

Nonlinear Inversion from Nonlinear Filters for Ocean Acoustics

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LONG TERM GOALS

The long term goals of this research are to develop practical and efficient algorithms for application to the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

OBJECTIVES

The specific objectives of this research are to adapt a specific nonlinear filter, known as a Daum filter, for acoustic inversion of shallow water environmental properties, and to assess the performance of this nonlinear filter relative to local linear inversion on the one hand and global methods, e.g. Monte Carlo methods on the other hand.

APPROACH

Many inverse problems of interest in ocean acoustics are intrinsically nonlinear, e.g. inverting measured pressure data for bottom and scattering properties. The solution to the nonlinear inversion problem is usually approached in one of two ways. The first way is to assume a starting model, which one hopes is near to the true model, then recursively solve a linearized version of the inverse problem for corrections to the starting model and model covariance. The advantage of this approach is that the numerical implementation of the solution algorithm is relatively straightforward and in a linear problem the statistical properties are well defined and will remain gaussian if they start out gaussian. However linearization of a nonlinear system can produce biased estimates for two reasons: 1. Linearization of the system and/or measurement equations may not be a good approximation, and 2. Nonlinear systems do not maintain gaussian statistics as they evolve even if they are initially gaussian. Another problem with linearizing a nonlinear system is that with a poor starting guess the solution algorithm may never converge to the true answer. If the starting model represents a point near a local minimum of the solution space, the final solution will be trapped in that local minimum, and never converge to the true answer. This can be circumvented by using Monte Carlo techniques to randomly sample the solution space for starting models.

The other class of solution methods attack the nonlinear problem directly by using simulated annealing or genetic algorithms. The disadvantage of these directly nonlinear methods, is that there is no way to conveniently propagate the statistical properties of the solution through to the final result. One solution

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to this problem is to find the global minimum in the solution space, if one exists, then linearize about the solution representing the global minimum and do a statistical analysis about that solution. This was done by Potty et al.(2000), who employed a genetic algorithm followed by linear analysis about the solution determined by the genetic algorithm.

The recursive algorithms commonly employed for the estimation of the model and covariance relative to some initial starting values bear a strong resemblance to Kalman filters, which are commonly employed in target tracking algorithms. The original Kalman filter was derived for strictly linear systems. However, the Extended Kalman Filter can be applied to systems which are weakly nonlinear. In the late 1980s Frederick Daum, a mathematician working at Raytheon Corporation, developed a fully nonlinear formulation to the filtering problem for target tracking (Daum, 1985, 1986, 1987). His theory is elegant, but impractical from an implementation point of view. Sometime later Schmidt (Schmidt, 1993) succeeded in deriving an approximate algorithm based on Daum's original theory, and developed a successful numerical implementation of a nonlinear filter that was a significant improvement to the Kalman and Extended Kalman filters for the type of tracking problem Schmidt was interested in.

Filter type algorithms are ideally suited to inverse problems with time dependent oceanography or range dependence. We do not anticipate attempting to include time dependent oceanography at this time, but we would like to look at the issue of range dependent inversion. The idea would be to sequentially update parameter estimates as a function of range. Also note that any inversion algorithm can be cast into a filter like algorithm by supplying the data sequentially and updating the model parameter estimates sequentially as data is added to the problem, or a smoother by considering the complete data set, and working both forwards and backwards through the data set. In the end, probably the best formulation to use for a given inverse problem depends on the noise statistics. This is also something we propose to investigate.

Linear inverse problems admit the construction of both data and model resolution matrices. These resolution matrices can be used as metrics with which to estimate model uniqueness and data predictability. We will be able to construct resolution matrices for the nonlinear problem and compare them with their fully linear equivalents.

Quantification of the resolution of an inversion can be used for experimental design. While the resolution of a linear problem is well defined, and described in basic texts such as Menke (1983), it is less so for a nonlinear problem. One of the objectives of this research is the quantification of the resolution for nonlinear problems. The resolution for the nonlinear problem can be defined formally: Given a true but unknown model m_{true} such that

$$d = g(m_{true}) \rightarrow m_{true} = g^{-1}(d)$$

where d is the data vector, and g is the operator connecting the data to the model, e.g. the wave equation, how close is a particular estimate m_{est} to m_{true} ?

$$m_{true} = g_{est}^{-1}(d) = g_{est}^{-1}[g(m_{true})]$$

$$m_{est} = r(m_{true})$$

where $r(.) = g^{-1}[g(.)]$ is the model resolution operator. The nonlinear model resolution $r(.)$ operator can be computed iteratively from the Neumann series representation for g with assumption that both the data functional and the model perturbation functional possess regular perturbation expansions. (An example of a problem which possesses a model perturbation functional with a regular perturbation series is normal mode acoustic propagation with “slow enough” perturbations such that the modes adjust adiabatically to the perturbations, and the mode eigenvalues are “far” from cut-off.)

$$d = g(\epsilon m) = \epsilon G^{(1)}m + \epsilon^2 G^{(2)}mm + \epsilon^3 G^{(3)}mmm + \dots$$

$$\epsilon m = \epsilon I^{(1)}d + \epsilon^2 I^{(2)}dd + \epsilon^3 I^{(3)}ddd + \dots$$

where the $I^{(i)}$ are the expansion operators of the g^{-1} . Substitute data into the model expansion and order by ϵ :

$$\epsilon^1: m = I^{(1)}G^{(1)}m$$

$$\epsilon^2: 0 = (I^{(1)}G^{(2)} + I^{(2)}G^{(1)}G^{(1)})mm$$

$$\epsilon^3: 0 = \dots$$

The remarkable thing to note about these expansions is that the nonlinear components of the model in the data do not contribute to the reconstruction. (Snieder, 1990). The term $I^{(1)}G^{(2)}$ is a linear inversion of the component of the data, that has a quadratic dependence on the data. If the nonlinear inversion is to reproduce the model m exactly, the $I^{(1)}G^{(2)}$ term must be canceled by the $I^{(2)}G^{(1)}G^{(1)}$ term. We can now define the nonlinear resolution. For the estimated model m_{est} we have

$$m_{est} = I^{(1)}G^{(1)}m + (I^{(1)}G^{(2)} + I^{(2)}G^{(1)}G^{(1)})mm + \dots$$

$$m_{est} = R^{(1)}m_{true} + R^{(2)}m_{true}m_{true} + \dots$$

where $R^{(1)}$ tells us how much smearing there is in the map between m and m_{est} , and $R^{(2)}$ tells us how much spurious nonlinear mapping from the true model there is to the model functional (Snieder, 1991).

WORK COMPLETED

This past summer Ganse explored the inversion of ocean acoustic reverberation for bottom loss and bottom scattering strength via the NATO Deployable Multistatic Sonar (DEMUS) experiment, supported under the [ARL project / John Tague]. This experiment was not optimized for the inversion of ocean bottom information since tracking was the focus of the experiment, but the resolution calculation helps one to quantify the limits of information gained about the ocean bottom in the problem, which is of interest in ultimately passing estimated environmental information on to other researchers' sonar tracking performance calculations. A particularly interesting development in this

more applied work was the discovery of another basic research issue – the need for a tradeoff between the stability of the inverse problem and the boundary conditions imposed on the regularization operator. The inverse problem without regularization cannot be solved due to its instability via the null space in the propagation model. But one's choice of regularization (e.g. to choose the smoothest among an infinite list of bottom profiles which all fit the data to within the noise equally well) may include a set of boundary conditions on the regularization. The choice of boundary conditions can affect whether the null space of the regularization overlaps with the null space of the propagation model. If it does overlap, the problem is still unstable and can require a different set of boundary conditions than initially specified, as was discovered in this work. The quantification of information content in inversion results has much to do with the choice of tradeoff points, whether between variance and resolution, data misfit and regularization, or stability and boundary conditions.

RESULTS

The quantification of inverse problem resolution allows for the possibility of new tools in the planning of ocean geoacoustic experiments. A pre-measurement inverse theory resolution analysis can be used as part of experiment planning regarding sensor placement and ship tracks, such that a desire for an experimental configuration giving the most information in bottom inversion can be quantitatively balanced with that for other needs like tracking and communication. This subject is one of the segments of Ganse's PhD work this year. The nonlinear geoacoustic inverse problem is ill-posed, so that one can only estimate the continuous function of seafloor properties to a limited resolution. This limited resolution varies with experiment geometry, frequency, and other such factors, and can be quantified in either a frequentist or Bayesian framework. Given statistics of the measurement noise (but without any new measurements themselves), the resolution can be quantified exactly for a linear inverse problem, and compared between different experiment geometries. Nonlinear problems complicate this picture, but if the problem can be transformed into a weakly nonlinear form then the resolution can still be explored in an approximate sense and used as a tool in the planning phase. The ideal situation is when previous seafloor estimates exist for the same region in which a new experiment with new geometry and configuration is being planned. For the scenario without previous results, a somewhat more ad-hoc approach can still compare changes in resolution across different seafloor models. An example resolution result is shown in Fig. 1.

With the aim of validating the use of standard linear tools for quantifying uncertainty and resolution on ocean geoacoustic nonlinear inverse problems, we continued development of Monte Carlo and nonlinear filter-based inversion techniques whose results will be compared against the linearized ones. This comparison is the ultimate goal of Ganse's PhD work, per his general exam which was passed in May, and during this year the Monte Carlo and filter-based tools were completed and tested. For example, in Fig. 2 a test problem result shows the difference between the form of estimation uncertainties calculated with a traditional linearized method and with a numerical Monte Carlo method. The numerical, Monte Carlo method is comprehensive in its treatment of uncertainty in nonlinear problems but can be extremely slow to compute. The idea behind the nonlinear filter-based work is to provide a much faster, if less comprehensive, way to address the non-Gaussian uncertainty of nonlinear problems and thus validate how accurate an approximation it is to use the very fast, traditional linear tools on a given inverse problem. This subject was presented in two fall 2006 AGU talks (Ganse and Odom, 2006a and Odom and Ganse, 2006a) and our fall 2006 ASA talk (Ganse and Odom, 2006a); the ASA talk received the 2nd place best student paper award. Odom and Ganse (2006b) was an invited talk at the Fall AGU Meeting.

IMPACT/APPLICATIONS

A nonlinear, well characterized filter-based inversion method and algorithm will have application to environmental estimation and target tracking. A practical method way to compute the resolution for a nonlinear inversion will have an impact on the characterization of uncertainty and uniqueness of environmental estimates required for acoustic propagation.

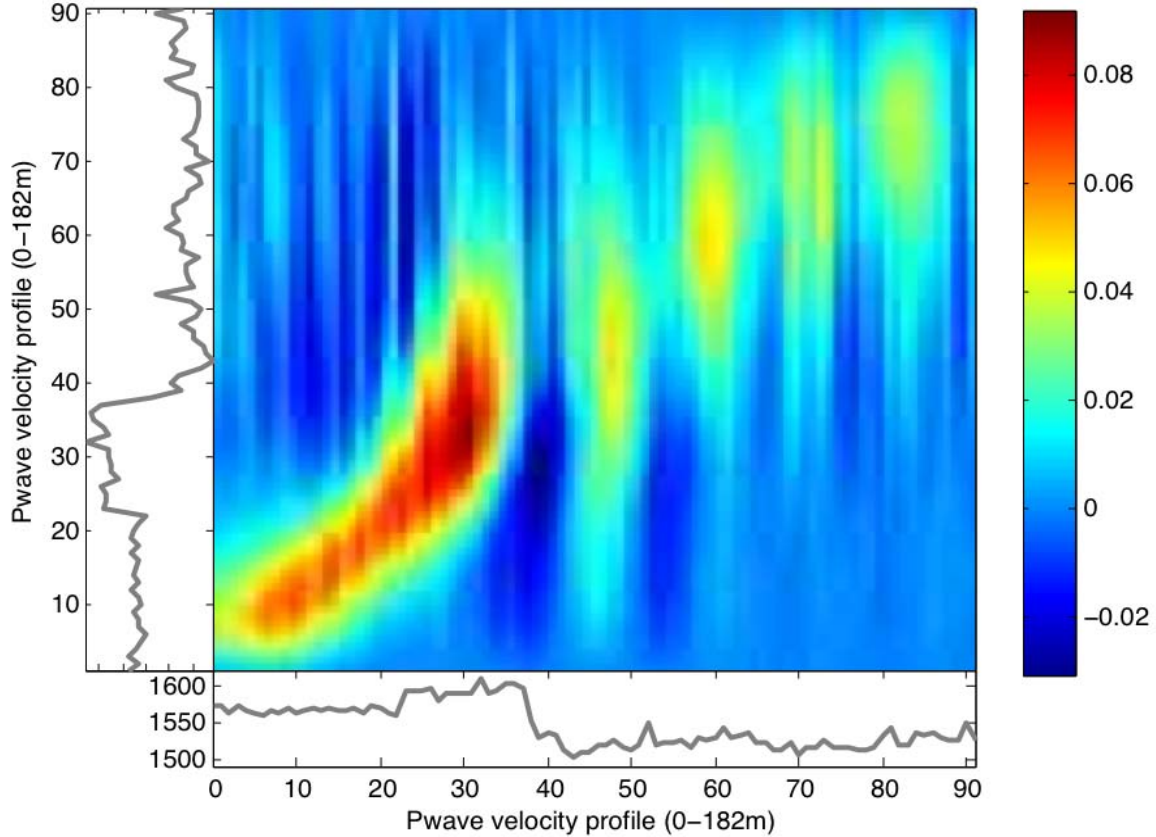


Figure 1: Resolution matrix quantifying the limited resolving power of the inverse problem in estimating the Pwave velocity profile shown in the margin plots (same profile on each axis). In the ideal, perfectly resolved problem – 100% information recovered about the ocean bottom – this matrix would show a red strip down the diagonal, and by optimizing experiment geometry one can attempt to move the problem toward that ideal. Frequentist inversion can only solve for weighted averages of the bottom profiles, and the weightings (the information this matrix contains) are generally clustered in neighboring parameters in the profile and thus represent resolving power as a function of depth. In this hypothetical experiment, in the top 30m of this profile this matrix shows that the inverse problem can only resolve Pwave information to within about 20m (10 parameters) depth resolution, and can resolve virtually nothing at all in the shadow zone around 70-80m (roughly 35th-40th parameter). Frequency was 50Hz, receiver VLA was 8 hydrophones from 10-150m deep at 1km range, SSP had minimum at ~40m and positive depth gradient (increasing soundspeeds) below, propagation code was OASES oasp.

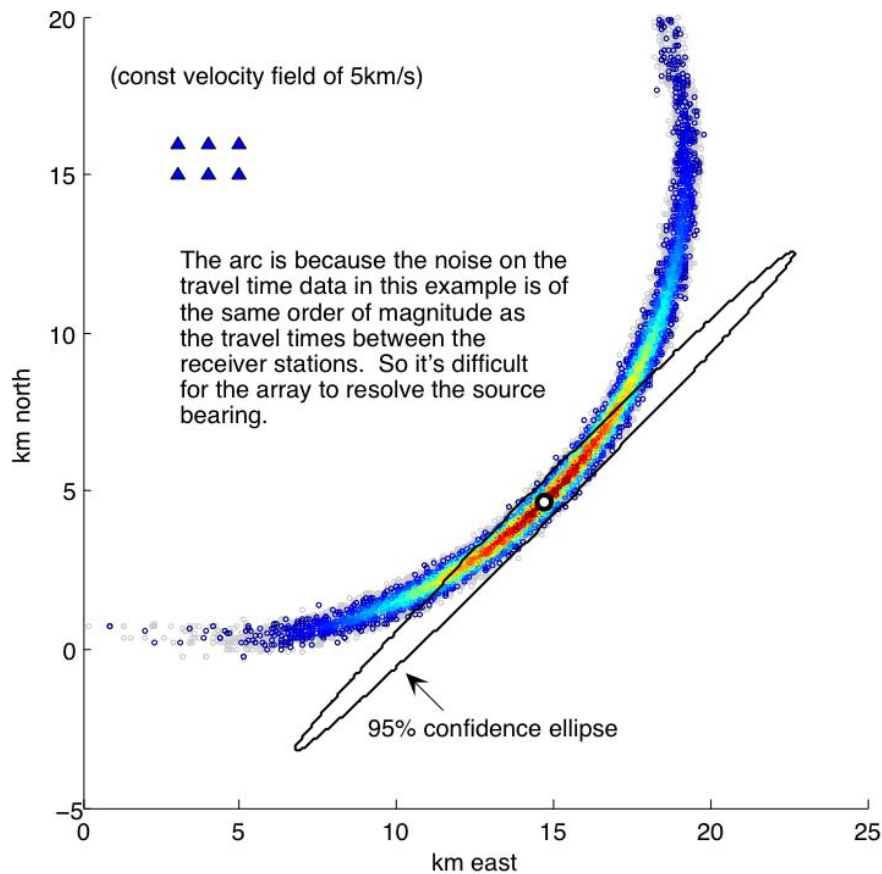


Figure 2. Solution statistics and probabilities for a simple, nonlinear, 2D source location estimation test problem to demonstrate the difference between linearized estimation results and fully nonlinear results. The blue triangles are the acoustic receiver array, and the black and white circle is the source location estimated by both a linearized estimator (Gauss-Newton) and a fully nonlinear, numerical Bayesian Markov-Chain Monte Carlo (MCMC) estimator. Noise on the receptions means the estimations have uncertainty on their solutions, but the linearized uncertainty can only be specified in terms of a multi-variate Gaussian, hence the ellipse, whereas the numerical MCMC result in color can show the full non-Gaussian shape of the uncertainty. In this simple test problem, the numerical MCMC result computes quickly, but in more complicated ocean geoaoustic estimations the result computes far more slowly, so our filter-based work aims to give some of the non-Gaussian information about the uncertainty in a much faster way. Part of researching the filter-based work will be comparing its results to MCMC results.

RELATED PROJECTS

Our research is directly related to other programs studying effects of uncertainty in the environment, measurements, and models on acoustic propagation, and target detection and characterization.

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