

REPORT DOCUMENTATION PAGE

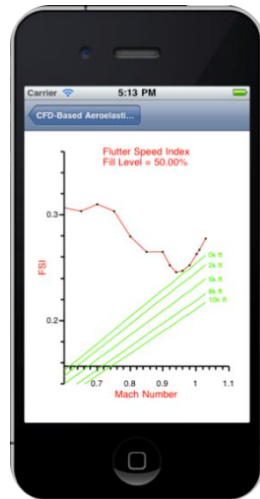
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CONTROL-ORIENTED AEROELASTIC REDUCED-ORDER MODELING OF FIGHTERS

FA9550-10-1-0539



Charbel Farhat and David Amsallem
Aeronautics and Astronautics
Stanford University



FSI: STATE OF THE ART

* Structural dynamics

- multibody dynamics
- geometrical nonlinearities (large displacements, rotations and strains)
- material nonlinearities (nonlinear constitutive models)
- crack propagation (failure)

* Computational fluid dynamics

- shocks
- turbulence

* Coupling

- static (steady) and dynamic (unsteady)
- eigen



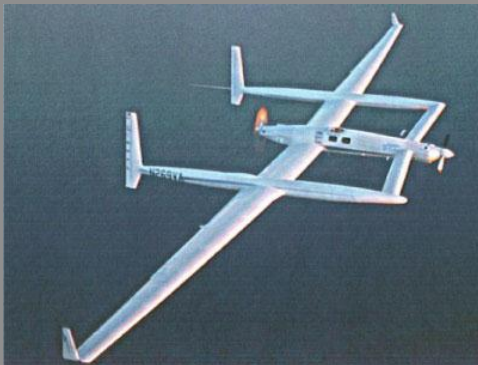
IMPACT ON ENGINEERING

✦ Strong

- analysis of carefully selected critical configurations

✦ Weak (or not as strong)

- routine analysis
 - design (and test)
 - control
- } significant CPU time issues



" [If I am not getting the NASTRAN answer after 4 hours on a Cray, then God is sending me the message I have the wrong design] "

Burt Rutan, 1993



Manuel Balce Ceneta / AP



REDUCED ORDER MODEL (ROM)

* Model reduction (MOR)

- build the lowest dimensional model that can capture the dominant behavior of the system of interest by projecting the high-fidelity model onto a well-chosen subspace

➔ drastic CPU time reduction

* Complex, time-dependent problems (with a CFD component)

o Perturbation problems (stability, trends, control, etc.)

- linearized ➔ linear ROMs

o Response problems (behavior, performance, etc.)

- nonlinear ➔ linearized ➔ linear ROMs
Newton



MODEL REDUCTION

* What this is NOT about

- building the simplest model
- building a variable (or multi) fidelity model
- adopting the coarsest mesh
- substructuring
- constructing a "surrogate" or "meta" model



LINEARIZED DYNAMICAL SYSTEMS



- * External description (input-output map)

$$y = S(u) = h * u = \int_{-\infty}^t h(t - \tau) \mu(\tau) d\tau$$

- * Internal description (model)

$$\dot{u} = A(\mu_1, \dots, \mu_p)u + f$$

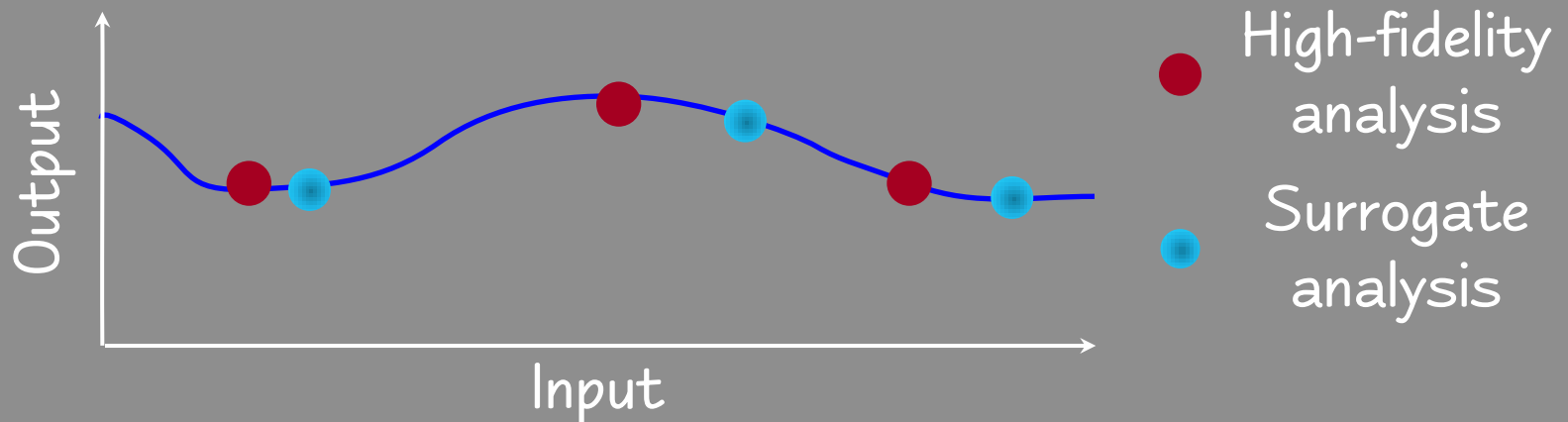
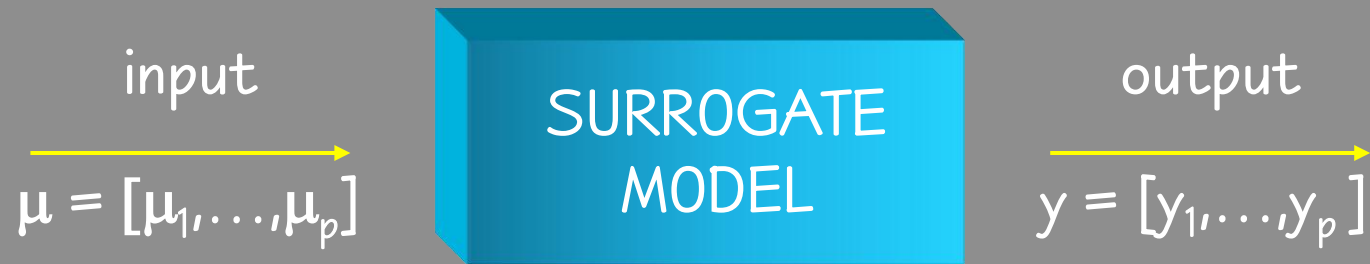
$$y = g(u, f, \mu_1, \dots, \mu_p)$$

$$u(t_0) = u_0$$



SURROGATE MODEL

* External description



- meta-model, surrogate model, response surface, kriging
- lower-dimensionality is not guaranteed
- it is not a model of the system but of the output



MODEL REDUCTION

* Internal description

$$\dot{u} = A(\mu_1, \dots, \mu_p)u + f \quad (\text{dimension} = N)$$

* Projection onto a subspace of dimension $k \ll N$

- right Reduced-Order Basis (ROB) V_k , $k \ll N$

$$u \sim V_k y \quad \longrightarrow \quad V_k \dot{y} = A(\mu_1, \dots, \mu_p)V_k y + f + r$$

- left Reduced-Order Basis (ROB) U_k , $k \ll N$

constraints \longrightarrow

$$\dot{y} = U_k^T A(\mu_1, \dots, \mu_p) V_k y + U_k^T f$$

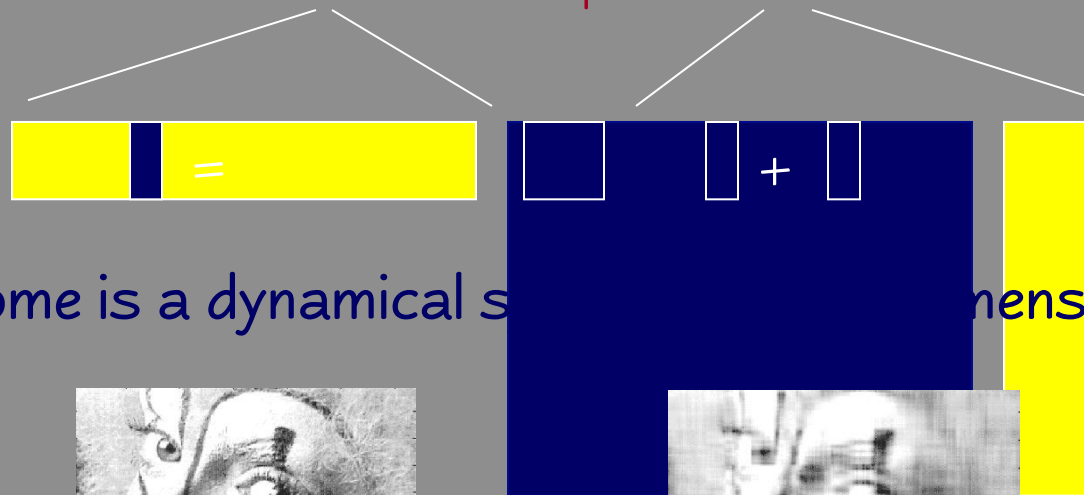
parameterized linear(ized) Reduced-Order Model (ROM)



MODEL REDUCTION

$$\dot{u} = A(\mu_1, \dots, \mu_p) u + f$$

$$\dot{y} = U_k^T A(\mu_1, \dots, \mu_p) V_k y + U_k^T f$$



* The outcome is a dynamical system of lower dimension



* Key issues

- choice of the lower dimension k and the ROBAs U_k and V_k
- dependence of the resulting ROM on the parameters $\{\mu_i\}$

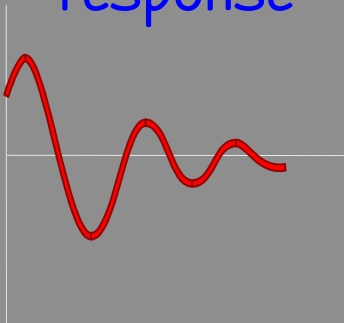


LINEAR(IZED) FLUID-STRUCTURE

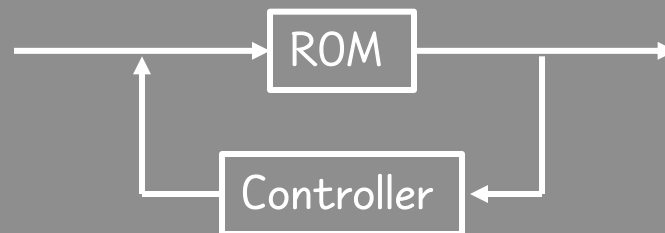
✦ General purpose linearized aeroelastic high-fidelity model (HFM)

$$\begin{pmatrix} \dot{w} \\ \ddot{u} \\ \dot{u} \end{pmatrix} = \underbrace{\begin{pmatrix} -H & -B & -C \\ M^{-1}P & 0 & -M^{-1}K \\ 0 & I & 0 \end{pmatrix}}_{A(\boldsymbol{\mu}); \boldsymbol{\mu} = (\mu_1, \dots, \mu_q)} \begin{pmatrix} w \\ \dot{u} \\ u \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}T_i^T \\ 0 \end{pmatrix} F$$

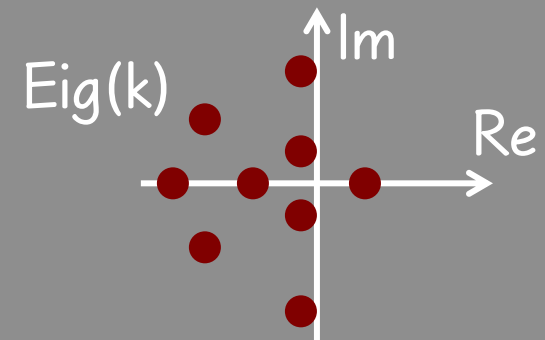
Aeroelastic response



Control



Flutter





MULTIDISCIPLINARY ROM

Fluid ROM

CFD-based HFM
Balanced POD-based ROB
(with stability guarantee)

Structural ROM

FE-based HFM
Eigen-based ROB
(with truncation)



FLUID-STRUCTURE ROM

Fluid

ROBs: $U_k, V_k / U_k^T V_k = I$

$$w = V_k w_r$$

ROM: $H_r = U_k^T H V_k$

Structure

ROB: $X_m \quad (X_m^T M X_m = I)$

$$u = X_m u_m$$

ROM: $\Omega_m^2 = X_m^T K X_m$

Coupling

$$B_r = U_k^T B X_m$$

$$C_r = U_k^T C X_m$$

$$P_r = X_m^T P V_k$$

✦ General purpose aeroelastic ROM (flutter, response, control, ...)

$$\begin{pmatrix} \dot{w}_r \\ \ddot{u}_m \\ \dot{u}_m \end{pmatrix} = \underbrace{\begin{pmatrix} -H_r & -B_r & -C_r \\ P_r & 0 & -\Omega_m^2 \\ 0 & I & 0 \end{pmatrix}}_{A_r(\mu)} \begin{pmatrix} w_r \\ \dot{u}_m \\ u_m \end{pmatrix} + \begin{pmatrix} 0 \\ X_m^T T_i^T F \\ 0 \end{pmatrix}$$

$A_r(\mu); \quad \mu = (\mu_1, \dots, \mu_q)$

- aeroelastic response
- control, flutter
- aeroservoelastic analysis, ...



OFFLINE FLUID SNAPSHOTS

- Natural mode shapes (ground vibrations) ($n_m = 9$ modes)



- Excitations in a sampled frequency range

$$0 < \kappa = \frac{\omega L_R}{v_R} \sim 1 \quad \longrightarrow \quad n_f = 5 \text{ frequencies}$$

- Responses of linearized flow (frequency domain)

$$(H + i\omega_q I) \mathbf{w}_{jq} = -(C + i\omega_q B) \mathbf{u}_j \quad j = 1, \dots, n_m, \quad q = 1, \dots, n_f$$

primal fluid snapshots $W \xrightarrow{\text{data compression}} V_k$

$$(-H^T + i\omega_q I) \mathbf{w}_{jq}^* = -P^T \mathbf{u}_j \quad j = 1, \dots, n_m, \quad q = 1, \dots, n_f$$

dual fluid snapshots $W^* \xrightarrow{\text{data compression}} U_k$



DATA COMPRESSION

- * Modal superposition (Fourier decomposition)
 - limited range of applications
- * Proper Orthogonal Decomposition (POD)
 - lacks stability
- * Balanced Proper Orthogonal Decomposition (POD)
 - more robust than POD but still lacks stability



GUARANTEED STABILIZATION METHOD

- * ROM stabilization method ([Amsallem and Farhat, 2011](#))
 - universal
 - non intrusive
 - computational complexity scales with the size of the ROM
 - preserves accuracy of original (unstable) ROM

input: ROM



Stabilization
Method

output: stable ROM

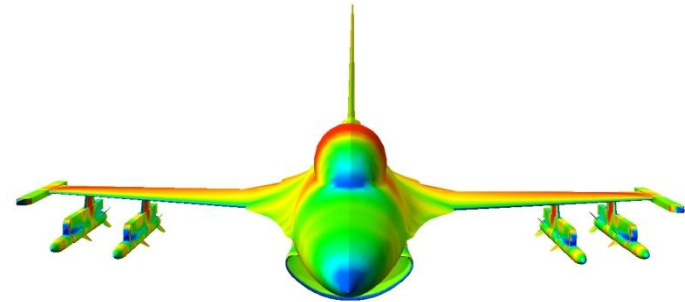
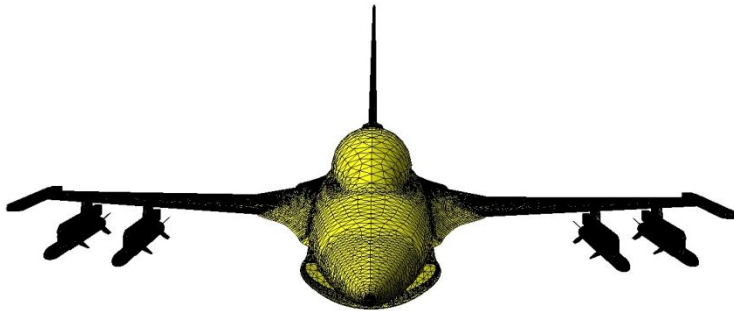




F-16 BLOCK 40

* Higher-fidelity (higher-dimensional) models (HFM)

- structure : FEM with 168,799 dofs
- fluid : Euler CFD model with 403,919 grid points



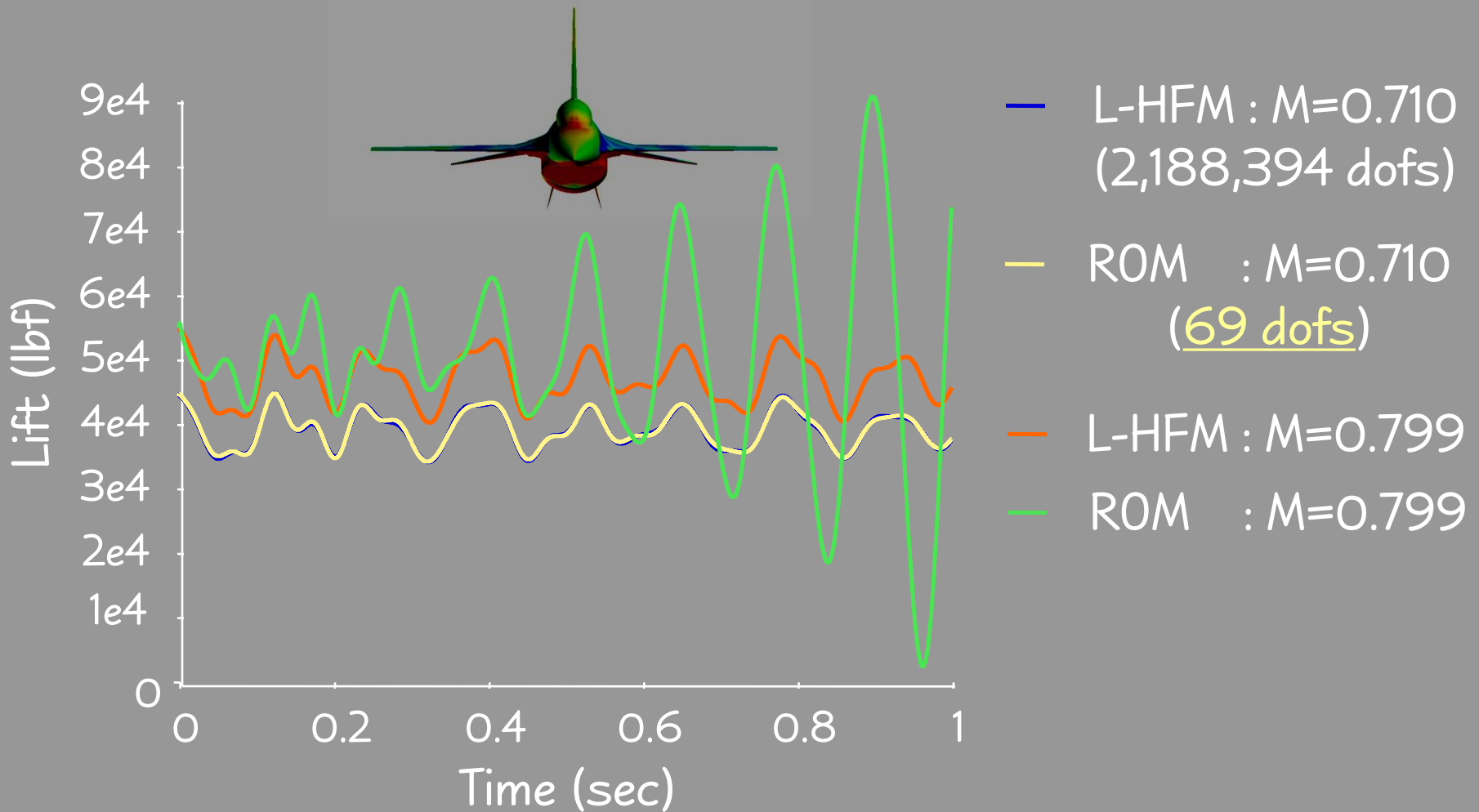
* ROMs

- structure : projection on ROB consisting of first 9 natural mode shapes
- fluid : projection on ROB of dimension 60 generated by POD using 99 snapshots (9 shapes x 5 ω s x 2 + 9 shapes at 0 Hz)



PARAMETER SENSITIVITY

POD-based fluid ROB (60) built at $M = 0.710$ (trimmed angle)





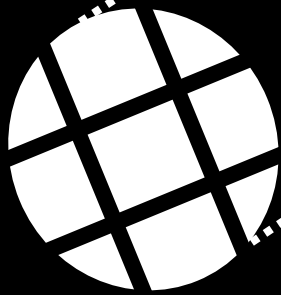
ANALOGY

Configuration 1

HFM₁



ROM₁



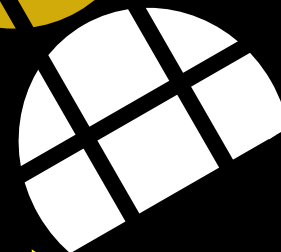
Reduced-order basis V_{k1}

Configuration 2

HFM₂



ROM₂

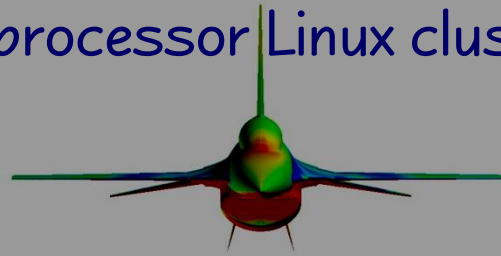


Reduced-order basis $V_{k2} = V_{k1}$



TURNAROUND TIME

- ✦ F-16 Block 40 — 1 operating point — 2nd-order discretization
64-processor Linux cluster



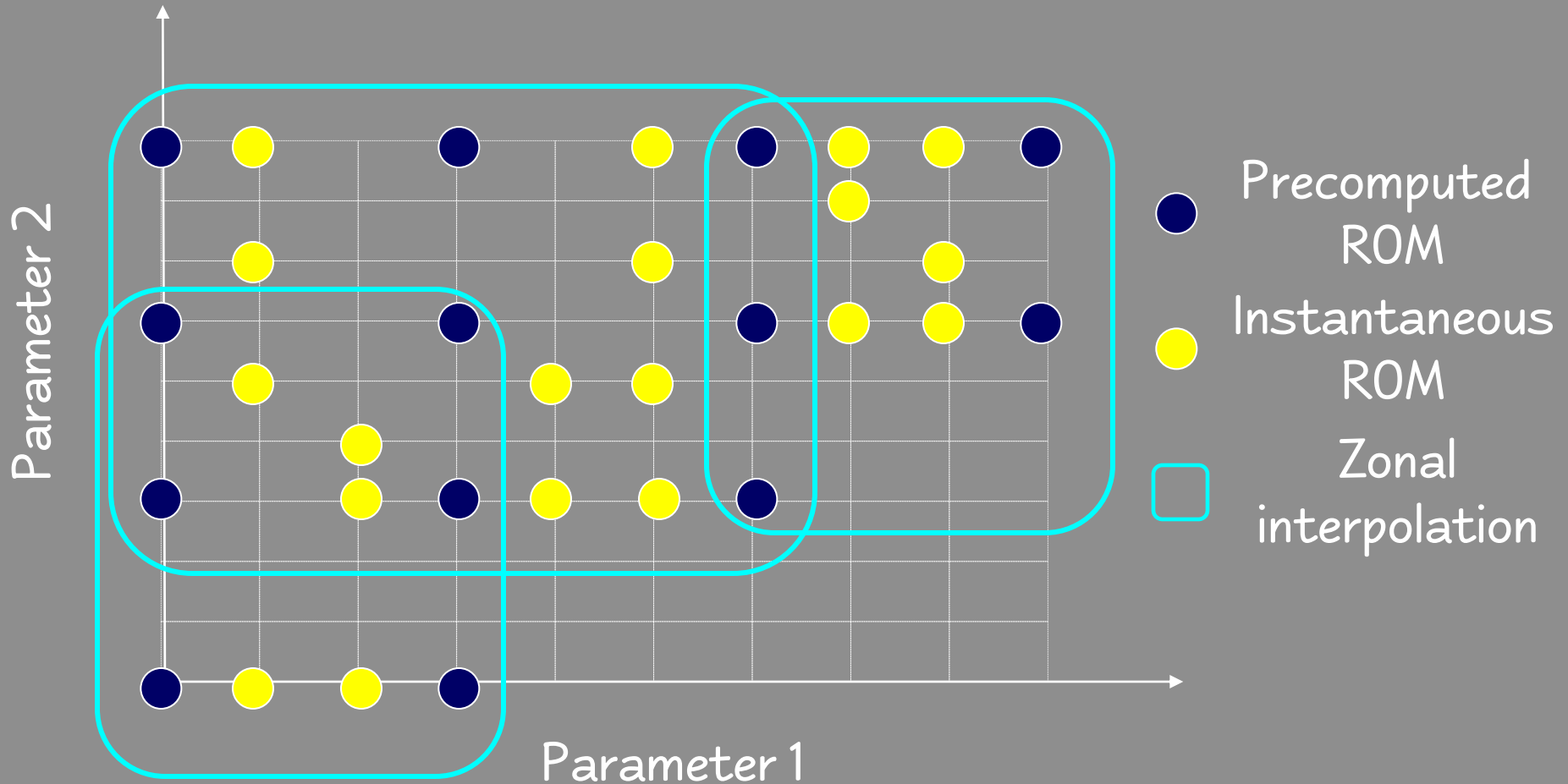
Construction and processing of aeroelastic ROM in $[0, 1.0]$ s

steady-state computation	6 minutes
generation of 99 fluid snapshots and data-based fluid ROM	50 minutes
construction of fluid ROM	0.25 minute
Processing aeroelastic ROM	0.10 minute
Total CPU time	57.25 minute



APPROACH

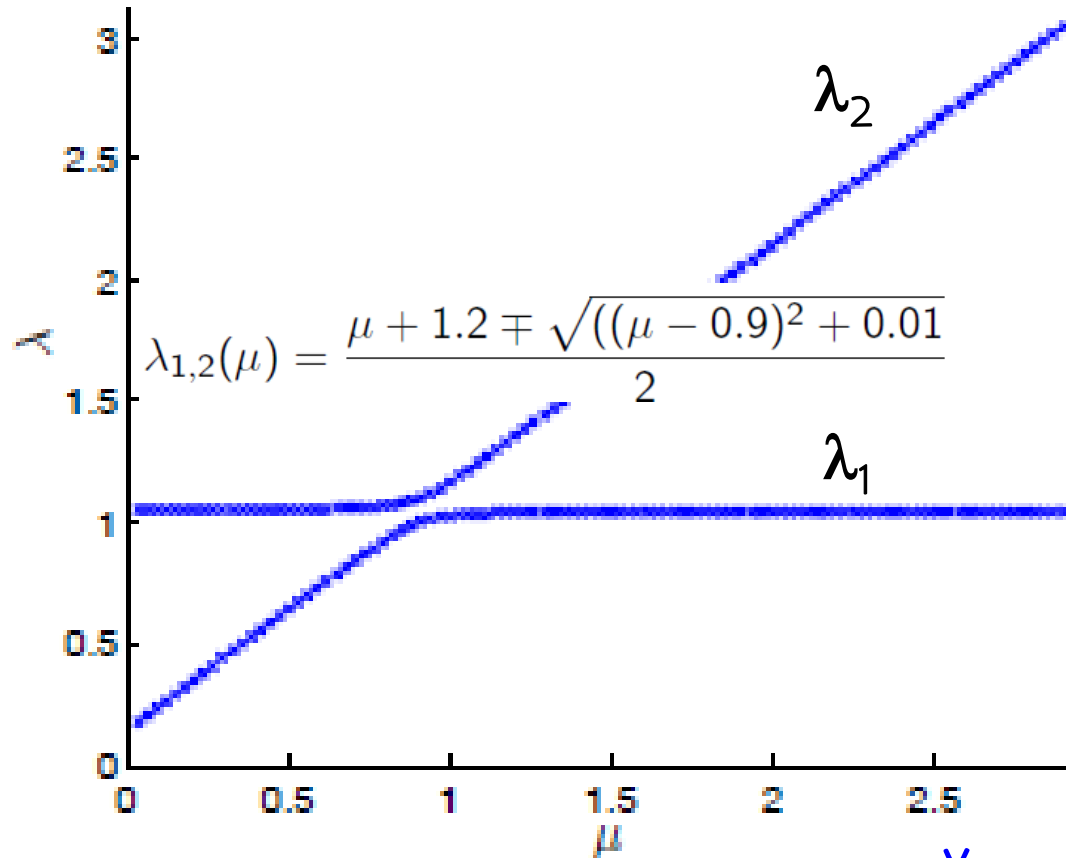
✦ Database of fixed-size stable ROMs



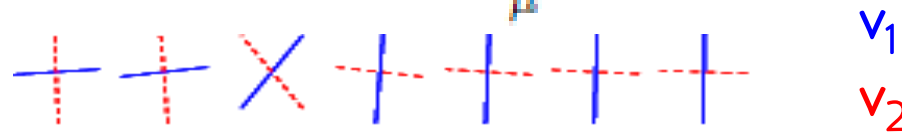


CONSISTENCY

Eigenvalues



Eigenvectors



➡ precomputed ROMs are not necessarily computed in a consistent set of generalized coordinates

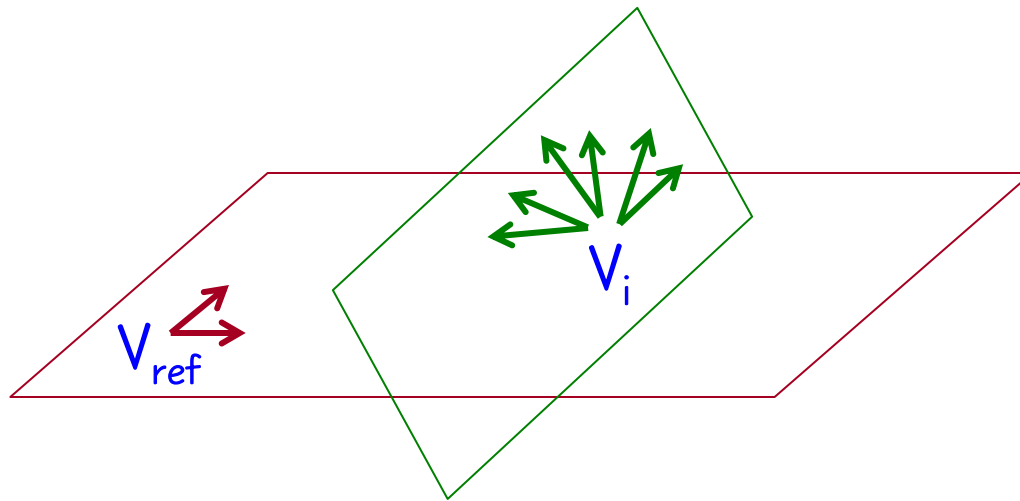


ROM TRANSFORMATION

* Congruence transformations

- reference ROB: $V_{\text{ref}} = V_k(\mu_{\text{ref}})$

- $V_i \longleftarrow V_i Q_i$ $Q_i = \arg \min_{Q \in O(k)} \|V_i Q - V_{\text{ref}}\|_F$



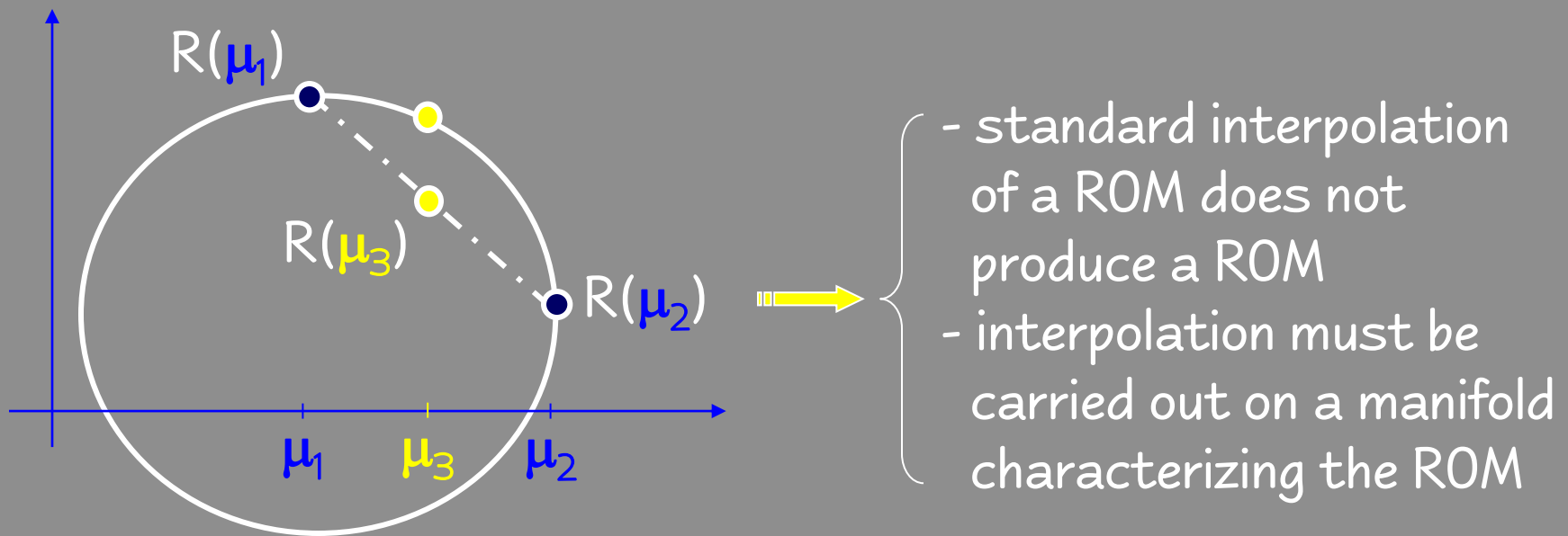
* Orthogonal Procrustes problem

- SVD: $V_i^T V_{\text{ref}} = U_i \Sigma_i Z_i^T$

$\implies Q_i = U_i Z_i^T$



QUOTIENT AND EMBEDDED MANIFOLDS



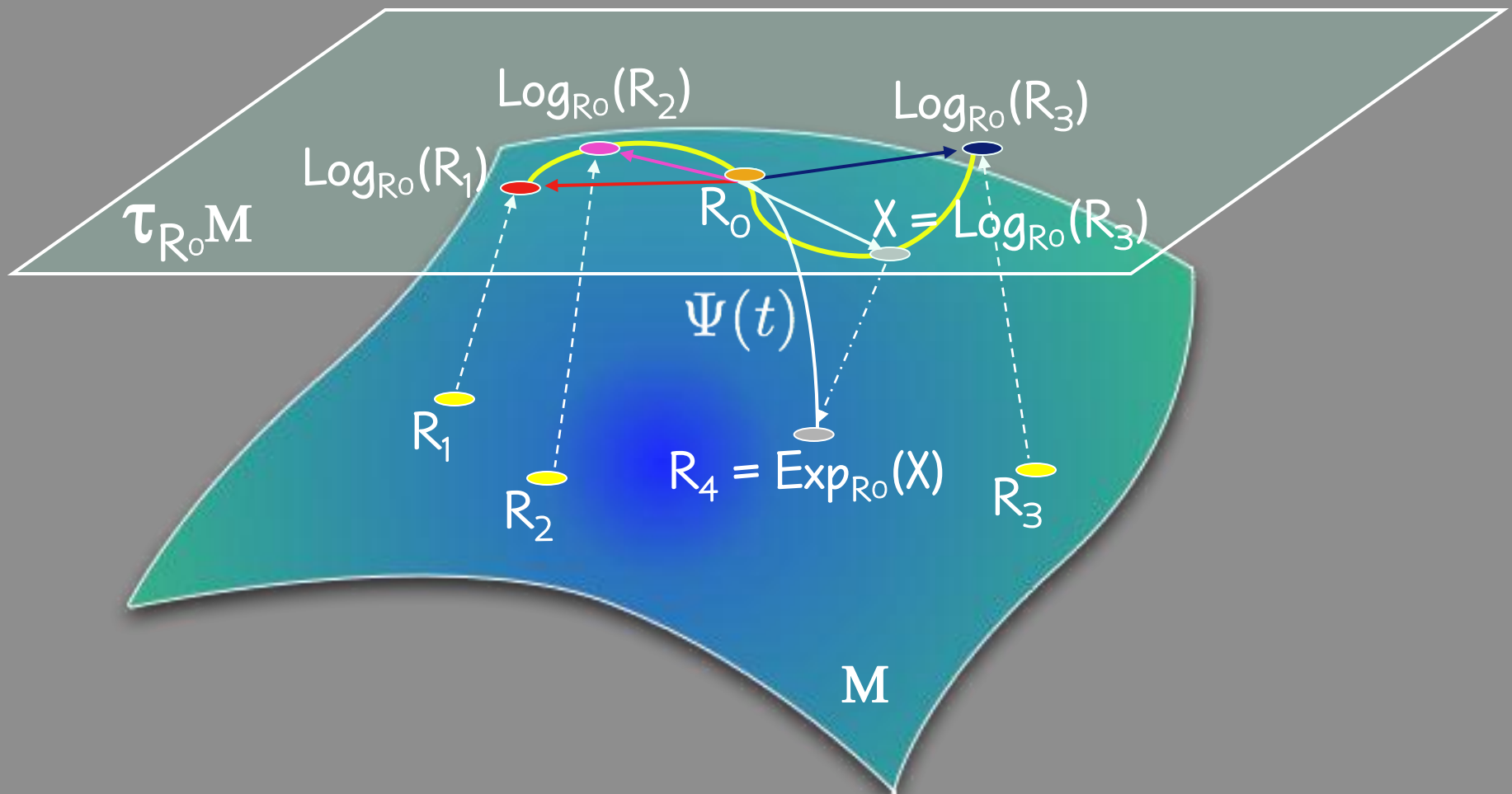
* Manifolds of interest (quotient = blue, embedded = yellow)

- $\text{span}(V_k)$ belongs to the Grassmann manifold $G(k, N)$
- $A_r(\boldsymbol{\mu}) = V_k^T A(\boldsymbol{\mu}) V_k$ belongs to
 - * manifold of invertible matrices $GL(k)$ [fluid]
 - * manifold of (reduced-order) symmetric positive definite matrices $SPD(k)$ [structure]



ONLINE INTERPOLATION ON A MANIFOLD

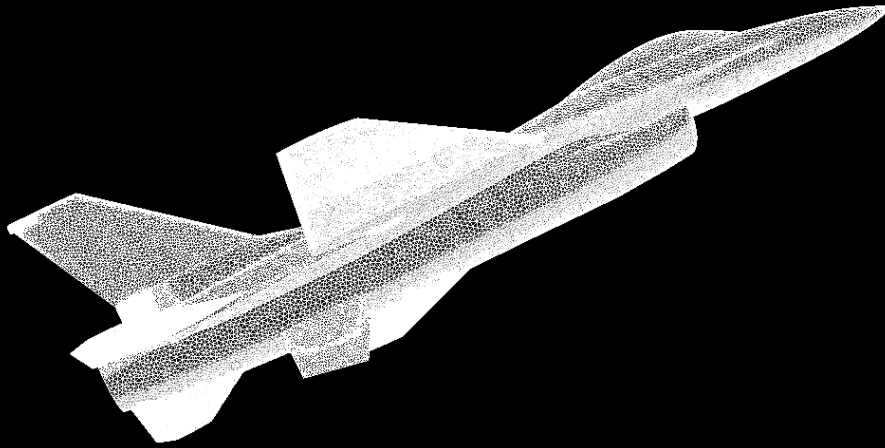
- Given a p -parameter system, the appropriate Riemannian manifold, and its logarithmic and exponential maps



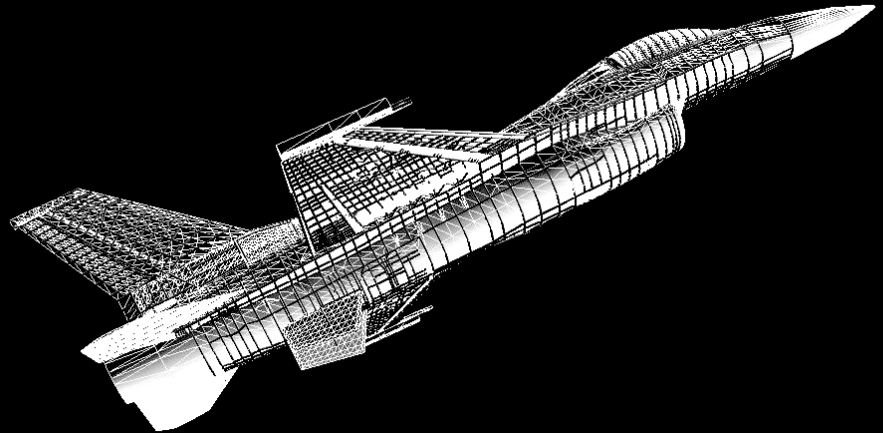


SHOWCASE APPLICATION

✦ F-16 Block 40



CFD model



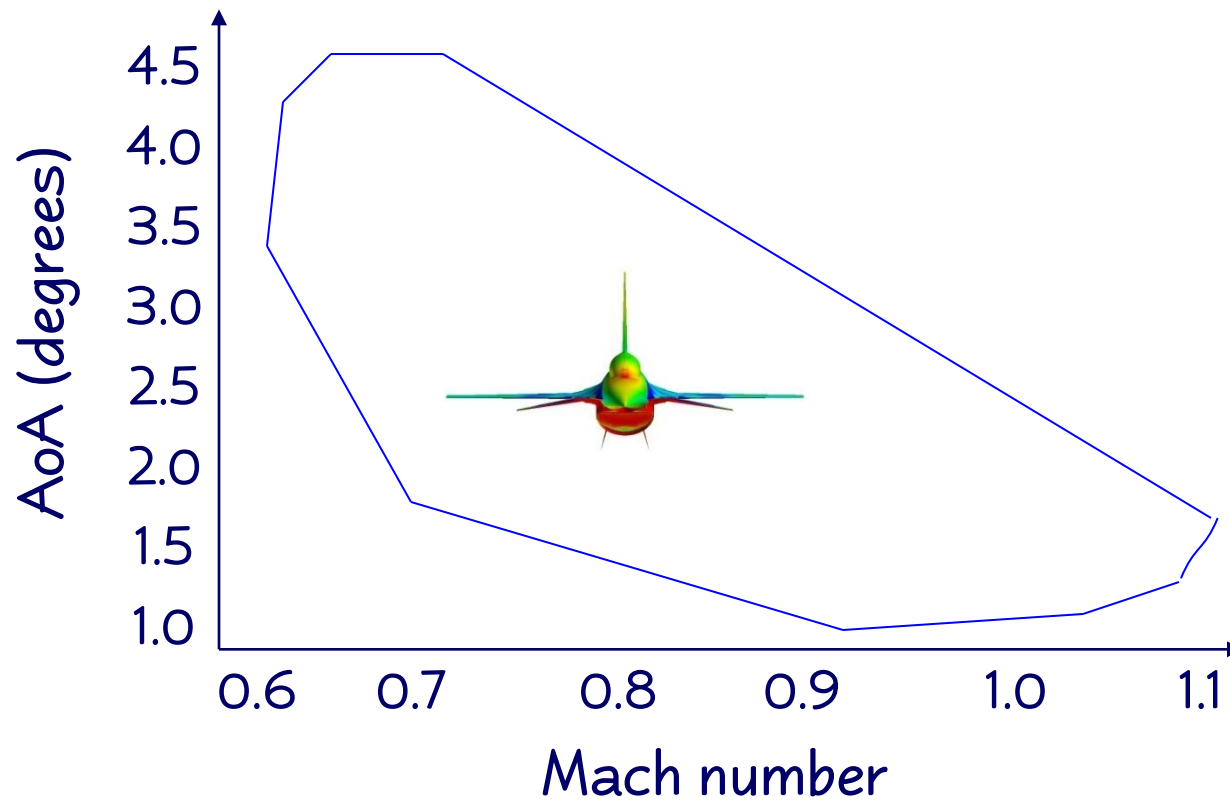
FEM structural model

✦ Assisting flight test

- single aircraft configuration → single structural ROM
- hundreds of flight conditions → database of fluid ROBs



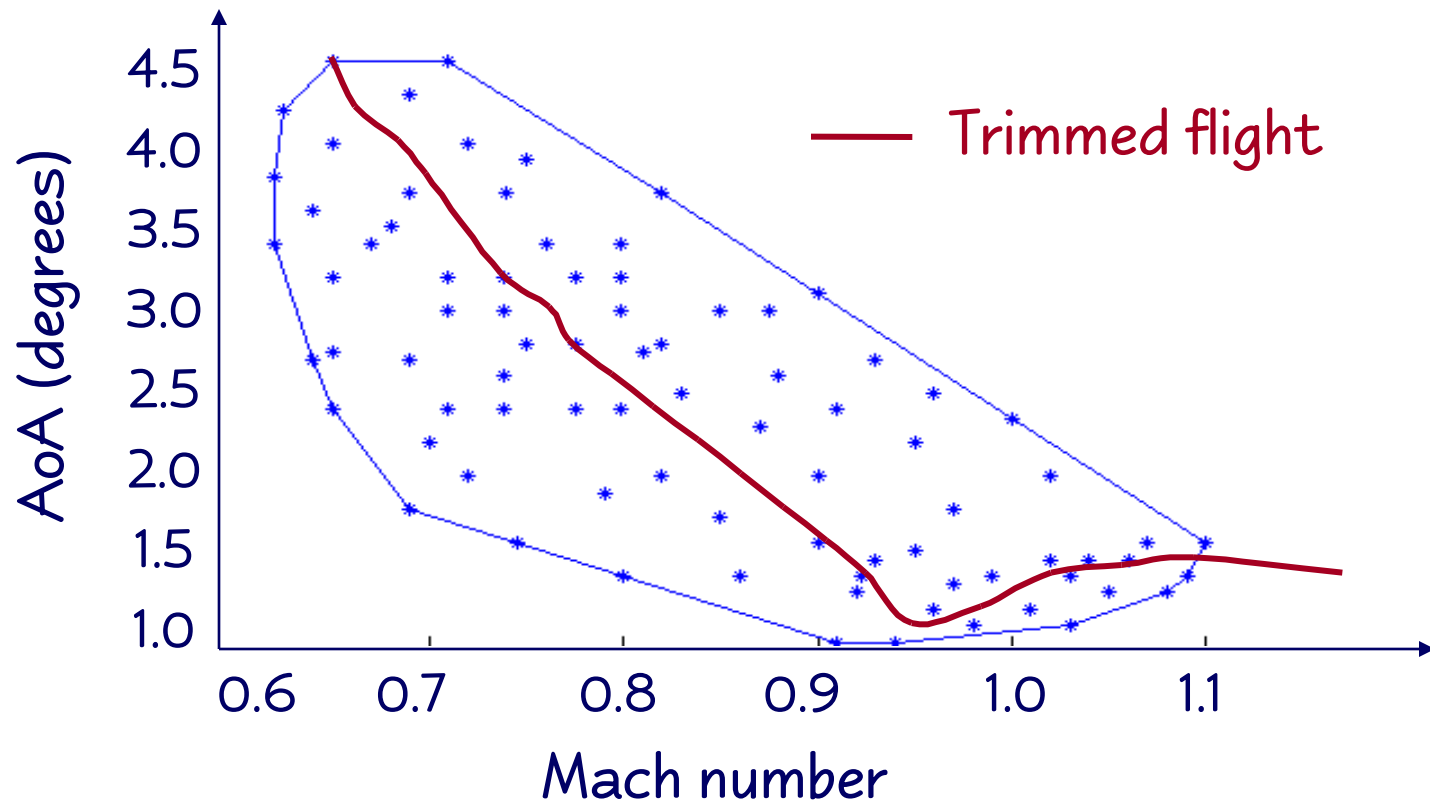
FLIGHT ENVELOPPE





ROM DATABASE

- * 83 pairs of flight conditions (operating points)
 - 70.6 hours CPU on a 64-processor Linux cluster

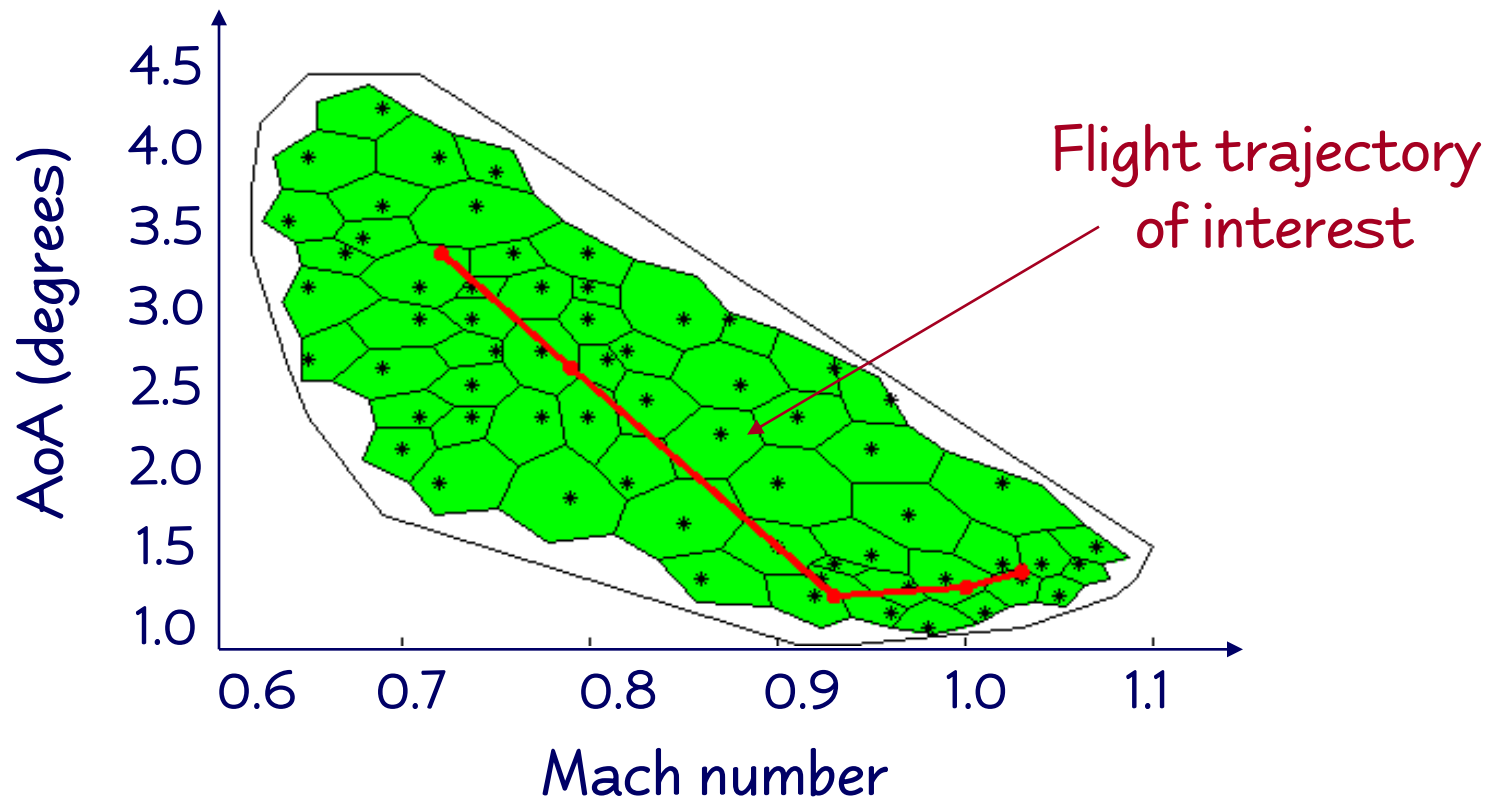




ON-DEMAND PREDICTIONS

✱ Fast responses to 5 queries

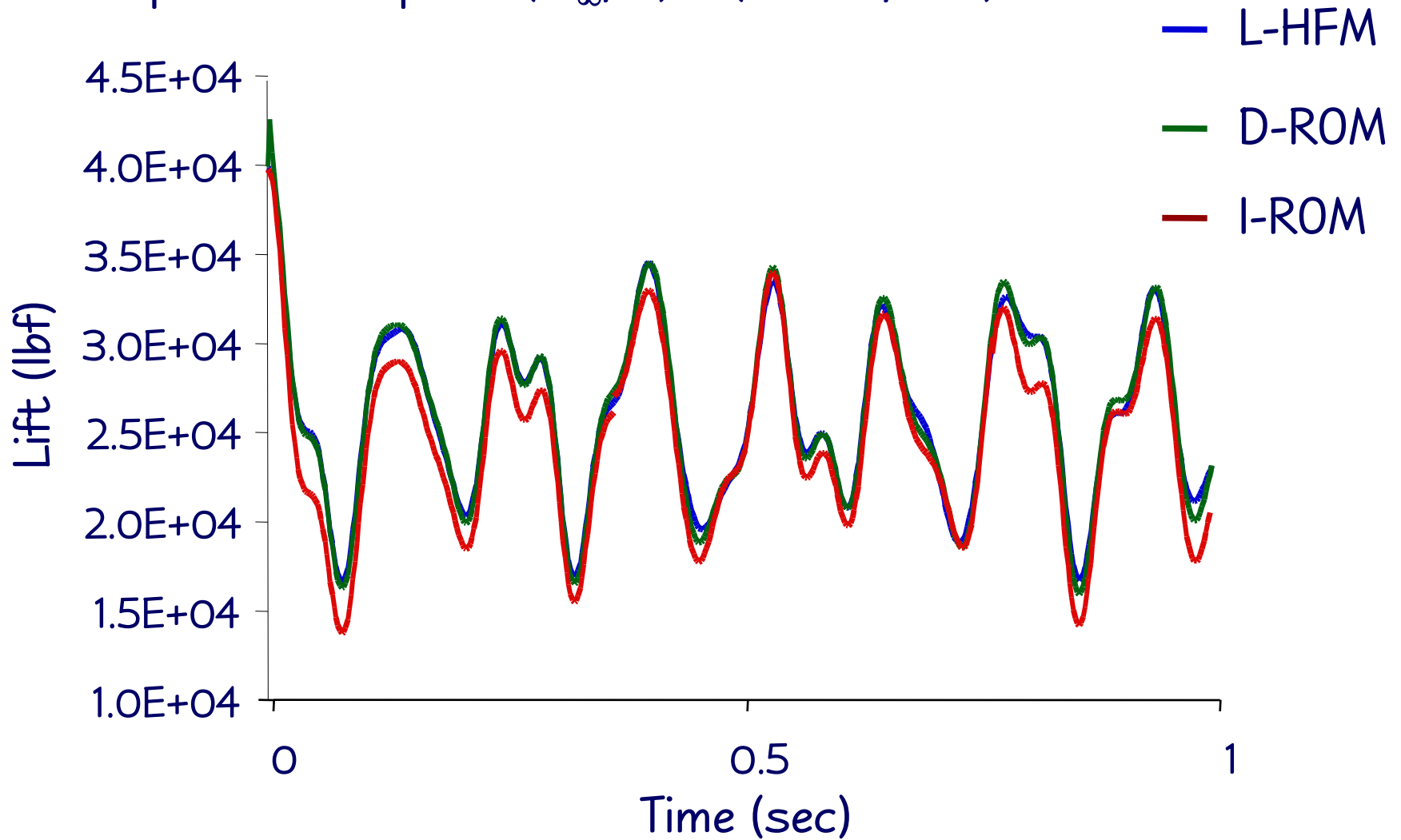
- possible scenarios: flight test, flutter clearance, optimization
- interpolation of fluid ROMs





DELIVERED ACCURACY

✱ Deep transonic point (M_∞, α) = (0.930, 1.3°)





TURNAROUND TIME

✦ F-16 Block 40 — 1 operation point — 1-processor (desktop)

CPU time for interpolating and processing
1 CFD-based aeroelastic ROM

Fluid ROM interpolation (5 points)

0.2 second

Aeroelastic ROM processing (FD)

0.3 second

Total CPU time

0.5 second

⇒ real-time, parameterized, CFD-based flutter analysis

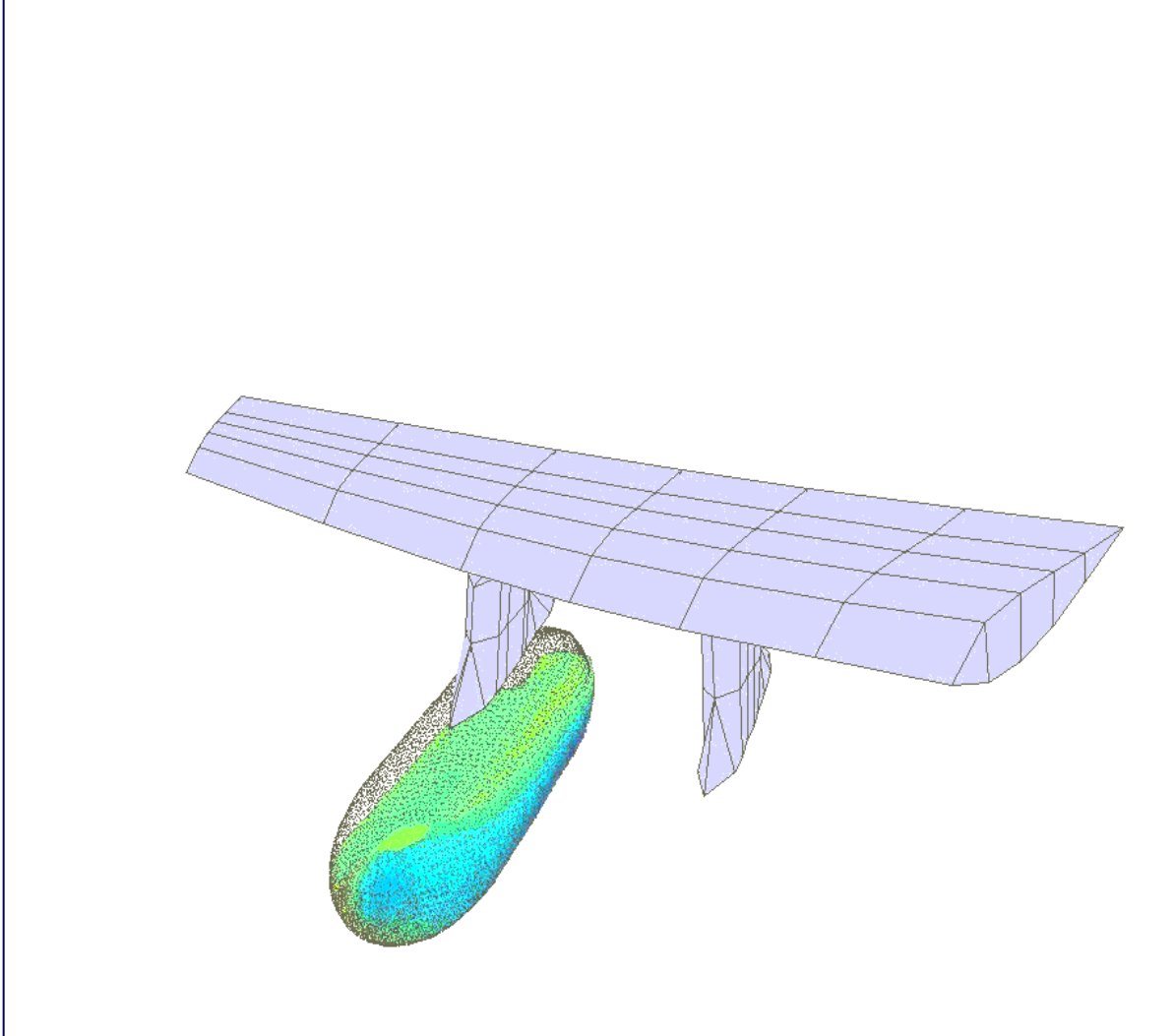
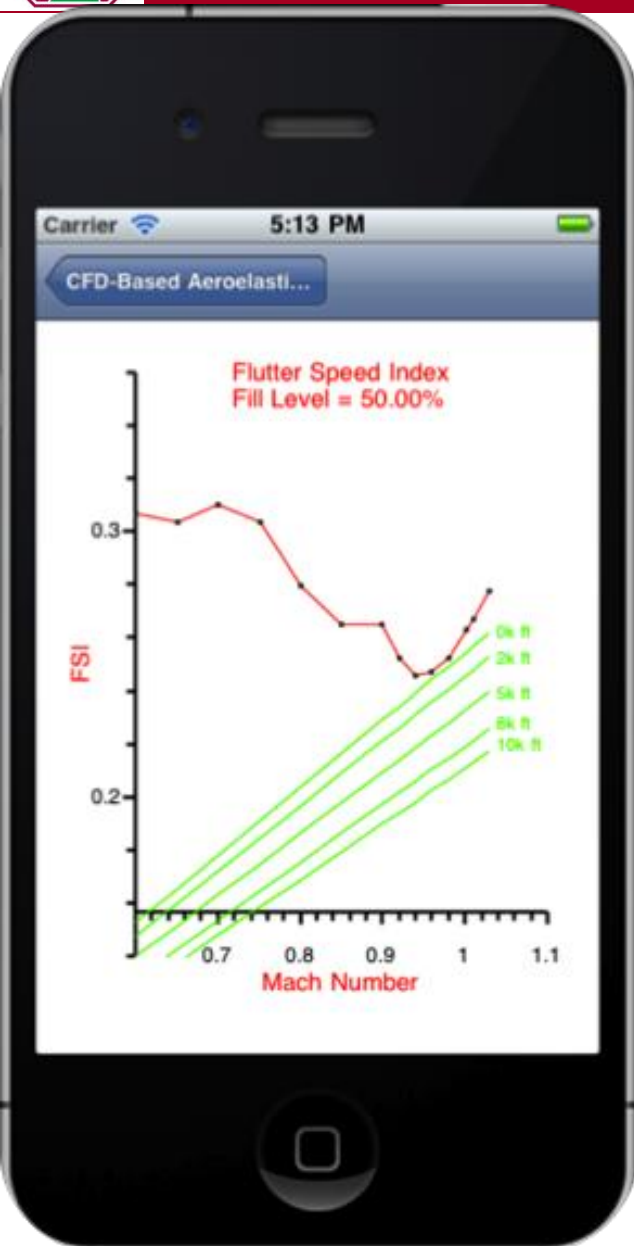


IMPLEMENTATION ON MOBILE DEVICES



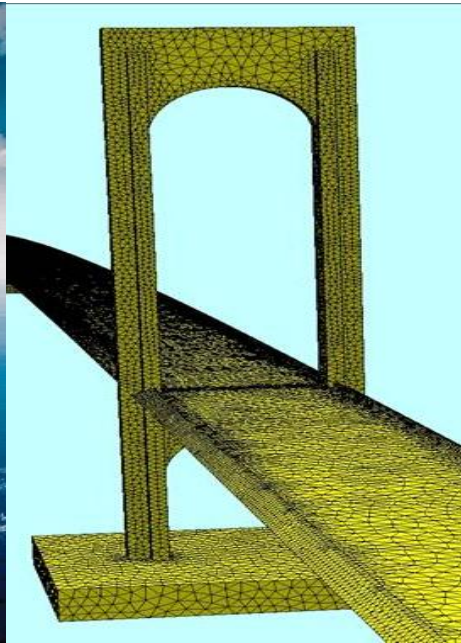
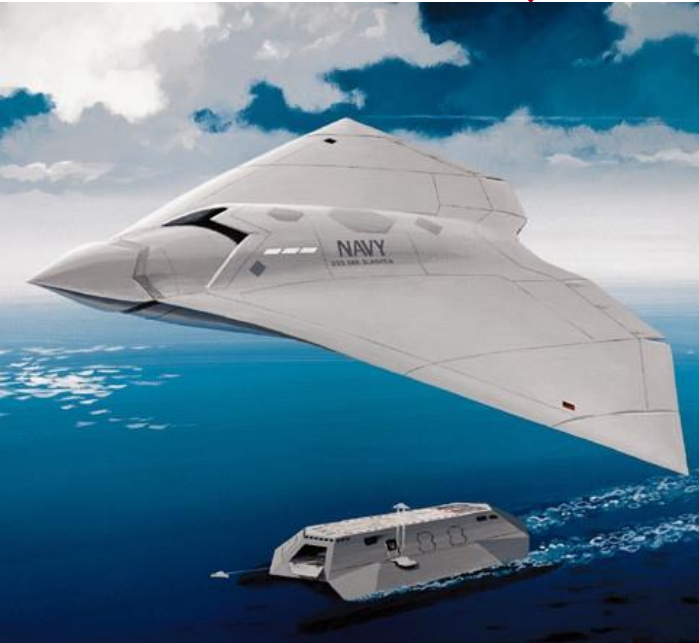


IMPLEMENTATION ON MOBILE DEVICES





VISION



Courtesy of Charlie Taylor



ACKNOWLEDGEMENTS

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