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14. ABSTRACT The use of krypton as an alternative to xenon for Hall thruster propellant is an interesting option for satellite system designers due to its lower cost. However, this cost-savings comes at the expense of thrust efficiency. Reduction in efficiency can be caused by energy losses from Joule heating, radiation, and the ionization process as well as degradation of plume quality from an increase in velocity distribution spread (most often from an increase in multiply charged ion populations) and geometric beam divergence. ¹ In order to quantify this performance reduction for the case of the flight model SPT-100 HET (1.35 kW), performance measurements were made using an inverted pendulum thrust stand. The plume was also characterized by a Faraday probe and RPA measurements to examine how plume qualities change with operating condition. Krypton operating conditions were tested over a large range of operating powers from 800 W to 3.9 kW. Analysis of how performance is impacted by propellant and operating condition is presented. A simple mission analysis was done based on the performance measurements to evaluate the practicality of krypton propellant for an SPT-100 subsystem using krypton propellant for north-south station keeping (NSSK) for a typical communications spacecraft in geosynchronous orbit.					
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Unclassified	Unclassified	Unclassified	SAR	30	W.A. Hargus Jr. N/A

A Performance and Plume Comparison of Xenon and Krypton Propellant on the SPT-100



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Introduction

• Motivation

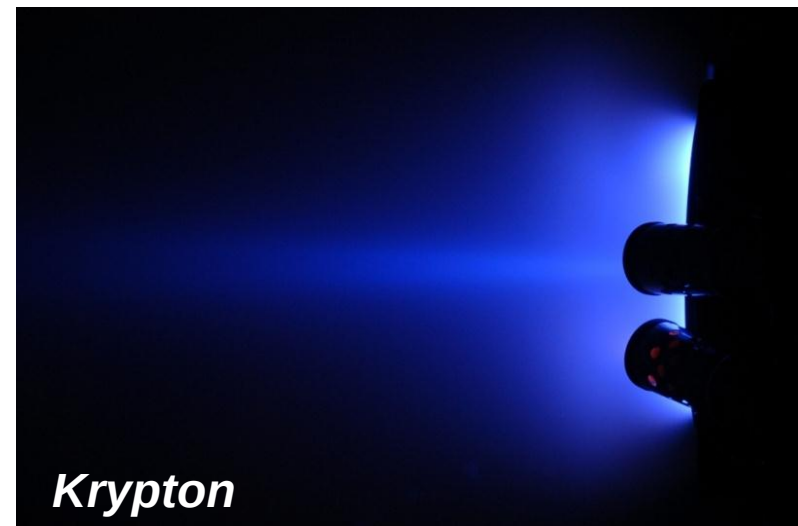
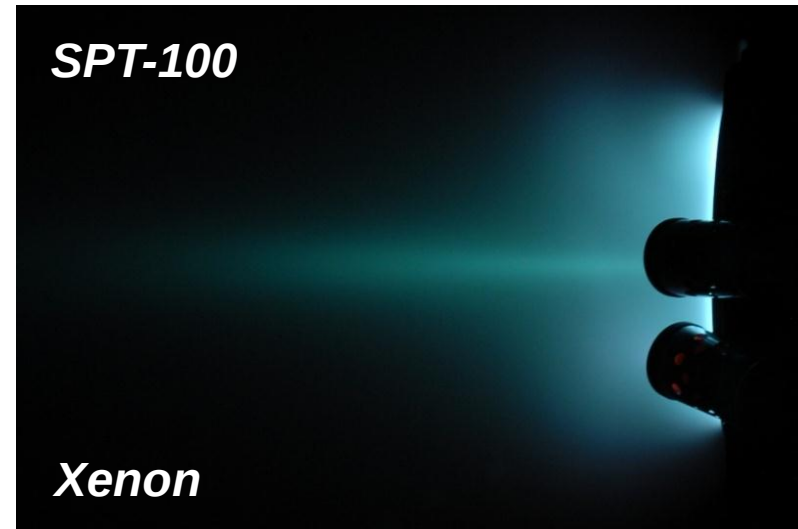
- Kr less expensive than Xe ($\sim 10\times$ by vol., $\sim 6\times$ by mass)
- Kr **may** increase I_{sp} (25% greater velocity for a given discharge voltage due to smaller atomic mass)
 - Beneficial for station-keeping missions
- Simple integration into existing Xe propellant management system

• Study Objectives

- Characterize differences between Xe and Kr performance and plume characteristics for a flight qualified thruster over a wide range of operating conditions
- Study how individual thruster efficiency components change with propellant and operating conditions
- Determine if Kr is a viable alternative propellant for the flight model SPT-100 for NSSK purposes

• Methodology

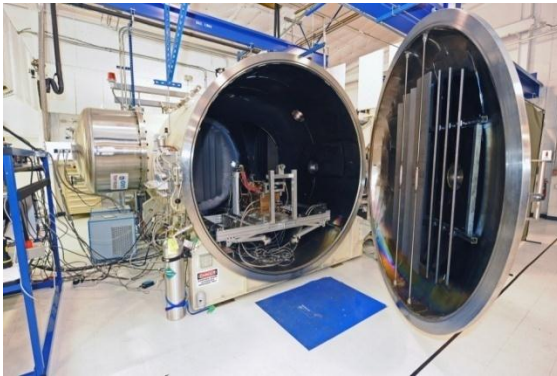
- Measure thrust for a wide range of operating conditions
 - Kr test matrix: 800 W - 3.9 kW
 - Smaller set of Xe test cases for comparison
 - Evaluate how measured performance translates into firing time and propellant mass required for typical NSSK duties for a GEO COMM satellite
- Measure plume characteristics with electrostatic probes
 - Calculate beam efficiency, current utilization efficiency, and voltage utilization efficiency from probe data





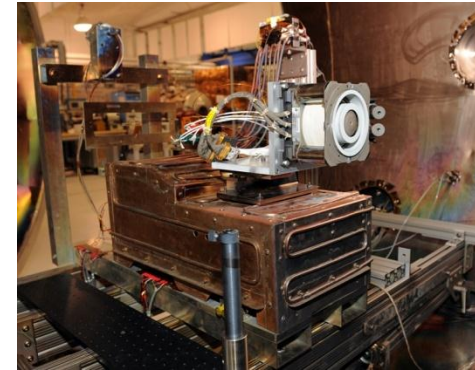
Testing Facilities

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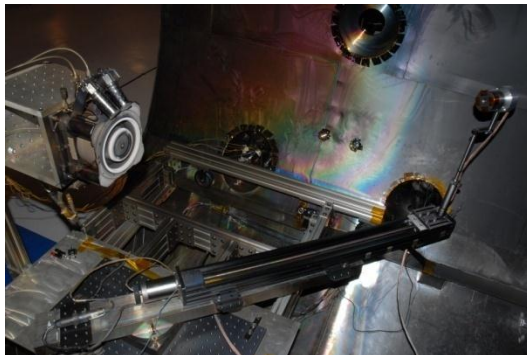
Chamber 1, AFRL Edwards AFB

- 2.4 m dia., 4.1 m length, stainless steel
- **Two cryogenic pumps**
- 1.2 m dia., LN2 baffled (70 K), 2 stage He (15 K)
- SPT-100 operation background pressure
 - Xe: 1.1×10^{-5} Torr (56.4 sccm flow rate)
 - Kr: 1.0×10^{-5} Torr (65.4 sccm flow rate)



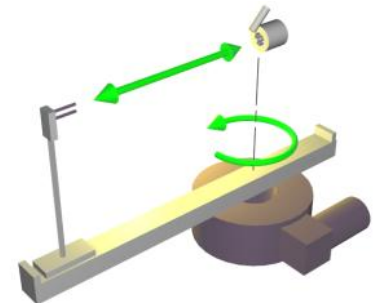
Inverted Pendulum Thrust Stand

- **Null-Type Design**
 - Electromagnetic force generated by PID feedback system counteracts displacement
 - Restoring force linear proportional to voltage applied to null coil
 - In-situ calibration performed with weights loaded and unloaded by mechanical pulley system
 - Design by Haag at NASA GRC



Probe Translation System

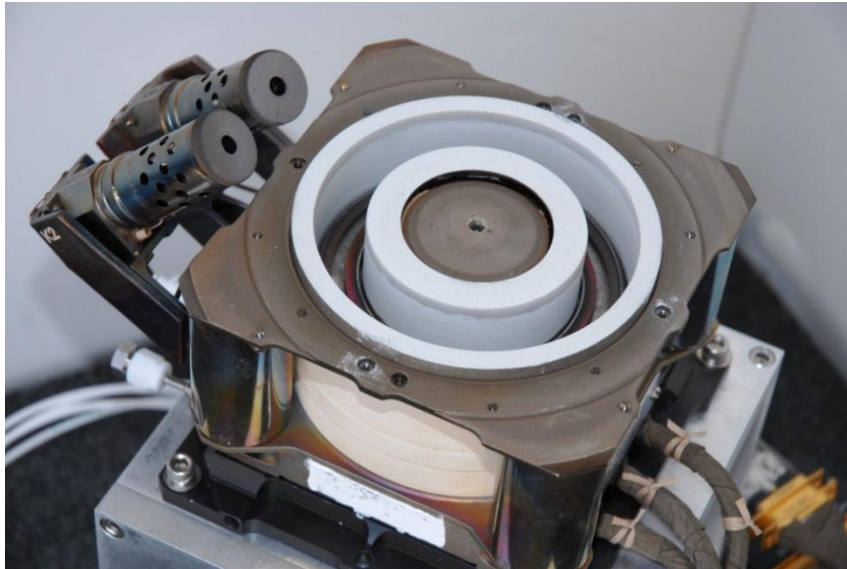
- **Two stage combination**
 - Rotary stage for angular movement
 - Linear stage for radial movement
- **Stepper motor driven**
 - r limit: 100 cm
 - θ limit: $\pm 90^\circ$





SPT-100 Hall Effect Thruster

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SPT-100 by Experimental Design Bureau Fakel

Thrust	83 mN
Thrust Efficiency	50%
I_{sp}	1600 s

NASA life and performance testing:

Garner, C. E., Brophy, J. R., Polk, J. E., and Pless, L. C., "Cyclic Endurance Test of a SPT-100 Stationary Plasma Thruster," Proceedings of the 30th AIAA/SAE/ASME/ASEE Joint Propulsion Conference and Exhibit, 1994, AIAA-94-2856.

Sankovic, J. M., Hamley, J. A., and Haag, T. W., "Performance evaluation of the Russian SPT-100 thruster at NASA LeRC," Proceedings of the 23rd International Electric Propulsion Conference, 1993, IEPC-93-094.

Kr and Kr/Xe mixture performance testing:

Kim, V., Popov, G., Kozlov, V., Skrylnikov, A., and Grdlichko, D., "Investigation of SPT Performance and Particularities of its Operation with Kr and Kr/Xe Mixtures," Proceedings of the 27th Electric Propulsion Conference, No. IEPC-01-065, 2001.

Thruster Characteristics

- Anode Power: 1350 W
- Conventional 5 magnetic core design
 - One inner, four outer connected in series
 - Magnetic circuit current supplied by anode current
- Acceleration channel: 100 mm outer dia., 69 mm inner dia., 28 mm depth
- 2 lanthanum hexaboride (LaB_6) cathodes
- Space Systems/Loral flight heritage
 - 7 spacecraft with SPT-100 propulsion subsystems in orbit
 - 13 years of cumulative on-orbit experience
 - single thruster has accumulated over 6 years of near-daily operation
 - 11 more spacecraft under construction



Operating Conditions Tested

Power Values (W)

-20%

Propellant Flow Rate

+40%

Krypton

Anode
Potential
-20%
+90%

	-20%	-10%	Nominal	+10%	+20%	+30%	+40%
	(3.27 mg/s)	(3.68 mg/s)	(4.09 mg/s)	(4.50 mg/s)	(4.90 mg/s)	(5.31 mg/s)	(5.72 mg/s)
-20% (242 V)	813	950	1083	1209	1339	1476	1606
-10% (272 V)	915	1069	1220	1363	1511	1671	1820
Nominal (302 V)	1015	1187	1356	1516	1681	1863	2033
+10% (332 V)	1121	1306	1491	1670	1852	2054	2258
+20% (363 V)	1242	1431	1628	1823	2023	2248	2468
+30% (393 V)	1356	1561	1769	1977	2208	2445	2675
+40% (423 V)	1468	1693	1913	2143	2376	2615	2878
+50% (453 V)	1567	1821	2061	2302	2547	2799	3078
+60% (483 V)	1662	1938	2213	2463	2720	2987	3276
+70% (513 V)	1751	2048	2347	2624	2895	3195	3481
+80% (544 V)	1857	2171	2480	2765	3063	3389	3690
+90% (574 V)	1960	2289	2605	2929	3245	3589	3913

Xenon

-30%

Propellant Flow Rate

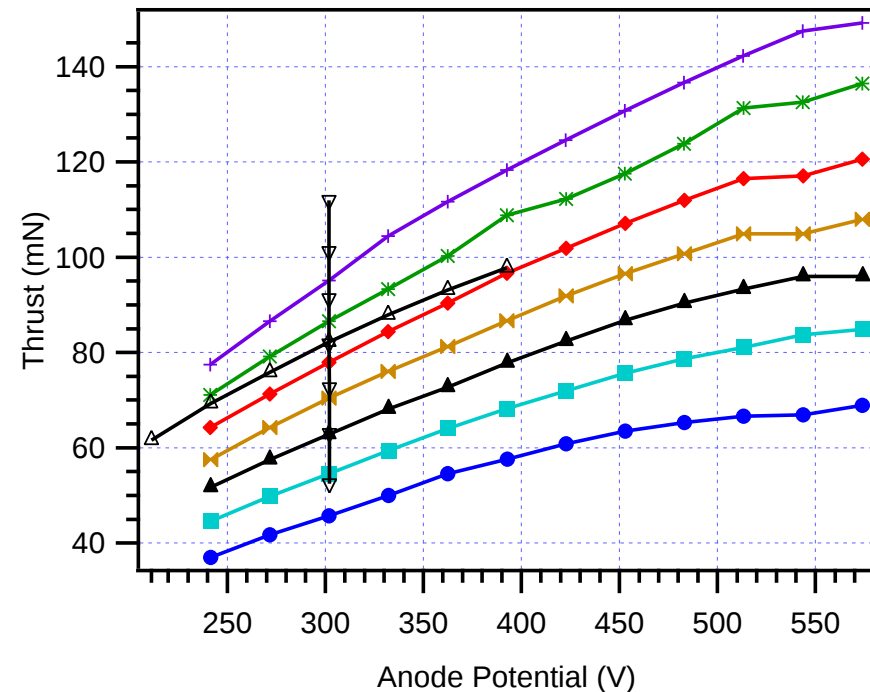
+30%

Anode
Potential
-30%
+30%

	-30%	-20%	-10%	Nominal	+10%	+20%	+30%
	(3.88 mg/s)	(4.44 mg/s)	(4.99 mg/s)	(5.54 mg/s)	(6.10 mg/s)	(6.65 mg/s)	(7.21 mg/s)
-30% (211 V)				945			
-20% (242 V)				1077			
-10% (272 V)				1213			
Nominal (302 V)	916	1061	1207	1351	1508	1666	1829
+10% (332 V)				1493			
+20% (363 V)				1638			
+30% (393 V)				1785			



Performance Results: Thrust

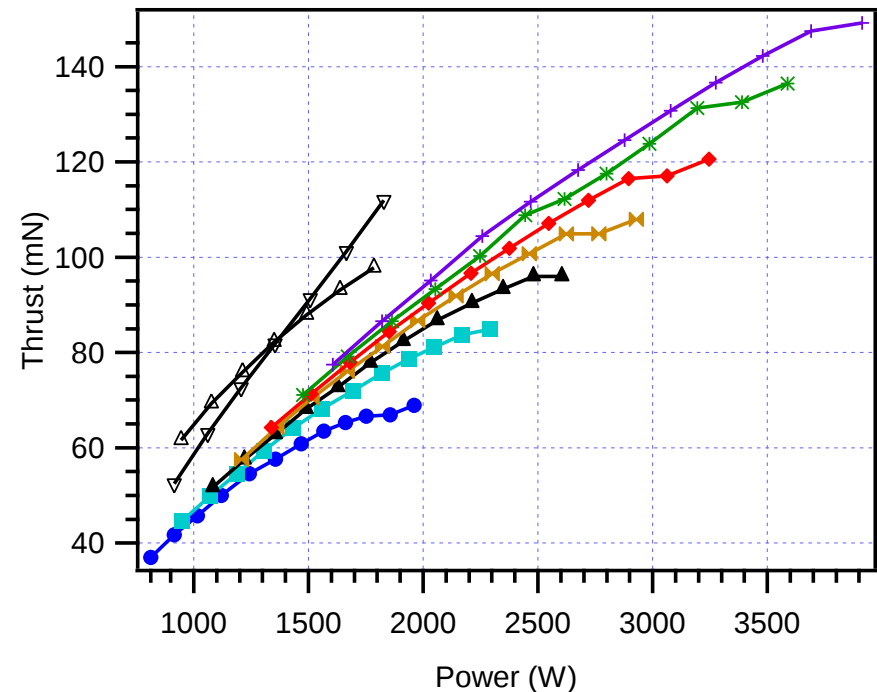


Krypton Data

- -20% Flow Rate (3.27 mg/s)
- -10% Flow Rate (3.68 mg/s)
- ▲ Nominal Flow Rate (4.09 mg/s)
- ✕ +10% Flow Rate (4.50 mg/s)
- ◆ +20% Flow Rate (4.90 mg/s)
- ✱ +30% Flow Rate (5.31 mg/s)
- ✚ +40% Flow Rate (5.72 mg/s)

Xenon Data

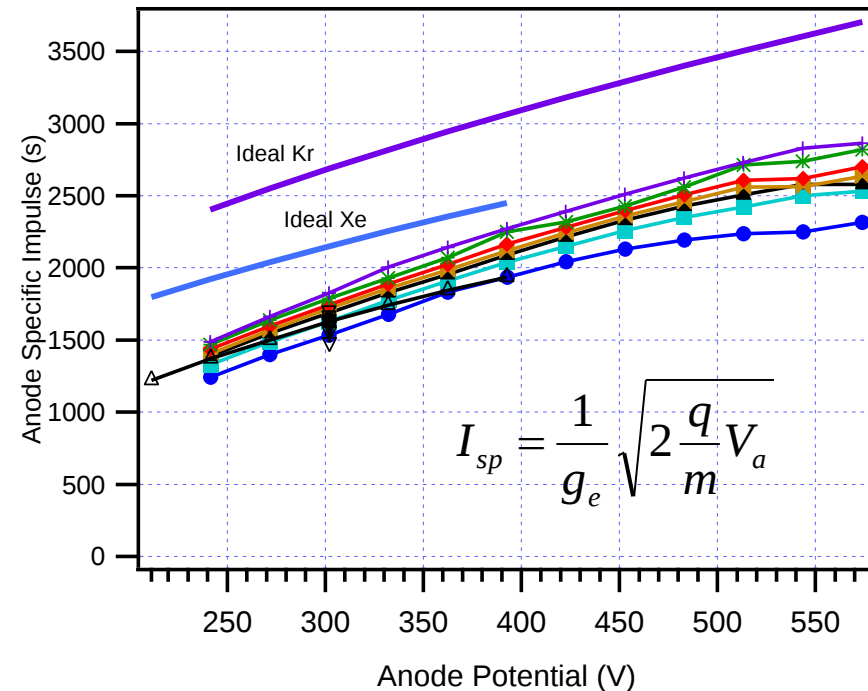
- △ Nominal Flow Rate (5.54 mg/s)
- ▽ Nominal Discharge Potential (302 V)



- Nominal Condition (300 V, 1350 W)
 - Xe: 82.1 mN
 - Kr: 62.9 mN
- Thrust increases linearly with discharge potential and propellant flow rate
- Thrust gains level out around 510 – 540 V



Performance Results: Specific Impulse

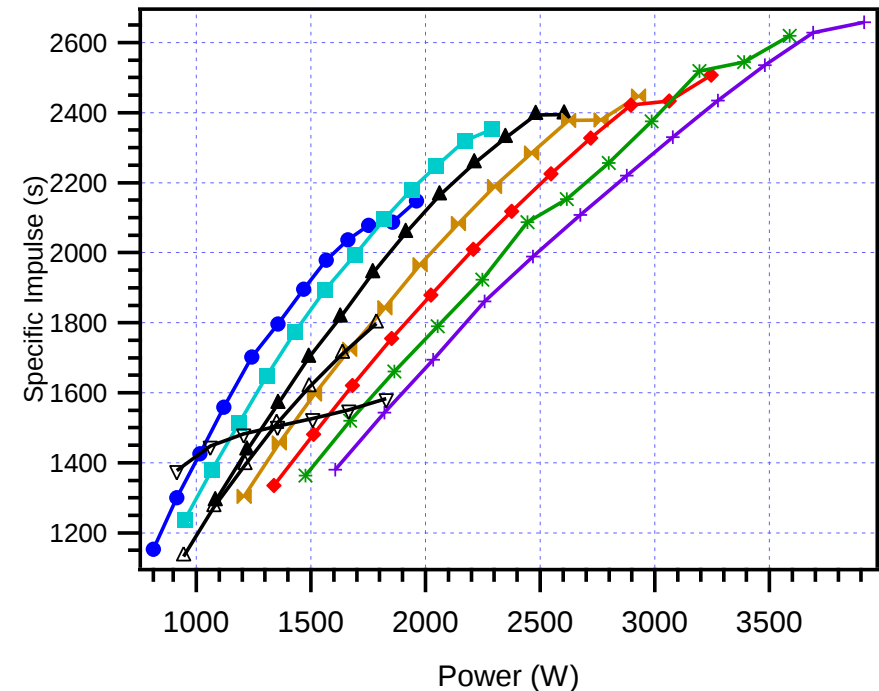


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- ✱ +40% Flow Rate (5.72 mg/s)

Xenon Data

- △ Nominal Flow Rate (5.54 mg/s)
- ▽ Nominal Discharge Potential (302 V)

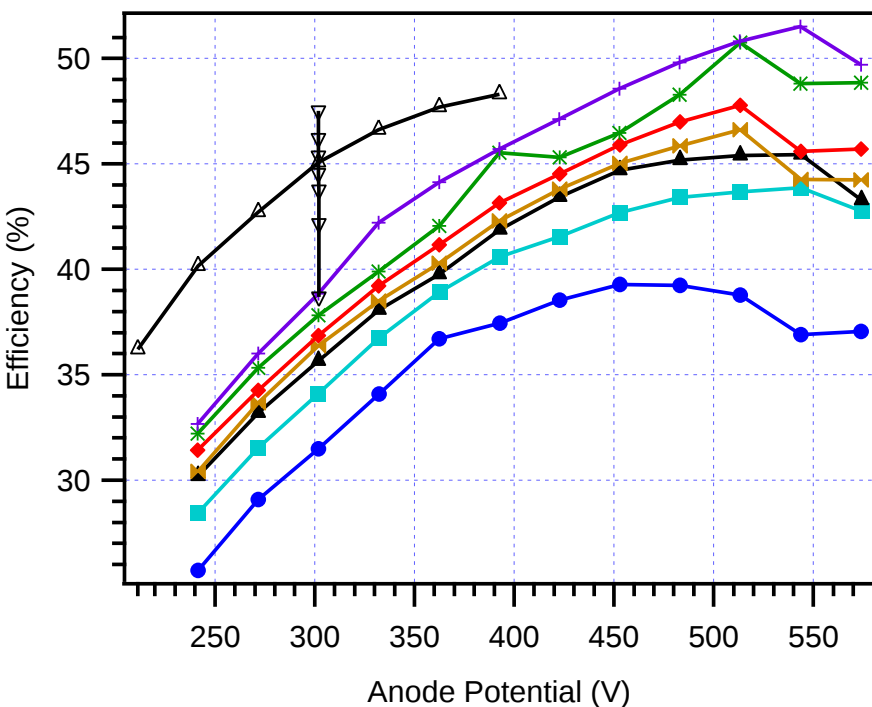


- Nominal Condition (1350 W)
 - Xe: 1511 s
 - Kr: 1568 s
- Performance Gap Between Actual and Ideal I_{sp}
 - Xe: 520 s
 - Kr: 1000 s
- Higher Kr I_{sp} not realized due to inferior propellant utilization fraction (m_i/m_a) and higher momentum divergence half-angle



Performance Results: Thrust Efficiency

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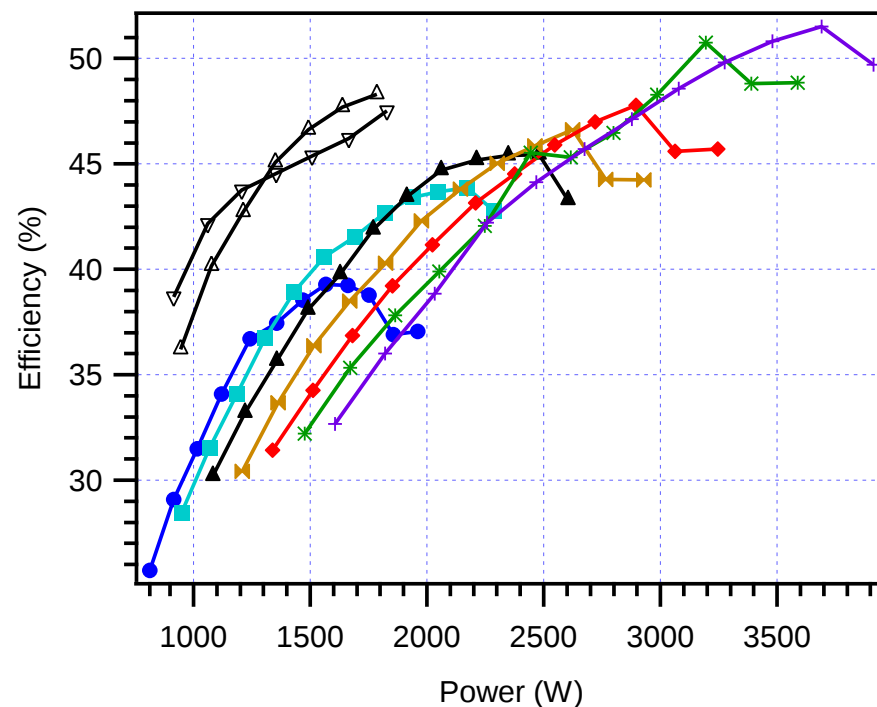


Krypton Data

- -20% Flow Rate (3.27 mg/s)
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- ▲ Nominal Flow Rate (4.09 mg/s)
- ◆ +10% Flow Rate (4.50 mg/s)
- ◆ +20% Flow Rate (4.90 mg/s)
- * +30% Flow Rate (5.31 mg/s)
- + +40% Flow Rate (5.72 mg/s)

Xenon Data

- △ Nominal Flow Rate (5.54 mg/s)
- ▽ Nominal Discharge Potential (302 V)



- Nominal Condition (300 V, 1350 W)
 - Xe: 45%
 - Kr: 36%
- Kr efficiency improves with higher operating powers



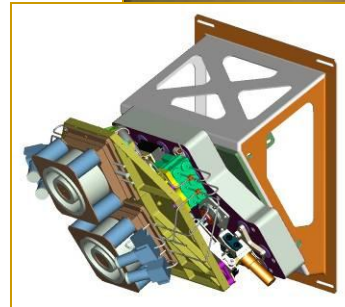
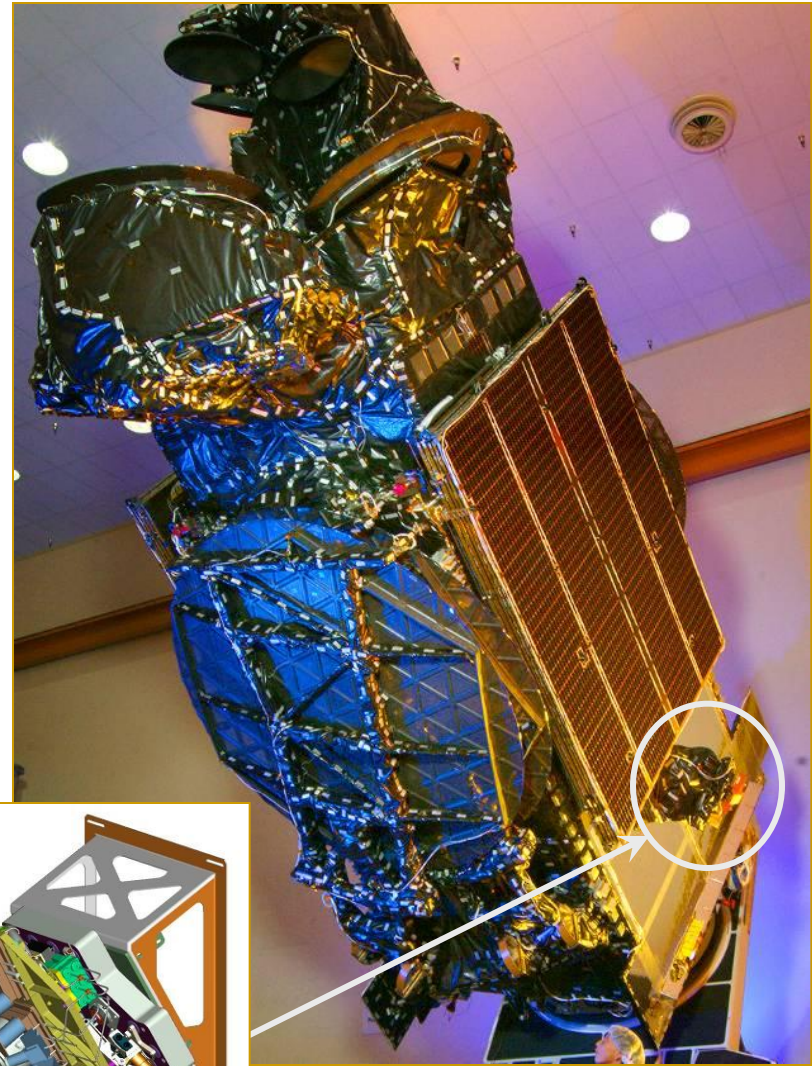
Krypton Mission Example: NSSK for GEO COMM S/C

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Can Krypton Propellant Satisfy
Mission Needs?

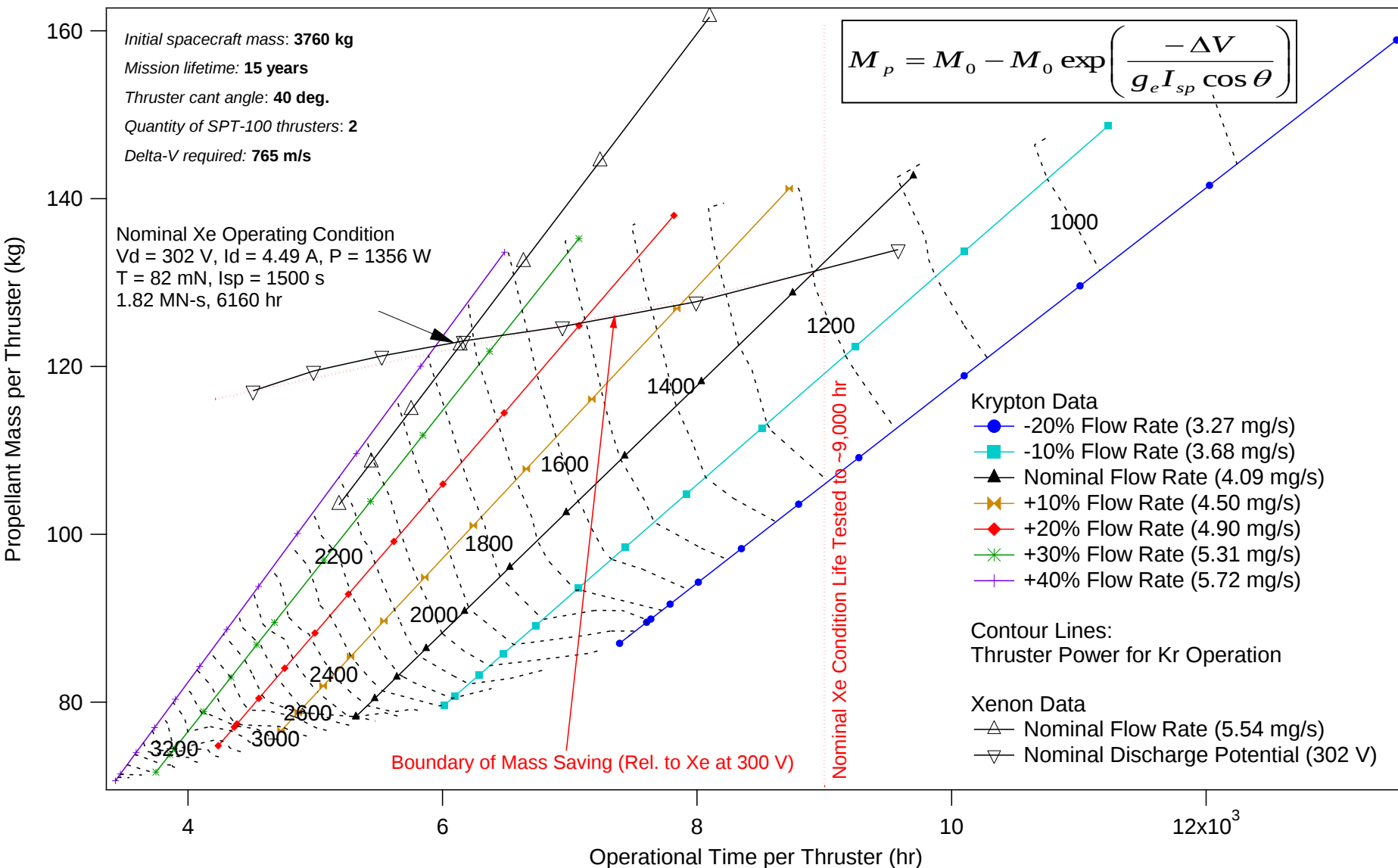
Mission Parameters

- *Initial spacecraft mass: 3760 kg*
- *Mission lifetime: 15 years*
- *Thruster cant angle: 40 deg.*
(directional cosine loss)
- *Quantity of SPT-100 thrusters: 2*
- *Delta-V required:*
 $(51 \text{ m/s/year}) \times 15 \text{ years} = 765 \text{ m/s}$





Krypton Mission Example: NSSK for GEO COMM S/C

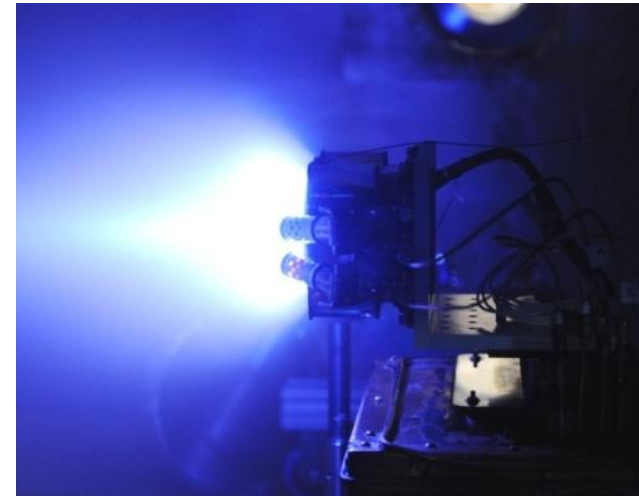




Krypton Mission Example: NSSK for GEO COMM S/C

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- **Kr appears capable of satisfying mission requirements (despite reduced performance)**
 - Higher power required
 - Longer firing time required
 - Erosion rates for Kr unknown
- **390 V, nominal flow rate operating point vs. nominal Xe condition**
 - 54 kg of total propellant mass saved
 - 400 additional Watts of power per thruster
 - 400 additional hours of firing time per thruster





Probe Study

Krypton Test Matrix

Discharge Potential	Propellant Flow Rate			
	F = Faraday Probe	Nominal	+20%	+40%
	R = RPA	(4.09	(4.90	(5.72
		mg/s)	mg/s)	mg/s)
	-20% (242 V)	F, R	F, R	F, R
	Nominal (302 V)	F, R	F, R	F, R
	+20% (363 V)	F, R	F, R	F, R
+40% (423 V)	F, R	F, R	F, R	F, R
+60% (483 V)	F, R	F, R	F, R	F, R
+80% (544 V)	F, R			

Faraday Probe

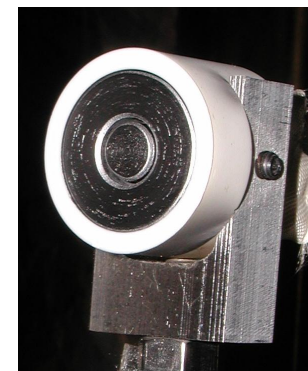
$r = 50$ cm to 100 cm, 5 cm increments

$\theta = -90$ to 90 deg, 1 deg. increments

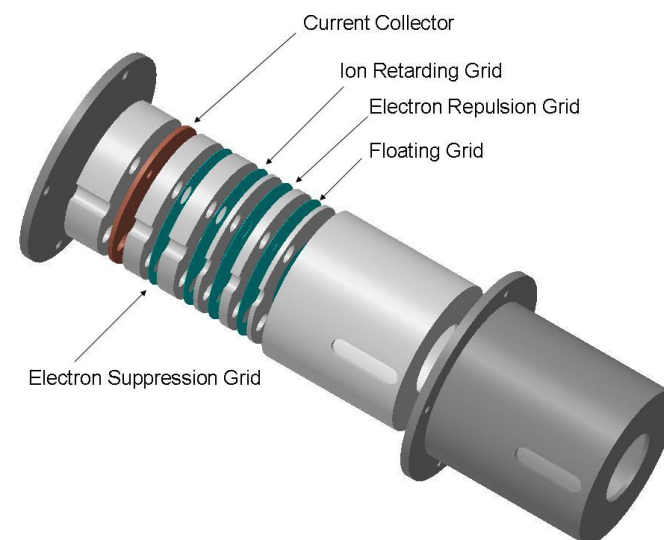
RPA

$r = 100$ cm

$\theta = -90$ to 90 deg, 5 deg. increments



Faraday Probe (22 mm outer dia.)

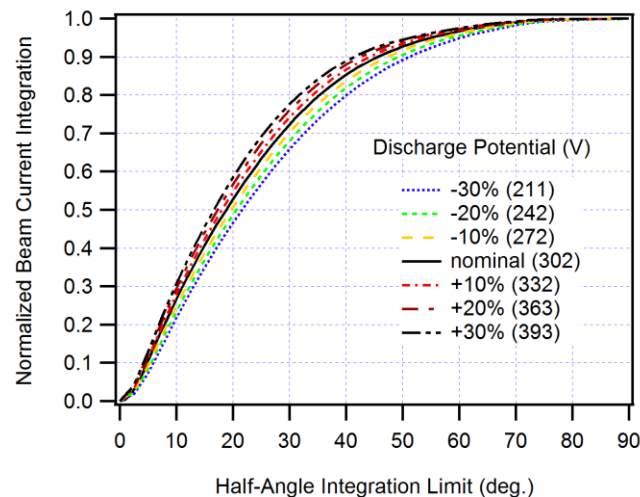
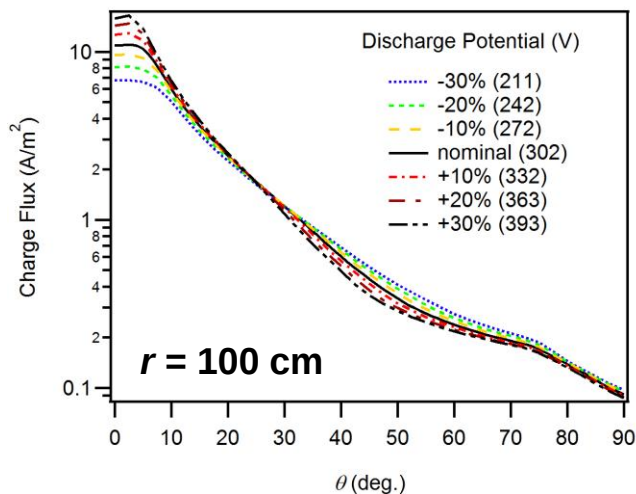


RPA (exploded view)

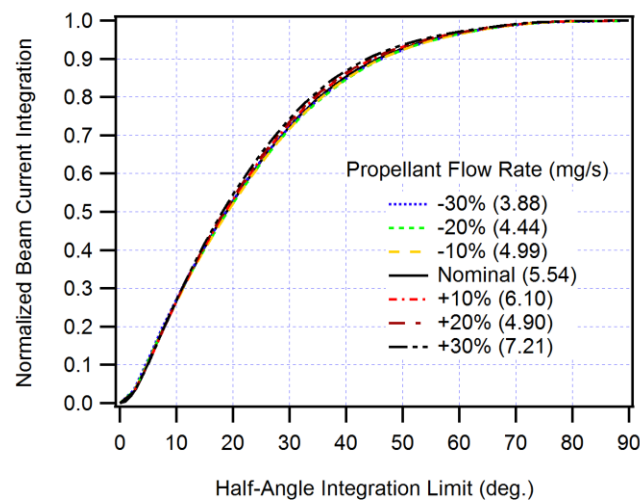
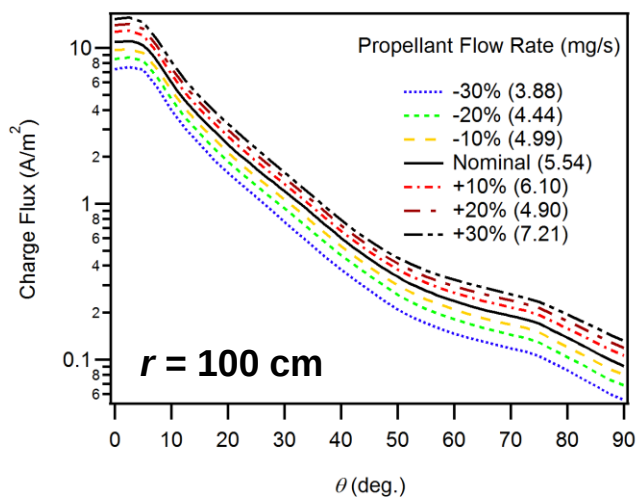


Faraday Probe Results: Xe

V_d Variations



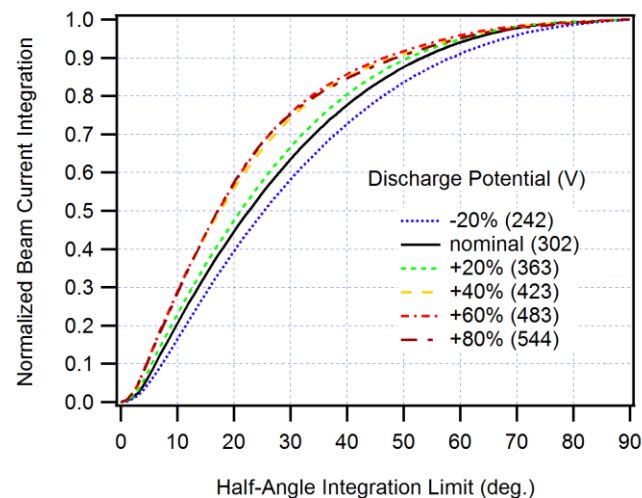
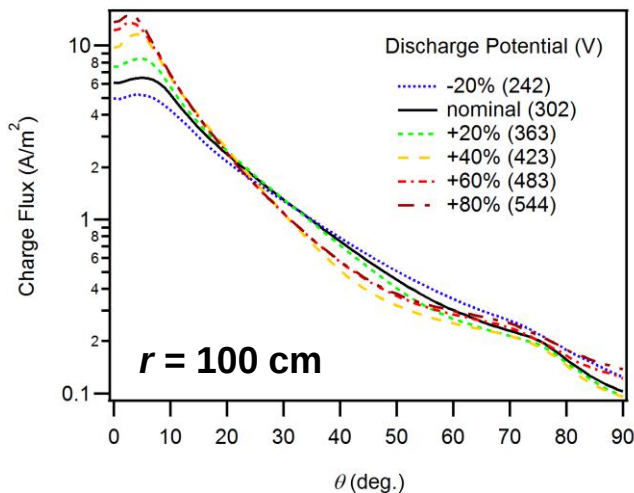
Flow Rate Variations



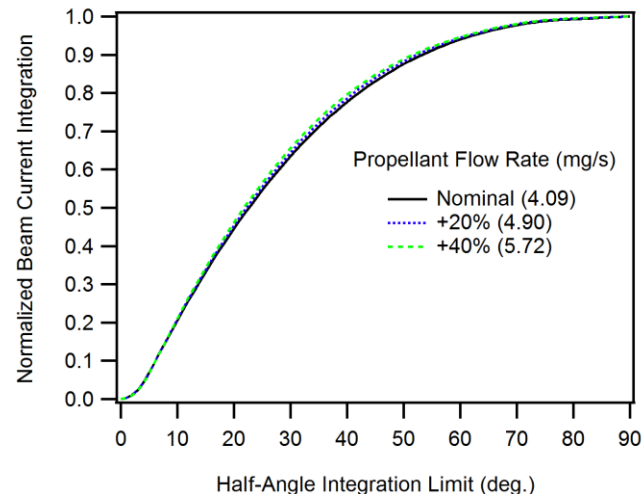
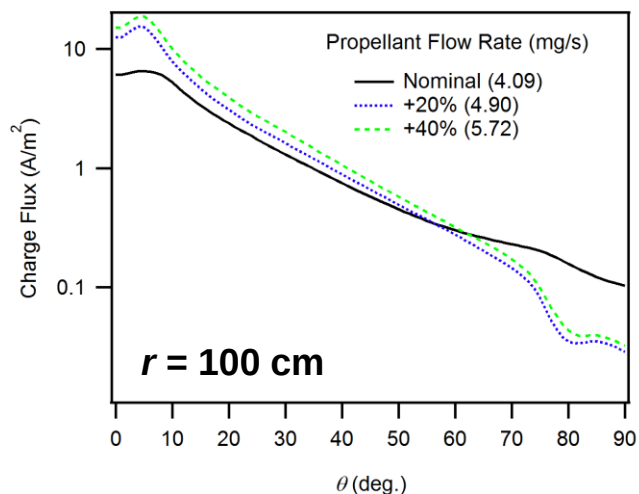


Faraday Probe Results: Kr

V_d Variations

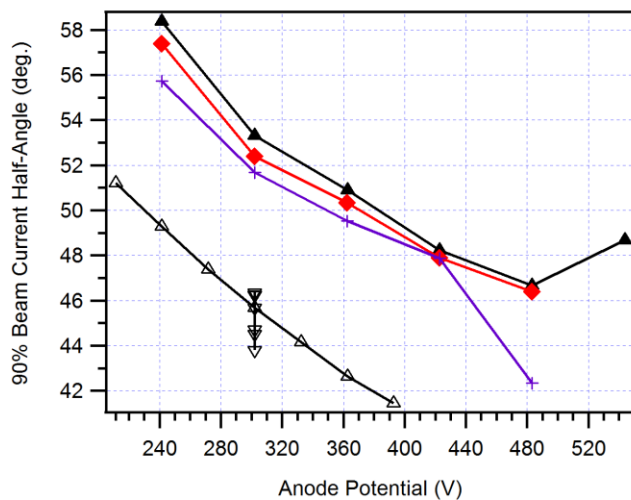
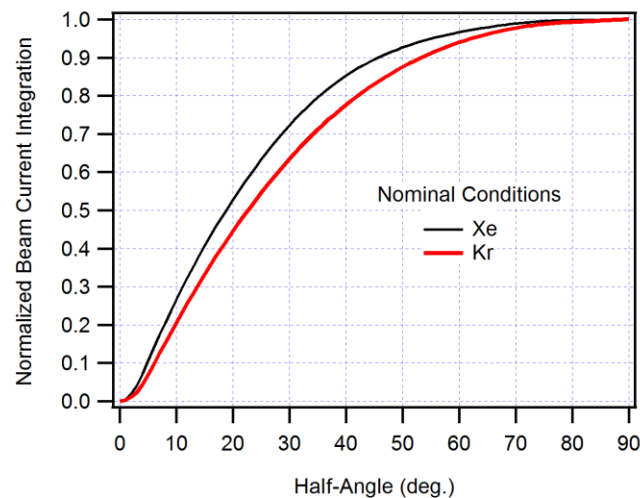
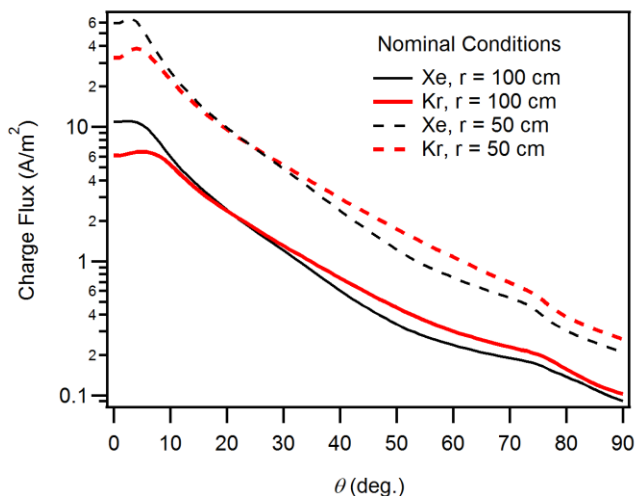


Flow Rate Variations





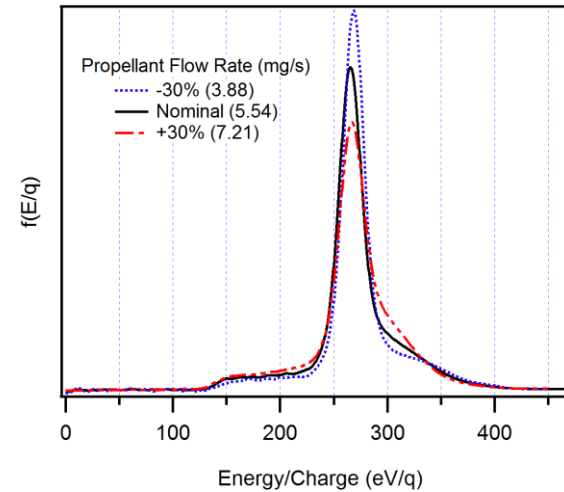
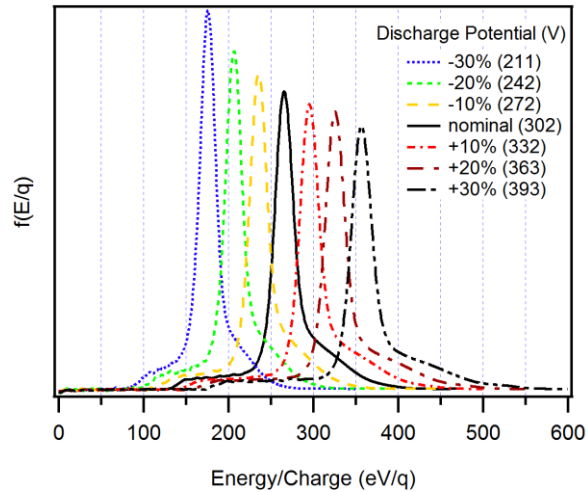
Faraday Results: Xe/Kr Comparison





RPA Results: Xe/Kr, 0 deg.

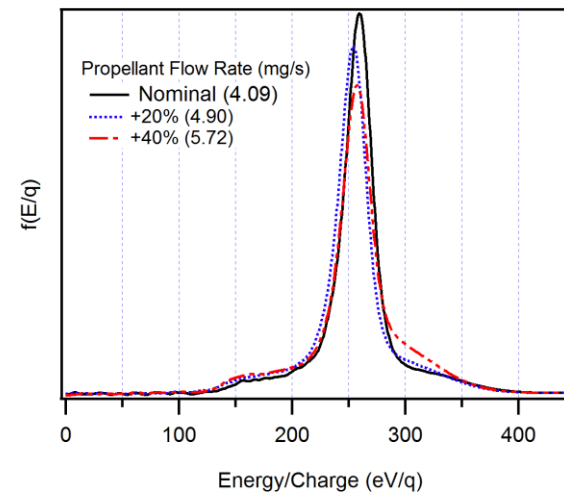
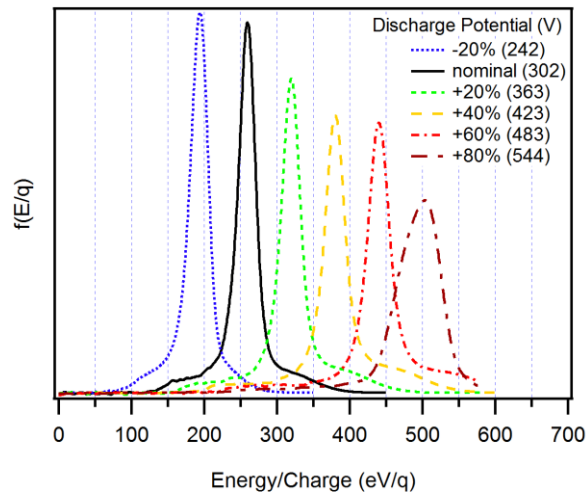
Xe



Nominal Flow Rate

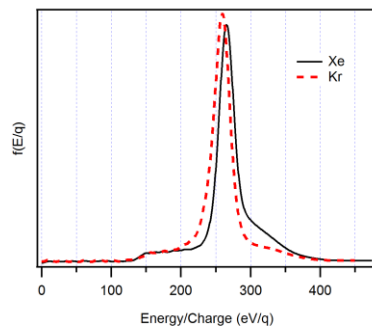
Nominal Discharge Potential

Kr

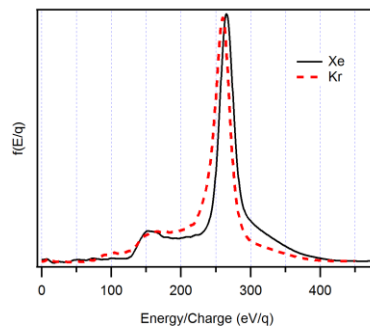




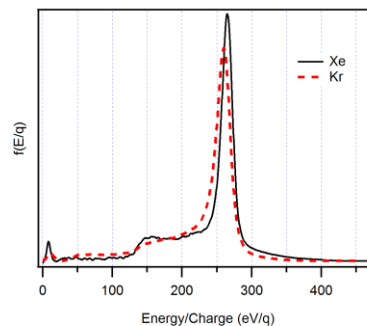
RPA Results: Xe/Kr, angular position



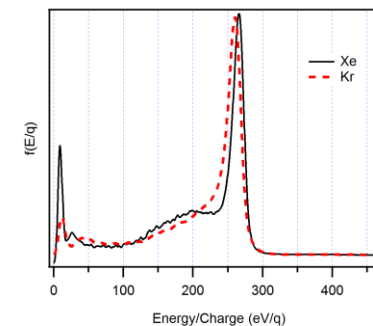
$\theta = 0$ deg.



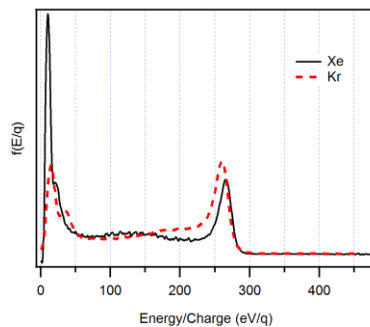
$\theta = 15$ deg.



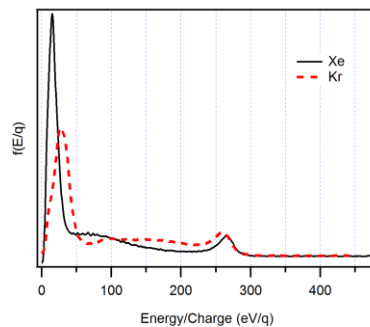
$\theta = 30$ deg.



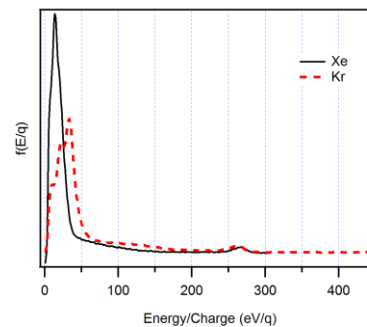
$\theta = 45$ deg.



$\theta = 60$ deg.



$\theta = 75$ deg.

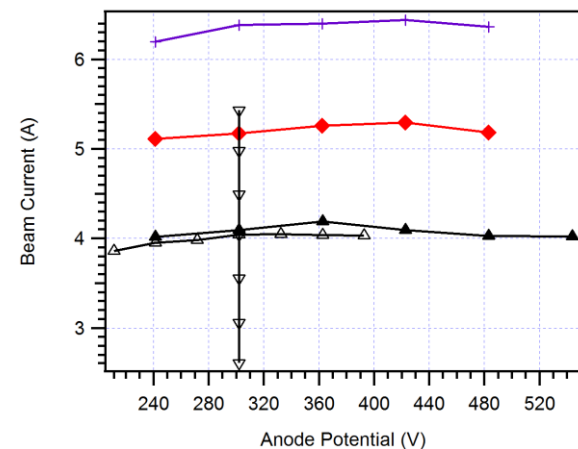


$\theta = 90$ deg.

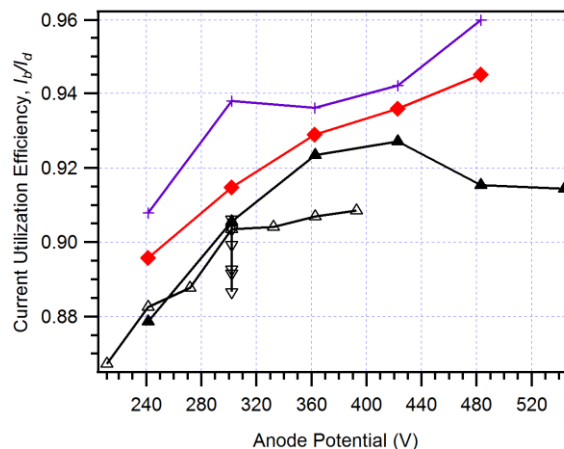
**EDF most
probable energy
6 V less for Kr**



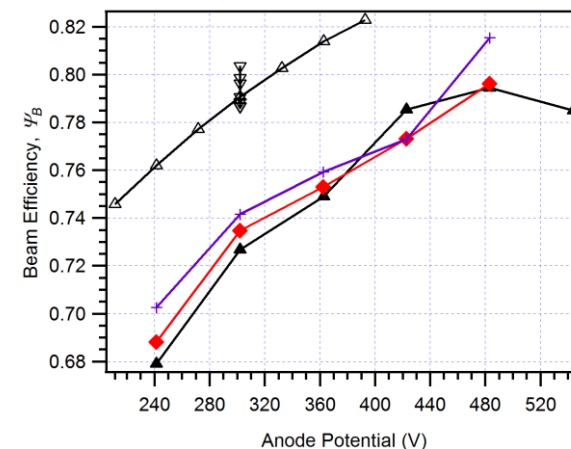
Probe Derived Efficiency Components: Faraday Probe



Total Beam Current



Current Utilization Efficiency



Beam Efficiency
(divergence loss factor)

$$I_b \approx \pi r^2 \int_{-\pi/2}^{\pi/2} j(\theta) |\sin \theta| d\theta \quad \left(\frac{I_b}{I_d} \right)$$

Krypton Data

- ▲— Nominal Flow Rate (4.09 mg/s)
- ◆— +20% Flow Rate (4.90 mg/s)
- +— +40% Flow Rate (5.72 mg/s)

Xenon Data

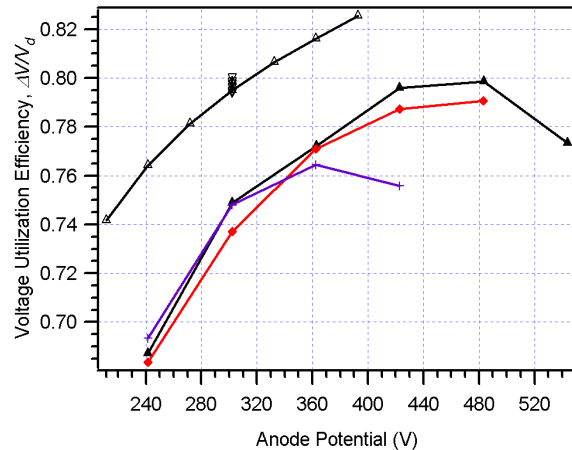
- △— Nominal Flow Rate (5.54 mg/s)
- ▽— Nominal Discharge Potential (302 V)

$$\Psi_B = \langle \cos \theta \rangle_{mv}^2$$

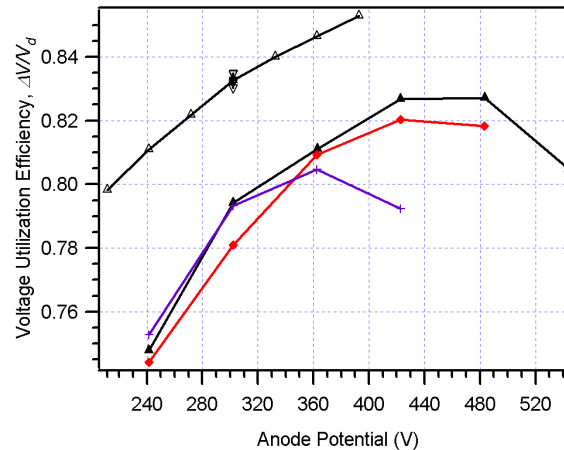
$$\approx \left(\frac{I_{axial}}{I_b} \right)^2 \approx \left(\frac{\pi r^2 \int_{-\pi/2}^{\pi/2} j(\theta) \cos \theta |\sin \theta| d\theta}{\pi r^2 \int_{-\pi/2}^{\pi/2} j(\theta) |\sin \theta| d\theta} \right)^2$$



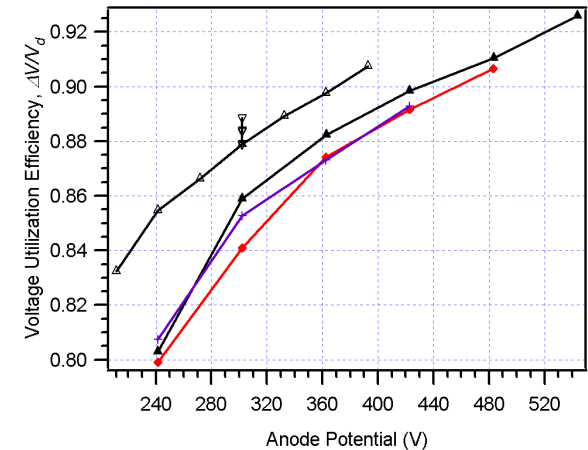
Probe Derived Efficiency Components: RPA – Voltage Utilization Efficiency



EDF mean method



EDF > 100 eV/q
method



EDF most probable
energy method

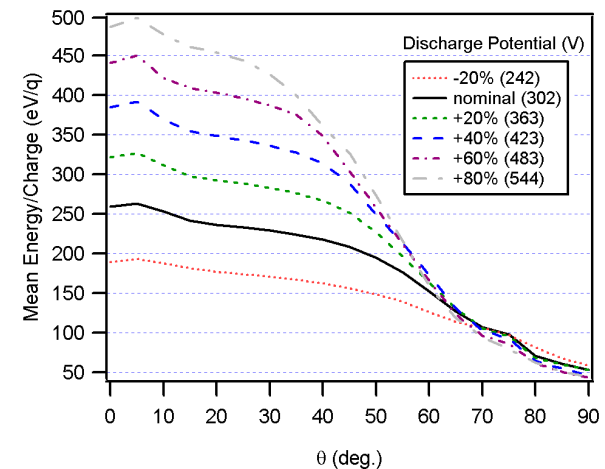
$$\left(\frac{\Delta V}{V_d}\right) = \left(\frac{\langle \bar{V}(\theta) \rangle_m}{V_d}\right) \approx \frac{1}{V_d} \left(\frac{\pi r^2 \int_{-\pi/2}^{\pi/2} j(\theta) \bar{V}(\theta) |\sin \theta| d\theta}{\pi r^2 \int_{-\pi/2}^{\pi/2} j(\theta) |\sin \theta| d\theta} \right)$$

Krypton Data

- ▲— Nominal Flow Rate (4.09 mg/s)
- ◆— +20% Flow Rate (4.90 mg/s)
- +— +40% Flow Rate (5.72 mg/s)

Xenon Data

- △— Nominal Flow Rate (5.54 mg/s)
- ▽— Nominal Discharge Potential (302 V)



Kr EDF mean values



Conclusions

- **Performance Study**

- **Thrust Efficiency**

- Kr is 9% less efficient at nominal operating power (36% vs. 45%)
 - Efficiency improves with increasing power

- **Specific Impulse**

- Kr not significantly higher than Xe
 - Due to lower propellant utilization fraction and higher plume divergence

- **On-orbit use**

- May be feasible for NSSK mission requirements despite lower performance
 - Life tests with erosion study would be necessary to validate thruster life time for inherent higher energy throughput required
 - Potential benefit is lower propellant cost, but not mass savings (due to similar I_{sp})

- **Plume Study**

- **Faraday Probe**

- Beam focusing improves with higher discharge potential (both Xe and Kr)
 - Kr has lower beam efficiency than Xe (by 0.06 for nominal condition)
 - Similar current utilization efficiency for Xe and Kr

- **RPA**

- Similar EDF shape for Xe and Kr (but higher low energy peak for Xe at oblique angles)
 - Most probable energy lower for Kr (6 V at nominal condition)
 - Kr has lower voltage utilization efficiency



Back Up Slides

SPACE SYSTEMS
LORAL

Back Up Slides



Krypton Mission Example: NSSK for GEO COMM S/C

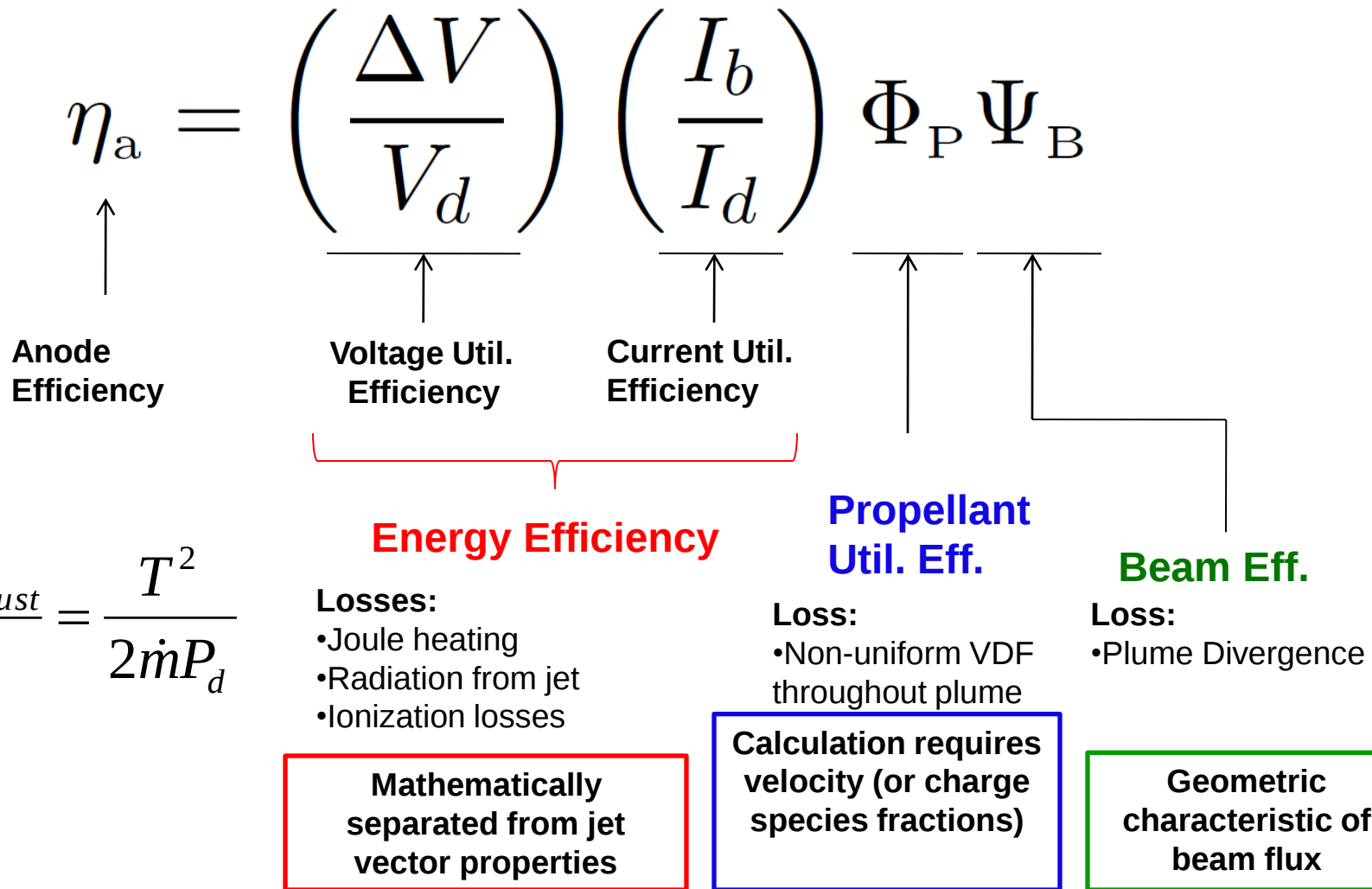
SPACE SYSTEMS
LORAL

- **Examine propulsion system requirements for each operating condition tested**
 - Total propellant mass
 - Total firing time
- **Calculate propellant mass with rocket equation:**
$$M_p = M_0 - M_0 \exp\left(\frac{-\Delta V}{g_e I_{sp} \cos \theta}\right)$$
- **Calculate total firing time with propellant mass and operating condition flow rate**

$$t = \frac{M_p}{\dot{m}_t}$$



Performance Evaluation Techniques



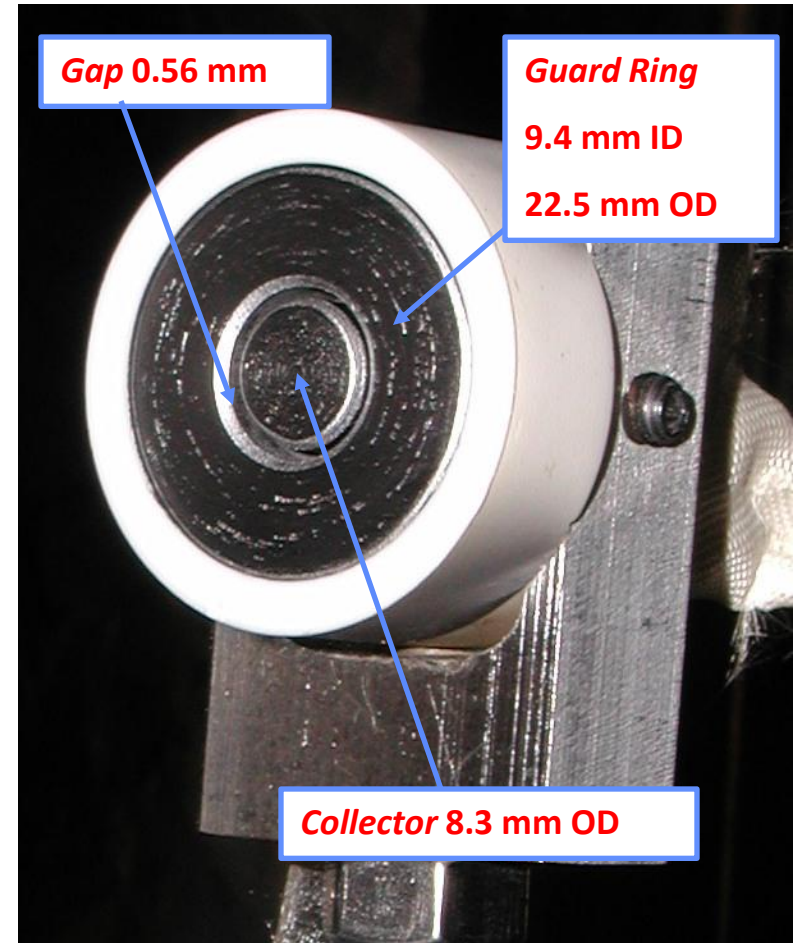
Brown, D. L., *Investigation of Low Discharge Voltage Hall Thruster Characteristics and Evaluation of Loss Mechanisms*, Ph.D. thesis, University of Michigan, 2009.

Faraday Probe

- Measures ion charge flux
- Molybdenum electrodes
- Guard ring ensures planar sheath over collection disk
- Biased to -30 V (relative to chamber ground) to ensure ion saturation
- Effective current collection area assumed to include a portion of the gap surface area

$$A_{\text{eff}} = A_c + A_{\text{gap}}(d_c/d_g)$$

(both electrodes have same thickness)

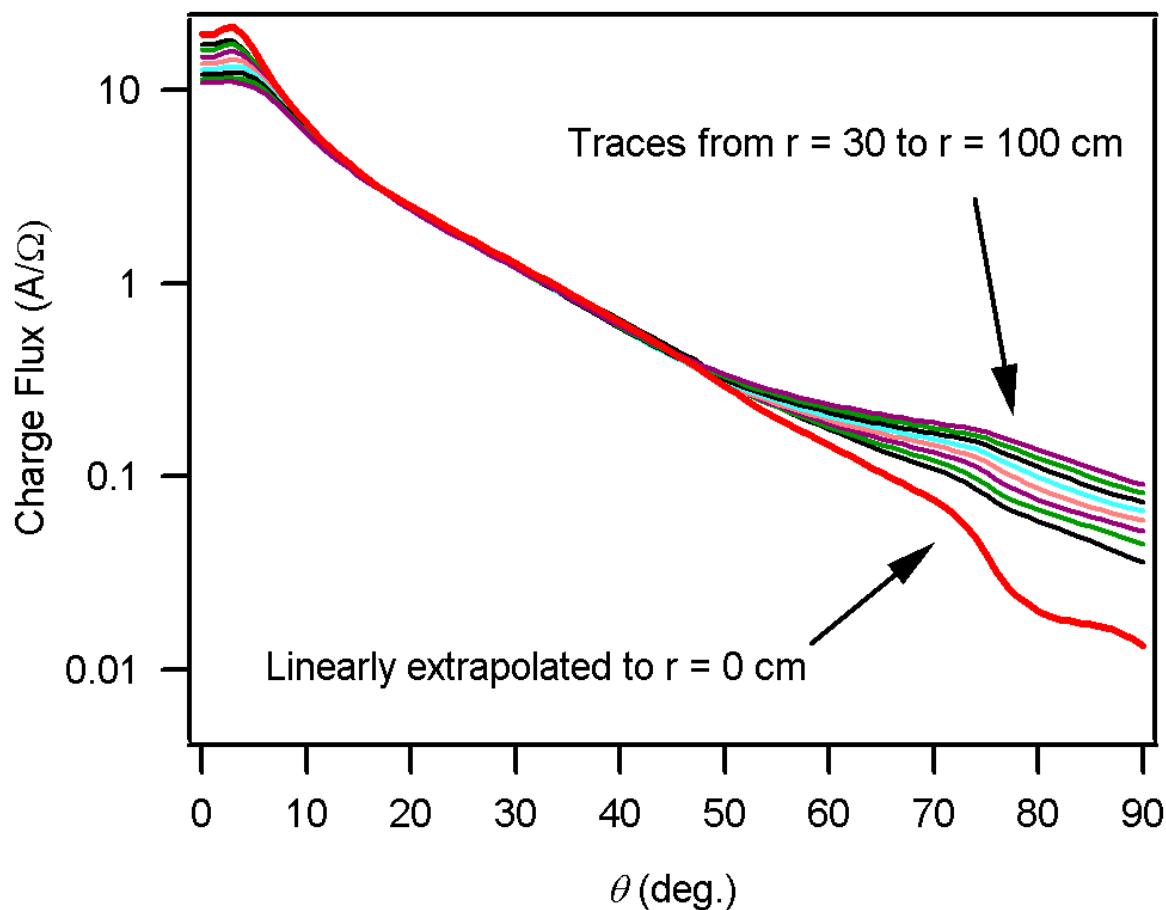


Probe Dimensions



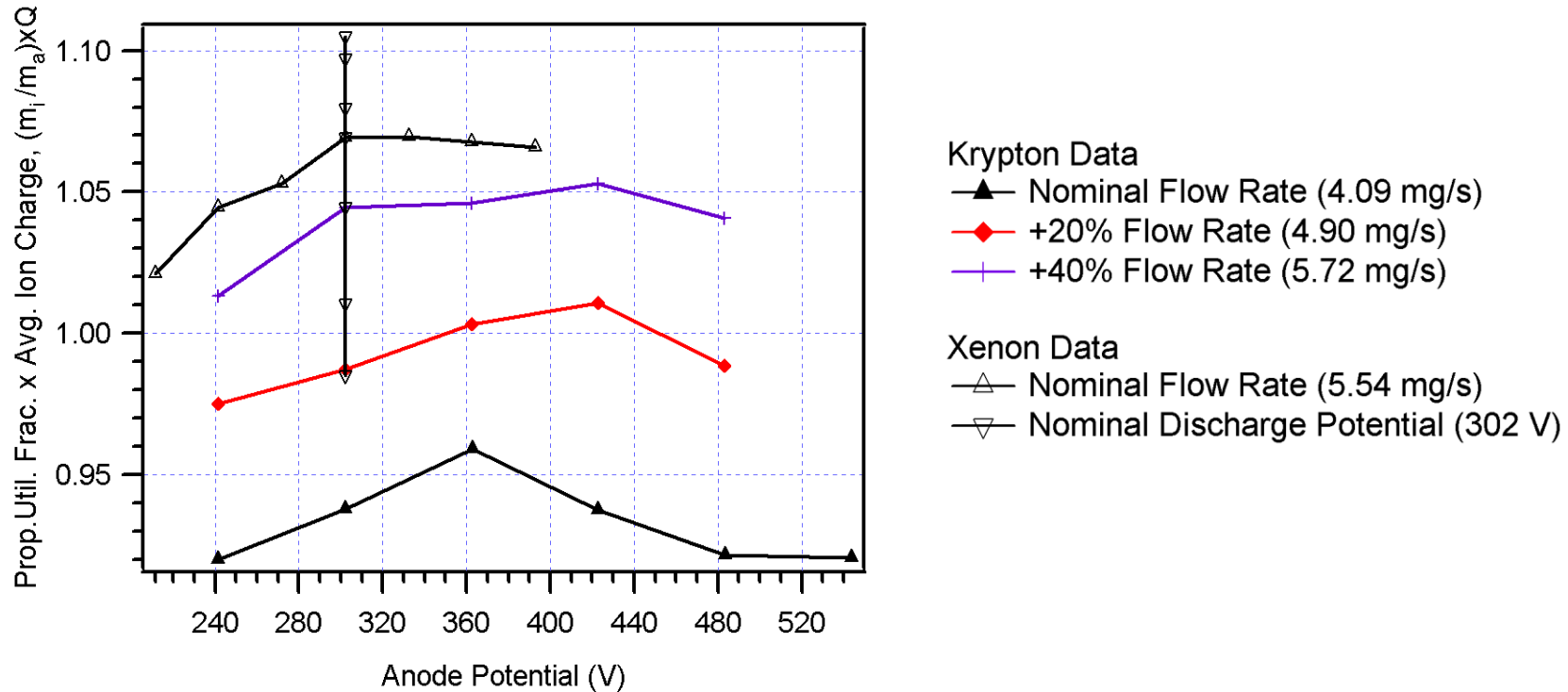
Faraday Probe: Chamber Pressure Correction

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Propellant Utilization Fraction



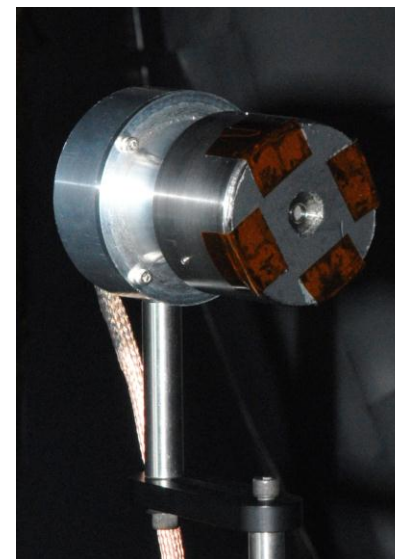
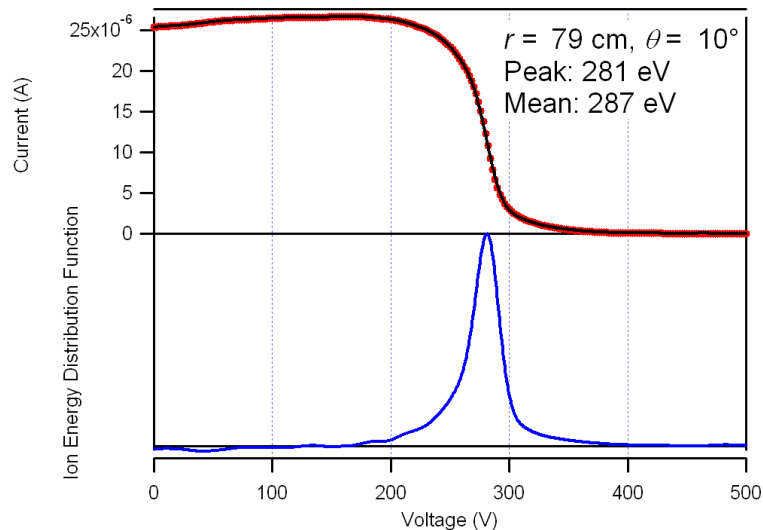
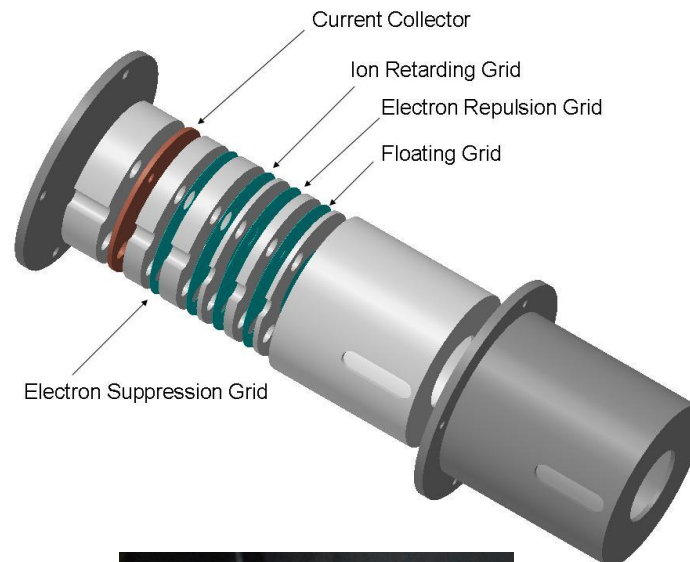
- Ion charge species fractions unknown
- Data suggests (m_i/m_a) significantly lower for Kr
- Increasing flow rate for Kr seems to improve propellant utilization



Retarding Potential Analyzer (RPA)

SPACE SYSTEMS
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- Energy filtered Faraday probe
- Four grid design
- I-V traces fit with smoothing spline
- Differentiated to calculate Energy-per-charge Distribution Function (EDF)





Methodology: Thrust

$$T = \dot{m} \langle \bar{v} \rangle_m \langle \cos(\theta) \rangle_{mv}$$

$$= \pi r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dot{m}(\theta) \bar{v}(\theta) \cos(\theta) |\sin(\theta)| d\theta$$

$$\bar{v} = \frac{\int f(v) v dv}{\int f(v) dv}$$

$$\langle \bar{v} \rangle_m = \frac{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) \bar{v}(\theta) |\sin(\theta)| d\theta}{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) |\sin(\theta)| d\theta}$$

$$\langle \cos(\theta) \rangle_{mv} = \frac{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) \bar{v}(\theta) \cos(\theta) |\sin(\theta)| d\theta}{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) \bar{v}(\theta) |\sin(\theta)| d\theta}$$



Methodology: Anode Efficiency

$$\begin{aligned}\eta_a &= \frac{P_{thrust}}{P_{input}} = \frac{\frac{1}{2}\dot{m}T^2}{P_d} = \frac{\frac{1}{2}\dot{m}(\dot{m} \langle \cos(\theta) \rangle_{mv} \langle \bar{v} \rangle_m)^2}{P_d} \\&= \frac{\frac{1}{2}\dot{m} \langle \cos(\theta) \rangle_{mv}^2 \langle \bar{v} \rangle_m^2}{P_d} \cdot \frac{\langle \bar{v}^2 \rangle_m}{\langle \bar{v}^2 \rangle_m} \\&= \frac{\frac{1}{2}\dot{m} \langle \bar{v}^2 \rangle_m}{P_d} \cdot \langle \cos(\theta) \rangle_{mv}^2 \cdot \frac{\langle \bar{v} \rangle_m^2}{\langle \bar{v}^2 \rangle_m} \\&= \frac{I_b \Delta V}{I_d V_d} \cdot \Psi_B \cdot \Phi_P \\&= \eta_E \cdot \Psi_B \cdot \Phi_P\end{aligned}$$

$$\langle \bar{v}^2 \rangle_m = \frac{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) \bar{v}^2(\theta) |\sin(\theta)| d\theta}{\pi r^2 \int_{-\pi/2}^{\pi/2} \dot{m}(\theta) |\sin(\theta)| d\theta}$$



Charge Flux Weighting Approximations

$$j(\theta) \approx \frac{I(\theta)}{A_{probe}} = \dot{m}_i(\theta) \frac{N_A Q(\theta)}{m A_{probe}} = C Q(\theta) \dot{m}_i(\theta)$$

- To approximate momentum weighting (beam efficiency):

$$\dot{m}(\theta) \bar{v}(\theta) \propto j(\theta)$$

Must assume $Q(\theta)$ AND $\bar{v}(\theta)$ are constant

- To approximate mass weighting (voltage util. efficiency):

$$\dot{m}(\theta) \propto j(\theta)$$

Only need to assume $Q(\theta)$ is constant