

Development and Application of Acoustic Metamaterials with Locally Resonant Microstructures

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Wave Propagation in Elastic Solids With Negative Mass Density or Modulus

What would happen if mass or modulus becomes negative?

- Dispersion equation: $q = \omega \sqrt{-\frac{\rho}{E}} = i\beta\omega$

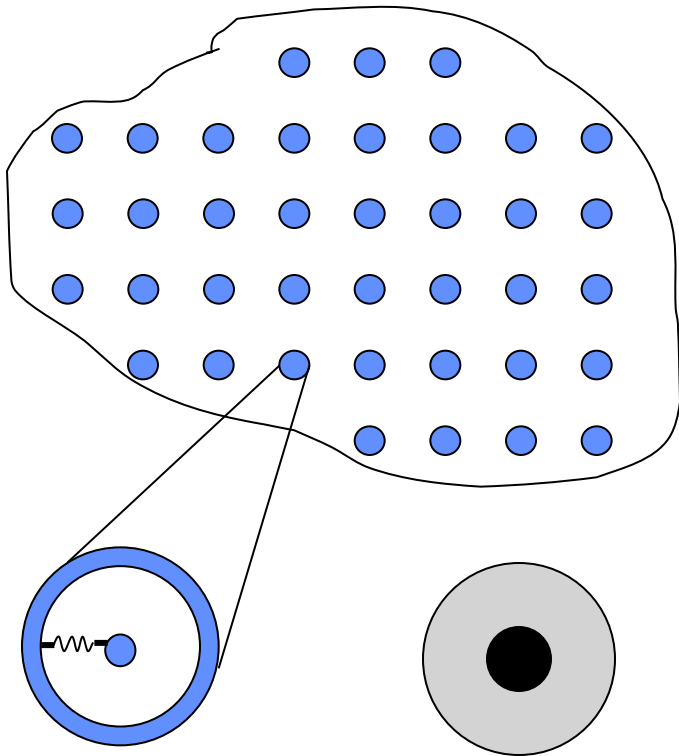
- Wave attenuates: $u = Ae^{i(qx - \omega t)} = Ae^{-\beta\omega x} e^{i\omega t}$

β Is attenuation factor

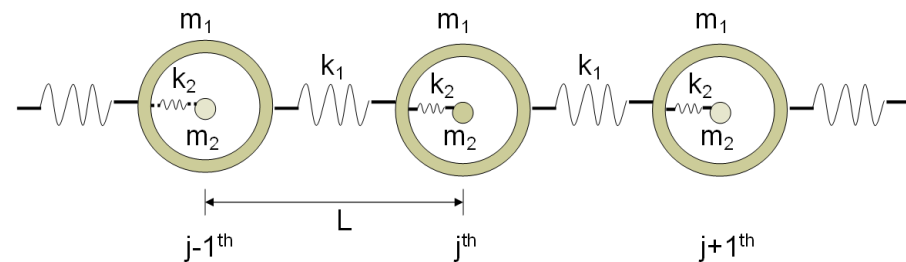
Wave cannot propagate without attenuation in elastic solids with negative mass density or modulus

Metamaterials with Local Resonators

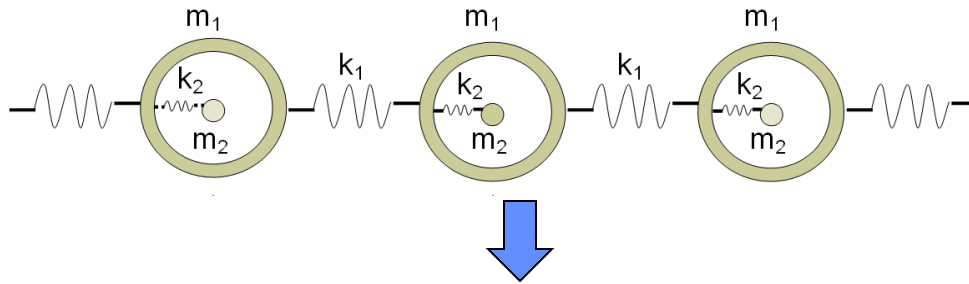
Composite with Resonators



1D Lattice Model

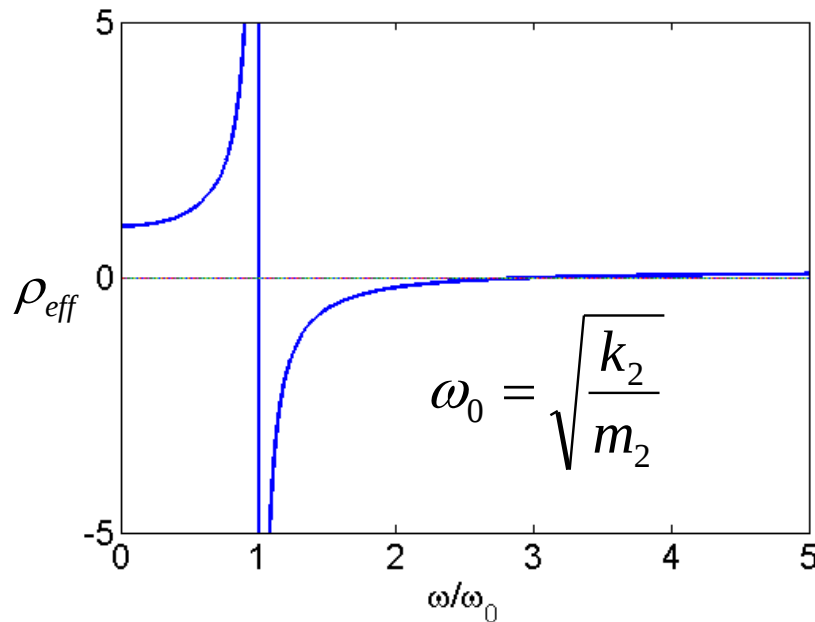


Metamaterials with Negative Effective Mass



ρ_{eff} , E_{eff}

Effective mass for mass-in-mass lattice

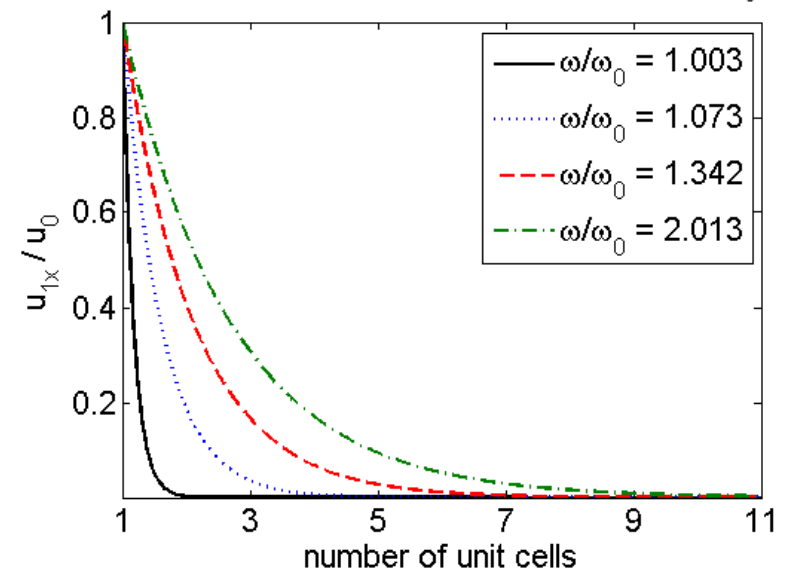


$$u = Be^{i(qx - \omega t)}$$

Negative effective mass

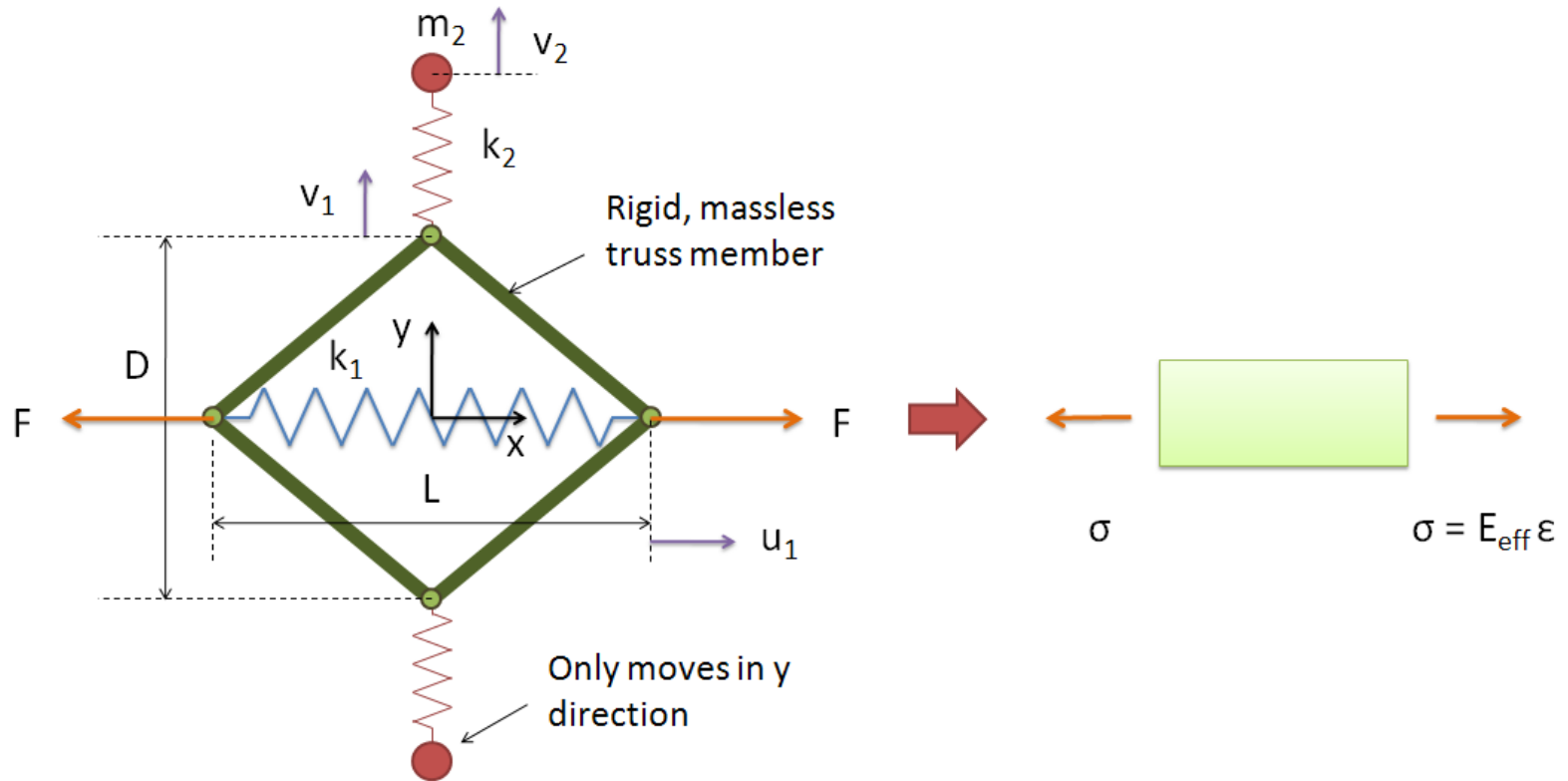
Wave attenuation

Displacement Envelope, (Local resonance ω_0)

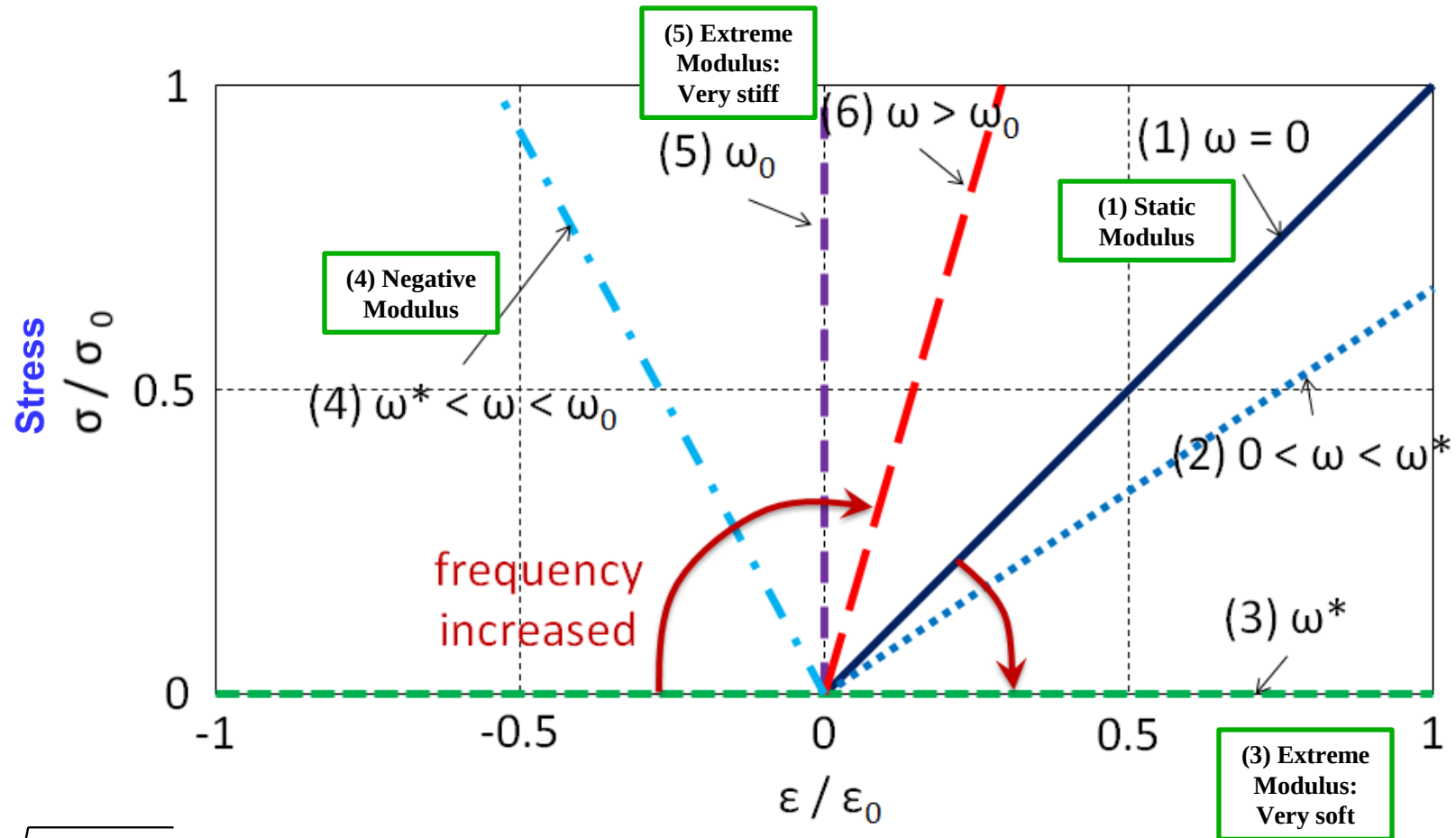


Acoustic Metamaterial with Negative Effective Young's Modulus

A Mechanical Unit Model and Its Representative Elastic Solid



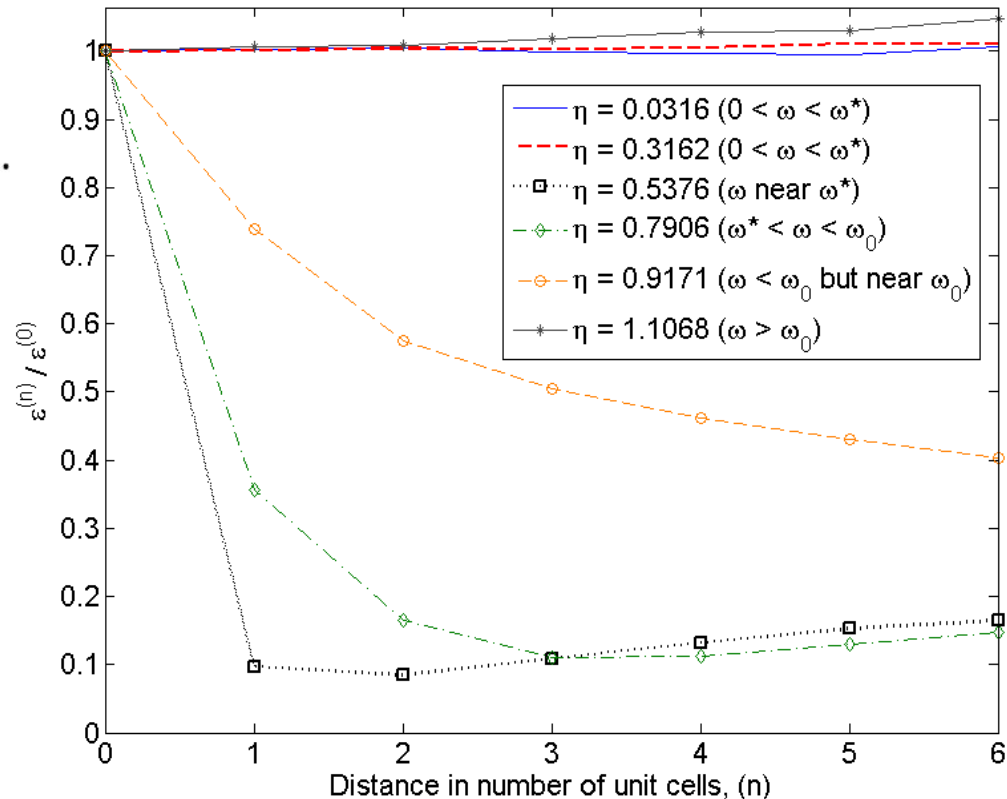
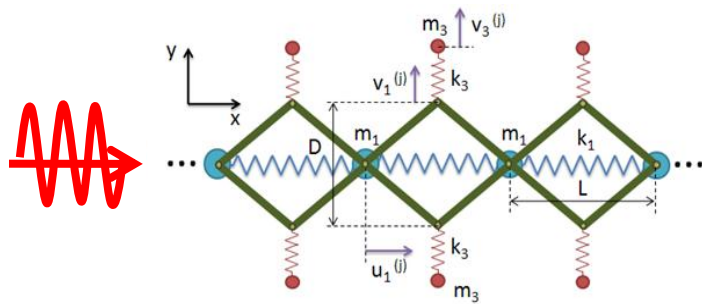
Frequency-dependent Modulus (stress-strain curves)



$\omega_0 = \sqrt{k_2 / m_2}$ Is local resonance frequency

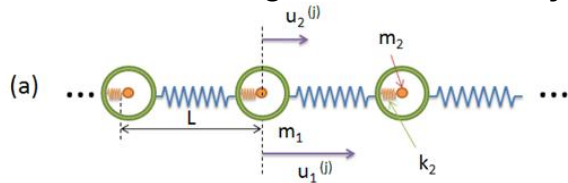
Wave Attenuation in Metamaterial with Negative Effective Modulus

- Wave amplitude decays when its frequency falls inside the band gap, especially if frequency is near the frequency

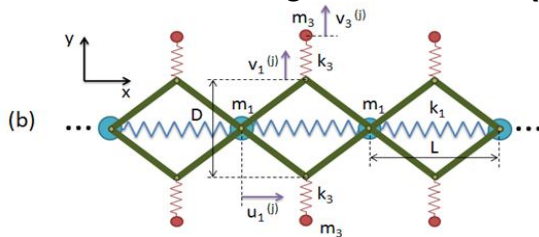


Metamaterial with Double Negativity (DN)

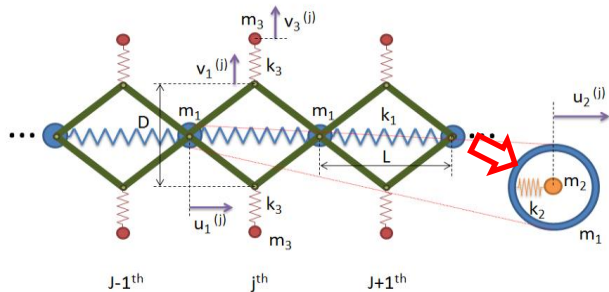
Metamaterial with negative mass density (NMD)



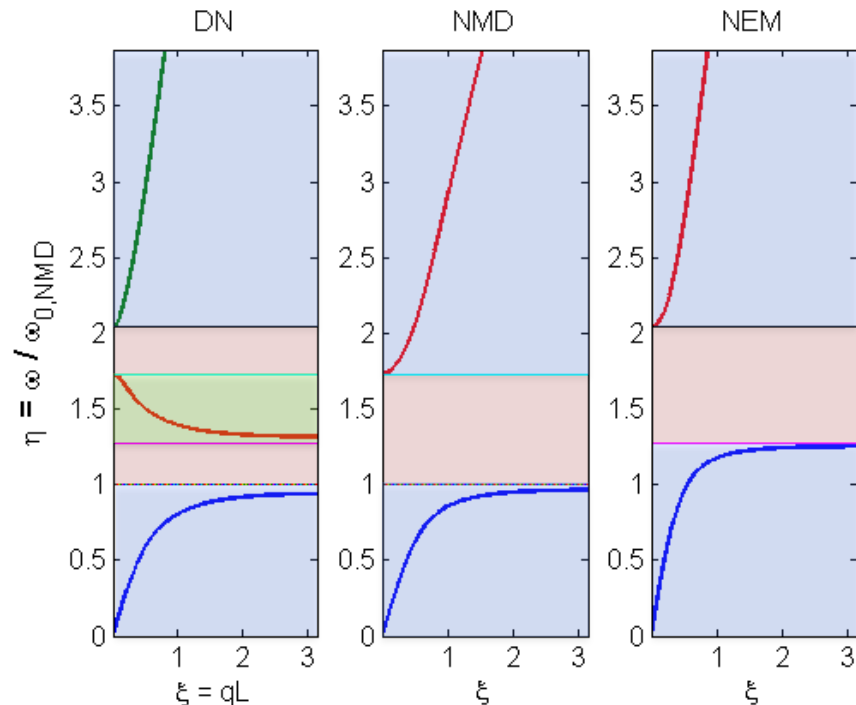
Metamaterial with negative modulus (NEM)



Metamaterial with Double Negativity (DN)

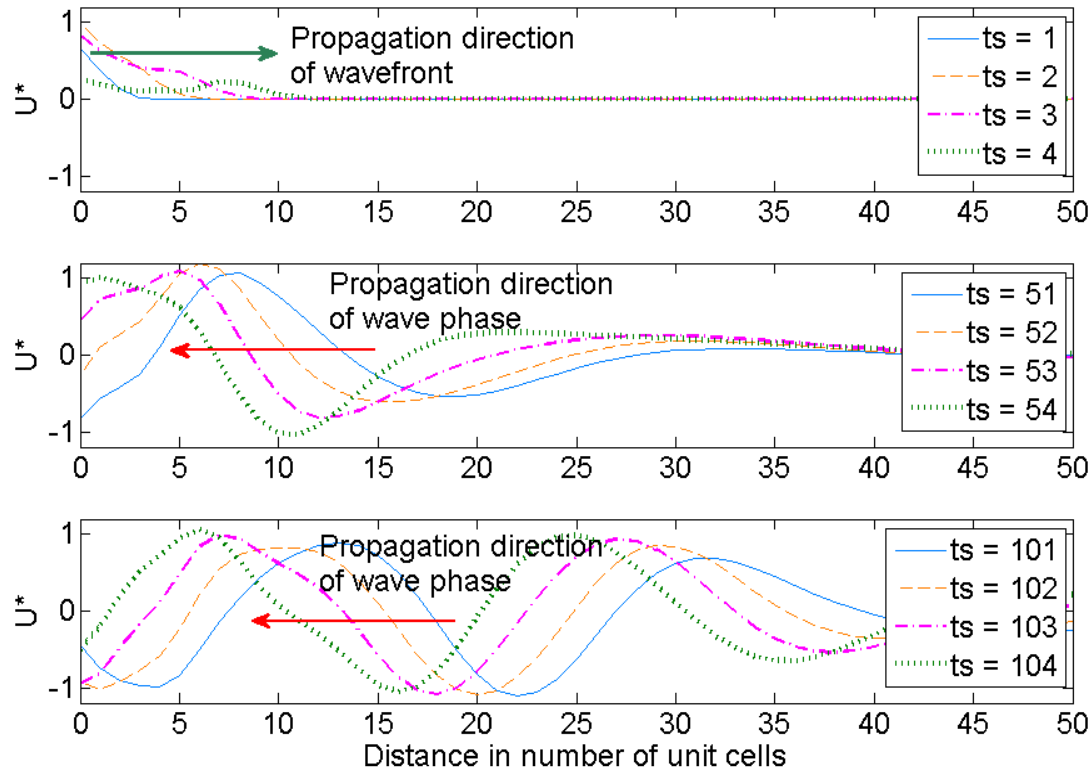
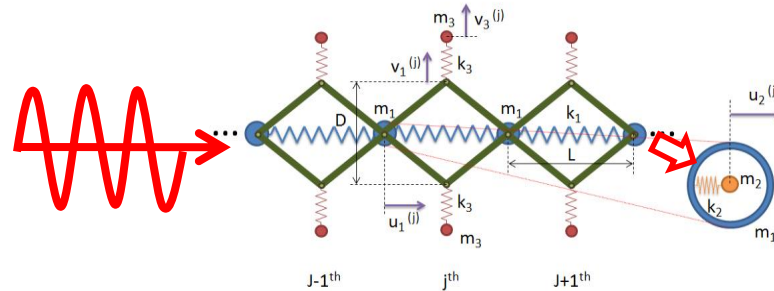


Double Negativity (green area)
Negative effective mass (Band Gap, red area)
Negative effective modulus (Band Gap, red area)

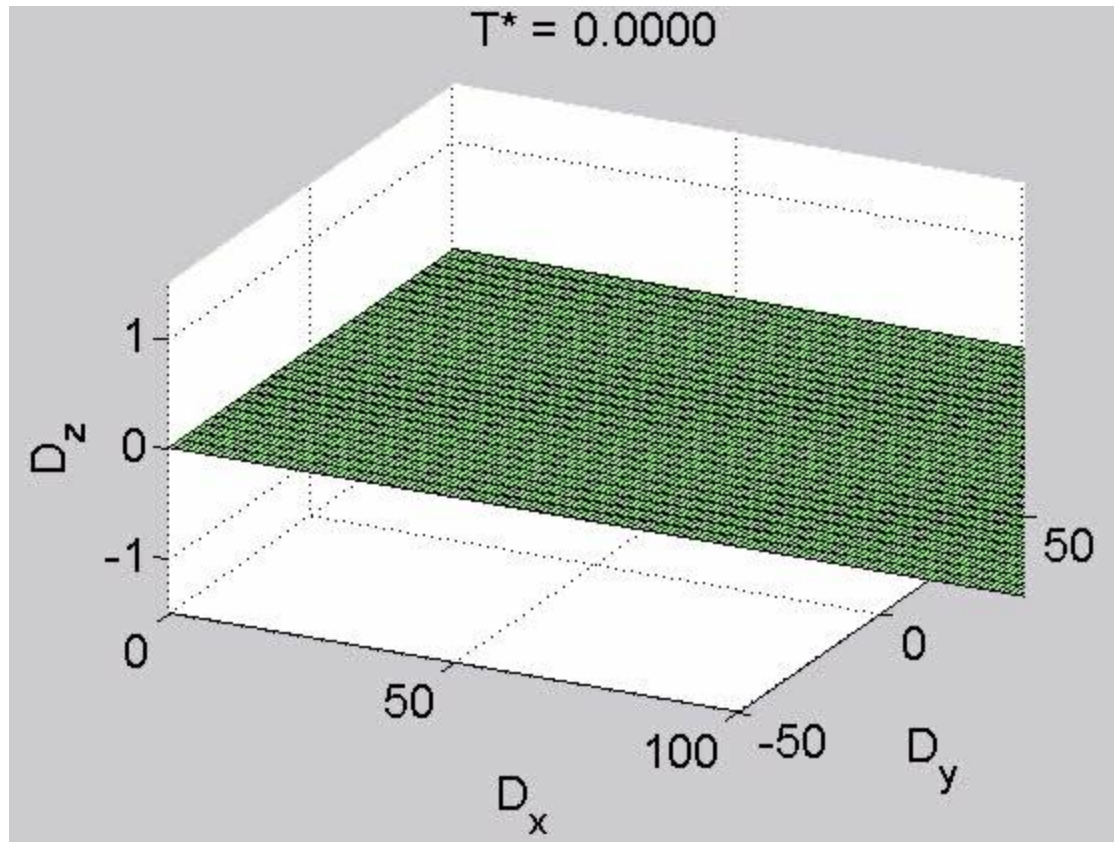


Wave Propagation in Metamaterial with Double Negativity

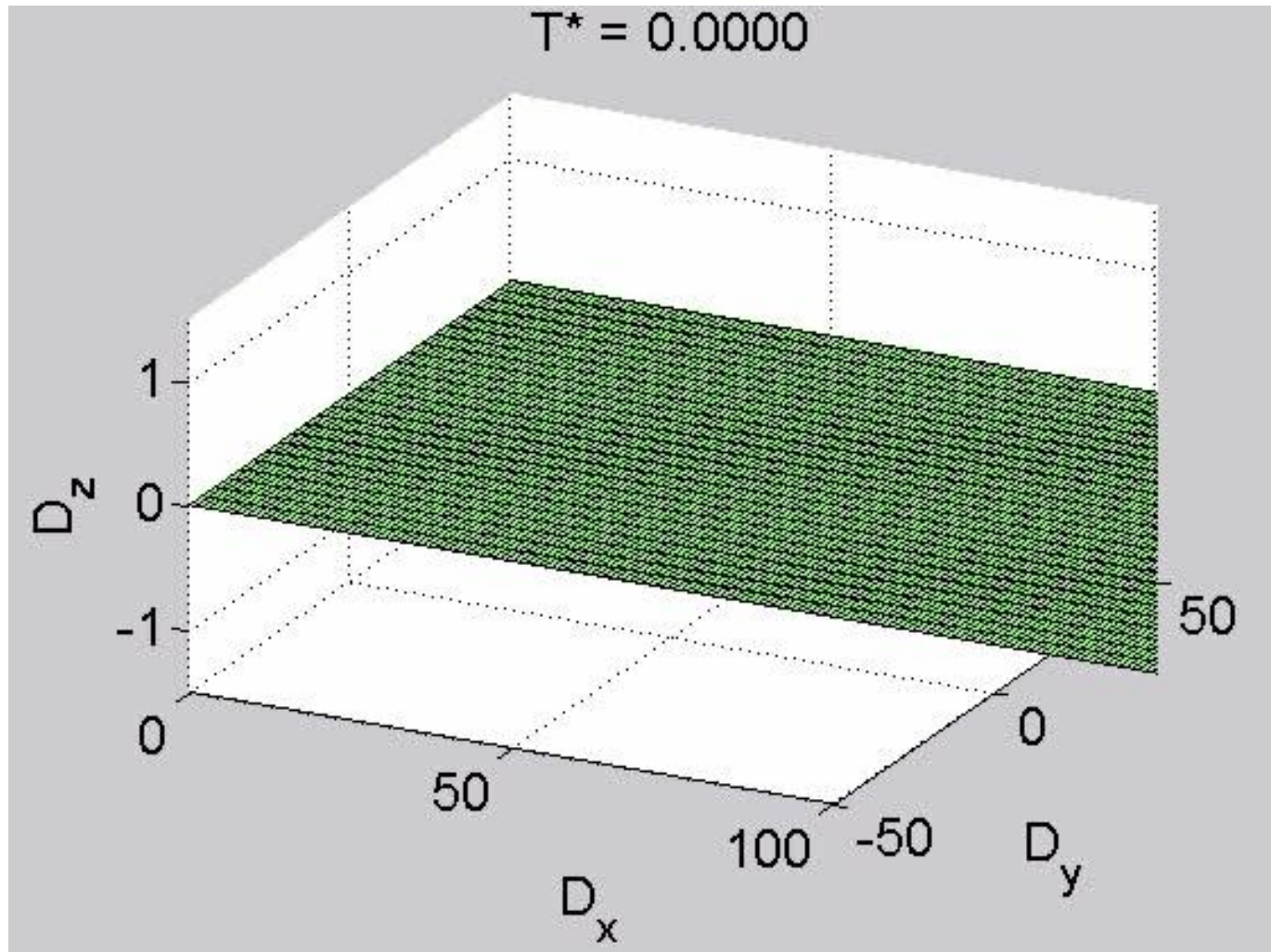
$$\omega / \omega_0^{NMD} = 1.56$$



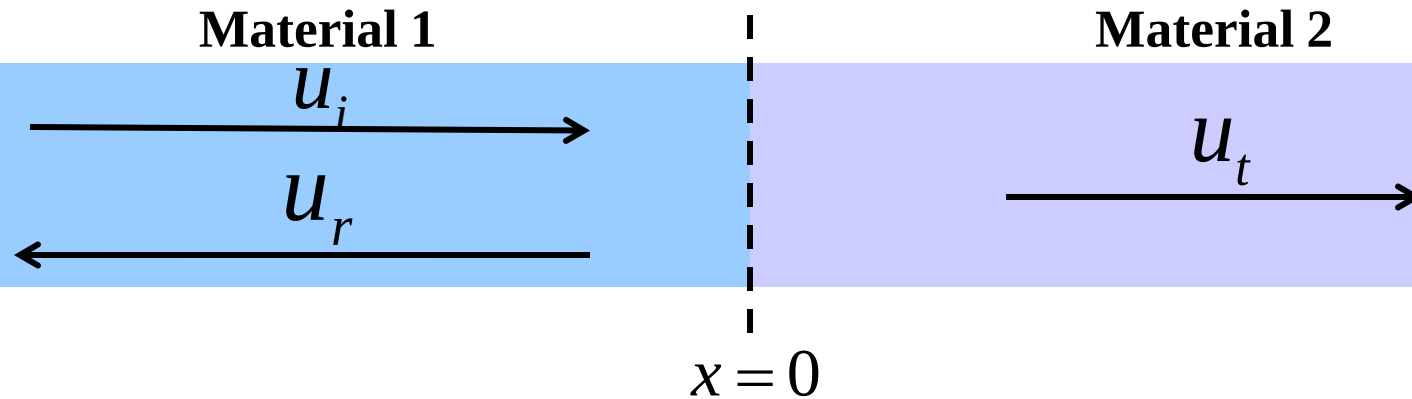
Double Positive Metamaterial



Double Negative Metamaterial



Derivation for Reflection and Transmission Coefficients



Assume

$$u_i = \hat{u}_i e^{i(\omega t - q_1 x)}$$

$$u_r = \hat{u}_r e^{i(\omega t + q_1 x)}$$

$$u_t = \hat{u}_t e^{i(\omega t - q_2 x)}$$

$$R \equiv \frac{\hat{u}_r}{\hat{u}_i} = \frac{E_1 q_1 - E_2 q_2}{E_1 q_1 + E_2 q_2}$$

$$T \equiv \frac{\hat{u}_t}{\hat{u}_i} = \frac{2E_1 q_1}{E_1 q_1 + E_2 q_2}$$

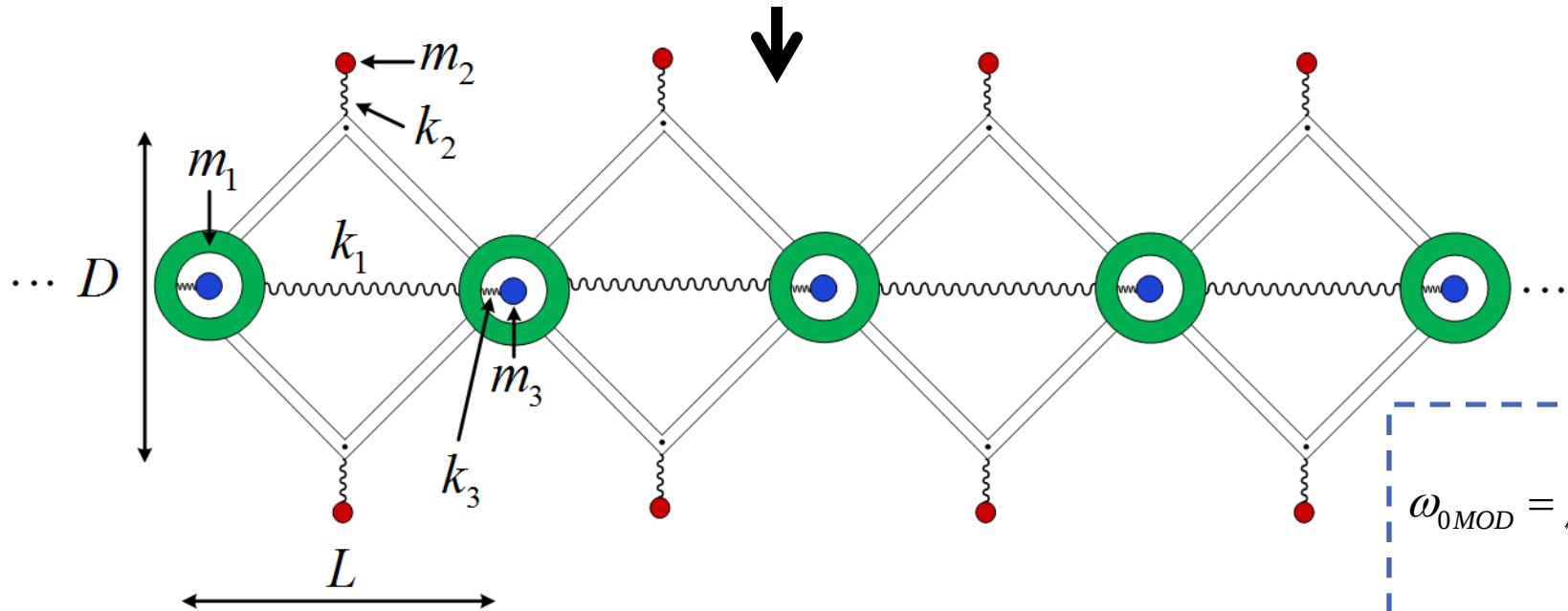
$\left\{ \begin{array}{l} \text{If } E_1 = E_2, \rho_1 = \rho_2, \text{ then } R = 0, T = 1 \\ \text{If } E_2 = -E_1, \rho_2 = -\rho_1, \text{ then } R = 0, T = 1 \end{array} \right.$

Material 2

Material 1
(Regular Material)

Material 2
(Metamaterial)

Material 1
(Regular Material)



$$E_{\text{eff}} = \frac{L}{A} \left[k_1 + \left(\frac{1}{2} \right) \left(\frac{k_2 \omega^2}{\omega^2 - \omega_{0\text{MOD}}^2} \right) \left(\frac{L}{D} \right)^2 \right]$$

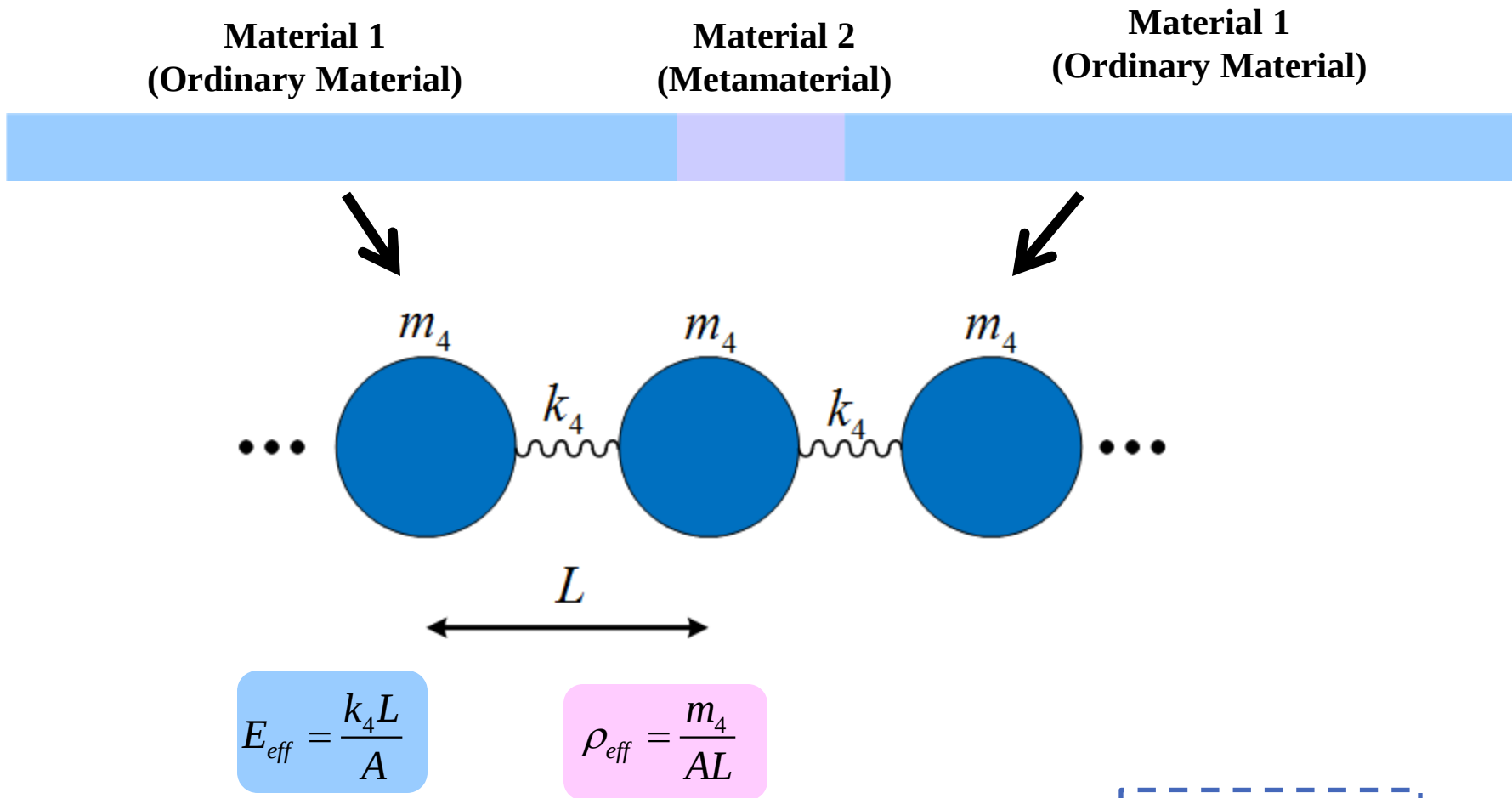
$$\rho_{\text{eff}} = \frac{1}{AL} \left[m_1 + m_3 \left(\frac{\omega_{0\text{MASS}}^2}{\omega_{0\text{MASS}}^2 - \omega^2} \right) \right]$$

$$\omega_{0\text{MOD}} = \sqrt{\frac{k_2}{m_2}}$$

$$\omega_{0\text{MASS}} = \sqrt{\frac{k_3}{m_3}}$$

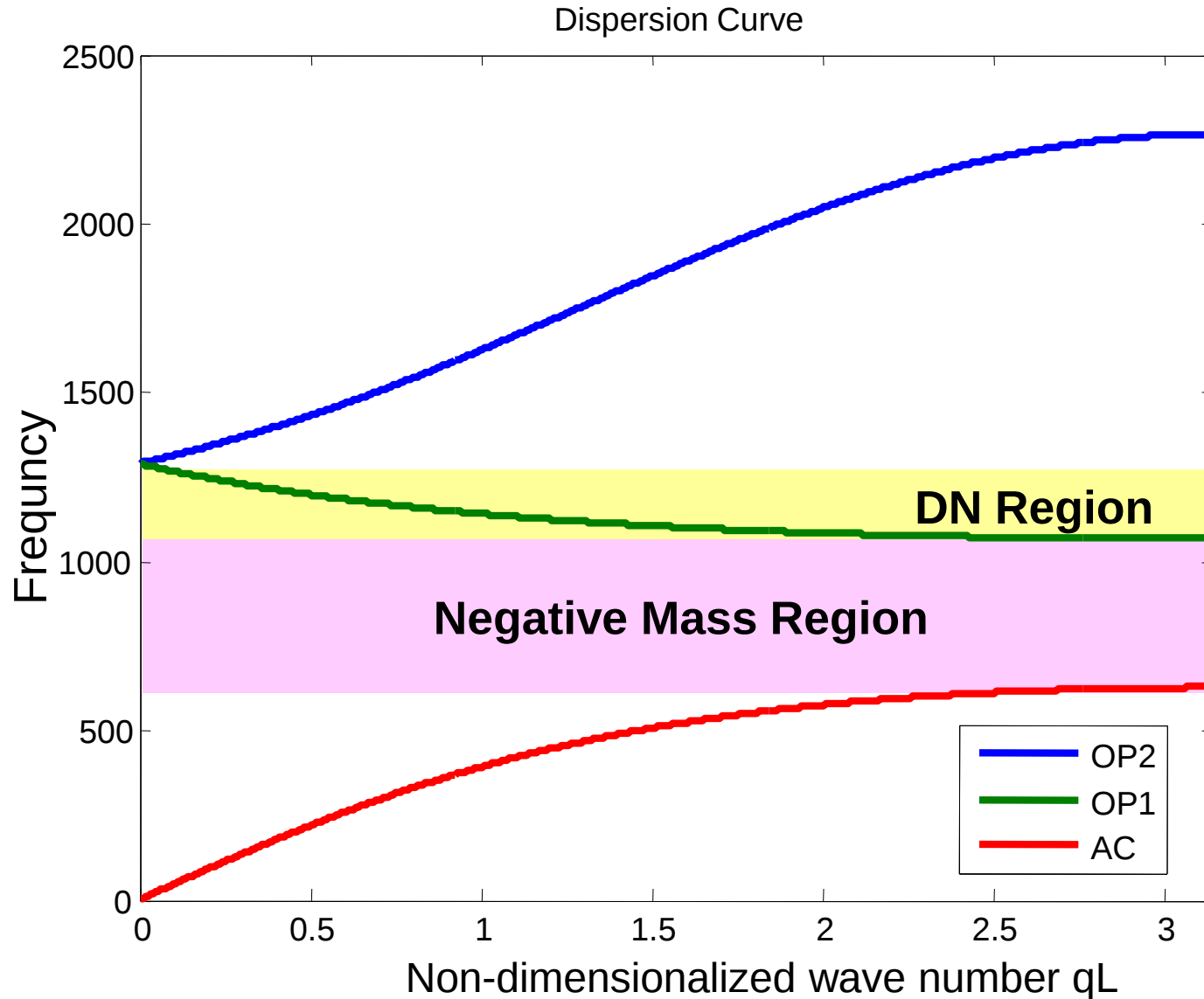
$$A=1$$

Material 1

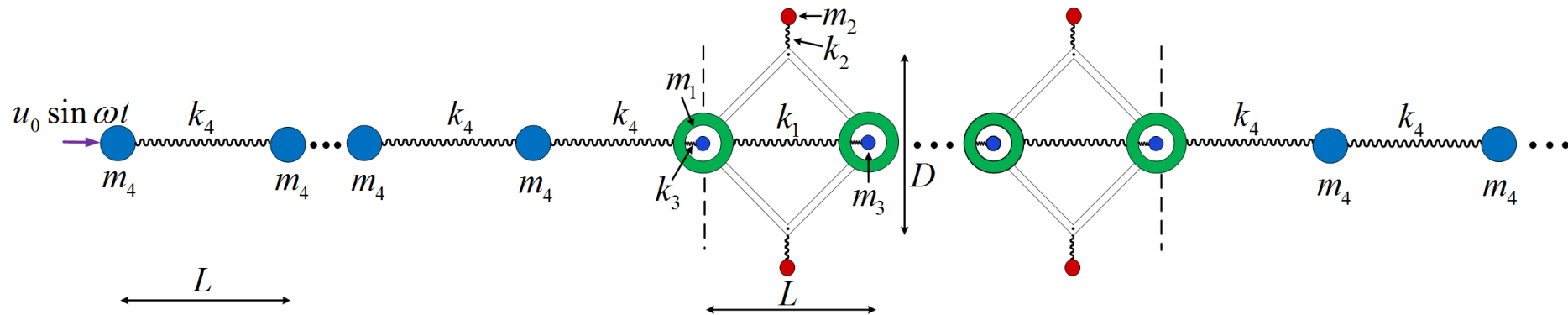


where $A=1$

Dispersion Curve for Metamaterial



Material Design



Case 1: $\omega = 1200$ (rad / s) $\rightarrow$ Frequency for double negativity

Case 2: $\omega = 650$ (rad / s) $\rightarrow$ Frequency for negative mass

$$m_1 = 2.4 \times 10^{-4} \text{ (kg)}$$

$$m_2 = 1.2 \times 10^{-4} \text{ (kg)}$$

$$m_3 = 2.4 \times 10^{-4} \text{ (kg)}$$

$$m_4 = 9.0 \times 10^{-5} \text{ (kg)}$$

$$k_1 = 100.0 \text{ (N / mm)}$$

$$k_2 = 200.0 \text{ (N / mm)}$$

$$k_3 = 200.0 \text{ (N / mm)}$$

$$k_4 = 535.3 \text{ (N / mm)}$$

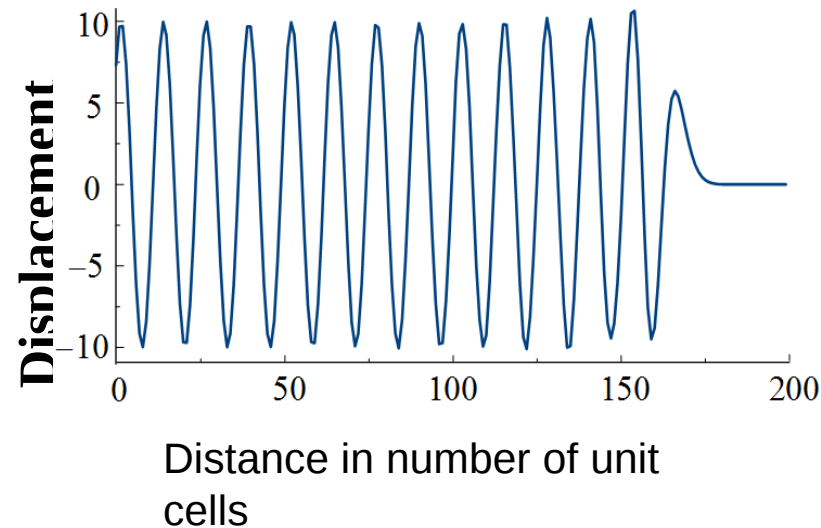
Case 1: Simulation Result in DN Region

$$\omega = 1200 \text{ rad} / s$$

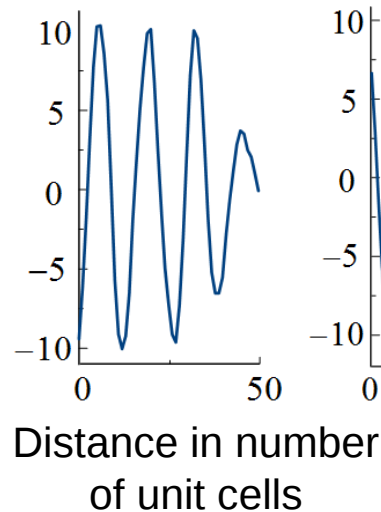
Material 1
(Regular Material)

Material 2
(Metamaterial)

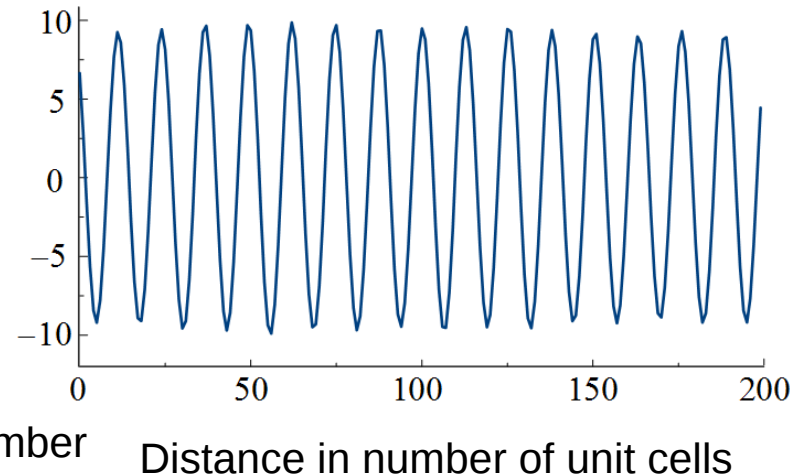
Material 1
(Regular Material)



$t = 0.07 \text{ s}$



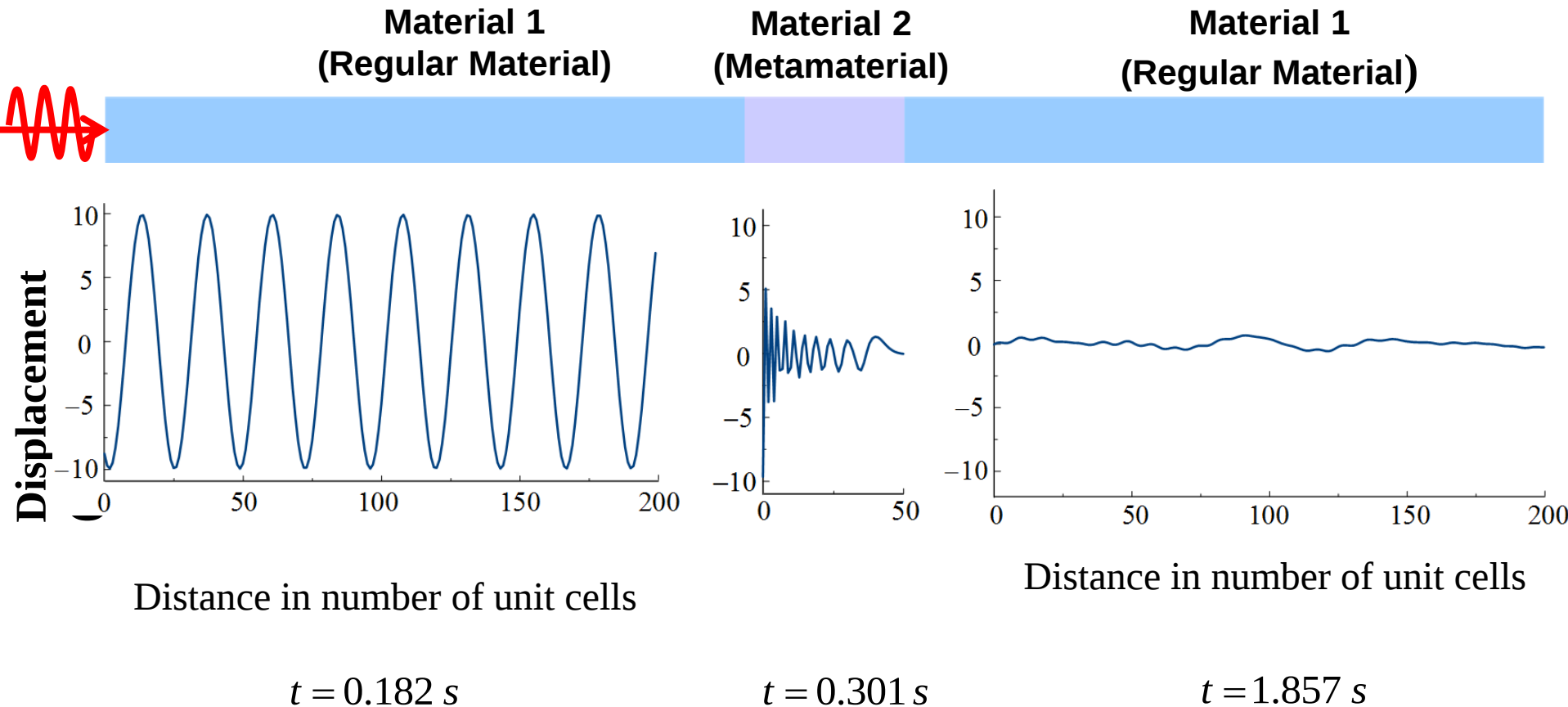
$t = 0.390 \text{ s}$



$t = 0.775 \text{ s}$

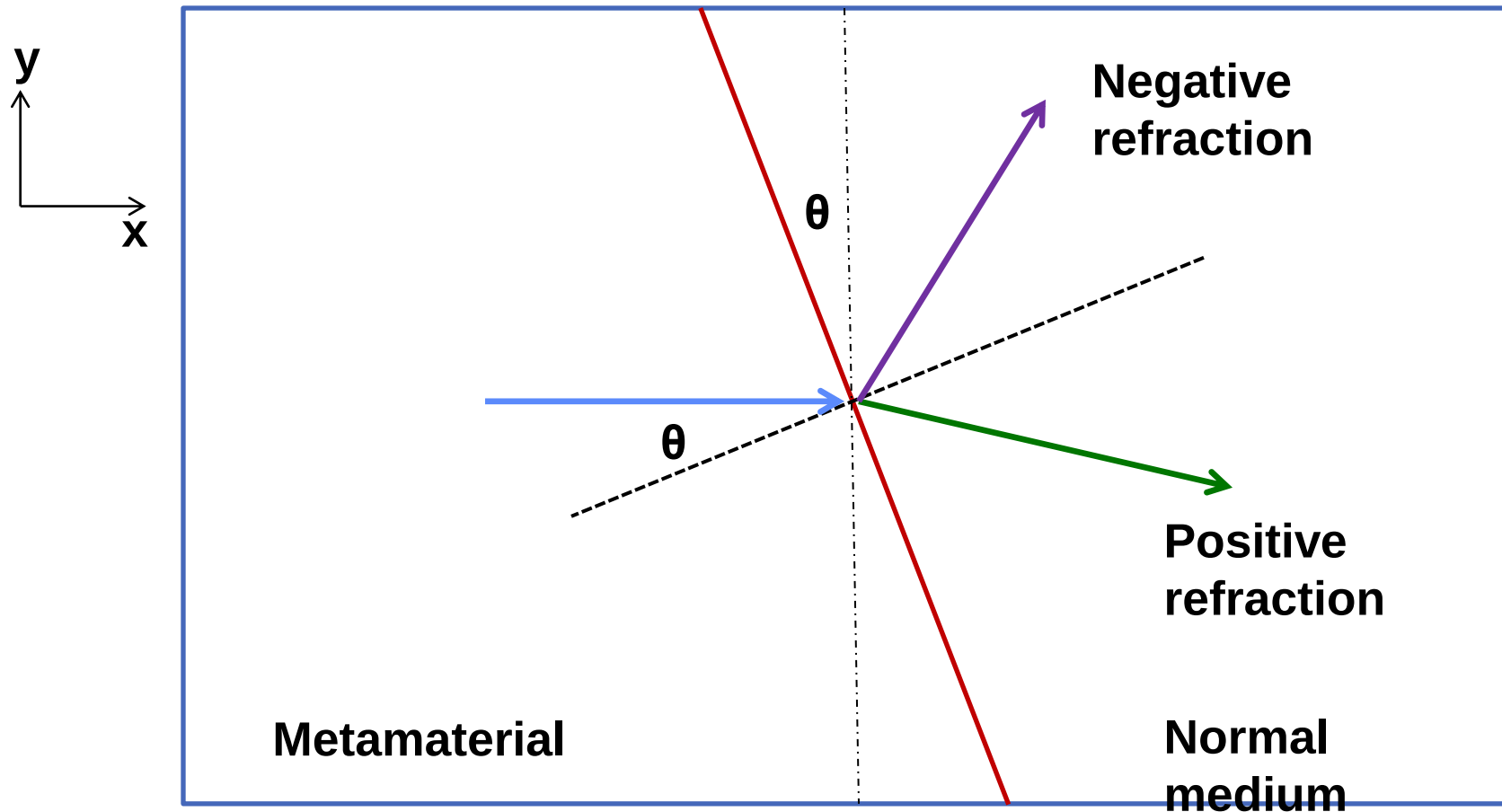
Case 2: Simulation Result in Negative Mass Region

$$\omega = 650 \text{ rad} / \text{s}$$

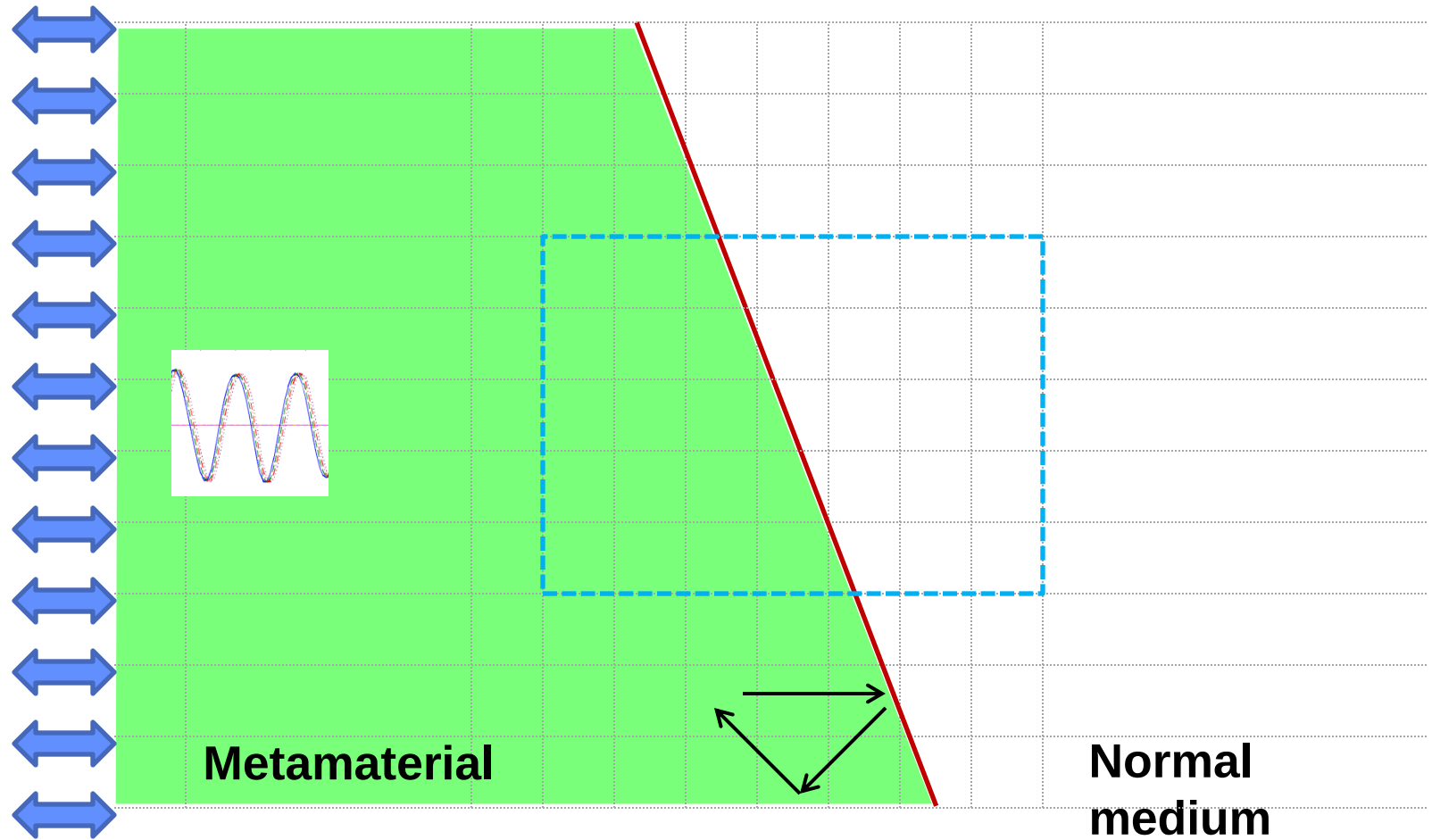


Refraction of Metamaterials

- 2D Double-Negativity Metamaterial

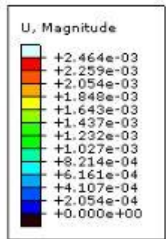


Boundary Condition: Plane wave

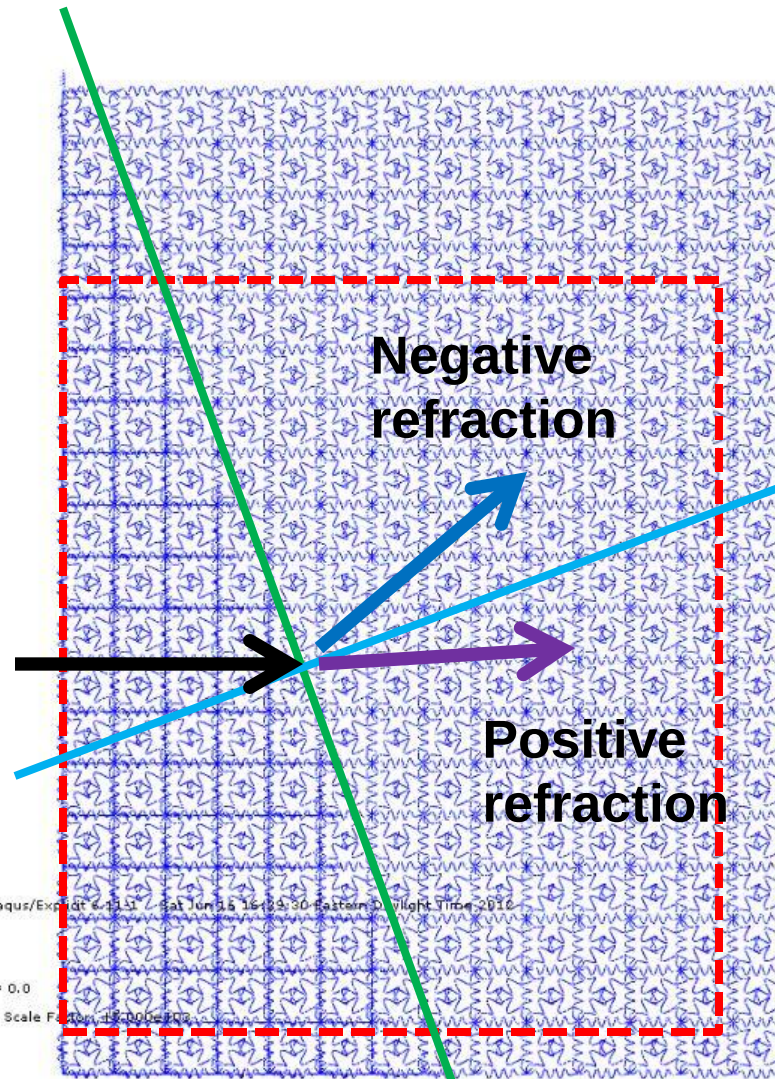


Simulation Window (15x20 units)

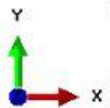
Interface



Step: Step-1 Frame: 0
Total Time: 0.000000

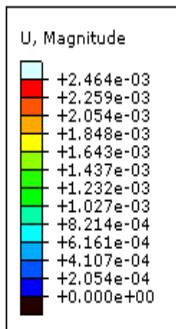


Normal to interface



ODB: WaveProp_Gen.odb Abaqus/Explicit Sat Jun 25 16:33:30 Eastern Daylight Time 2016

Step: Step-1
Increment: 0: Step Time = 0.0
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor: 1.000e+00



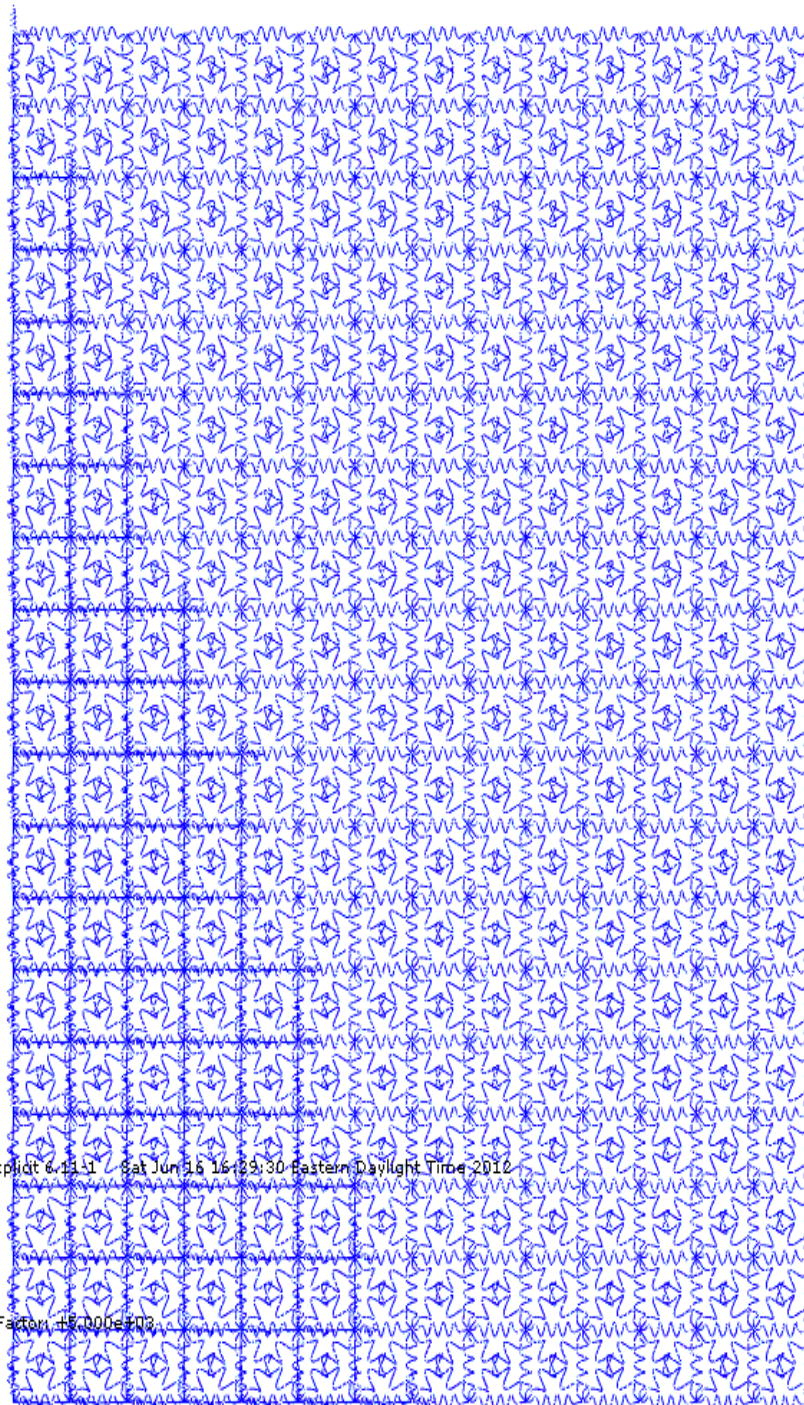
Step: Step-1 Frame: 1
Total Time: 0.000000

DN



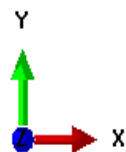
ODB: WaveProp_Gen.odb Abaqus/Explicit 6.11.1 Sat Jun 16 16:29:30 Eastern Daylight Time 2012

Step: Step-1
Increment 0: Step Time = 0.0
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor: +5.000e+03



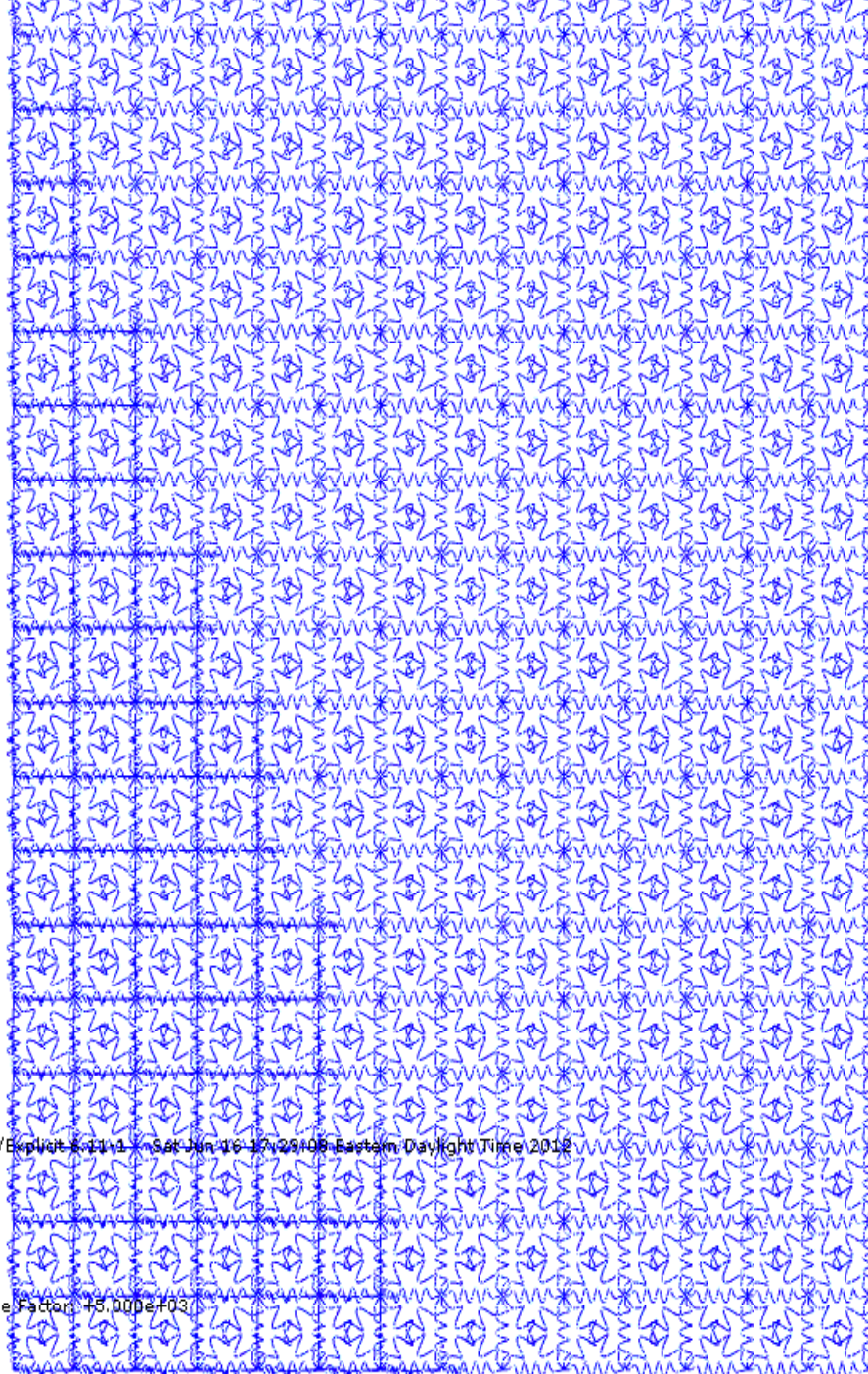
071e-03
898e-03
726e-03
553e-03
381e-03
208e-03
035e-03
628e-04
903e-04
177e-04
451e-04
726e-04
000e+00

DP

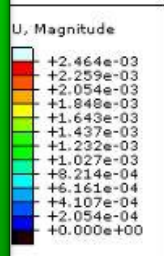


ODB: WaveProp_Gen.odb Abaqus/Explicit: Job-1: Sat Jun 16 17:29:08 Eastern Daylight Time 2012

Step: Step-1
Increment 0: Step Time = 0.0
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor: 45.000e+03



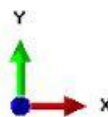
Simulation: Plane wave (DN region: 1)



Interface

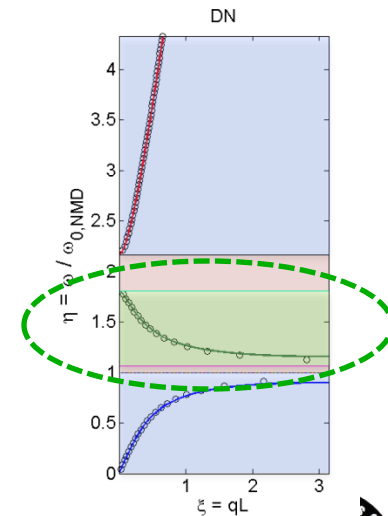
Step: Step-1 Frame: 100
Total Time: 0.010000

Normal to
interface

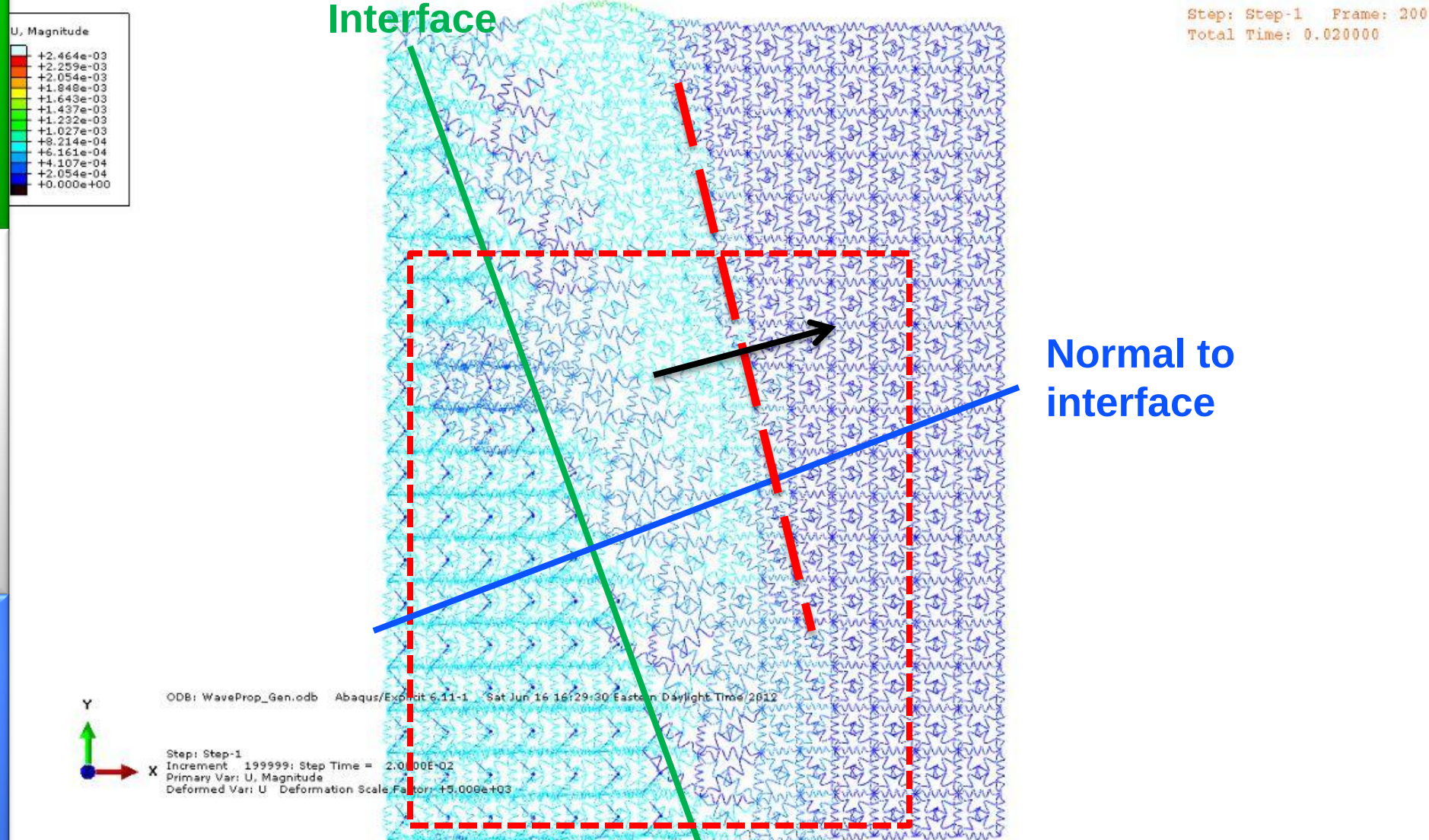


ODB: WaveProp_Gen... Aquas/Exp... dt: 6.11e-1 Sat Jun 16 16:29:30 Eastern Daylight Time 2016

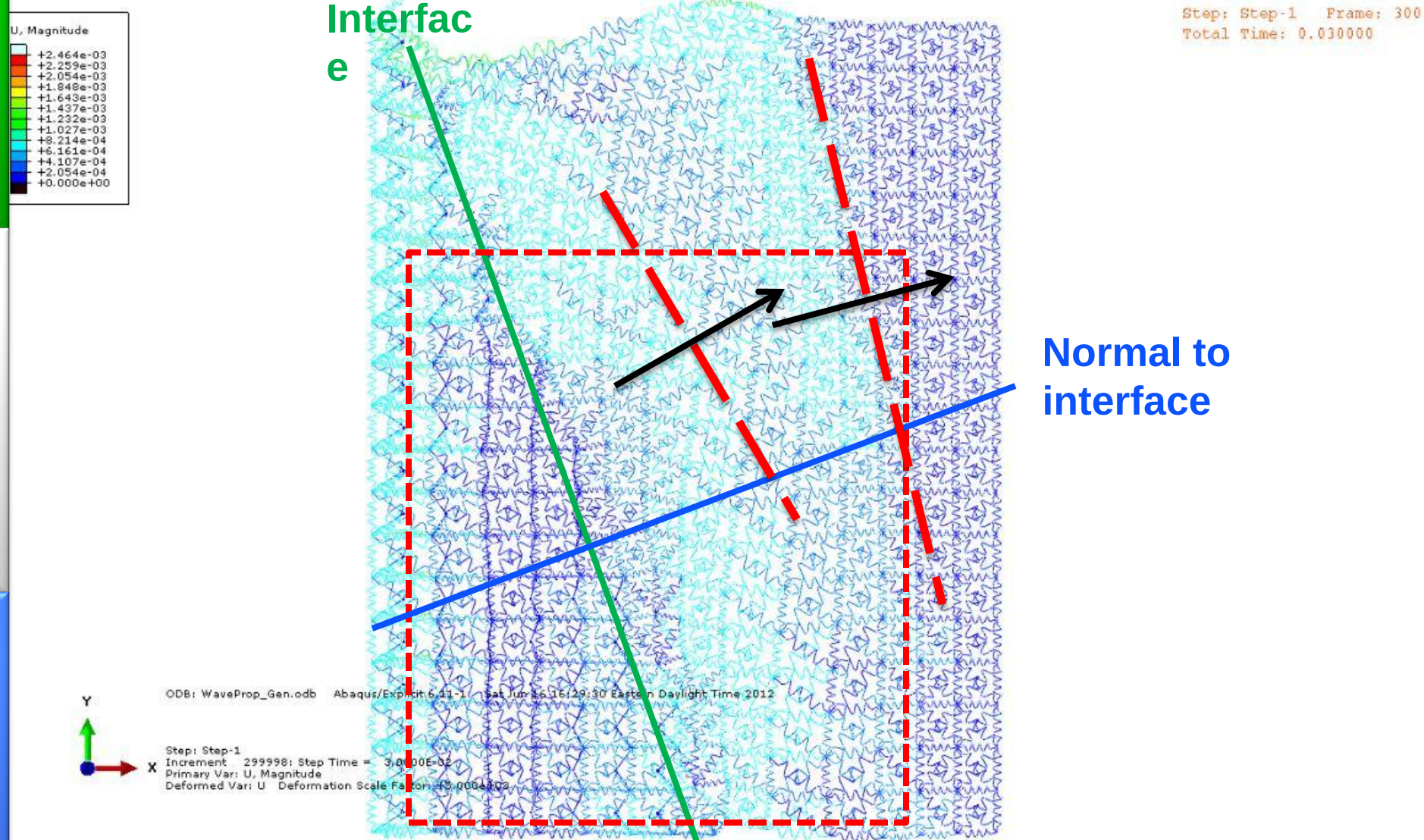
Step: Step-1
Increment: 100000 Step Time = 1.0000E-02
Primary Vari: U, Magnitude
Deformed Vari: U Deformation Scale Factor: +5.000e+03



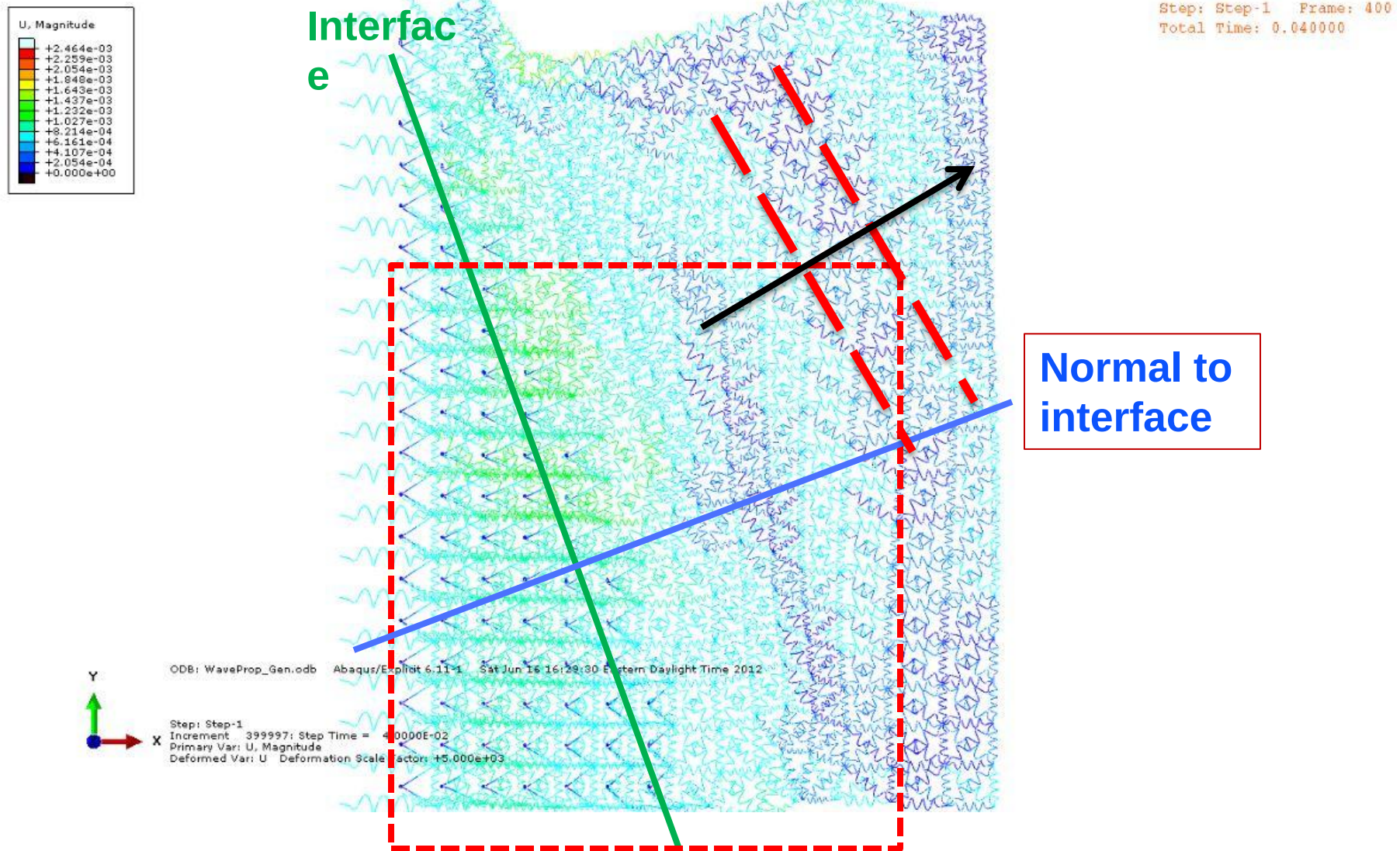
Simulation: Plane wave (DN region: 2)



Simulation: Plane wave (DN region: 3)

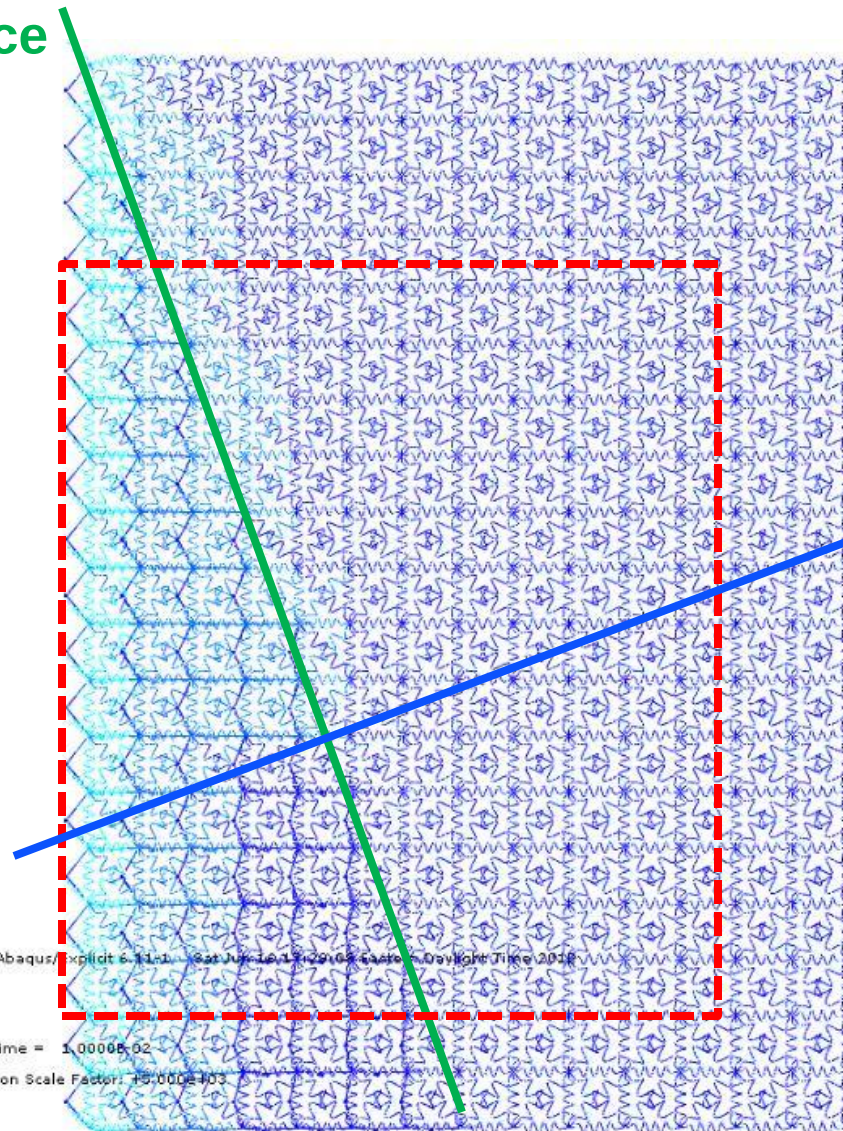
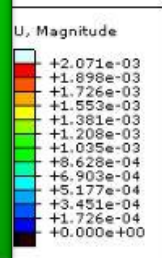


Simulation: Plane wave (DN region: 4)



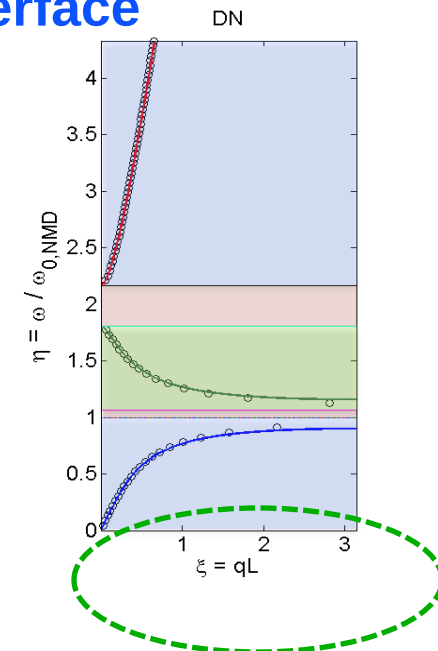
Simulation: Plane wave (DP region: 1)

Interface

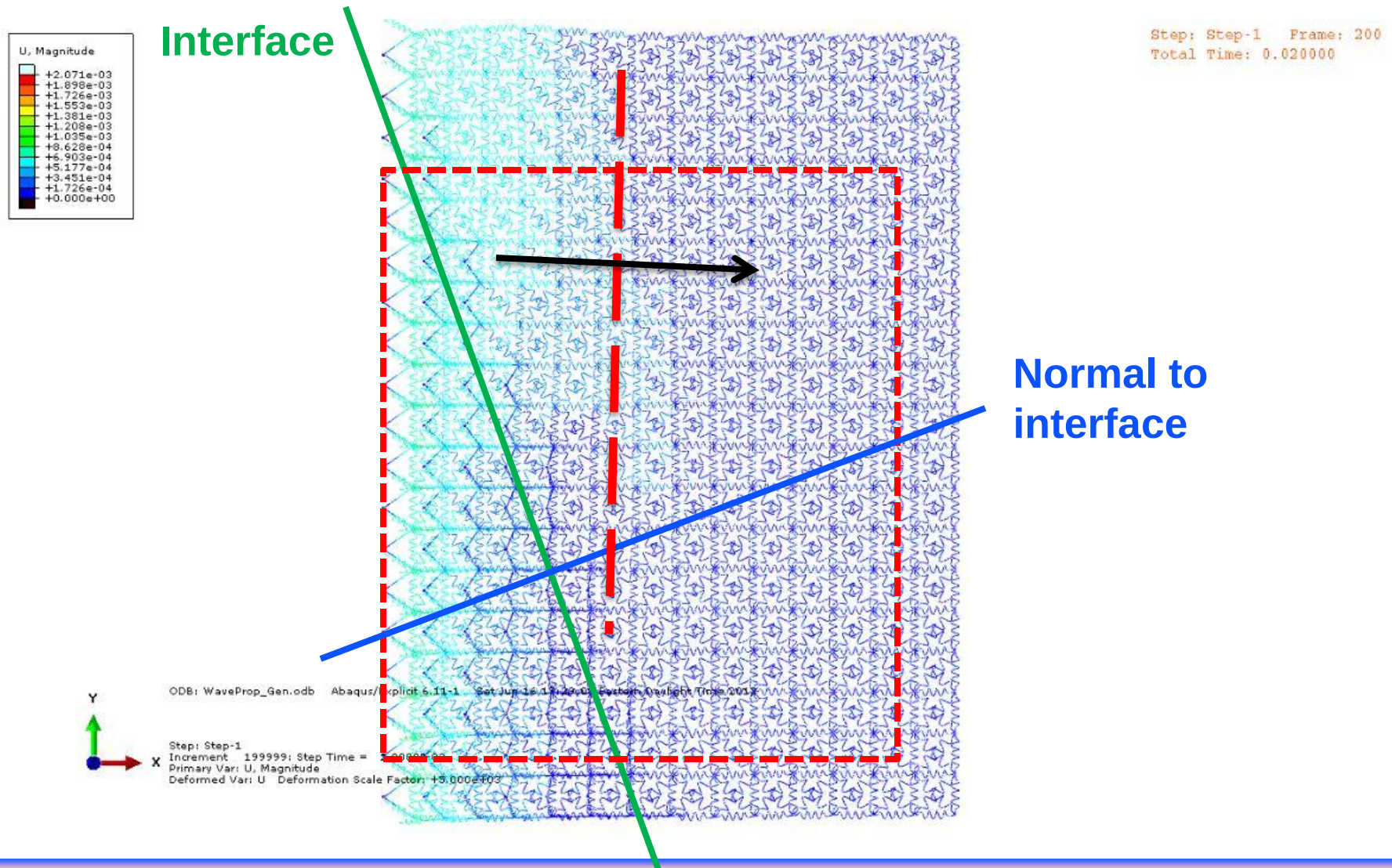


Step: Step-1 Frame: 100
Total Time: 0.010000

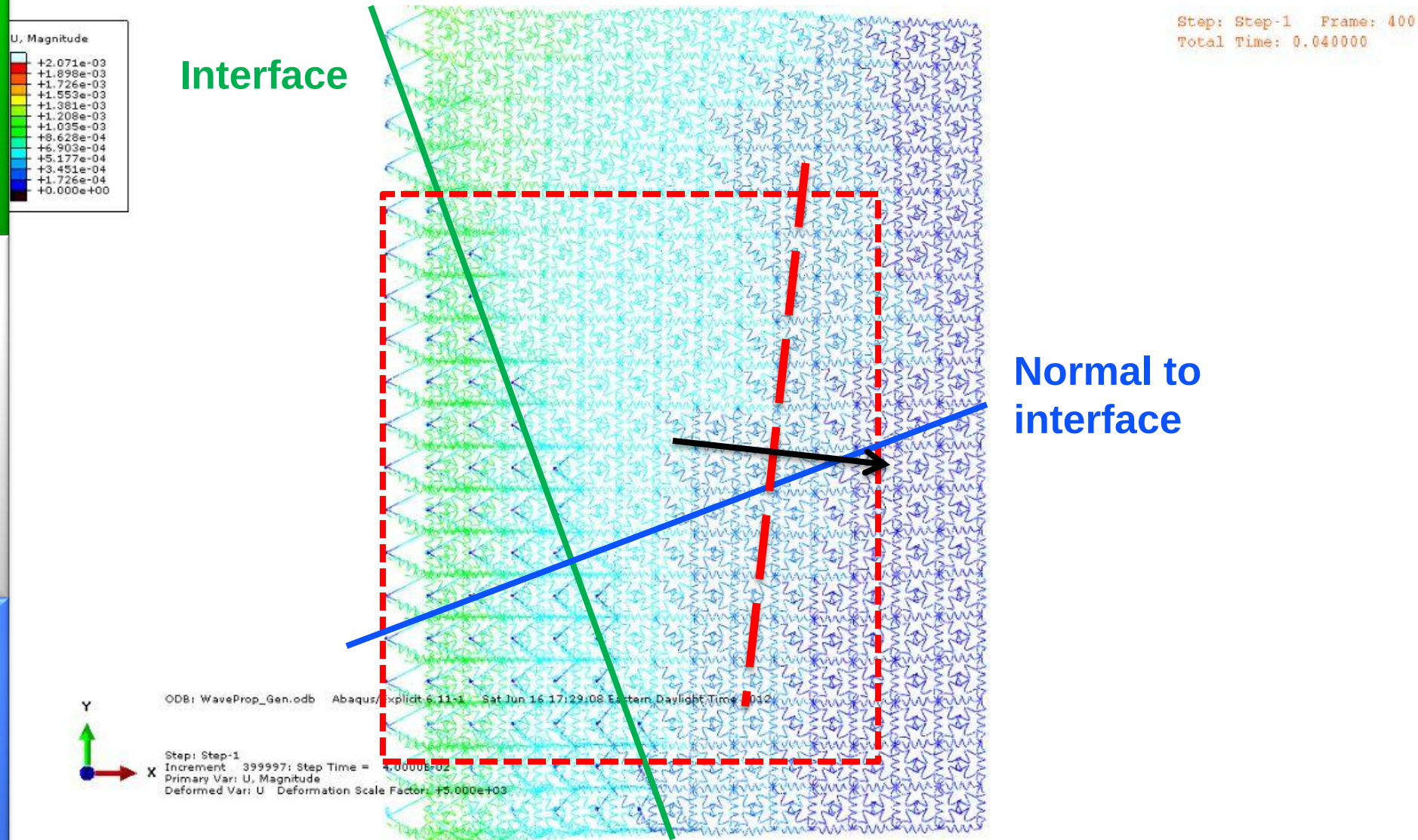
Normal to
interface



Simulation: Plane wave (DP region: 2)

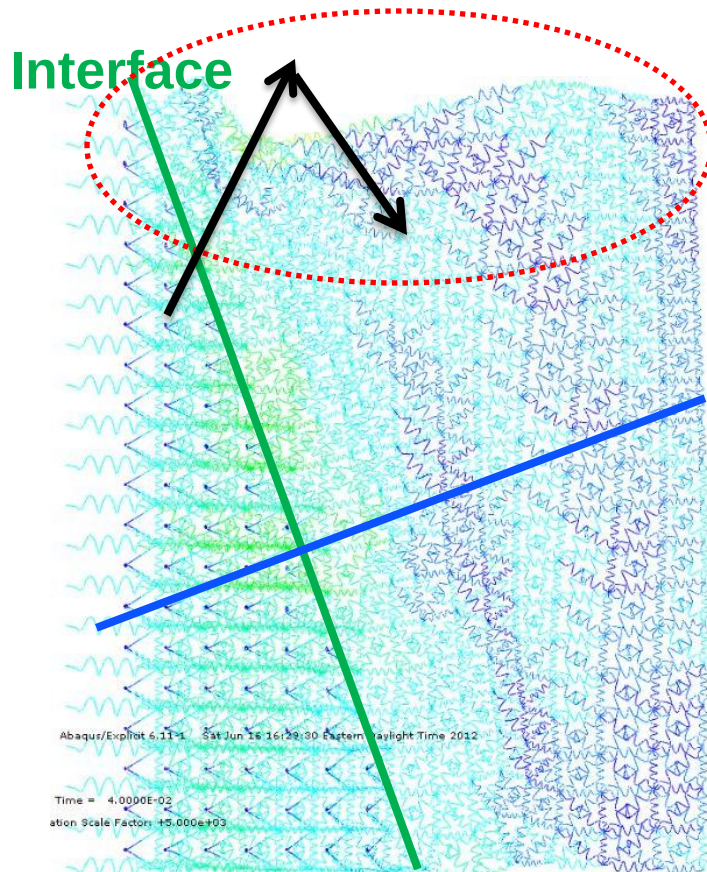


Simulation: Plane wave (DP region: 4)

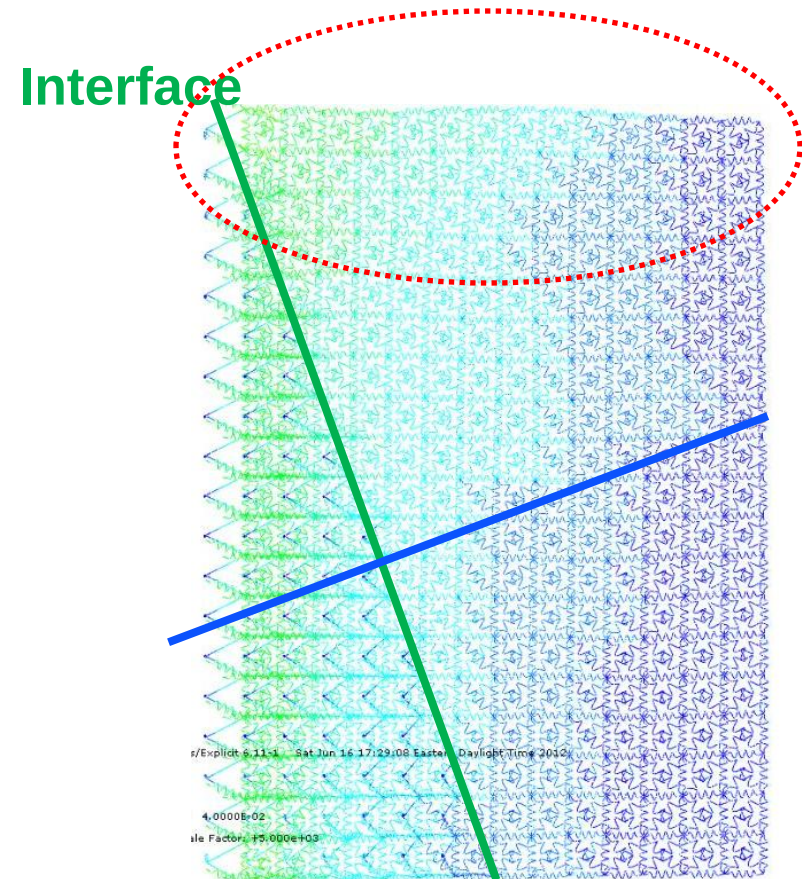


Plane Wave Comparison: DN vs. DP

Double Negativity



Double Positivity



List of Publications

- H. H. Huang and C. T. Sun, "Locally Resonant Acoustic Metamaterials with 2D Anisotropic Effective Mass Density," *Philosophical Magazine*, Vol. 91, No.6, 2011, pp. 981-996.
- H. H. Huang and C. T. Sun, "A study of Band-gap Phenomena of Two Locally Resonant Acoustic Metamaterials," *J. Nanoengineering and Nanosystems*, 2011.
- X. N. Liu, G. K. Hu, C.T. Sun, and G. L. Huang, "Wave Propagation Characterization and Design of Two-Dimensional Elastic Chiral Metacomposite," *J. of Sound and Vibration*, 330, pp. 2536-2553, 2011
- X.N. Liu, G. K. Hu, G. L. Huang, and C.T. Sun, "An Elastic Metamaterial with Simultaneously Negative Mass Density and Bulk Modulus," *Applied Physics Letters*, 98, 251907, 2011.
- H.H. Huang and C.T. Sun, "Behavior of an Acoustic Metamaterial with Extreme Young's Modulus," *J. Mechanics and Physics Solids*, doi:10.1016/j.jmps.2011.07.002, 2011.
- R. Zhu, G. L. Huang, H.H. Huang, and C. T. Sun, "Experimental and Numerical Study of Guided Wave Propagation in a Thin Metamaterial Plate," *Physics Letters A*, 375, , 2011, pp. 2863-2867
- H.H. Huang and C.T. Sun, "Continuum Modeling of a Composite Material with Internal Resonators," *Mechanics of Materials*, 46, 2012, pp.1-10.
- Hsin-Haou Huang and C. T. Sun, "Anomalous Wave Propagation in a One-dimensional Acoustic Metamaterial Having Simultaneously Negative Mass Density and Young's Modulus," to appear in the *Journal of the Acoustical Society of America*, 2012