

# Numerical Studies of Coastal Internal-Wave Mixing

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Award number: N00014-05-1-0491

## LONG-TERM GOALS

To develop parameterizations of internal-wave driven turbulent mixing in a coastal environment suitable for use in regional numerical models.

## OBJECTIVES

1. To investigate the dependence of coastal diapycnal mixing on aspects of low-mode, low-frequency internal waves that may feasibly be resolved by a regional numerical model, including:
  - internal wave frequency (generally tidal or wind-generated near-inertial waves)
  - vertical wave structure
  - wave strength
  - latitude (some wave instabilities are strongly latitude dependent)
2. To test the underlying assumptions and kinematic scalings of various proposed coastal mixing parameterizations, including KPP, the Gregg-Henyey scaling [Gregg, 1989], the scaling proposed by *D'Asaro and Lien* [2000], and the MacKinnon-Gregg scaling [MacKinnon and Gregg, 2003].

## APPROACH

Away from surface and bottom boundary layers turbulent mixing is primarily driven by breaking internal waves. While internal wave energy is generated mostly in the form of large-scale waves (typically internal tides), it is the smallest-scale internal waves that break through shear or convective instabilities to produce turbulence. When waves are reasonably linear (as opposed to, for example, strongly nonlinear solitons or bores), the rate of small-scale wave breaking is controlled by and equal to the rate at which energy is transferred from large to small-scale waves through wave-wave interactions. In the open ocean, theories have been developed that use concepts of wave-wave interaction to predict turbulent mixing rates in terms of larger-scale shear and stratification. Such theories, in particular the class known as Gregg-Henyey parameterizations, compare well to observed turbulent dissipation rates.

However, in the coastal ocean empirical evidence suggest that the Gregg-Henyey (GH) model fails to reproduce observed turbulence [Kunze *et al.*, 2002; MacKinnon and Gregg, 2003]. We plan to use an idealized numerical model of coastal internal waves to determine how to best represent and parameterize the relationship between turbulent mixing and shear from the internal waves that ultimately supply the energy for mixing. Using the pseudo-spectral model of *Winters et al.* [2004], we

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

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1. REPORT DATE <b>30 SEP 2007</b>		2. REPORT TYPE <b>Annual</b>		3. DATES COVERED <b>00-00-2007 to 00-00-2007</b>	
4. TITLE AND SUBTITLE <b>Numerical Studies Of Coastal Internal-Wave Mixing</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Scripps Institution of Oceanography, 9500 Gilman Drive, M/C 0213, La Jolla, CA, 92037</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>code 1 only</b>					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>8</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

are able to explicitly resolve the cascade of energy from large to small-scale internal waves, and hope to understand why this cascade is different in coastal waters.

Specifically, the Gregg-Henyey theory predicts that the turbulent dissipation rate scales quadratically with the spectral energy level of the internal wave field,

$$\epsilon \propto \hat{E}^2.$$

However, coastal observations *MacKinnon and Gregg* [2003]; *Carter et al.* [2005] show dissipation to empirically scale with the magnitude of low-frequency shear, or equivalently with the square root of low-mode energy,

$$\epsilon_{\text{coast}} \propto \sqrt{E}.$$

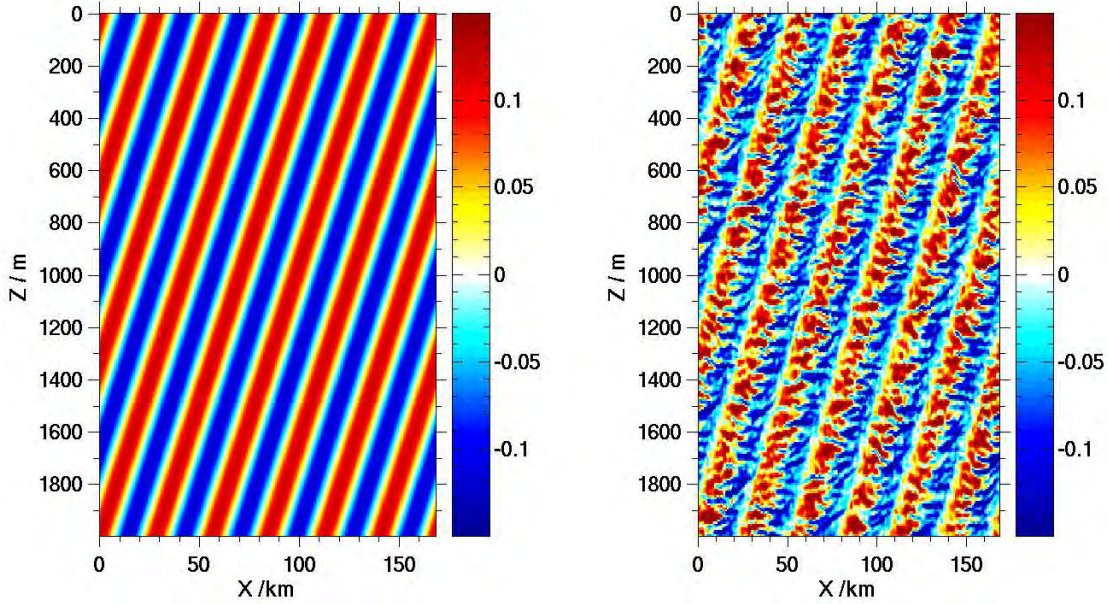
Studies on the New England Shelf, Monterey Shelf, and Oregon Shelf (G. Avicola, OSU, pers. comm.) all show similar scaling, but with different coefficients. Thus far, there has not been a good theoretical basis for a different coastal turbulence scaling.

The present work set out to understand why GH fails on the coast, why the empirical scaling actually observed in coastal areas comes about, and what underlying physics governs the difference in regimes. I started this work with three main hypotheses as to why internal-wave driven coastal mixing might be different from that in the open ocean.

- 1. Vertical bandwidth:** The GH model assumes that there is a large vertical scale separation between the energy containing waves (large vertical scales) and the breaking waves (small vertical scales). This translates to a large vertical bandwidth in spectral space for wave interactions to occur that are not directly influenced by either wave forcing or breaking. However, in coastal waters the largest scale waves have vertical scales limited by the water depth (hundreds instead of thousands of meters), and the scale separation between generated and breaking waves is an order of magnitude smaller.
- 2. Random Phase:** The GH model assumes that even at the largest scales, internal waves are incoherent with a random phase distribution. Mathematically, this means that when deriving the rate of energy transfer through nonlinear interaction, the first order perturbation term disappears and all energy transfer comes from the second order perturbation term, of much smaller magnitude. However, internal waves in coastal areas are often in the form of internal tides that have been generated nearby, and hence are still largely coherent.
- 3. Statistical Steady State:** The GH model assumes that a statistical steady state has developed in which the largest scale waves maintain a steady energy level, and energy flow to smaller scales is balanced by dissipation. However, in coastal waters wave generation changes significantly over spring-neap cycles and as shifting mesoscale conditions subtly alter wave generation strength. Hence the wave field may never be in a steady state.

## WORK COMPLETED and RESULTS

I started by trying to step back and understand as simple a case as possible, the decay of a single monochromatic internal wave. Simulations were performed with the pseudo-spectral model of *Winters et al.* [2004]. The run was initialized with a single monochromatic, relatively large scale internal wave and infinitesimal noise at all scales (Fig. 1, left). The domain is periodic in all directions.



**Figure 1: Decay of a simple monochromatic internal wave. Left: horizontal velocity of initial condition. Right: horizontal velocity after substantial energy has been transferred to small-scale waves.**

To understand what dynamics govern this energy transfer to small scales, consider the equations of motion. The zonal momentum equation is given by

(1)

Consider a quasi-linear solution with energy changing on a slow time-scale:

(2)

Because the nonlinear term in (1) is quadratic, energy exchange occurs between triads of waves, where one wave's amplitude is altered by nonlinear interaction of two other waves:

(3)

When the wave numbers and frequencies add up to form triads ( $k_b + k_c - k_a = 0$ , etc), there is a net exchange of energy. If one assumes that the three interacting waves all obey the linear polarization relations (relating the amplitudes of  $u$ ,  $v$ ,  $w$ ,  $b$ ), then one can explicitly write out the change in energy of one wave in terms of the other two,

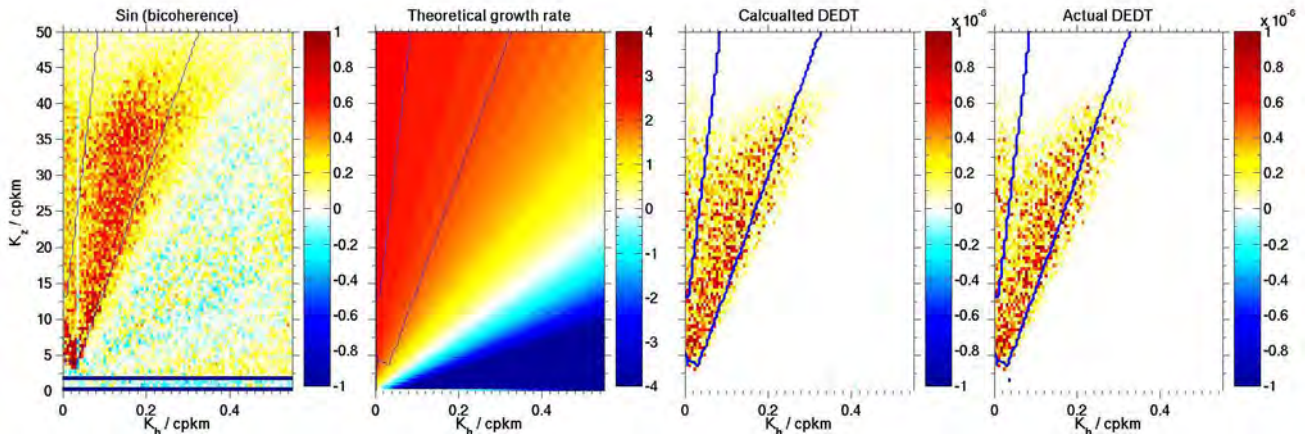
(4)

where  $\Gamma$  includes all combinations of wave numbers that fall out of the advection terms above, and the  $\sin$  of phase difference is the bicoherence between three interacting waves. In the case considered here, we have a primary wave (let's call it 'c') interacting with two small-scale waves ('a' and 'b'). If we consider energy transfer to one of the small scale waves, and due to symmetry assume the energy of the two small-scale waves is similar ( $Ea \approx Eb$ ), then we can write the growth rate of the small-scale wave as

(5)

Physically, the rate of energy transfer to small scales is the product of three things, the square root of energy in the low-mode wave, a composite coefficient ( $\Gamma$ ), and the bicoherence between the three waves. **For a given low-amplitude wave energy, the term  $\sqrt{E_c}\Gamma$  gives the theoretical upper bound of rate of energy transfer to small scales.** This upper bound is achieved only if the interacting waves are perfectly bicoherent, that is if the sin term is 1. For reference, recall that the Gregg-Heney model assumes that interacting waves have random phase, so this term becomes zero energy transfer is calculated from the next term in the perturbation expansion ( $\delta^2$ ). However, **note that the square root dependence on low-mode energy is just what is predicted by the empirical coastal observations described above.**

Let's return to our simple simulation and see how well the numerically simulated energy transfer matches these predictions and upper bounds. For the case of a decaying monochromatic wave, the rhs terms from (5) are shown in Figure 2. The left-most panel shows the sin of bicoherence, plotted versus horizontal and vertical wave numbers. In this framework, lines of constant frequency radiate like spokes from the origin. Bicoherence is high only in a particular 'wedge' of the diagram, corresponding to a particular frequency band. This band corresponds to waves that are *close* to being an exact frequency triad (in addition to wave number triads). The blue boundaries drawn correspond to waves for which  $|\omega_b + \omega_c - \omega_a| \leq 0.5\omega_c$ , chosen somewhat arbitrarily to indicate how close waves are to resonance. Interestingly, within this frequency band, the bicoherence is on the order of 1. The theoretical upper bound growth rate (second panel) is of the order of days within this frequency band. The most important conclusion here is that the product of these two terms (panel 3) looks very similar to the actual difference in spectral energy density over the spin-up time period (panel 4), indicating both that the weakly nonlinear theory does a good job at describing energy transfer to small scales, and that the actual energy transfer is of the same order as the theoretical upper bound. Again, this makes the situation of a simple decaying wave very different from the random-phase energy transfer assumed by models like Gregg-Heney.

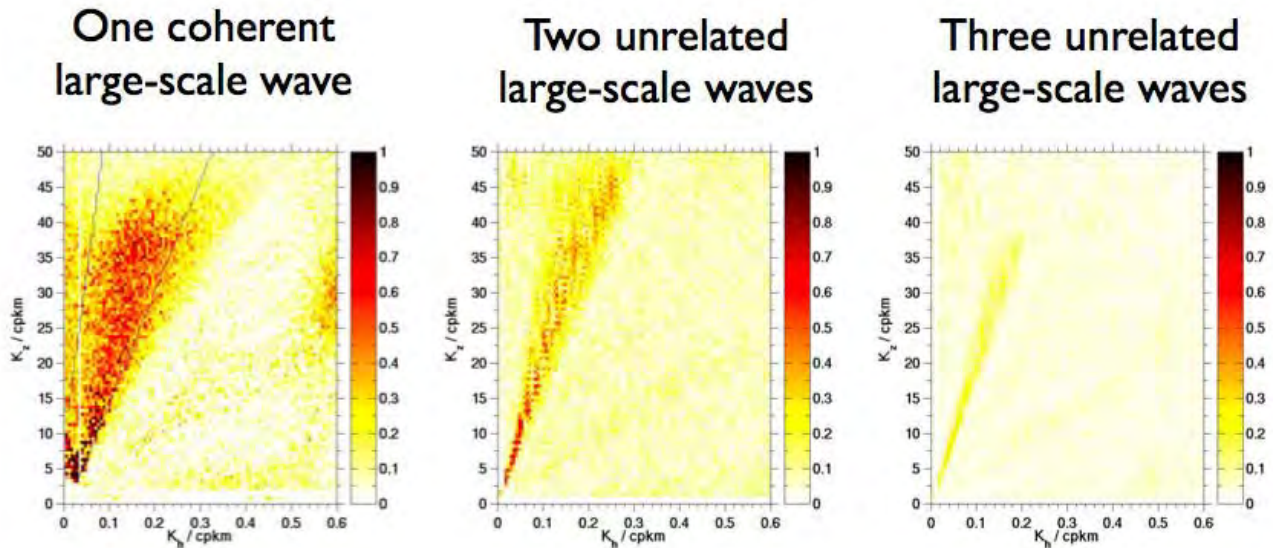


**Figure 2: Calculations of the terms in (5) from the decay of the monochromatic wave pictured in Figure (1). From L to R the panels are: sin of bicoherence; the theoretical maximum upper bound growth rate ( $\sqrt{E_c} \times \Gamma$ ), measured in inverse days; expected change in small-scale wave energy as the product of these two terms; and the actual difference in energy spectra over the time period. The blue lines in each panel indicate the boundaries of the frequency band discussed in the text.**

To bring the discussion back to tangible coastal mixing concepts, consider the list above of ways in which coastal waves may differ from open-ocean waves. The first difference had to do with vertical bandwidth, or how scale separated the primary wave(externally generated internal tide) and breaking waves are. In Figure 2, the bicoherence is largest at the low-wave number end of the wedge, where the scale-separation with the primary wave is small, and decays towards zero at the high-wave number end of the wedge (large scale-separation). The physical reasons for this are unclear, but may relate to more rapid interaction and phase distortion of smaller-scale waves through local interactions or violation of weak nonlinearity assumptions where wave phase speeds become comparable to wave velocities. The mechanisms at work will be further investigated in the coming year, but the preliminary implications are that wave interactions with less scale-separation have a higher bicoherence.

The second hypothesis for why coastal wave interactions are different had to do with coherency of the primary wave. So far we have considered decay of a single monochromatic wave. Now let's look at similar situations initialized with one or two additional large scale waves, as you would get, for example, in an open ocean setting in which large-scale inertial tides or near-inertial waves were propagating in from different directions from different far-away generation sites. Figure 3 shows the effect of adding additional large-scale waves to the bicoherence of energy transfer. By the time two additional primary waves have been added (right-most panel), bicoherence has decayed by almost a factor of 10. This result is in agreement with the thinking behind the GH derivation, namely that with incoherent energy-containing waves, bicoherence goes to zero and you move to the next perturbation term.

My conclusion so far is that due to both lower vertical bandwidth and higher wave coherency, coastal wave interactions are likely to be much closer to the upper bound of energy transfer described by (5). As such, the appropriate scaling for the rate of energy transfer to small-scales, and hence the dissipation rate, is a square root dependence on low-mode energy, or equivalently a linear dependence



**Figure 3: Sine of the bicoherence for the primary example of a single decaying wave (left), two large-scale waves (middle) and three large-scale waves (right).**

on low-mode shear magnitude ( $|du/dz|$ ). This prediction is in agreement with the empirical results described in previous papers.

## **FUTURE PLANS**

Work for the coming year will proceed on two fronts and is more fully described in the accompanying planning letter. First, I plan to continue work on the idealized numerical simulations to flush out the circumstances in which energy transfer can be described by the equations presented here, the nature of the transition to more open-ocean dynamics and scalings, and how these features change in a world with non-constant stratification. Second, and perhaps more importantly at this point, I plan to work closely with the observations collected as part of AESOP to validate the dynamics I've looked at numerically and explore the implications in regional models.

## **COLLABORATIONS**

Over the next year I hope to work with a variety of other AESOP PIs. Comparison of wave properties and dynamics can be made with data collected by Klymak/Pinkel on FLIP. Their measurements have high enough vertical resolution to not only look at wave spectra, but to explicitly calculate triple products that comprise the energy transfer terms. Similar comparisons can be made with data collected by Johnston/Rudnick and Girton/Kunze. Once a parameterization has been developed, it can be applied to output from AESOP-models that resolve low-mode internal tides (Fringer/Street/Wang), and resulting dissipation compared to patterns of measured dissipation from Johnston/Rudnick/Gregg.

## **RELATED PROJECTS**

On a related topic, I am working with Dr. Jerome Smith (SIO) on an NSF-funded project to investigate how Langmuir Cells and internal waves interact at the base of the surface mixed layer. Using LES modeling we've found that wind stress produces both Langmuir cells in the mixed layer and a large-scale inertially rotating current -not new results. More interestingly, we found that when the water beneath the mixed layer is stratified, advection of langmuir rolls by the inertial flow generates high-frequency waves that propagate downward. In essence this is an upside-down version of the lee-wave mechanism of wave-generation by flow over bumpy topography. The resultant waves have a high enough frequency that they are constrained to break in the highly stratified transition zone beneath the surface mixed layer and may be an additional important source of mixing in the coastal ocean.

## **IMPACT / APPLICATIONS**

Accurate representation or parameterization of turbulence from breaking internal waves is necessary for regional and larger-scale numerical models to successfully reproduce the evolution of regional currents, water properties, and particulate matter distribution on timescales of weeks to months. This work attempts to understand the dynamics leading from internal waves to turbulence and develop parameterizations that can be used in regional numerical models. A further expected outcome of this analysis will be a demarcation of which parameterization is appropriate for different model resolutions.

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