

Exploring the Low-Frequency Performance of Thermal Converters Using Circuit Models and a Digitally Synthesized Source

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Abstract—Low-frequency errors of thermal voltage converters are described and estimated using a circuit model that includes easily measured parameters. A digitally synthesized source is used to confirm the estimated ac-dc differences in the 0.01 Hz to 40 Hz range.

I. INTRODUCTION

A CALCULABLE digitally synthesized source (DSS) has been used as an independent method of verifying the ac-dc differences of thermal voltage converters (TVCs) below 1 kHz [1]. A DSS has also been used to probe the very-low-frequency (below 1 Hz) performance of TVCs [2]. Results of that study show that low frequency errors are on the order of 5% for typical TVCs that use single junction thermoelements (TE's). This paper describes these low-frequency errors, which are quite difficult to measure directly, in terms of a simple circuit model and easily measured circuit parameters.

II. TVC FUNDAMENTALS

Most TVCs consist of a resistor, used to limit the test current to a few milliamperes, connected in series with a low-current TE. A TE consists of a low-resistance heater to which a temperature sensor is attached. The sensor is generally a thermocouple that produces a small output emf E proportional to the heater temperature. The heater temperature is proportional to the input power $P = I^2R$, where I is the TVC current and R is the TE heater resistance. If the amplitude of the test voltage is constant and its period is small compared to the time constant of the TE, the heater temperature and corresponding output emf are constant. In this operating mode, the TVC is a mean-squared voltage-to-dc voltage converter.

As the period of the input voltage approaches the TVC time constant, the heater temperature begins to track the square of the instantaneous input voltage. At very low frequencies, the output emf waveform looks like a squared version of the input voltage. However, the TE heater loses heat through radiation and along its input leads. Since the loss is greater at higher temperature, the positive peaks of the output emf waveform will be somewhat reduced, resulting in a lower average value

for E than that produced by an equivalent voltage at a higher frequency where the temperature tracking is negligible. This tracking is the source of the low frequency errors that will be addressed in this paper.

III. TVC MODEL

The low frequency ac-dc differences of a TVC (based on a single junction TE) were measured using a programmable DSS [2]. Results of those measurements indicated that the differences at very low frequencies were constant at about 5% rolling off toward zero between 0.01 Hz and 1 Hz. To explore the nature of this phenomenon, a model was developed to relate the low frequency ac-dc differences of a TVC to its time constant τ , dc transfer function, and ac-dc difference at higher frequencies. These parameters are easily measured using commercial instrumentation, whereas measuring the low frequency errors directly requires a special low frequency source like the DSS.

A. Time Constant

This parameter is measured by applying a voltage step to a TVC and sampling the output emf at fixed time intervals using a fast digital multimeter (DMM). The time constant τ is defined as the time required for the output emf to reach 63.2% of its final value.

B. DC Transfer Function

The dc transfer function is determined by applying a known dc voltage to the TVC and measuring the output emf. The measurements are made at M voltages using a programmable dc voltage source to generate voltage steps between \pm full scale ($\sqrt{2}$ times the rms value of the sinusoidal operating voltage). The steps are defined as the sine of M equally spaced angles between -90 and 90 degrees. The dc input voltage V and the corresponding output emf E for each step are measured using precision DMMs, after allowing the TVC to settle for at least 10τ .

The model is constructed of the three elements shown in Fig. 1. The first element represents the ideal power dissipated in the heater as a function of I^2R . The second is a frequency dependent term that describes the variations in the heater temperature as a function of the input signal frequency. This element is modeled as an ideal first order low pass filter (LPF)

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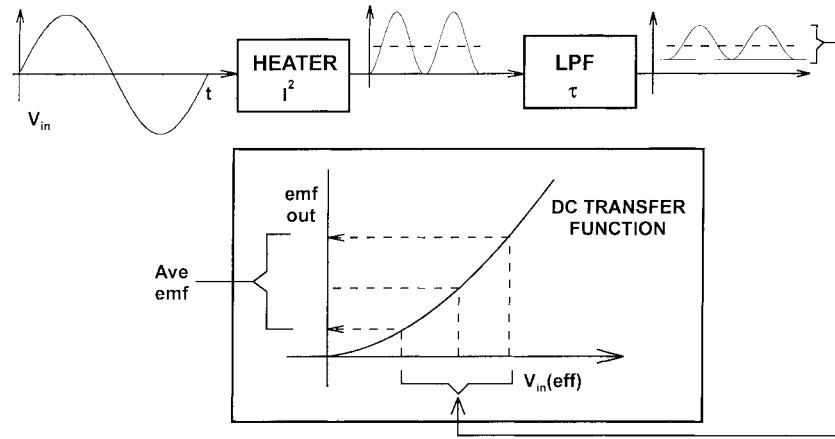


Fig. 1. Block diagram of the TVC model which uses measured parameters to simulate the low frequency errors due to temperature tracking.

with a corner frequency $f_c = 1/(2\pi\tau)$. Its output represents the effective input voltage seen by the TVC. This voltage is then passed into the third element which describes the dc transfer function of the TVC. This element defines the actual output emf as a function of the effective input voltage. The emf is then averaged to give the predicted output.

To determine the ac-dc difference of a TVC at voltage V and frequency F , the measured parameters are entered into an analysis program that does the following:

- 1) Fits an n th order polynomial (L) to the dc transfer function data to predict the actual output emf at any input voltage within the operating range.
- 2) Generates and filters a \sin^2 signal in a simulated low pass filter with a corner frequency $f_c = 1/(2\pi\tau)$. The important information here is the peak-to-peak deviation relative to the average value of the signal.
- 3) Maps the filtered signal (which represents the effective input voltage V_{in}) onto the dc transfer function defined by L to generate the predicted output emf and its average value E_{ave} .

The ac-dc difference δ is approximated by

$$\delta = (E_{dc} - E_{ave})/nE_{dc} \quad (1)$$

where E_{dc} = the average output emf for input voltages $+V$ and $-V$ (computed using L), and n is the slope of L at V .

C. Digitally Synthesized Source

To test the model, a calculable DSS was used to determine the ac-dc differences of a 2 V TVC based on a single junction TE. The DSS is a programmable source that uses sampled data (stored in memory) together with a digital-to-analog converter (DAC) to generate an amplitude- and frequency-adjustable staircase approximation of a sinusoidal voltage with N steps per period. The rms value of the synthesized waveforms is given by

$$V_{rms} = \left(\sum v_i^2 / N \right)^{1/2} \quad (2)$$

where the summation is made over one period of the N -step waveform and v_i is the dc voltage of the i th step. The source

can also be programmed to a static dc voltage. The short-term (10-min) stability of the ac and dc voltages produced by the DSS is on the order of $\pm 1 \mu V/V$ and its flatness errors (changes in rms value vs frequency) below 100 Hz are negligible. The difference was measured between the low frequency (0.01 Hz to 20 Hz) TVC response and its response at 40 Hz (where the tracking errors are negligible) using the following procedure:

- 1) The TVC is connected to the DSS which is programmed to output an N -step sine wave at approximately voltage V and a frequency of 40 Hz. The TVC output emf is measured using a digital multimeter (DMM).
- 2) The DSS is then programmed to each frequency in the test range and the output emf of the TVC is measured. The DMM is triggered by the DSS clock signal so that it measures TVC output emf at each step and the average value of the emf is computed.
- 3) The measurement process is completed by performing ac-dc difference measurements using the DSS and the following procedure:
 - a) With the TVC connected to the DSS, the steps of the synthesized waveform are measured using a DMM, and the rms value is computed using (2).
 - b) The DSS is programmed to produce a 40 Hz sine wave and the average value of the TVC output emf E_a is measured using a DMM.
 - c) The DSS is programmed to output a positive direct voltage V and adjusted to bring the TVC output E_{+d} as close as possible to E_a . The adjusted voltage V_{+d} is measured.
 - d) The DSS is programmed to output a negative direct voltage V and adjusted to bring the TVC output E_{-d} as close as possible to E_a . The adjusted voltage V_{-d} is measured.
 - e) The ac-dc difference δ is given by

$$\delta = (V_a - V_d)/V_d - (E_a - E_d)/nE_d \quad (3)$$

where $V_d = (V_{+d} - V_{-d})/2$, $E_d = (E_{+d} + E_{-d})/2$, and n = power exponent at V [3], [4].

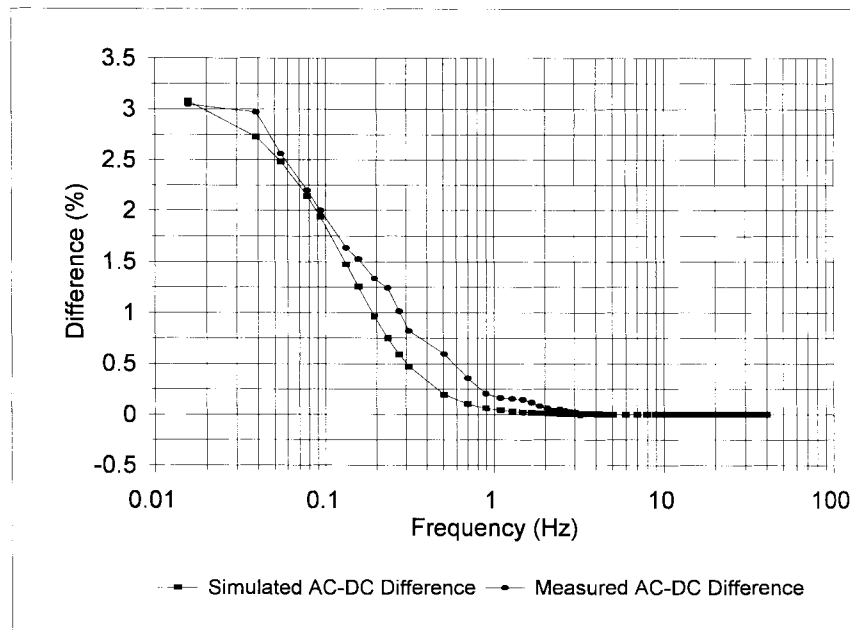


Fig. 2. Simulated and measured ac-dc differences of a single-junction TVC in the 0.01 Hz to 40 Hz range.

The mean ac-dc difference of the 2 V TVC at 40 Hz measured using the DSS was $0.2 \mu\text{V}/\text{V} \pm 4 \mu\text{V}/\text{V}$ (2-sigma). This compares to a difference of $2 \mu\text{V}/\text{V} \pm 5 \mu\text{V}/\text{V}$ (2-sigma) determined by comparing the TVC to the national standards [5]. The uncertainties of these measurements are several orders of magnitude better than required for this application (verification of the model), but they are included here to show the potential of the DSS as means of providing low frequency ac-dc difference measurements that are independent of present TVC standards.

The measured and model-simulated low frequency ac-dc differences of the 2 V TVC are shown plotted in Fig. 2.

IV. CONCLUSIONS

A simple circuit model has been developed to predict the low frequency ac-dc differences of a TVC based on a knowledge of its dc transfer function, time constant, and ac-dc difference at a reference frequency, where the ac-dc difference is well known. Once the TVC parameters have been measured, the model can predict ac-dc differences at any voltage and frequency (below the reference frequency) within the operating range of the TVC. This evaluation was done using TVCs based on single junction TEs at very low frequencies where the errors are large enough to clearly demonstrate the model. A digitally synthesized source was used as a low frequency reference to measure ac-dc differences between 0.01 Hz and 40 Hz. It remains to be seen whether the technique will prove useful in evaluating multijunction TVCs which have very small tracking errors. For TVCs which use TEs with thin film heaters (where time constants are measured in milliseconds rather than seconds), the technique could prove useful in predicting differences at much higher frequencies.

The DSS has also been used to calculate the ac-dc difference of 2 V and 8 V single-junction TVCs at 40 Hz. These differ-

ence agreed with differences assigned from TVC standards to within $2 \mu\text{V}/\text{V}$. The procedure which uses the DSS to generate both the ac and dc signals, provides an independent method of verifying TVC ac-dc differences below 100 Hz.

APPENDIX

Programming code used to define and implement the low-frequency TVC model:

1. Fit an n th order polynomial to the dc transfer function data to predict actual output emf at any input voltage.

Create sinusoidal input voltage for measured output emf.

```
LOAD file_with_measured_output_emf.txt
input_resolution=(pi/2)/number_of_steps
(for example: 128)
input_steps=from 0 to pi/2 with step=input_resolution
input_voltage=measured_peak_value*SIN(input_step)
```

Routine **POLYFIT** finds the coefficients of the polynomial that fits measured data in the least square error sense. Input arguments are input voltage and measured output emf and the polynomial order that the data has to be fit. Output arguments are the polynomial coefficients for the specified order. In this example the polynomial order is 3.

```
[coefficients_for_the_polynomial_that_fits_the_measured_data: a3 a2 a1 a0] = POLYFIT(input_voltage, measured_output_emf, polynomial_order=3)
```

Create sinusoidal input voltage for the fitted dc transfer function.

```
resolution_for_the_fitted_dc_transfer_function
=(pi/2)/number_of_steps (for example:8000)
input_steps_for_the_fitted_data= from 0 to pi/2 with
step = resolution_for_the_fitted_dc_transfer_function
input_voltage_for_the_fitted_dc_transfer_function=measured_peak_value*SIN(input_steps_for_the_fitted_dc_transfer_function)
```

Create output emf for the fitted dc transfer function.

$$\begin{aligned} [\text{output_emf_for_fitted_dc_transfer_function}] = & \\ & + a_3^* (\text{input_voltage_for_the_fitted_dc_transfer_function})^3 \\ & + a_2^* (\text{input_voltage_for_the_fitted_dc_transfer_function})^2 \\ & + a_1^* (\text{input_voltage_for_the_fitted_dc_transfer_function})^1 \\ & + a_0 \end{aligned}$$

2. Pass the signal through the low pass filter.

Create the output signal from the low pass filter. The output signal is proportional to current squared and trigonometric identity used: $[\sin(x)]^2 = [1 - \cos(2 * x)]/2$.

$$\text{LPfilter_output} = (1 - \text{COS}(2*[2*\pi*\text{operating_frequency}]) + \text{ATAN}(2*[2*\pi*\text{operating_frequency} * \text{time_constant}]) / \text{SQRT}(1 + (2*[2*\pi*\text{operating_frequency} * \text{time_constant}])^2)) / 2$$

Find peak to peak variations for the signal at the output of the low pass filter.

$$\begin{aligned} \text{minimum} &= \text{MIN}(\text{LPfilter_output}) \\ \text{maximum} &= \text{MAX}(\text{LPfilter_output}) \end{aligned}$$

3. Map the output signal from the low pass filter to dc transfer function and find the average value.

Mapping the filtered signal onto dc transfer function. Input signal is normalized.

$$\begin{aligned} [\text{vector_index_for_which_the_mimimum_value_is_achieved}] &= \text{MIN}(\text{minimum} - (\text{input_voltage_for_the_fitted_dc_transfer_function}/\text{measured_peak_value})^2) \\ [\text{vector_index_for_which_the_maximm_value_is_achieved}] &= \text{MAX}(\text{maximum} - (\text{input_voltage_for_the_fitted_dc_transfer_function}/\text{measured_peak_value})^2) \\ \text{segment_of_the_output_emf} &= \text{output_emf_for_fitted_dc_transfer_function} \text{ (values from vector_index_for_which_the_mimimum_value_is_achieved to vector_index_for_which_the_maximum_value_is_achieved)} \end{aligned}$$

Find the average value.

$$\text{average_value} = \text{MEAN}(\text{segment_of_the_output_emf})$$

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