

EXISTENCE AND ESTIMATION OF IMPROPRIETY IN REAL RHYTHMIC SIGNALS

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ABSTRACT

Impropriety in complex signal processing has been studied and used primarily in a communications context, but also in some cases where complex signals are generated by adding real signals in quadrature. We discuss the meaning of impropriety, and the associated use of complementary statistics, when a real-valued random process is improper in the frequency domain. Through the use of modulation signal models, spectral impropriety can be connected explicitly to the frequency and phase of components belonging to a periodic, or more generally rhythmic, modulator waveform. We give theoretical signal models and provide an example of complementary analysis on underwater propeller noise from a merchant ship.

Index Terms— Modulation, nonstationary process, improper, periodical correlation, spectral analysis

1. INTRODUCTION

Many signals can be characterized as a sum of interacting frequency components. Examples are overtones in voiced speech, rhythms of brains waves, or subbands in machine noise. Such signals are non-stationary in the sense that temporal information, like syllables in speech or modulations in propeller noise, arises from correlativity in the frequency domain. In other words, non-stationarity is not always evident in the complex amplitudes of the frequency components themselves, but rather in the temporal co-variation between different frequencies.

Spectral variation is essential to many signal processing applications, and the spectrogram is a familiar tool in this regard. For example, amplitude modulation appears as the collective rise and fall of frequency amplitudes over time. Another example is the rise in one frequency bin coordinated with the fall in another bin, signifying a shifting resonance or harmonic in speech analysis.

To second order, nonstationarity in a temporal signal manifests as spectral correlations. As a complex random variable, the amplitude $X(\omega)$ at frequency ω has two distinct

second-order statistics: the real-valued Hermitian variance $E\{|X(\omega)|^2\}$, and the lesser-known, complex complementary variance $E\{X^2(\omega)\}$. If the latter is equal to zero, then $X(\omega)$ is proper [1], or second-order circular [2]. Otherwise, it is improper or non-circular. In the communications literature, it has been shown that optimal detection requires the use of all second-order statistics, Hermitian and complementary (e.g., [3]). It is also increasingly common to use complementary statistics for data which is bivariate and naturally represented by complex numbers, such as fMRI analysis [4].

Impropriety in the spectrum and analytical signal calculated from real physical signals, like those mentioned at the beginning of this section, has comparatively little coverage in the literature. Speech can be empirically improper in the frequency domain [5], but impropriety is poorly understood in terms of its relation to physical properties of real-world signals. A necessary, but not sufficient condition for impropriety is that the real-valued random process be at least non-stationary [6].

More specifically, it would seem that impropriety is related to the phase characteristics of a signal. The complex variance $E\{X^2(\omega)\}$ parameterizes the eccentricity and angle of an elliptical probability distribution in the complex plane [7], which suggests that impropriety in the frequency domain corresponds to deterministic-like, relative timing of sinusoidal components in the time domain. Phase information becomes important in the analysis of approximately periodic signals, which we define here as “rhythmic.” In this paper, we study spectral impropriety as it relates to periodically and rhythmically modulated random processes, with possible applications in speech and machine noise.

The paper is organized as follows. We define spectral impropriety via the Fourier transform and time-varying systems in Section 2, and then give a motivating theoretical example of periodic modulation in Section 3. Finally, we develop a framework for rhythmic analysis in Section 4, with application to maritime ship noise data, and conclude in Section 5.

2. DEFINITION OF SPECTRAL IMPROPRIETY

Our definition of impropriety for a real, univariate signal occurs in the frequency domain, or similarly in the complex ana-

This research was supported by the Office of Naval Research.

Report Documentation Page				Form Approved OMB No. 0704-0188	
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1. REPORT DATE MAY 2012		2. REPORT TYPE N/A		3. DATES COVERED -	
4. TITLE AND SUBTITLE Existence And Estimation Of Impropriety In Real Rhythmic Signals				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Electrical Engineering Department, University of Washington, Seattle, USA				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release, distribution unlimited					
13. SUPPLEMENTARY NOTES See also ADA561051. AOARD-CSP-111007 International Conference on Acoustics, Speech and Signal Processing (37th) (ICASSP 2012) Held in Kyoto, Japan on March 25-30, 2012. U.S. Government or Federal Purpose Rights License.					
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15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT SAR	18. NUMBER OF PAGES 4	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

lytic signal. We then treat impropriety as a condition resulting from the action of a generative system. These concepts are the basis for our discussions of modulation frequency in Section 3.

2.1. Complex Signals from Real Data

To understand impropriety, we must consider the origin of complex numbers in a real-valued world. In addition to Fourier transforming a real-valued time series, we can obtain a complex-valued time series by calculating its analytical signal. We review Fourier synthesis and the analytic signal here.

In continuous time, a harmonizable random process has the Fourier-Stieltjes integral

$$x(t) = \int dX(\omega) e^{j\omega t} \quad (1)$$

where $dX(\omega)$ is a zero-mean, complex increment process which equals $X(\omega)d\omega$ when $x(t)$ has finite energy. For weakly stationary processes, $dX(\omega)$ is an orthogonal and proper increment process. It is also well-known that the analytic signal

$$x_a(t) = 2 \int_0^\infty dX(\omega) e^{j\omega t} \quad (2)$$

is strictly proper [6].

On the other hand, suppose the analytic signal is improper so that by definition $E\{x_a(t)x_a(t')\} \neq 0$. Equivalently, since

$$E\{x_a(t)x_a(t')\} = 4 \int_0^\infty \int_0^\infty E\{dX(\omega)dX(\omega')\} e^{j(\omega t - \omega' t')} \quad (3)$$

it follows that the spectral increment process must also be improper. In Section 3, we will use the analytic signal as an indicator of impropriety in the particular case of amplitude modulation.

2.2. Generative Model for Impropriety

Our focus is not only impropriety in the frequency domain, but also how impropriety relates back to the properties of the real time-domain signal. Given $x(t)$, assume that its increment process $dX(\omega)$ is improper. To understand the physical meaning of impropriety, we treat $dX(\omega)$ as the output of a system driven by a proper, orthogonal increment process $d\varepsilon(\omega)$. Impropriety is then related to the properties of the system.

By the Cramér-Wold decomposition, $x(t)$ is the output of a time-varying convolution

$$x(t) = \int L(t, t - \tau) \varepsilon(\tau) d\tau \quad (4)$$

where $\varepsilon(t)$ is a stationary, white random process. Converting to a bi-frequency representation [8], we equivalently have

$$dX(\omega) = \int L(\omega, \lambda) d\varepsilon(\lambda) \quad (5)$$

Since $\varepsilon(t)$ is real, its Fourier increment process is conjugate symmetric and (5) is equivalent to

$$dX(\omega) = \int_0^\infty A(\omega, \lambda) d\varepsilon(\lambda) + \int_0^\infty B(\omega, \lambda) d\varepsilon^*(\lambda) \quad (6)$$

which in our case is defined for $0 \leq \omega \leq \infty$ since $dX(\omega)$ is also conjugate-symmetric. Expression (6) is a widely linear system [9] with respect to $d\varepsilon(\omega)$ and its complex conjugate. Although it does not satisfy the scaling property of a truly linear system, (6) is notable in that superposition still holds with respect to the input $d\varepsilon(\omega)$.

Since $d\varepsilon(\omega)$ is white and proper, we have, respectively,

$$\begin{aligned} E\{d\varepsilon(\omega_1)d\varepsilon^*(\omega_2)\} &= \delta(\omega_1 - \omega_2)d\omega_1d\omega_2 \\ E\{d\varepsilon(\omega_1)d\varepsilon(\omega_2)\} &= 0 \end{aligned} \quad (7)$$

It follows that the output complementary covariance is

$$\begin{aligned} E\{dX(\omega_1)dX(\omega_2)\} &= \int_0^\infty A(\omega_1, \lambda)B(\omega_2, \lambda)d\lambda \\ &+ \int_0^\infty B(\omega_1, \lambda)A(\omega_2, \lambda)d\lambda \end{aligned} \quad (8)$$

which vanishes if either system function, $A(\omega, \lambda)$ or $B(\omega, \lambda)$, is identically zero. With respect to (5), the impropriety of $dX(\omega)$ results from the mixing of $d\varepsilon(\omega)$ and its complex conjugate, or equivalently, from mixing positive- and negative-frequency spectral elements from a stationary excitation process. Widely-linear mixing in the frequency domain is essential to the concept of over-modulation, which we discuss next.

3. MODULATION FREQUENCY AND IMPROPRIETY

The concept of modulation frequency is useful for studying the intelligibility of speech [10][11] and for classifying underwater propeller noise [12], to name a few examples. In the present context, we formalize modulation frequency in terms of periodical correlation (PC) [13], or cyclostationarity [14].

Suppose $x(t)$ is a continuous-time PC process defined by the point-by-point multiplication

$$x(t) = m(t) \cdot c(t) \quad (9)$$

where all components are real-valued, $c(t)$ is stationary white noise, and $m(t)$ is periodic. With respect to (4), $L(t, \tau) = m(t)$ has no convolution component. For now we present this simple model as a tool for understanding impropriety, but we will expand upon this model to account for aperiodicity and temporal correlation later in Section 4.

There are two cases within the PC signal model, sinusoidal and generally periodic modulation.

3.1. Sinusoidal Over-Modulation

The simplest PC case is a sinusoidal modulator, $m(t) = r_0 \cos(\omega_0 t + \theta_0)$, where $\omega_0 = 2\pi f_0$ is the modulation frequency. In the following, we show that the spectral impropriety of $x(t)$ is directly related to ω_0 .

First decompose $c(t)$ into real, lowpass and highpass components $c_L(t)$ and $c_H(t)$, having non-overlapping power spectra, such that $c(t) = c_L(t) + c_H(t)$. Now, suppose that the upper and lower cutoff frequency of $c_L(t)$ and $c_H(t)$ is equal to the modulation frequency ω_0 , as shown in Figure 1.

Then by the Bedrosian product theorem [15],

$$x_a(t) = m_a(t) \cdot c_L(t) + m(t) \cdot c_{H,a}(t) \quad (10)$$

where subscript- a in every case denotes an analytic signal. The complementary variance of the analytic signal is

$$E\{x_a^2(t)\} = \sigma_L^2 m_a^2(t) \quad (11)$$

where σ_L^2 is the variance of $c_L(t)$. The above result follows from the fact that $c_H(t)$ is stationary and its analytic signal $c_{H,a}(t)$ is proper. Cross-terms similarly vanish due to orthogonality of $c_L(t)$ and $c_H(t)$.

We conclude that the analytic signal is improper for any modulation frequency $\omega_0 > 0$. Equivalently, $x(t)$ is spectrally improper. This result links impropriety to over-modulation, or when the frequency of modulation overlaps the spectral content of a multiplicative carrier signal. From the perspective of the widely linear system (6), the impropriety of $dX(\omega)$ is a result of the modulator shifting positive- and negative-frequency spectral increments of $c_L(t)$ into the range $0 \leq \omega \leq \pi$.

3.2. Generally Periodic Modulation

When $m(t)$ is periodic with a Fourier series consisting of multiple harmonics, the Bedrosian approach to the analytic signal leads to a sum of non-orthogonal cross terms. Another way to decompose the signal is a subband approach. For example, the short-time Fourier transform (STFT) is

$$X(t, \omega) = \int g(t - \tau) x(\tau) e^{-j\omega\tau} d\tau \quad (12)$$

where $g(t)$ is a lowpass filter. Since $x(t)$ is temporally white, the complementary variance of the subband array is

$$E\{X^2(t, \omega)\} = \int g^2(t - \tau) m^2(\tau) e^{-j2\omega\tau} d\tau \quad (13)$$

Since $m^2(t)$ has the same fundamental frequency as $m(t)$, a subband centered on frequency ω_k will be improper if and only if $2\omega_k$ is in the vicinity of a harmonic in $m^2(t)$.

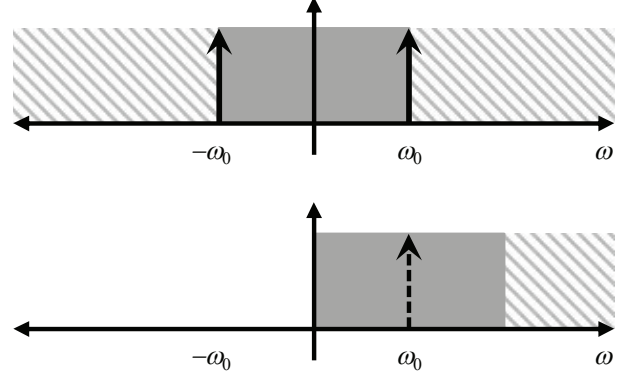


Fig. 1. Schematic for Bedrosian over-modulation, where the dark shaded and pattern-shaded power spectra correspond to $c_L(t)$ and $c_H(t)$, respectively (top). The bottom plot shows the components of the analytic signal after the frequency shift.

4. RHYTHMIC SIGNAL ANALYSIS

The PC model is insufficient to explain most real-world signals, since $x(t)$ in (9) is both white and perfectly periodic. However, many signals of interest are rhythmic in the sense that they are approximately periodic. Notable examples are syllabic rhythms in speech, propeller modulations in ship noise, and brain waves. In this section, we propose a rhythmic analysis approach using complementary statistics.

Consider the rhythmic process $y(t)$ given by the model

$$y(t) = \int h(t, t - \tau) x(\tau) d\tau. \quad (14)$$

We assume that $h(t, \tau)$ varies slowly in the t dimension, at rates less than the fundamental frequency of $m(t)$. Consequently, $h(t, \tau)$ is a time-varying perturbation on the otherwise PC $x(t)$. Additionally, the convolution component of $h(t, \tau)$ accounts for non-white $y(t)$. All components are real-valued, and (14) is a particular version of (4).

Equivalently,

$$y(t) = \int H(t, \omega) e^{j\omega t} dX(\omega). \quad (15)$$

Assuming that $H(t, \omega)$ is sufficiently smooth in ω , we have

$$x(t) \approx \sum_k H_k(t) \cdot x_k(t) \quad (16)$$

where $H_k(t)$ is the average complex value of $H(t, \omega)$ in the k th rectangular interval in ω , and $x_k(t)$ is the k th analytic subband component of $x(t)$. From (13), $x_k(t)$ can be improper if $m^2(t)$ contains harmonics in the vicinity of two times the subband center frequency.

Defining $Y(t, \omega_k)$ as the STFT of $y(t)$, we claim that $Y(t, \omega_k)$ contains the k th modulated component $H_k(t) \cdot x_k(t)$

for appropriate subband frequencies ω_k . The Hermitian envelope $|Y(t, \omega_k)|^2$ is related to the Hilbert envelope and the spectrogram. Our contribution with this paper is the complementary envelope $Y^2(t, \omega_k)$, which contains the phase of $H_k(t)$ and $x_k(t)$.

Both envelopes can be analyzed in the modulation-frequency domain by Fourier transforming over t . The resulting modulation spectra are the usual Hermitian DEMON (Demodulated Noise) spectrum [12] and the new complementary DEMON spectrum introduced here. Figure 2 displays both DEMON spectra for merchant ship noise in a subband centered on 450 Hz. As seen, the 2-Hz blade rate appears as a ripple pattern in the complementary spectrum, which is consistent yet different from the harmonics seen in the Hermitian spectrum. Further work is needed to disentangle the propeller blade modulation $m(t)$ from the aperiodicities in $h(t, \tau)$, but the preservation of phase in the complementary variance suggests that aperiodic timing information can be demodulated as a complex term from $Y^2(t, \omega_k)$.

5. CONCLUSION

Using a periodically correlated signal model, we proved the connection between modulation frequency and impropriety in the frequency domain and in the analytic signal. Specifically, the phenomenon of over-modulation, where the modulating frequency overlaps the spectrum of a stationary carrier signal, can be estimated directly by complex subband analysis of the signal. We developed a framework for analyzing a rhythmic signal, which is a complex-modulated version of a periodically correlated signal. Preliminary results from propeller noise data begin to verify the existence of impropriety in real-world signals. For future work, spectral impropriety necessitates new estimators and filters for processing rhythmic nonstationary signals.

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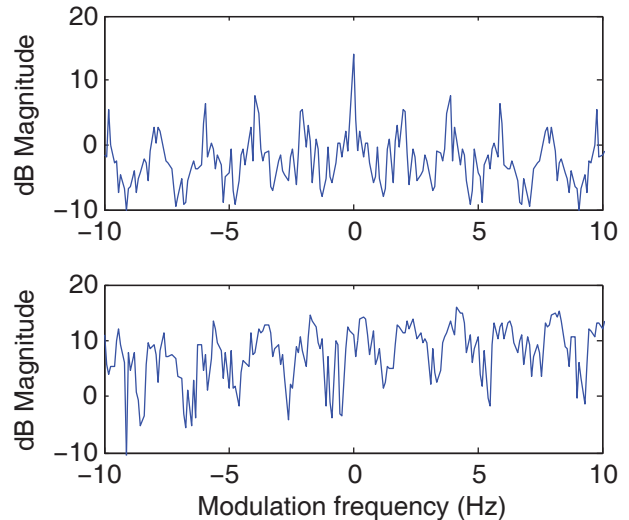


Fig. 2. Hermitian (top) and complementary (bottom) DEMON spectra, estimated for propeller noise from an ocean-going merchant vessel.