally, we say $f(n) = O(g(n))$ w.h.p. if $P(f(n) \leq kg(n)) \rightarrow 1$ for some k .

of two values, the first depending only on the number of base stations b and the second depending on the wired link bandwidth f and/or b . If, as assumed in [13], f is taken to be infinity, then we are left with only the first expression.

Second, an important condition is whether $b \log b = O(n)$ is true or not. Roughly, this means that we have some throughput expression for a wide range of b values, but when b scales *almost on the same order* with n , the throughput value switches to another phase. In that case, the simple b/n value is no longer true and we have a more complicated expression.

The only difference in 2-D is the fact that pure ad-hoc communication already achieves a scaling of $1/\sqrt{n}$ and when $b = O(\sqrt{n})$ or $\sqrt{b}f = O(\sqrt{n})$ is true, the added infrastructure does not improve on this value. Only when these thresholds are exceeded (by a fast enough scaling of f, b), do we start to see throughput values that depend on b, f and in that case we have similar results with 1-D.

III. PROOF OVERVIEW

Our main results provide both lower and upper bounds on the per-node throughput. The upper bounds are proved using a cut-set technique, and the lower bounds are proved by showing a construction which achieves the stated throughput. For the upper bounds, we briefly explain the idea behind the cut-set method. For the lower bounds, we sketch the constructions used for both proofs, and motivate why they achieve the stated throughput values.

A. Cut-set Upper Bounds on Throughput

Consider a graph representing a communication network where the vertices are the nodes, and the edges are the communication links with their associated bandwidth values. If the set of vertices is partitioned into two subsets (cuts), the cut-set is the set of edges connecting one subset to the other, i.e., the set of edges *crossing* the cut. Then one can add the available rate (bandwidth) on all crossing edges, which gives a quick upper bound on how many bits can be carried from the nodes in the first subset to the nodes in the second subset.

In our case, an edge in the graph is either a wired link between two base stations or a wireless link. The wired links can be easily represented by edges with their given bandwidth values. For wireless links, however, one should consider the interference caused by simultaneous transmissions. This can be done by considering the communication model given in Section II.B. In particular, “wireless” edges crossing a given cut can be drawn between node pairs which *collectively* satisfy the SINR condition on each link. A more formal definition of cut-set for an ad hoc network can be found in [14]. Here, it suffices to say that for a wired edge, the bandwidth is $f(n)$ bps, whereas a constant rate is achieved on a crossing wireless link (if it is active), which can be taken as one without affecting the scaling results. An example is shown in Fig. 3.

B. Constructions for Achievability Results

In our constructions, the network is considered as being divided by the base stations into b regions, called “cells”.

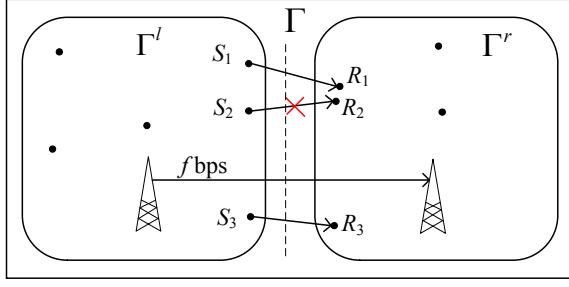


Fig. 3. The cut Γ divides the network into two parts, Γ^l, Γ^r . We are interested in how many bits can be carried from nodes in Γ^l to Γ^r . One wired edge crosses the cut resulting in $f(n)$ bits per second. Although there may be three wireless connections across the cut, due to interference, the connection between nodes S_2, R_2 is not active as it interferes with the connection from S_1 to R_1 . Each wireless edge has a constant rate. Hence, the *cut capacity* for Γ is $2 + f$, which is an upper bound on how many bits can be carried from nodes in Γ^l to nodes in Γ^r .

Hence, there are b cells each with roughly n/b nodes. A source communicates to its destination in three steps. It first delivers the packet to the closest base station through multihop connection (upload phase). The message is then carried through the wired network to the destination base station, i.e., the closest base station to the destination node (wired phase), which then delivers the message to the destination node (download phase).

What determines the throughput achieved by this construction is the *relaying load* in each phase. Here we take the 1-D case as an example. In the upload and download phases, the nodes have to relay packets from nodes only in that cell, so the number of nodes in a cell determines that cell's relaying load. Each cell has roughly n/b nodes, and it can be shown that for most values of b , even the busiest cell has at most a constant factor of n/b nodes. What serves this load is a wireless link of constant rate, which translates to a per-node rate on the order of b/n bits per second. In the wired phase, the information belonging to the whole network is carried. Hence, a single wired link of capacity f bps may need to carry data for all n nodes, which gives an achievable rate of f/n bps. The smaller of these two values, $\min\{b/n, f/n\}$, determines the overall achievable per-node throughput.

The only exception to the above result is when the number of base stations grow such that it is no longer true that $b \log b = O(n)$. In that case, one can show that the number of nodes in the busiest cell deviates from the mean, which results in the throughput value stated in Theorem 1 for the second case. A very similar three-step construction is used for the 2-D case, resulting in the same expressions for the upload and download phases. The wired phase brings the term $\sqrt{b}f/n$ in the 2-D case. Finally, in the 2-D case, the capacity of pure ad hoc communication is already $1/\sqrt{n}$, hence the construction described is used only when it achieves a throughput better than $1/\sqrt{n}$.

IV. ONE-DIMENSIONAL NETWORK

Theorem 1 given in Section II-C is our main result for the one-dimensional hybrid network. Before we prove the upper

and lower bounds, we state and prove a key result which is used for both 1-D and 2-D cases.

The regularly placed b base stations can be thought of as dividing the network into b equal size “cells”. In 1-D, each cell is a subinterval of length n/b , $[0, n/b]$ being the first cell (see Fig. 5). In 2-D, the region $[0, \sqrt{n}] \times [0, \sqrt{n}]$ is divided into b cells, where each cell is a square of size $\sqrt{n/b} \times \sqrt{n/b}$ (see Fig. 7). Consider each base station as serving the nodes inside its corresponding cell. As noted in Section III, the overall achievable per-node throughput depends on the maximum number of nodes in any cell, i.e., the number of nodes in the busiest cell, which we denote by M_b . The following lemma states how this number scales with b . This key result is what enables us to improve the previous achievable results in [13] for the case $b \log b = w(n)$.

Lemma 1: Suppose there are $b(n)$ base stations with $b(n) \rightarrow \infty$, as $n \rightarrow \infty$, and $b(n) = O(n)$. Let X_i be the number of nodes in the i th cell, $1 \leq i \leq b$, and let $M_b = \max\{X_1, X_2, \dots, X_b\}$. Then,

- 1) If $b \log b = O(n)$, then for some $c < \infty$ independent of n , $P(M_b \leq cn/b) \rightarrow 1$.
- 2) Let

$$h(b) = \frac{\log b}{\log(\frac{\log b}{n/b})} \quad (2)$$

If $b \log b = \omega(n)$, then for any $\varepsilon > 0$, $P((1 - \varepsilon)h(b) < M_b \leq (1 + \varepsilon)h(b)) \rightarrow 1$.

Proof:

- 1) Note that $\{X_i, 1 \leq i \leq b\}$ are i.i.d. Poisson random variables with mean n/b . Then, using a Chernoff bound argument, for any $c > 0$, and $s > 0$

$$P(X_i > cn/b) \leq E(\exp(sX_i)) / \exp(cn/b), \quad 1 \leq i \leq b$$

where $E(\exp(sX_i)) = \exp((e^s - 1)n/b)$. Then, using $s = 1$,

$$P(X_i > cn/b) \leq \frac{1}{\exp((c + 1 - e)n/b)}, \quad 1 \leq i \leq b$$

Then,

$$P(M_b \leq cn/b) \geq \left(1 - \frac{1}{\exp((c + 1 - e)n/b)}\right)^b \quad (3)$$

Given $b \log b = O(n)$, by definition, there exists some $k > 0$ independent of n , such that for some $n_0 > 0$, $b \log b \leq kn$, for $n > n_0$. Hence, for b sufficiently large, $n/b \geq \log b/k$. Choosing $c = k + 2$,

$$\begin{aligned} P(M_b \leq cn/b) &\geq \left(1 - \frac{1}{\exp((c + 1 - e)n/b)}\right)^b \\ &\geq \left(1 - \frac{1}{\exp((k + 3 - e) \log b/k)}\right)^b \\ &= \left(1 - \frac{1}{b^{(k+3-e)/k}}\right)^b \\ &\rightarrow 1, \quad b \rightarrow \infty. \end{aligned}$$

2) Note that the sequence of random variables $\{X_i, 1 \leq i \leq b\}$, are i.i.d. Poisson random variables with mean $\lambda_b = n/b$, i.e., the mean changes with b . Hence, a triangular array approach is more appropriate. For a given b , denote the sequence $\{X_i^{(b)}, 1 \leq i \leq b\}$, and $M_b = \max\{X_i^{(b)}, 1 \leq i \leq b\}$. Let $F_b(x) = P(X_i^{(b)} \leq x)$ be their common distribution function. It is known that in the case where $\lambda_b = \lambda$ is constant, there is no limiting distribution and M_b converges to one of two consecutive integers [15]. On the other hand, when λ_b grows with b , it is shown in [16] that the maximum M_b exhibits two different behaviors depending on the growth rate of λ_b . The proof in the first part shows that M_b is bounded by a constant factor of λ_b when $\lambda_b = \Omega(\log b)$, which is consistent with the result in [16]. In the case where $\lambda_b = o(\log b)$, [16] shows that M_b converges to one of two integers, as would be with constant mean $\lambda_b = \lambda$ [15]. In other words, when $\lambda_b = o(\log n)$, there is a sequence of integers I_b such that

$$P(M_b \in \{I_b, I_b + 1\}) \rightarrow 1, \text{ as } b \rightarrow \infty. \quad (4)$$

However, it is of interest here to consider a question left open in the literature on the maximum of Poisson random variables, which is how M_b , i.e., I_b scales when $\lambda_b = o(\log b)$. Note that for constant $\lambda_b = \lambda$ it is shown in [17] that $I_b \sim \log b / \log \log b$, which is independent of the value of λ .

Here, we study the asymptotic behavior of I_b by adopting a similar technique to that used in [17] for constant λ_b . This proof is given in the Appendix. ■

Proof of Theorem 1:

The proof is divided into two parts. The first part shows the upper bound by cut-set arguments. The second part shows the achievability result by presenting a construction.

A. Upper Bound on Capacity

We prove the upper bound by a cut argument on the graph representing the hybrid network. Consider the network in Fig. 4 (a), where the cut Γ_1 partitions the nodes into two subsets: one to the left and the other to the right of the point $n/2$, denoted by Γ_1^l, Γ_1^r , respectively. Γ_1^l includes the ad hoc nodes and the $b/2$ base stations in the left half of the interval, Γ_1^r includes the rest of the nodes. The “cut capacity” of Γ_1 in the direction from left to right is the sum of the bandwidths on the links crossing the cut in this direction, and it gives an upper bound on the rate of information that can be carried from left to right in the network. The wired network crosses the cut by a single link with bandwidth f . The ad hoc wireless network, on the other hand, can achieve only one transmission across the cut at a given time, resulting in a bandwidth of 1 [14]. There are $\Theta(n)$ source-destination pairs that need to cross the cut from left to right w.h.p. Hence, the per-node throughput $T(n)$ is upper bounded by $\Theta(1/n + f/n) = \Theta(f/n)$.

For the same network, consider the cut Γ_2 (Fig. 4 (b)). Γ_2 partitions the network into two subsets Γ_2^l, Γ_2^r , where Γ_2^l

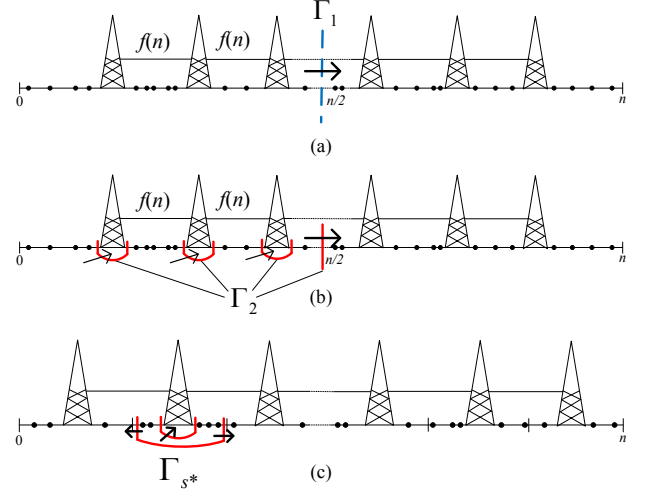


Fig. 4. Three cuts are shown on the 1-D dimensional network. The first cut Γ_1 partitions the network into two halves: the ad hoc nodes and base stations to the left and to the right of the point $n/2$, respectively. This cut is crossed by one wired and one wireless link from left to right, hence the cut capacity is $\Theta(1 + f)$ which serves $\Theta(n)$ nodes w.h.p. The cut Γ_2 has the ad hoc nodes to the left of the point $n/2$ on one side, and the rest of the ad hoc nodes and all base stations on the other side. This cut is crossed by $b/2$ node-to-base-station wireless edges, and one node-to-node wireless edge resulting in a capacity of $\Theta(1 + b/2)$, which again serves $\Theta(n)$ nodes w.h.p. The last cut is drawn around the ad hoc nodes in the cell with the most number of nodes. This cut can be crossed by at most three wireless edges resulting in a capacity of $\Theta(3)$, which serves M_b nodes. When $b \log b = O(n)$, M_b scales as n/b giving the same per-node throughput upper bound as Γ_2 .

includes the ad hoc nodes in the interval to the left of the point $n/2$. Γ_2^r includes the remaining ad hoc nodes and all base stations. Consider the cut capacity of Γ_2 in the direction from Γ_2^l to Γ_2^r . All the links crossing the cut are wireless links with bandwidth capacity of 1. For the parts of the cut surrounding the $b/2$ base stations, note that there can be at most one transmission at a given time to cross the cut to reach from a node to a base station, resulting in a capacity of $b/2$. In addition, there exists one link crossing the cut through the ad hoc network. There are $\Theta(n)$ source-destination pairs that need to cross the cut from Γ_2^l to Γ_2^r w.h.p. Hence, Γ_2 upper bounds the per-node throughput by $\Theta(1/n + b/2n) = \Theta(b/n)$.

Finally consider a third cut drawn around the nodes in the busiest cell. In particular, let $s_i = [(i-1)n/b, in/b]$ be the i th cell $1 \leq i \leq b$. For a given placement of nodes, let s^* be the cell with the maximum number of nodes, denoted by M_b . Let the cut Γ_{s^*} divide the network into two regions (see Fig. 4 (c)): $\Gamma_{s^*}^i$, which includes the ad hoc nodes inside s^* , and $\Gamma_{s^*}^o$ which includes the rest of the nodes and all base stations. The cut capacity of Γ_{s^*} in the direction from $\Gamma_{s^*}^i$ to $\Gamma_{s^*}^o$ upper bounds the rate of information that can be carried away from the nodes inside s^* . The cut can be crossed in the direction to the base station or by ad hoc transmission to neighboring cells, with each crossing having a capacity of 1. Hence, Γ_{s^*} upper bounds the per-node throughput by $\Theta(3/M_b)$. As proved in Lemma 1, in the case $b \log b = O(n)$, M_b scales with n/b , hence Γ_{s^*} ,

brings the same upper bound (in the order sense) with Γ_2 . On the other hand, when $b \log b = w(n)$, M_b deviates from the mean and scales with $h(b)$ defined in (2), which asymptotically dominates n/b and brings a per-node throughput upper bound that scales with $1/h(b)$ for these nodes. In other words, when $b \log b = w(n)$ the throughput achieved by nodes in the busiest cell determines the overall per-node throughput. Finally, the overall per-node throughput upper bound can be found by considering all cuts giving the values in Theorem 1.

B. Achievability

We present a construction that describes how information is carried between source-destination pairs, and then calculate the throughput achieved by this construction, which gives a lower bound. Note that this construction is similar to the one presented in [13].

First, the interval $[0, n]$ is divided into small “segments” of length $\log n$, the first segment being $[0, \log n]$ (see Fig. 5). It can be easily verified that each segment contains at least one node w.h.p. For ad hoc communication, data is carried through multihop where each time data is delivered to a node inside the next segment on the route. Note that segments are different from “cells”, the subintervals of length n/b . Data is delivered from a source node to a destination node in three steps.

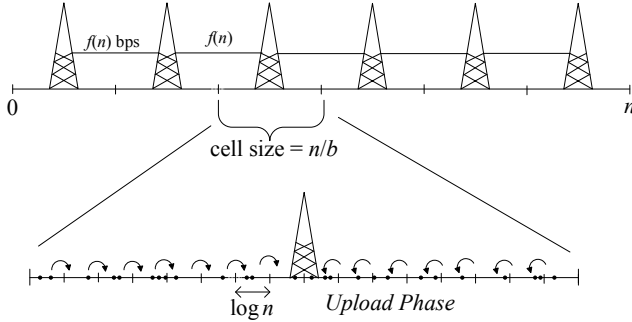


Fig. 5. The 1-D network consists of b cells of each length n/b (top figure). A source node sends its packet to a destination node in three steps. In the upload phase, the packet is delivered to the closest base station through multihop communication (bottom figure). The packet is then carried through the wired network to the destination base station, which delivers the packet to the destination node following the reverse of the operation in the upload phase (not shown).

- 1) *Upload Phase*: The source sends the packet to the closest base station through multihop, where the packet is delivered to the next segment at each hop until it reaches the base station (see Fig. 5). Note that the cells become smaller than the segments in the case $n/b \leq \log n$, and nodes reach the base station in one-hop.
- 2) *Wired Phase*: The packet is carried through the wired network until it reaches the base station which is closest to the destination node.
- 3) *Download Phase*: The base station closest to the destination node delivers the packet to the destination node using multihop transmission by nodes in each segment. Note that if the base stations have enough power to reach

the whole cell, this phase can be done with broadcast instead. However, this does not change the scaling result.

Having defined the routing part of the construction, we next describe how to schedule transmissions for each phase using time division multiplexing. In the upload and download phases, nodes relay information for other nodes. Due to interference, nodes inside adjacent segments cannot transmit simultaneously. However, a standard spatial reuse scheme can be used where time is divided into slots and segments sufficiently far apart can be active in the same time slot. In particular, it can be shown that there exists a constant (independent of n) integer d , such that time every segment can transmit once at the end of a total of d time slots (e.g., see Appendix A in [13]). The upload phase is completed once the segments finish relaying all the packets. A segment needs to relay information for at most all the nodes inside the cell. Hence, the number of nodes in a cell determines the number of time slots needed to finish its upload phase. Therefore, overall the upload phase can be finished in dM_b time slots, where M_b is the number of nodes in the busiest cell. Hence, the throughput constraint achievable in the upload phase is $\Theta(1/M_b)$. As shown in Lemma 1, $1/M_b$ scales as b/n when $b \log b = O(n)$, and scales with $1/h(b)$ otherwise, where $h(b)$ is defined in (2). Note that the download phase brings the same throughput constraint.

In the wired phase, the links between base stations can be active simultaneously. Hence, the time needed to complete the wired phase is solely determined by the relaying load on the base stations. A base station needs to deliver information for at most all nodes in the network, which is sent at a rate $f(n)$ bits per second. Hence, the throughput achieved in the wired phase is $\Theta(f(n)/n)$. Finally, the throughput achieved by this construction as a whole is given by considering the throughput constraint coming from all three phases, giving a lower bound of $\Omega(\min\{1/M_b, f/n\})$ w.h.p. ■

V. TWO-DIMENSIONAL NETWORK

Our main result for the two-dimensional hybrid network is given in Theorem 2 in Section II-C. The upper bounds and the achievability construction are very similar to the 1-D case, and here we only emphasize the differences. Remember that Lemma 1, which states how the maximum number of nodes in any cell scales, also applies to the 2-D case.

Proof of Theorem 2:

A. Upper Bound

Consider the cut Γ_1 given in Fig. 6 which divides the region into two halves. On one side of the cut we have the set Γ_1^l , which consists of ad hoc nodes and $b/2$ base stations that are inside the left half region of size $\sqrt{n}/2 \times \sqrt{n}$. On the other side, Γ_1^r includes the rest of the ad hoc nodes and the remaining $b/2$ base stations. Consider the cut capacity in the direction from Γ_1^l to Γ_1^r . The wired network crosses the cut with $\sqrt{b}$ links each with capacity f . On the other hand, wireless links between ad hoc nodes can be shown [14] to achieve at most $\sqrt{n}$ bps across the cut Γ_1 . There are $\Theta(n)$ nodes which need

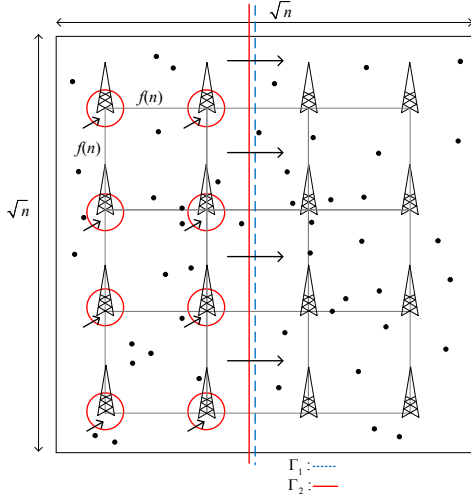


Fig. 6. Cuts Γ_1 and Γ_2 shown on the two-dimensional network. The cuts are defined as similar to the case in 1-D shown in Fig. 4.

to cross the cut in this direction w.h.p. Hence the per-node capacity upper bound due to Γ_1 is $\Theta(\sqrt{n}/n + \sqrt{b}f/n)$.

The second cut Γ_2 is defined in a similar way to the 1-D case and is shown in Fig. 6. On one side of the cut, we have the subset Γ_2^l , with the ad hoc nodes in the left half of the square. On the other side, the subset Γ_2^r consists of *all* base stations and the remaining ad hoc nodes. The cut capacity in the direction from Γ_2^l to Γ_2^r consists of only wireless connections. Across the part of the cut surrounding the $b/2$ base stations, the ad hoc nodes can achieve a constant rate to each base station resulting in a capacity of $b/2$. For the pure ad hoc communication across the rest of the cut, again we have a capacity of $\sqrt{n}$ as for the first cut. Hence, the cut capacity due to Γ_2 is $\Theta(\sqrt{n}/n + b/2n)$.

A third cut can be drawn around the ad hoc nodes in the busiest cell, which includes M_b nodes, as done in the 1-D case (see Fig. 4 (c) for the 1-D cut, the cut for 2-D is not shown). However, the difference in the 2-D case is that these nodes can cross the cut to outside the cell with a total rate proportional to the edge length of the cell, $\sqrt{n/b}$, as opposed to the constant rate in the 1-D case. This results in a per-node capacity of $\Theta(\sqrt{n/b}/M_b)$ for these nodes, where M_b scales as stated in Lemma 1, giving the upper bound stated in Theorem 2 for the case $b \log b = w(n)$.

B. Achievability

The construction giving the lower bounds to the 2-D capacity is similar to the 1-D case, where the source delivers a packet to a destination in three phases. The only difference in the 2-D case is that, this time we divide the whole network into “squarelets” of size $\sqrt{\log n} \times \sqrt{\log n}$ as shown in Fig. 7. It can be shown that each squarelet includes at least one node w.h.p. In the upload phase, the source delivers the packet to the closest cell by multihop communication which follows a route consisting of straight lines as shown in Fig. 7. By a time division multiplexing scheme similar to the 1-D case, it can

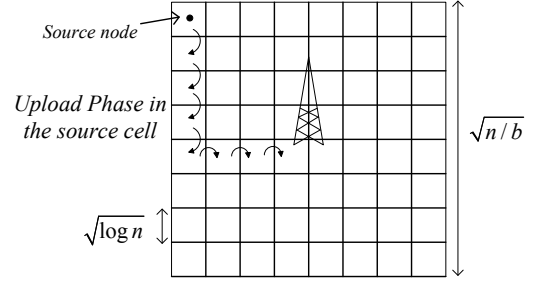
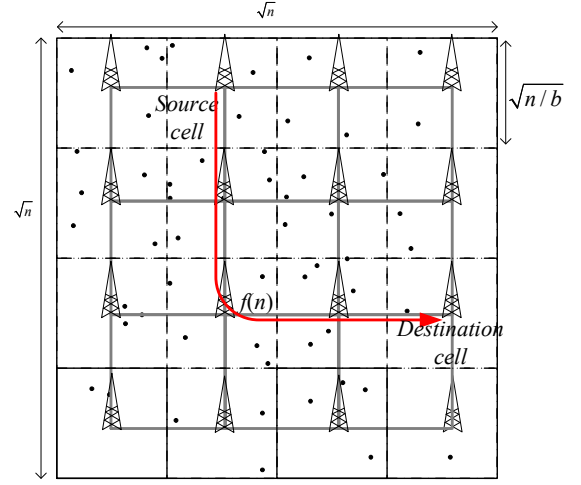


Fig. 7. The region is divided into b cells, each of size $\sqrt{n/b} \times \sqrt{n/b}$ (top figure). A source node sends its packet to the destination in three steps. In the upload phase, the source node delivers the packet to the closest base station through multihop communication (bottom figure). In the wired phase, the packet is delivered from the source base station to the destination base station on a vertical route followed by a horizontal route (top figure). The download phase follows the reverse operation of the upload phase (not shown).

be shown that the upload phase can be finished in $M_b d$ time slots, for some integer d . Hence, the throughput achievable in the upload phase is $\Theta(1/M_b)$, which is the same as the download phase. In the wired phase, the packets are delivered from the source base station to the destination base station on a route which consists of a vertical path followed by a horizontal path as shown in Fig. 7. Therefore, a base station has to deliver packets from cells that are in the same row or column with it. Hence, the relaying load on a base station is equal to the number of nodes inside the union of a horizontal strip and a vertical strip, each of size $\sqrt{n/b} \times \sqrt{n}$. It can be shown that, these strips have at most a constant factor of $n/\sqrt{b}$ nodes w.h.p. (proof is omitted due to space constraints). The packets coming from these nodes are served by a link of bandwidth f , hence the throughput achievable in the wired phase is $\Theta(\sqrt{b}f/n)$. Therefore, this construction achieves a throughput of $\Theta(\min\{1/M_b, \sqrt{b}f/n\})$, which improves on the pure ad hoc capacity only if $\min\{1/M_b, \sqrt{b}f/n\}$ scales faster than $1/\sqrt{n}$. Hence, the lower bounds in 2-D capacity is achieved by using pure ad hoc communication [18] when $b = O(\sqrt{n})$ or $\sqrt{b}f = O(\sqrt{n})$, and by using the above three-step construction otherwise. ■

VI. CONCLUSION

We study the capacity of hybrid networks, where n wireless nodes share information in a network supported by a wired infrastructure of b base stations. Our work establishes an upper bound on per-node throughput that can be achieved by the nodes using recent cut-set results shown for pure ad hoc networks. Our upper bounds match previously shown lower bounds for a wide range of values of b where $b \log b$ grows more slowly than n . We further study the extreme case where b grows almost on the same order with n . Through a theoretical study of the maximum of a sequence of Poisson random variables, we show that, in that case, there exists a base station which has to serve too many nodes compared to the rest of the base stations. Hence, in that case the overall per-node throughput is determined by the throughput that can be achieved by nodes serviced by this busiest base station. With this result, we obtain matching upper and lower bounds on the capacity of hybrid networks for all values of b in the 1-D case, and for the values $b \log b = O(n)$ in the 2-D case. Other interesting directions involve the cases of random/arbitrary placement of base stations, and the better modeling of packet transmission through the wired network with existing protocols.

APPENDIX I

First, define the tail function $\mathcal{F}_b = 1 - F_b$. As in [17], we associate the following continuous function to the exact function $\mathcal{F}_b$, which agrees with $\mathcal{F}_b$ at integer values.

$$\mathcal{F}_{c,b}(x) = e^{-\lambda_b} \lambda_b^x \sum_{j=1}^{\infty} \lambda_b^j / \Gamma(x + j + 1), \quad (5)$$

where Γ is the gamma function. Finally, define the sequence of real numbers $\{\beta_b, b = 1, 2, \dots\}$ by the following relation:

$$1/b = \mathcal{F}_{c,b}(\beta_b) \quad (6)$$

Note that the function $\mathcal{F}_{c,b}$ is strictly decreasing, hence β_b is a growing sequence. In [15], it is shown that the integer sequence I_b is given by $I_b = \lfloor \beta + 1/2 \rfloor$. Therefore, the asymptotic behavior of I_b can be found by finding the growth rate of β_b .

By taking logarithms of both sides in (6),

$$\log b = \lambda_b - \beta_b \log \lambda_b - \log \sum_{j=1}^{\infty} \lambda_b^j / \Gamma(\beta_b + j + 1) \quad (7)$$

We do an asymptotic analysis on the above equation to analyze the growth rate of β_b . We first make the observation that, as b grows, the first term in the summation in (7) dominates the sum of the rest of the terms. This can be found by using the fact that for any $y > 0$, $\mathcal{F}_{c,b}(x)/\mathcal{F}_{c,b}(x+y) \rightarrow \infty$, as $x \rightarrow \infty$ [16]. The dominance result then follows by using $y = 1, x = \beta_b$. Hence, we may keep only the first term in the summation above and get

$$\log b \sim \lambda_b - \beta_b \log \lambda_b - \log \lambda_b + \log \Gamma(\beta_b + 2) \quad (8)$$

Then we proceed similarly. First, keeping the most dominant term in Stirling's approximation to the gamma function,

$$\log b \sim \lambda_b - \beta_b \log \lambda_b - \log \lambda_b + \beta_b \log \beta_b. \quad (9)$$

Finally, for the terms on the right hand side above, we look for the most dominant term(s). It is shown in [16] that β_b is asymptotically dominant to λ_b . (Note that $\log \beta_b$ does not necessarily dominate $\log \lambda_b$). Hence we are left with

$$\log b \sim \beta_b \log \beta_b - \beta_b \log \lambda_b \quad (10)$$

From above, finally it can be shown that β_b satisfies

$$\beta_b \sim \frac{\log b}{\log(\frac{\log b}{\lambda_b})}. \quad (11)$$

REFERENCES

- [1] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, March 2000.
- [2] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, pp. 477–486, August 2002.
- [3] A. Ozgur, O. L  v  que, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3549–3572, 2007.
- [4] B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks," in *INFOCOM 2003*, vol. 2, march-3 april 2003, pp. 1543 – 1552 vol.2.
- [5] O. Dousse, P. Thiran, and M. Hasler, "Connectivity in ad-hoc and hybrid networks," in *INFOCOM 2002*, vol. 2, 2002, pp. 1079 – 1088 vol.2.
- [6] U. Kozat and L. Tassiulas, "Throughput capacity of random ad hoc networks with infrastructure support," in *Proc. of MobiCom*. ACM, 2003, pp. 55–65.
- [7] A. Zemplianov and G. de Veciana, "Capacity of ad hoc wireless networks with infrastructure support," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 3, pp. 657 – 667, march 2005.
- [8] S. Toup  pis, "Capacity bounds for three classes of wireless networks: asymmetric, cluster, and hybrid," in *Proceedings of the 5th ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2004, pp. 133–144.
- [9] A. Agarwal and P. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," *ACM SIGCOMM Computer Communication Review*, vol. 34, no. 3, pp. 71–81, 2004.
- [10] X. Mao, X. Li, and S. Tang, "Multicast capacity for hybrid wireless networks," in *Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2008, pp. 189–198.
- [11] X. Chen, W. Huang, X. Wang, and X. Lin, "Multicast capacity in mobile wireless ad hoc network with infrastructure support," in *Proc. of INFOCOM*, 2012.
- [12] W. Shin, S. Jeon, N. Devroye, M. Vu, S. Chung, Y. Lee, and V. Tarokh, "Improved capacity scaling in wireless networks with infrastructure," *IEEE Trans. Inf. Theory*, vol. 57, no. 8, pp. 5088–5102, 2011.
- [13] B. Liu, P. Thiran, and D. Towsley, "Capacity of a wireless ad hoc network with infrastructure," in *MobiHoc '07*. New York, NY, USA: ACM, 2007.
- [14] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the throughput gain of network coding in unicast and multicast wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 27, no. 5, pp. 582 –592, june 2009.
- [15] C. Anderson, "Extreme value theory for a class of discrete distributions with applications to some stochastic processes," *Journal of Applied Probability*, pp. 99–113, 1970.
- [16] C. Anderson, S. Coles, and J. H  sler, "Maxima of poisson-like variables and related triangular arrays," *The Annals of Applied Probability*, vol. 7, no. 4, pp. 953–971, 1997.
- [17] A. Kimber, "A note on poisson maxima," *Probability Theory and Related Fields*, vol. 63, no. 4, pp. 551–552, 1983.
- [18] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Inf. Theory*, vol. 53, no. 3, pp. 1009 –1018, March 2007.