

**Optimization of Purchase, Storage and Transmission Contracts for Natural Gas Utilities**



William Avery; Gerald G. Brown; John A. Rosenkranz; R. Kevin Wood

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## OPTIMIZATION OF PURCHASE, STORAGE AND TRANSMISSION CONTRACTS FOR NATURAL GAS UTILITIES

**WILLIAM AVERY**

*Life Fitness Limited, Franklin Park, Illinois*

**GERALD G. BROWN**

*Naval Postgraduate School, Monterey, California*

**JOHN A. ROSENKRANZ**

*Boston Energy Associates, Boston, Massachusetts*

**R. KEVIN WOOD**

*Naval Postgraduate School, Monterey, California*

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Natural gas utilities supply about a quarter of the energy needs of the United States. From wellhead to consumer, operations are governed by an astounding diversity of purchase, transport, and storage contract agreements which prepare a complex physical distribution system to meet future demands no more predictable than next year's weather. We present a decision support system based on a highly detailed optimization model used by utilities to plan operations which minimize cost while satisfying regulatory agencies. Applications at Southwest Gas Corporation are presented along with a case study at Questar Pipeline Corporation.

"But thou, contracted to thine own bright eyes,  
Feed'st thy light's flame with self-substantial fuel"  
*William Shakespeare, First Sonnet*

Natural gas utilities purchase gas from suppliers, arrange for its transport and storage, and deliver it to most end users on an as-needed basis. Gas is distributed from the wellhead to consumers in extensive systems of pipelines and storage facilities. The pipelines can be utilized at relatively high capacity even during low demand periods because gas can be injected into underground storage facilities or liquefied and stored in tanks for periods of high demand. (Some gas is stored in above-ground gas holders, but these are minor volumes used to buffer hour-to-hour load swings.)

In 1988, the United States used about 19 quadrillion British Thermal Units ( $1.9 \times 10^{16}$  BTUs<sup>1</sup>) of natural gas, almost a quarter of the total energy consumption in the country (American Gas Association 1989b).

Natural gas utilities supplied 48 million residential customers, as well as 5 million commercial, industrial and other end users. The average cost to end users was about \$5 per MMBTU—10% less than oil, and less than a quarter the cost of electricity. Most of this natural gas was produced domestically. Natural gas produces energy—by combustion, or via catalysis and indirect chemical conversion—with less undesirable by-products than competing conventional fuels: carbon dioxide, nitrous oxides, sulfur oxides, and complex hydrocarbon byproducts are minimized (Burnett and Ban 1989).

Structural and regulatory changes in the natural gas industry have increased the importance of systematic analysis of supply options by natural gas purchasers. As a result of deregulation, gas purchasers, including

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gas pipeline companies, local distribution companies (LDCs or utilities), and large end users are able to take advantage of new supply alternatives. Developing an optimal gas supply strategy in this new market requires the evaluation of a wide range of purchase, transportation and gas storage options over a multiyear time horizon (Stewart 1987, American Gas Association 1989a).

We describe a decision support system called Gas Contract Analyzer which employs a linear programming model to assist gas supply planners in making both strategic and operational gas supply and transportation decisions. This system has been used successfully to develop both short-term and long-term gas supply strategies by a number of pipeline and natural gas distribution companies.

## 1. NATURAL GAS INDUSTRY STRUCTURE

The natural gas industry consists of three distinct segments:

*Producers*, both major oil companies and smaller independents, are primarily involved in exploration for, development of, and production from gas reserves; *gas pipeline companies* aggregate gas in areas of production and transport the gas to areas of consumption; *LDCs* provide natural gas service to end users in a particular geographic region.

Despite recent moves toward deregulation, government regulatory bodies still have a great deal to say about how natural gas is bought and sold. Although production itself is not regulated, producer sales of natural gas have been subject to federal price regulation since 1954. The activities of interstate gas pipeline companies are regulated by the Federal Energy Regulatory Commission (FERC) under the Natural Gas Act of 1938. Intrastate pipelines and LDCs come under the regulatory authority of state and local public utility commissions.

LDCs have traditionally purchased the majority of their gas supplies from interstate pipeline companies under long-term contracts. Contract terms can extend twenty years or longer. These contracts allow the LDC to purchase up to a specified contract demand quantity each day, and up to a maximum seasonal quantity of gas each year. Under these contracts the LDC pays a monthly demand charge based on the daily and seasonal contract quantities, and a commodity charge for each unit of gas purchased. Sales rates and terms of service are established in pipeline tariffs approved by the FERC.

An LDC, as a public utility company, has a legal obligation to provide natural gas service to residential, commercial, and industrial customers within its service territory. Service to larger commercial and industrial customers is provided on either a *firm* or *interruptible* basis. Interruptible sales customers generally have the capability of switching on short notice between natural gas and an alternate source of fuel, such as propane or fuel oil. In exchange for a lower delivered price, interruptible customers face the risk that gas supplies will be curtailed during periods of peak demand.

LDCs meet peak demand requirements through a combination of contract purchases and withdrawals from storage. Gas is stored in underground or liquefied natural gas facilities operated by the LDC, or through storage service contracts with pipeline suppliers. In either case, the LDC incurs a cost related to the amount of storage capacity available, a cost related to the maximum quantity that can be withdrawn from storage on a given day, and a cost for each unit of gas which is injected into or withdrawn from the storage facility.

## 2. OPEN ACCESS

The FERC has implemented a series of regulatory changes (beginning with Order 436 in 1985) to encourage pipeline companies to unbundle their traditional merchant services and allow buyers to transport their own gas through the pipeline system. This open access to gas transportation services gives gas purchasers the opportunity to buy gas directly from nonpipeline sources, including natural gas producers and independent marketing companies, and has contributed to the development of an active spot market for natural gas.

As part of this market restructuring, pipeline customers have been given the opportunity to convert all or part of their existing gas purchase entitlements to firm transportation service. A firm transportation contract allows the shipper to reserve a portion of the pipeline's total delivery capacity for his own use. The shipper pays a monthly demand charge based on the maximum daily delivery quantity contracted for, and a transportation charge for each unit of gas delivered. Additional pipeline transportation service is available on an interruptible basis. For interruptible transportation services the shipper generally pays only for the gas transported.

Open access has had perverse consequences for some pipeline companies and LDCs. Major industrial

customers of an LDC have traditionally contracted for inexpensive interruptible gas from the LDC which obtains the gas from some pipeline company. In the past, interrupting these customers has provided the LDC with a handy method of leveling demand peaks. Under open access, however, many of these customers have contracted directly with a producer to supply gas, with a pipeline company being obligated to provide interruptible transportation. Typically, the LDC will have concurrent sales agreements with the industrial customers which stipulate that if the producer has trouble meeting its commitments, the LDC will make up the deficit at "best efforts." Thus, the industrial customers have become a part of the peak-load problem rather than contributing to its solution and, in fact, in the last few years the proportion of interruptible demand as a fraction of total demand has decreased by one third.

Finally, because gas buyers are now able to swing from one supplier to another based on price, many gas contracts include pricing provisions to encourage buyers to maintain high purchase load factors. At the pipeline level, many companies have proposed new gas inventory charges. These charges are structured either as a demand charge based on the buyer's gas purchase entitlement, or as a penalty charge for each unit that total monthly or seasonal purchases fall short of a contractual target quantity. Direct sales contracts arranged with producers or independent marketers often contain similar provisions. In the extreme, the buyer is required to pay for a specified quantity of gas whether it is taken or not.

### 3. GAS SUPPLY OBJECTIVES

Purchased gas costs can account for 60% or more of the total costs of an LDC. The objective of gas supply planning is to minimize gas supply costs while maintaining sufficient supply to meet potential peak requirements and provide for future growth in demand.

Over the short term, this means optimally dispatching the available gas supply to meet variable demand. A gas dispatch plan reconciles gas purchases, storage inventory targets for key points in the injection/withdrawal cycle, and the expected level of firm and interruptible sales.

Over the long term, the objective of supply planning is to construct an optimal portfolio of gas sources including gas purchases from pipelines, storage, and transportation of gas purchased directly from the producer. As the menu of gas supply options has increased, the problem of sorting through these

options to develop a coherent gas supply strategy has become much more complex. Among the strategic issues that supply planners must now address are the following:

#### Contract Restructuring

- How much of the company's existing pipeline purchase entitlement should be converted to firm transportation?
- Should the overall entitlement level be reduced or increased?

#### Direct Purchases

- How much gas should be purchased from nonpipeline suppliers under long-term contracts?
- What role should short-term spot purchases play in the company's supply portfolio?

#### Storage

- Should the company develop or lease additional storage capacity?
- What storage capacity and deliverability levels are best?

#### Marketing

- What types of gas suppliers should support each customer market segment?
- How much gas should the company make available for interruptible sales?

## 4. PROBLEM CONSTRUCTS

We seek a flexible optimization system designed to address both contract restructuring decisions and the optimal utilization of gas supply sources. The key elements of the optimization problem are outlined below.

#### Time Periods

The dispatch periods can be daily, weekly, monthly, or any aggregation of these. The planning horizon can range from one year to over a decade.

#### Gas Contracts

All sources of gas are modeled as contracts. The model sets dispatch flow for all contracts, each of which is assumed to have its own *commodity rate*, i.e., a cost proportional to the amount of gas purchased. In addition, the model determines optimal maximum daily and maximum seasonal purchase levels within predefined upper and lower bounds. For potential contracts (as opposed to existing contracts), a *D1 charge* is incurred proportional to the maximum daily

purchase level and a *D2 charge* is similarly incurred for the maximum seasonal purchase. Both existing and potential contracts can also be assigned a minimum seasonal purchase quantity, and a penalty rate, called a *deficiency-based gas inventory charge*, to be applied to any purchase shortfall. Groups of contracts may share joint daily and seasonal purchase limits.

### Storage

Existing gas storage facilities or potential storage service contracts have a maximum daily injection volume, a maximum daily withdrawal volume, and a maximum storage capacity. For potential contracts, within preset lower and upper bounds, the maximum daily withdrawal level can be determined taking into account the *storage service demand charge* and the maximum storage capacity can be determined taking into account the *storage service capacity charge*. Optional storage relationships can be used to control the rate of withdrawals and/or injections as a function of gas inventory. These relationships reflect the fact that the higher inventory is the easier it is to withdraw, but the harder it is to inject.

### Gas Transportation

The dispatch of gas from sources to dispositions can be characterized by defining a network of nodes and arcs. Each arc can be given minimum and maximum daily flow values, a charge per unit of throughput called the *transportation rate*, and a loss percentage on each unit transported. For potential transportation contracts, the maximum throughput capacity is set for both firm and interruptible transportation service based on costs called the *firm demand rate* and the *interruptible demand rate*, respectively.

### Side Constraints

Additional dispatch constraints can be defined linking flows across network arcs and across time periods. This feature is used to model contractual limitations on the sources used to inject into storage, and to segregate transportation for end users from system supply volumes in a commingled gas flow.

### Peak-Day Constructs

The model determines optimal gas flows for each dispatch period. In addition, the sizing of supply, storage, and transportation must satisfy a parallel system of peak-day constraints. The peak-day constraints require that total system deliverability, adjusted for the expected peak-day reliability of existing and potential gas sources, must be sufficient to meet peak-day

demands. Separate peak-day constraints can be specified for any aggregate time period, such as a month.

The parallel peak-day submodel provides a managerially appealing mechanism to account for future demands which cannot be forecasted. Reliability factors express in common parlance the probability that agreed limits will actually be met under peak conditions: On a peak day some contractual partners are more trustworthy than others, and some contracts are easier to fulfill than others. Using the reliability factor, such quantities as maximum daily purchase for a pipeline contract are replaced by their expected value on the peak day. More elegant stochastic programming methods might require more data than can be certified comfortably and typically yield optimization problems which are much larger and much harder to solve than our corresponding deterministic problems.

### Objective Function

The present value of all costs is minimized. This includes variable costs on all gas sources, storages, and transportation arcs; committed charges on all contracts, storages, and arcs for which a sizing decision is made; and penalty costs associated with failure to meet gas deliverability requirements and minimum purchase levels. A slight overcounting occurs in the objective function because peak-day variables also have costs associated with them. However, such costs are necessary to yield sensible peak-day flow patterns; leaving peak-day costs at zero could lead to peak-day flows from the most distant, most expensive supplier with the greatest amount of transmission loss.

We assume, for the sake of tractability, that all costs are linear; we ignore the fact that nonlinear costs can arise when constructing storage facilities and when purchasing gas. In the first case, the modeler will normally be considering discrete options, such as opening two caverns of a salt dome, or four, or maybe six. Typically, the different options will be modeled as existing storages in separate scenarios rather than modeling the options in a single scenario as a storage service contract. Thus, construction costs are computed completely outside of the model. The second case occurs when there are quantity discounts for gas purchase contracts. In this case, the modeler would normally apply the lowest price and determine if the model purchases sufficient gas to achieve that discounted price. If it does, the solution is optimal. If not, the next higher price could be used and the model rerun to see if the appropriate level of purchases is made, etc. While optimality cannot be guaranteed, this technique seems to work well in practice and we have not felt compelled to introduce the binary

variables and additional constraints necessary to model these quantity discounts exactly.

## 5. MODEL CONSTRUCTS

Our model of gas transmission by pipeline and distribution by LDCs is deceptively simple to visualize. Superficially, a single commodity, base-demand natural gas, is purchased at supply points, held in storage facilities, and transported through a pipeline network to meet the demands of different customer classes over many time periods. Losses resulting from consumption of gas in compressors and other line losses are modeled both on transportation links and in storage facilities.

However, this simple single-commodity generalized network flow model of base demand fails to capture a host of essential details. For instance, providing for peak-day capacity requires a coupled, parallel model for meeting extreme demands, incorporating “peak-day gas” flowing through a “peak-day network.” In addition, contract stipulations may require that the gas provided be treated distinctly from other gas. Variables and constraints related to setting the size and other parameters of purchase, storage and transmission contracts complicate the model further. Thus, the resulting model has multiple commodities, complicating constraints and variables, and bears a distant resemblance, at best, to a simple generalized network flow model.

Incorporation of all these complicating features into a single, unified optimization model (linear program) is a distinguishing feature of the work reported here. The objective is to minimize cost, while meeting all firm demands and interruptible demands with best efforts. We assume that all data are deterministic although we discuss how uncertainty in demand, availability and price is incorporated in Section 6.

The model consists of a template of numerous standard constructs which may be selected, specified, and connected in various ways to represent a gas supply system. Below, we describe these building blocks, first those associated with the period-by-period aspects of the model, and then those associated with the peak-day aspects. The parenthetical mnemonics key and a complete mathematical formulation are given in the Appendix.

### Period-By-Period Constructs

**Spot Purchase Node.** A spot purchase node is a simple supply node with upper and lower limits on the amount of gas which can be purchased in any period

as specified by maximum and minimum daily purchase levels (*MXMP*). A cost proportional to the amount purchased is incurred.

**Existing Pipeline Contract Node.** In addition to the features of a spot purchase node, over a specified season (*MXMP*), an upper bound on total purchases for a season, i.e., a maximum seasonal purchase, is specified together with a fraction of the maximum seasonal purchase which must be purchased (*MXSP*). This minimum purchase may be violated at a specified cost per unit of violation.

**Potential Pipeline Contract Node.** A pipeline contract may be fixed for the next few years, and then be subject to renegotiation. During the fixed period, its specification is identical to that of an existing pipeline contract node. After that time, the following characteristics associated with a potential contract are determined:

- A maximum daily purchase level is chosen and enforced (*MXMM*) and a cost proportional to the level chosen is incurred.
- A fixed minimum daily purchase may be specified (*MNMP*).
- A maximum seasonal purchase level is chosen and enforced (*SEAS*) and a cost proportional to the level chosen is incurred.
- Also, a minimum seasonal purchase may be specified as a given fraction of the maximum seasonal purchase level being chosen (*MNPF*). This constraint may be violated at a specified cost per unit of violation.
- The maximum daily purchase and the maximum seasonal purchase may be related to each other (*DSLO* and *DSUP*).

**Parent-Children Pipeline Contract Nodes.** In addition to the above single-node contracts, it is possible to specify contracts which cover several nodes in a parent-to-children relationship. In addition to the constraints associated with the individual nodes, the following constraints may be specified:

- A joint maximum daily purchase level (over the parent and its children) is chosen and enforced (*JMXXM*), while a cost proportional to that level is incurred.
- A joint maximum seasonal purchase level is chosen and enforced (*JSEAS*), while a cost proportional to that level is incurred.
- A maximum daily purchase level over the children in the contract is enforced as a fraction of the joint maximum daily purchase (*JCPM*).

- The joint maximum daily purchase and the joint maximum seasonal purchase may be related to each other (*JDSL* and *JDSU*).

**Existing Storage Node.** This is structurally modeled as an injection node, an inventory node and a withdrawal node with bypasses allowed, i.e., gas may be transferred through the storage location without actually entering and leaving storage. Loss fractions for both injections and withdrawals may be specified along with minimum and maximum levels for these quantities.

Withdrawals in any period may be limited to a fraction of the total available inventory plus an offset. This approximates the phenomenon that withdrawals from inventory become more difficult as inventory levels drop (*WINV*).

Injections in any period may be limited by an inverse function of the inventory. This approximates the phenomenon that injections into inventory become more difficult as inventory levels rise (*IJIV*).

Flow into the “injection node” equals the amount of gas injected into storage plus the amount bypassed (*INPT*). A cost proportional to the amount injected is incurred.

Inventory during any time period equals total injections less total withdrawals up to, but not including, that period, and is limited by total inventory capacity. A minimum inventory target level may be specified with a per-unit penalty for violating the target (*STOR*).

The amount of gas leaving a storage facility equals withdrawals plus gas which bypasses actual storage (*OUTP*). A cost proportional to the amount withdrawn is incurred.

**Potential Storage Node.** A potential storage node has the same physical structure as an existing storage node and the same constraints dealing with maximum daily withdrawals (*WINV*), maximum daily injections (*IJIV*), input (*INPT*), and output (*OUTP*).

In contrast, the maximum inventory level is not fixed but is chosen by the model within specified limits (*STOR*). A cost proportional to this level is incurred.

A separate constraint enforces inventory targets if desired (*INVT*).

The maximum daily withdrawal is also a variable which must be selected by the model within specified limits (*VMDW*). A cost proportional to the maximum daily withdrawal is incurred.

**Junction Node.** A junction node requires that inflow equal outflow (*JUNC*).

**Demand Node.** A demand node simply requires that inflow less outflow must equal demand, except that demand may be shortened at a specified cost per unit (*DMDM*).

**Existing Transportation Arc.** This is simply a link connecting two nodes in the network with minimum and maximum daily throughput constraints. A loss factor may also be specified along with a cost per unit of gas transported.

**Potential Transportation Arc.** Two parallel links are used to model this situation, representing firm and interruptible transportation. These links have the properties of existing transportation arcs along with the following additional properties:

- The total flow over the two links is limited by a joint capacity constraint (*JCAP*).
- A maximum daily throughput is chosen and enforced individually for both firm and interruptible transportation (*MXMF* and *MXMI*). Separate costs, proportional to the maximum throughputs are incurred.
- Interruptible transportation may be reduced by a multiplier ranging from 0 to 1 in periods when it is partially or completely unavailable.

**Side Constraints.** The flow on any number of transportation arcs may be linked together over a specified set of time periods (*ALNK*).

### Peak-Day Constructs

The parallel peak-day submodel ensures that demands can be met on a peak day as well as on an average day. This submodel dispatches gas on the peak day not in addition to the period-by-period requirements but subject to the contract constraints described above. Additionally, the withdrawals made from storage on the peak day are limited by the inventory levels obtained at the end of the period. Peak-day constructs are automatically generated when their parallel non-peak constructs are selected, although the peak-day constructs may be limited to specific periods when peaking capabilities are important, i.e., during the winter. The individual constructs are described below.

**Peak-Day Spot Purchase Node.** Total purchases are limited by minimum and maximum daily purchase levels possibly adjusted downward by a “reliability factor” (*PMDP*). A cost proportional to the amount purchased is incurred.

**Peak-Day Existing Pipeline Contract Node.** This is the same as a peak-day spot purchase node (*PMDP*).

**Peak-Day Potential Pipeline Contract Node.** Peak purchases are limited by the maximum daily purchase level chosen, possibly adjusted downward by a reliability factor (*PMMD*). A cost proportional to the amount purchased is incurred.

The peak purchases must exceed the minimum daily purchase requirement (*PMND*).

For a joint (parent-children) contract, the total purchases may not exceed the joint maximum daily purchase level chosen, possibly adjusted downward by a reliability factor (*PJMM*).

For a joint contract, the total purchases from the children may not exceed a given fraction of the joint maximum daily purchase level chosen (*PJCC*).

**Peak-Day Existing Storage Node.** Unlike the non-peak portion of the model, a peak-day existing storage node is structurally a single node.

Outflow from the node equals inflow plus withdrawals (*PSTR*).

Withdrawals are limited to a fraction of the period-ending inventory plus an offset (*PINV*). A cost proportional to the withdrawal is incurred.

In no case may withdrawals exceed inventory (*PTIV*) or the maximum daily withdrawal which is possibly adjusted downward by a reliability factor.

**Peak-Day Potential Storage Node.** A potential storage node has the constraints (*PSTR*), (*PINV*) and (*PTIV*) as in an existing storage node. However, the fixed upper bound on withdrawals is replaced by a variable maximum daily withdrawal possibly adjusted downward by a reliability factor (*PVMW*).

**Peak-Day Junction Node.** Flows into the node equal flows out of the node on the peak day (*PJNC*).

**Peak-Day Demand Node.** Demand on the peak day equals gas flowing into the node less gas flowing out of the node. Demand can be shorted at a specified cost per unit (*PDMD*).

**Peak-Day Existing Transportation Arc.** This is simply a single link connecting two nodes in the peak-day submodel. Flow is limited by the minimum daily throughput and the maximum daily throughput, possibly adjusted downward by a reliability factor. A cost per unit of gas transported is incurred.

**Peak-Day Potential Transportation Arc.** This is represented as two links, in parallel, connecting two nodes in the peak-day portion of the model. Costs proportional to the amount of flow on each link are incurred.

The sum of the flow on the links is limited by their joint capacity (*PTCP*).

The peak-day flow on the firm transportation link is limited by the maximum daily (firm) throughput chosen by the model. This may be adjusted downward by a reliability factor (*PMXF*).

The peak-day flow on the interruptible transportation link is limited by the maximum daily (interruptible) throughput chosen by the model. This may be adjusted downward by a reliability factor which is distinct from the "firm" reliability factor (*PMXI*).

## 6. DEALING WITH UNCERTAINTY

While Contract Analyzer handles variability of demand and peak-day surge it is essentially a deterministic model. It is meant to be used for analyzing a number of future demand/price scenarios posited by an LDC. Typically, a number of scenarios are developed and run under Contract Analyzer and solutions are compared. If there is little difference among the scenarios in, say, the purchase level of a particular contract, the analyst can be fairly certain of the correct purchase level. If instead, significant variability arises, the analyst can determine a composite purchasing strategy, fix it and then run the various scenarios against that strategy to see how solutions vary. Sensitivity analyses on the strategy can then be carried out if desired. (See Wagner 1969, Chapter 16 for a classic discussion of this type of analysis.)

While it would be possible to create a two- or multistage stochastic programming model with recourse (e.g., Walkup and Wets 1967) which would simultaneously incorporate different scenarios, the size of such a problem (it would grow at least linearly with the number of scenarios included) would make its solution very difficult. Scenario analysis gives good results in practice and its use is well accepted by LDCs and by their regulators.

Note that a stochastic model, called the "Contract Mix Model" (Fancher, Wilson and Mueller 1985) can be used for analysis of contract purchases under uncertain demand and prices. While this model may be useful in some circumstances it has a number of severe limitations. In particular, the model does not consider storage and transportation contracts, it cannot model a gas distribution network, it works only at a yearly level of detail, and requires strong assumptions about the independence of demands and prices.

To develop scenarios for analysis, LDCs must try to predict likely levels of future demand and costs. Predicting demand for natural gas is neither easier nor

more reliable than long-range climatological and econometric forecasts: Demand is dominated by domestic space heating and industrial applications. However, utilities normally have available climatological data for a past "normal" year, a cold year and a very cold year. Using these data, along with historical demand for those years and the best econometric models available, an LDC can develop reasonable scenarios for future demands.

Price scenarios are more difficult to develop in a logical fashion. Natural gas prices in the future will depend on demand, gas availability, availability and price of alternate energy sources, and politics. LDCs have used statistical forecasting models for predicting future prices, but these are usually tempered by managerial judgment to consider such things as spikes in prices caused by unforecastable economic and political events. At least one LDC is planning on developing price scenarios using a Delphi method to incorporate the judgment of its best analysts and managers.

## 7. DECISION SUPPORT SYSTEM SHELL

Our system provides a user-friendly interface for the optimization model. Data entry, scenario management, report writing, and ad hoc analysis of results are supported via full-screen interfaces and command menus.

Data entry is supported by full-screen templates which prompt the required data and automatically perform range and consistency checks. This reduces false starts when data are being debugged. Data are organized in a compact form so that changes can be made without extensive regeneration.

Scenarios are generated by pointing to, rather than copying data. Thus, alternate excursions are easy to assemble via command macros to set conditions for each run. By contrast, traditional data management schemes generate multiple copies of input data sets, one for each scenario. Each of the data sets must be individually edited, validated, and debugged making coordination of multiple scenarios difficult. For instance, if one fundamental constant is changed, traditional systems require the editing of the data for each of the multiple scenarios. An additional advantage of our system is that the user is not expected to master editors, job control language or other system-specific details. In fact, the user sees very few differences between implementations on different computers or operating systems.

Report writing is easy. Standard reports display most of the interesting details. Report modifications

are made with simple English commands to format output displays. These commands are interpreted and then applied to a standard output data template to render the required report.

## 8. APPLICATIONS AT SOUTHWEST GAS CORPORATION

To date, Gas Contract Analyzer has been installed at eight LDCs around the country. The use of this model at Southwest Gas Corporation is typical.

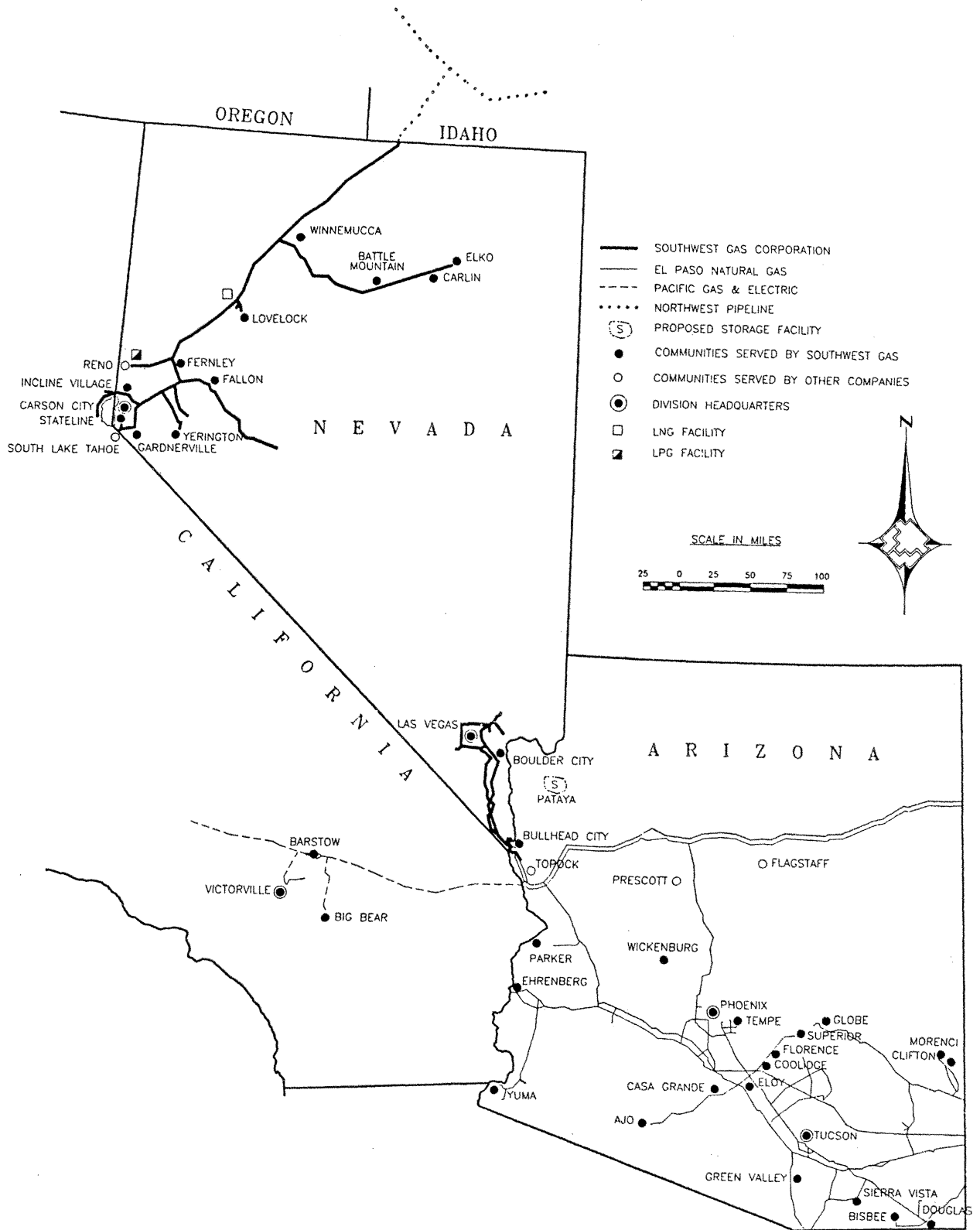
Southwest Gas Corporation operates gas transmission and distribution facilities in the states of Arizona, Nevada, and California. In 1989, Southwest Gas delivered 133,000 MMCF to 797,000 customers. Sales comprised 68% of this volume, while transportation of customer-owned gas comprised the remaining 32%. Revenues from gas operations totaled \$570 million.

Southwest's gas distribution system, depicted in Figure 1, is divided into the Northern System, covering northern Nevada and a small portion of California, and the Southern System, which serves southern Nevada, Arizona, and extreme southeastern California. Each system is connected to a single interstate pipeline. Northwest Pipeline System serves the Northern distribution system through Southwest's interstate gas transmission subsidiary, Paiute Pipeline Company. El Paso Natural Gas (EPNG) serves the Southern System. Peak deliverability on the Northern System is supplemented by a liquefied natural gas storage facility and a liquid petroleum gas plant. There is no connection between the two Southwest Gas distribution systems at the present time.

With open access transportation, Southwest Gas has been able to purchase an increasing portion of its gas from nonpipeline sources. Early in 1989, Southwest Gas converted 100% of its sales entitlement on Northwest Pipeline to firm transportation. Spot market purchases now account for approximately 60% of total gas purchases system-wide.

Southwest Gas is currently restructuring its gas supply arrangements for the Southern System. The gas supply options the company is considering include:

**New EPNG Sales Service.** EPNG has proposed a new sales service that includes a monthly demand charge, a commodity charge based on the quantity of gas purchased, and a gas inventory charge (GIC). A GIC penalty could be assessed whenever gas purchases fall below a target level tied to the buyer's maximum daily purchase entitlement. Southwest Gas can choose a new daily purchase entitlement between zero and its



**Figure 1.** Southwest Gas Corporation's major facilities and transmission lines. Also shown are the principal communities where the corporation supplies gas either as a wholesaler or distributor.

current entitlement level. FERC action is required on EPNG's tariff proposal before the exact provisions of this service will be known.

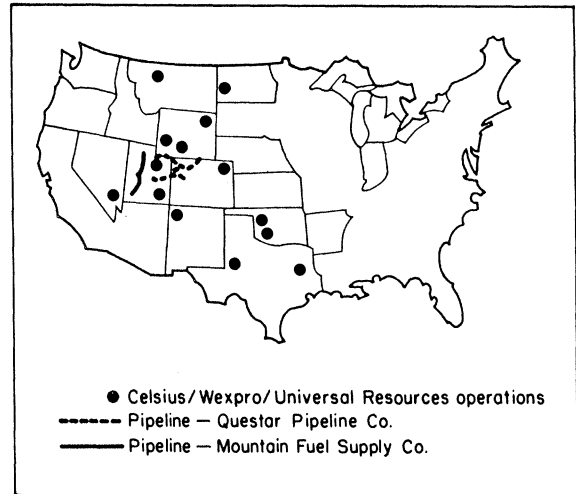
**Long-Term Direct Purchases.** Southwest Gas has received proposals from gas producers for long-term gas sales over periods of up to five years. Contract provisions typically include a commodity rate tied to an index of spot market prices, and either a monthly demand charge or a minimum purchase requirement with some form of deficiency penalty. Gas purchased directly from producers would be transported under firm transportation arrangements with EPNG.

**Pataya Storage.** Southwest Gas is considering development of a new gas storage field in northwest Arizona. Storage would allow Southwest Gas to reduce its peak-day entitlements with suppliers (and associated demand charges) and purchase additional spot gas in the off-peak season. The storage injection, inventory capacity and withdrawal sizing have been optimized with the model under various postulated future market conditions. The results indicate that building the storage could save the company's customers \$5 to \$40 million per year. The company is closely monitoring market conditions which affect this analysis and when forecasts firm up the company will decide whether or not to go ahead with construction. If the decision is to build, the model will be used to determine optimum injection, size and withdrawal specifications. The Pataya storage could become available as early as 1993.

The Southwest Gas Supply Planning Group is currently testing both near-term and long-term supply options for the Southern System. One analyst is employed full-time to make model runs and interpret results. To date, over 50 gas contract options and combinations of options have been tested over one, three, and five-year timeframes. Promising supply alternatives are then tested against a range of forecasts for gas sales requirements and spot price conditions. A typical five-year scenario involves 4,600 constraints, 12,900 variables and 61,000 nonzero coefficients. Optimization of each scenario takes one to ten minutes of CPU time to run on an IBM 3090 operating under MVS/XA.

**9. A CASE STUDY AT QUESTAR CORPORATION**

Questar Corporation is a regional, integrated energy company headquartered in Salt Lake City, Utah

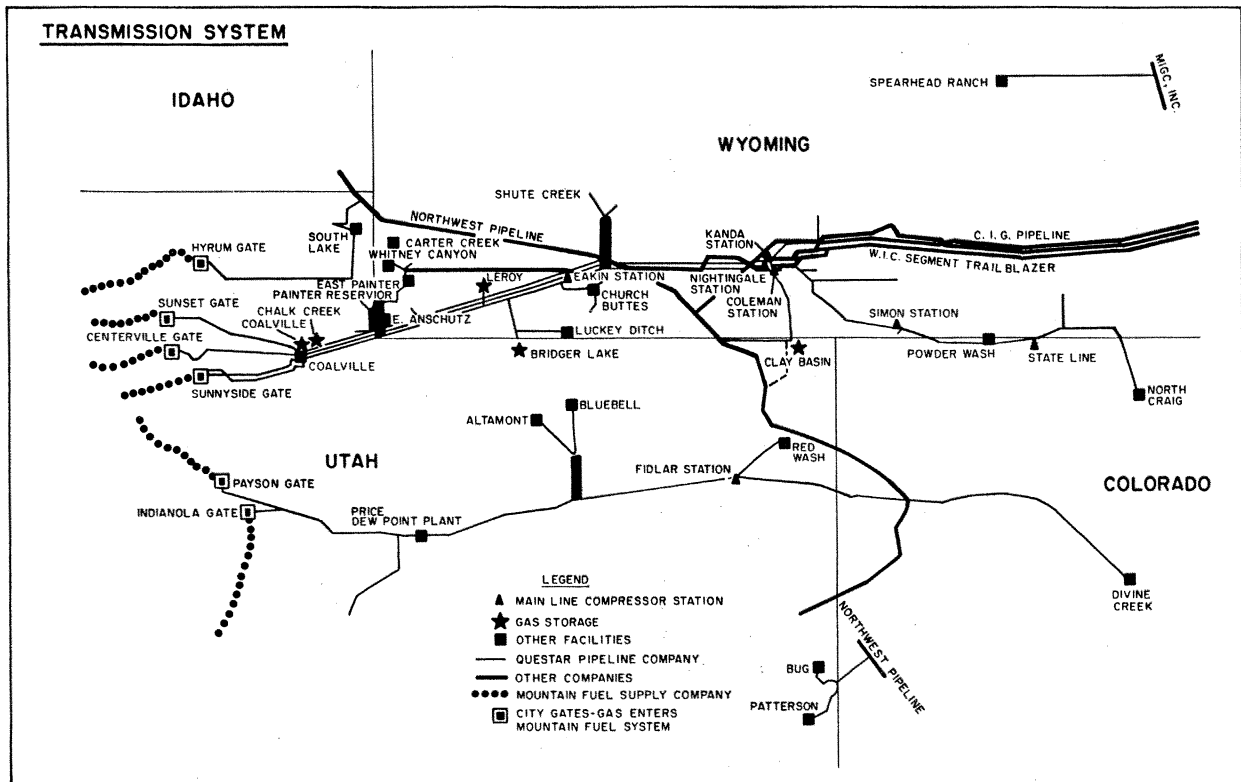


**Figure 2.** Questar oil and gas production areas. Much of the natural gas is transported by Questar Pipeline Company to Mountain Fuel Supply Company.

(Questar 1989). Questar engages, via subsidiaries Wexpro/Celsius, in oil and gas exploration and production throughout the Rocky Mountain region and elsewhere (see Figure 2). Questar Pipeline operates an interstate pipeline transmission system in Colorado, Wyoming and Utah, and owns underground storage facilities in Wyoming and Utah (see Figure 3). Questar Corporation also has a marketing affiliate, Questar Energy Company, which pursues unregulated gas acquisition, gathering, compression and sales. In addition, Questar operates an LDC, Mountain Fuel Supply Company, which provides retail gas distribution to about one-half million customers in Utah, southwestern Wyoming and southern Idaho. Thus, Questar engages in virtually all aspects of the natural gas industry.

Questar's three principal subsidiaries—engaged in exploration and production, storage, transmission and local distribution—have all been affected profoundly by deregulation. Wexpro wants to develop and produce more gas than it now contributes to Questar customers. Questar Pipeline seeks to provide reliable and profitable transmission and storage services, and Mountain Fuel, as an LDC, seeks to maximize return on stockholders' equity subject to existing contractual commitments with suppliers and subject to approval from regulatory agencies. These goals sometimes conflict.

A case study serves to illustrate some of the issues raised by deregulation. Major industrial customers



**Figure 3.** Questar Pipeline transmission system. Producers and other pipelines supply natural gas which is stored or distributed to the local distribution company, Mountain Fuel.

have traditionally contracted for inexpensive, interruptible gas from Mountain Fuel—interruptible customers provide a handy way to balance demand. However, under open access, most of these customers have been able to contract directly with gas suppliers other than Mountain Fuel, and have obtained interruptible transportation from Questar Pipeline and Mountain Fuel. If the other gas suppliers have trouble meeting their commitments to the industrial customers, Mountain Fuel has firm and interruptible concurrent sales agreements with many customers to make up the deficit.

The evolution from a single source of gas for end-use customers to a plethora of providers coupled with accessible transportation has had adverse consequences for Questar Pipeline. Questar Pipeline must negotiate a portfolio of supply contracts to ensure long-term availability of baseload gas, and must provide storage and peak-day capacity to Mountain Fuel for gas from many sources of varying reliability. In contrast, Mountain Fuel's major industrial customers who have converted to direct producer contracts with firm concurrent fallback have essentially become part of the peak-load problem rather than contributing to

its solution. In the past few years, the proportion of interruptible demand as a fraction of total demand has decreased significantly.

The three Questar subsidiaries decided to rationalize their individual competitive strategies in the context of the entire gas supply system. For this planning exercise, a nine-year horizon was chosen at a monthly level of detail. Analysis and survey of industry literature resulted in three alternate future cost and demand load factor (the proportion of maximum contracted gas which is take-or-pay) scenarios: nominal price and demand increases, moderate increases in price and demand, and a scenario with a moderate increase in demand and a sharp increase after a few years. (The last scenario, in which a sharp price increase occurs, is a "gas bubble" scenario which many industry analysts predict. Evaluation of such a scenario makes good business sense.)

With these scenarios, and the existing portfolio of gas contracts, Questar Pipeline provided modeling support to render a viable joint policy. The key goals were providing projected reliable gas supplies at reasonable prices while balancing the requirements of both customers and subsidiaries. Fifty to sixty model

excursions were evaluated, each with about 6,200 constraints, 12,000 variables and 81,000 nonzero coefficients. Each scenario ran in 9 megabytes (2 megabytes for the decision support shell, 2 megabytes for optimization, and the remainder for operating system overhead) for about 3 minutes on an IBM 3084-Q running under MVS/XA-SP.

Surprisingly, the solutions indicated that a modest amount of additional storage and peak-day capacity will provide a great deal of leverage to defer and reduce the magnitude of contract purchases for the purpose of meeting future peak demands. In particular, the solutions involve expanded use of the Clay Basin storage field (see Figure 3), which is a depleted "cap-rock" gas field in northeastern Utah with 7.5 BCF base and 125 MMCF peak capacity dedicated to Questar Pipeline. This can be augmented at attractive cost by a 2–3 BCF base and 50 MMCF peak capacity, thus deferring and reducing the need for additional purchase agreements over the next several years. While this conclusion may not appear controversial, many competing proposals of varying cost and complexity required evaluation to the satisfaction of competing executive constituents. It is in this environment that good models, especially good optimization models, excel.

## 10. CONCLUSIONS

This model formulation has several important advantages over those restricted to generalized networks (e.g., McBride 1986, Planmetrics 1988). First, a complete representation of the supply planning problem is provided. At the strategic level, the relationships among gas purchase contracts, storage, and transportation in a gas supply portfolio are captured. At the dispatch level, the model includes important operating and contractual constraints. These include the system network, contract pricing terms, and the relationships between inventory levels and injection and withdrawal capabilities. The diversity of available contractual terms mandates incorporation of many complicating constraints and variables. That the model accurately incorporates the right amount of detail is demonstrated by the model's wide acceptance in the natural gas industry (Rosenkranz 1989).

From the perspective of the gas supply planner, it is also important that the problem is actually solved optimally. In contrast, it has been fashionable to suggest Lagrangian relaxation techniques to render easier problems (i.e., generalized networks, or even pure networks assuming no transmission losses) and attempt to satisfy remaining complicating constraints

heuristically, i.e., with no assurance of success. The Lagrangian approach allows the use of much simpler optimization packages, e.g., Bradley, Brown and Graves (1977), and Brown and McBride (1984). However, the complicating constraints in natural gas contracts constitute the core of the business aspects of the problem—indirect treatment of these constraints inevitably admits infeasible and suboptimal solutions to the problem at hand. At best, this greatly complicates comparisons of competing scenarios—at worst, comparisons are meaningless. The gas supply planner is well advised to depend upon complete, feasible, optimal analyses of complex scenarios for presentation before company management and regulatory agencies.

## NOTES

<sup>1</sup>The U.S. gas industry commonly uses BTUs, THERMs ( $10^5$  BTUs), MMBTUs ( $10^6$  BTUs), cubic feet (denoted cf, approximately  $10^3$  BTU), MCF ( $10^3$  cf), MMCF ( $10^6$  cf), and BCF ( $10^9$  cf). A BTU may better be expressed outside the U.S. as 1,055 joules or 252 gram calories.

## APPENDIX

### Subscripts

- $j, k$  nodes;
- $J_j$  the set of child nodes associated with parent node  $j$ ;
- $jk$  the transportation agreement from node  $j$  to node  $k$ ;
- $t, \tau$  time period;
- $\bar{t}$  the last time period in the planning horizon;
- $l$  side constraint;
- $S$  season, i.e., a contiguous set of periods;
- $\underline{S}$  the first time period of season  $S$ ;
- $T$  a contract period, a contiguous set of periods;
- $\underline{T}$  the first time period of contract period  $T$ ;
- $\bar{T}$  the last time period of contract period  $T$ ;
- $T_l$  the set of periods associated with side constraint  $l$ ;
- $L$  the set of side constraints;
- $A$  the set of transportation agreement pairs  $jk$ .

### Constants

- $DAYS_t$  the number of days in time period  $t$ ;
- $DMDM_{jt}$  the demand at node  $j$  during time period  $t$ ;
- $LOAD_{jt}$  peak-day load factor at demand node  $j$  during time period  $t$ ;

- $MNDW_{jt}$  minimum daily withdrawal from storage node  $j$  during time period  $t$ ;
- $MXDW_{jt}$  maximum daily withdrawal from storage node  $j$  during time period  $t$ ;
- $MNDI_{jt}$  minimum daily injection at storage node  $j$  during time period  $t$ ;
- $MXDI_{jt}$  maximum daily injection at storage node  $j$  during time period  $t$ ;
- $TINV_{jt}$  minimum (target) inventory level at storage node  $j$  at end of time period  $t$ ;
- $SCAP_{jt}$  maximum inventory level at storage node  $j$  at end of period  $t$ ;
- $MNCP_{jT}$  minimum allowable capacity at potential storage node  $j$  over contract period  $T$ ;
- $MXCP_{jT}$  maximum allowable capacity at potential storage node  $j$  over contract period  $T$ ;
- $MNMW_{jT}$  minimum maximum daily withdrawal from potential storage node  $j$  over contract period  $T$ ;
- $MXMW_{jT}$  maximum maximum daily withdrawal from potential storage node  $j$  over contract period  $T$ ;
- $MXDP_{jt}$  maximum daily purchase from pipeline contract node  $j$  during time period  $t$ ;
- $MNDP_{jt}$  minimum daily purchase from pipeline contract node  $j$  during time period  $t$ ;
- $MXSP_{jS}$  maximum seasonal purchase from pipeline contract node  $j$  during season  $S$ ;
- $MDPF_{jS}$  minimum daily purchase fraction from pipeline contract node  $j$  over contract period  $T$ ;
- $MNPF_{jS}$  minimum purchase fraction from pipeline contract node  $j$  in season  $S$ ;
- $MNPP_{jS}$  minimum purchase penalty for pipeline contract node  $j$  in season  $S$ ;
- $MXMD_{jT}$  maximum allowable maximum daily purchase from potential pipeline contract node  $j$  over contract period  $T$ ;
- $SDAY_{jT}$  the number of days in a season for potential contract period  $T$  if minimum seasonal purchase is based on maximum daily purchase  $u_{jT}$  (constraint  $(MNPF')$  is used); otherwise,  $SDAY_{jT} \leq 0$  indicates that constraint  $(MNPF)$  is used, i.e., minimum seasonal purchase is based on maximum seasonal purchase  $z_{jT}$ ;
- $JCAP_{jkt}$  the joint firm and interruptible daily capacity on potential arc  $jk$  during period  $t$ ;
- $MXMF_{jk}$  maximum firm daily capacity on potential arc  $jk$ ;
- $MXMI_{jk}$  maximum interruptible daily capacity on potential arc  $jk$ ;
- $MNDT_{jkt}$  minimum daily throughput on existing arc  $jk$  during time period  $t$ ;
- $MXDT_{jkt}$  maximum daily throughput on existing arc  $jk$  during time period  $t$ ;
- $RELF_{jt}$  reliability factor for node  $j$  during time period  $t$ ;
- $RELF_{jkt}$  reliability factor for firm arc  $jk$  during time period  $t$ ;
- $RELI_{jkt}$  reliability factor for interruptible arc  $jk$  during time period  $t$ ;
- $INTA_{jkt}$  interruptibility factor for interruptible arc  $jk$  during time period  $t$ ;
- $JMXM_{jT}$  maximum joint maximum daily purchase on potential joint contract node  $j$  over contract period  $T$ ;
- $FCMX_{jT}$  maximum fraction of joint maximum daily purchase children of parent  $j$  may have over contract period  $T$ ;
- $LHSV_l$  the left-hand side value for side constraint  $l$ ;
- $RHSV_l$  the right-hand side value for side constraint  $l$ ;
- $m_{jk}$  the flow multiplier  $(1 - \text{loss})$  on arc  $jk$ ;
- $i_{j0}$  the initial inventory at storage node  $j$ ;
- $m_j^v$  the flow multiplier on injection into storage node  $j$ ;
- $m_j^w$  the flow multiplier on withdrawal from storage node  $j$ ;
- $\underline{d}_{jT}$  the lower maximum daily purchase/maximum seasonal purchase constant for node  $j$  over contract period  $T$ ;
- $\bar{d}_{jT}$  the upper maximum daily purchase/maximum seasonal purchase constant for node  $j$  over contract period  $T$ ;
- $\tilde{d}_{jT}$  the joint contract lower maximum daily purchase/maximum seasonal purchase constant seen at parent node  $j$  over contract period  $T$ ;
- $\bar{\tilde{d}}_{jT}$  the joint contract upper maximum daily purchase/maximum seasonal purchase constant seen at parent node  $j$  over contract period  $T$ ;
- $f_j$  the daily fraction of inventory at storage node  $j$  which can be withdrawn (withdrawal function slope);

- $h_j$  the least daily withdrawal amount at storage node  $j$  (withdrawal function  $y$ -axis intercept);
- $f'_j$  the daily fraction of inventory at storage node  $j$  which can be injected (negative of injection function slope);
- $h'_j$  the maximum daily injection amount at storage node  $j$  given that MXDI is not binding (injection function  $y$ -axis intercept) at storage node  $j$ ;
- $r_t$  the discount factor for period  $t$ ;
- $a_{jkl}$  the coefficient associated with arc  $jk$  in side constraint  $l$ ;
- $CCOM_{jt}$  the cost of commodity at supply node  $j$  during time period  $t$ ;
- $CTFM_{jkt}$  the cost of firm transportation on potential arc  $jk$  during time period  $t$ ;
- $CTIN_{jkt}$  the cost of interruptible transportation on potential arc  $jk$  during time period  $t$ ;
- $CDFM_{jkt}$  the demand charge for firm transportation on potential arc  $jk$ , contract period  $T$ ;
- $CDIN_{jT}$  the demand charge for interruptible transportation on potential arc  $jk$  over contract period  $T$ ;
- $CTRN_{jkt}$  the cost of transportation on existing arc  $jk$  during time period  $t$ ;
- $CIN_j$  the cost of injections at storage node  $j$ ;
- $CINV_j$  the daily inventory holding cost at storage node  $j$ ;
- $CWDR_j$  the withdrawal charge at storage node  $j$ ;
- $CCAP_{jT}$  the cost of capacity at potential storage node  $j$  over contract period  $T$ ;
- $CMDW_{jT}$  the cost of maximum daily withdrawal at potential storage node  $j$  over contract period  $T$ ;
- $CDMD_{jT}$  the demand charge at potential supply node  $j$  over period  $T$ ;
- $CDMS_{jT}$  the seasonal demand charge at potential supply node  $j$  over period  $T$ ;
- $CJDD_{jT}$  the demand charge for joint contract seen at parent potential supply node  $j$  over contract period  $T$ ;
- $CJDS_{jT}$  the seasonal demand charge for joint contract seen at parent potential supply node  $j$  for period  $T$ ;
- $CPEN_j$  the penalty charge for not meeting demand at node  $j$ ;
- $CLPN_l$  the penalty for violating left-hand side of side constraint  $l$ ;

$CRPN_l$  the penalty for violating right-hand side of side constraint  $l$ .

**Variables**

- $x'_{jkt}$  the flow from node  $j$  to node  $k$  in time period  $t$  on potential firm arc;
- $x''_{jkt}$  the flow from node  $j$  to node  $k$  in time period  $t$  on potential interruptible arc;
- $x_{jkt}$  the flow on existing arc  $jk$  in time period  $t$ ;
- $x_{jkt}$  the total flow on all arcs from node  $j$  to node  $k$  in time period  $t$ ;
- $\hat{x}'_{jkt}$  the peak-day flow from node  $j$  to node  $k$  in time period  $t$  on potential firm arc;
- $\hat{x}''_{jkt}$  the peak-day flow from node  $j$  to node  $k$  in time period  $t$  on potential interruptible arc;
- $\hat{x}_{jkt}$  the peak-day flow on existing arc  $jk$  in time period  $t$ ;
- $\hat{x}_{jkt}$  the total peak-day flow on all arcs from node  $j$  to node  $k$  in time period  $t$ ;
- $u_{jt}$  injections at storage node  $j$  in time period  $t$ ;
- $w_{jt}$  withdrawals from storage node  $j$  in time period  $t$ ;
- $\bar{v}_{jT}$  the variable capacity on potential storage node  $j$ , contract period  $T$ ;
- $\bar{w}_{jT}$  the variable upper bound on withdrawals from node  $j$ ; contract period  $T$ ;
- $b_{jt}$  the flow directly bypassing storage node  $j$  in time period  $t$ ;
- $\hat{w}_{jt}$  withdrawals from storage node  $j$  on peak day of time period  $t$ ;
- $y'_{jkt}$  the variable upper bound on firm capacity in arc  $jk$ , contract period  $T$ ;
- $y''_{jkt}$  the variable upper bound on interruptible transportation capacity in arc  $jk$ , contract period  $T$ ;
- $u_{jT}$  the variable maximum daily purchase for potential pipeline contract node  $j$ , contract period  $T$ ;
- $z_{jT}$  the variable maximum seasonal purchase for potential pipeline contract node  $j$ , contract period  $T$ ;
- $\tilde{u}_{jT}$  the variable maximum daily purchase for potential joint pipeline contract whose parent is node  $j$ , contract period  $T$ ;
- $\tilde{z}_{jT}$  the variable maximum seasonal purchase for potential joint pipeline contract whose parent is node  $j$ , contract period  $T$ ;
- $s_{jt}^D$  the unmet demand at node  $j$  in time period  $t$ ;
- $s_{jS}^M$  the violation of minimum purchase (minimum bill) for contract node  $j$  for season  $S$ ;
- $\hat{s}_{jt}^D$  the unmet peak-day demand at node  $j$  in time period  $t$ .

**Period-by-Period Constraints**

Spot purchase node  $j$ :

$$MNDP_{jt} DAYS_t \leq \sum_k x_{jkt} \leq MXDP_{jt} DAYS_t \quad \text{for all } t. \quad (MXMP)$$

Existing pipeline contract node  $j$ :

$$MNDP_{jt} DAYS_t \leq \sum_k x_{jkt} \leq MXDP_{jt} DAYS_t \quad \text{for all } t. \quad (MXMP)$$

$$-s_{jS}^M + MXSP_{jS} MNPF_{jS} \leq \sum_k \sum_{t \in S} x_{jkt} \leq MXSP_{jS} \quad \text{for all } S. \quad (MXSP)$$

Potential pipeline contract node  $j$ :

$$\sum_k x_{jkt} - DAYS_t u_{jT} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (MXMM)$$

$$0 \leq \sum_k x_{jkt} - MDPF_{jt} DAYS_t u_{jT} \quad \text{for all } t \in T, \text{ for all } T \quad (MNMP)$$

$$-s^M \leq \sum_k \sum_{t \in S} x_{jkt} - MNPF_{jS} z_{jT} \quad \text{for all } S \in T, \text{ for all } T \text{ if } SDAY_{jT} \leq 0 \quad (MNPF)$$

$$-s^M \leq \sum_k \sum_{t \in S} x_{jkt} - SDAY_{jT} MNPF_{jS} u_{jT} \quad \text{for all } S \in T, \text{ for all } T \text{ if } SDAY_{jT} > 0 \quad (MNPF')$$

$$\sum_k \sum_{t \in S} x_{jkt} - z_{jT} \leq 0 \quad \text{for all } S \in T, \text{ for all } T \quad (SEAS)$$

$$0 \leq z_{jT} - d_{jT} u_{jT} \quad \text{for all } T \quad (DSLO)$$

$$z_{jT} - \bar{d}_{jT} u_{jT} \leq 0 \quad \text{for all } T \quad (DSUP)$$

$$MNDP_{jT} \leq u_{jT} \leq MXMD_{jT}, 0 \leq z_{jT} \quad \text{for all } T.$$

$$\sum_{j' \in J_j} \sum_k x_{j'kt} + \sum_k x_{jkt} - DAYS_t \tilde{u}_{jT} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (JMXXM)$$

$$\sum_{j' \in J_j} \sum_k \sum_{t \in S} x_{j'kt} + \sum_k \sum_{t \in S} x_{jkt} - \tilde{z}_{jT} \leq 0 \quad \text{for all } S \in T \text{ for all } T \quad (JSEA)$$

$$\sum_{j' \in J_j} \sum_k x_{j'kt} - DAYS_t FCMX_{jT} \tilde{u}_{jT} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (JCPM)$$

$$0 \leq \tilde{z}_{jT} - \bar{d}_{jT} \tilde{u}_{jT} \quad \text{for all } T \quad (JDSL)$$

$$\tilde{z}_{jT} - \bar{d}_{jT} \tilde{u}_{jT} \leq 0 \quad \text{for all } T \quad (JDSU)$$

$$0 \leq \tilde{u}_{jT} \leq JMXXM_{jT}, 0 \leq \tilde{z}_{jT} \quad \text{for all } T.$$

Existing storage node  $j$ :

$$\frac{1}{f_j DAYS_t} w_{jt} + \sum_{\tau=1}^{t-1} -m_j^v v_{j\tau} + \sum_{\tau=1}^{t-1} w_{j\tau} \leq i_{j0} + \frac{h_j}{f_j} \quad \text{for all } t \quad (WINV)$$

$$\frac{1}{f'_j DAYS_t} v_{jt} + \sum_{\tau=1}^{t-1} m_j^v v_{j\tau} + \sum_{\tau=1}^{t-1} -w_{j\tau} \leq -i_{j0} + \frac{h'_j}{f'_j} \quad \text{for all } t \quad (IJIV)$$

$$\sum_k -m_{kj} x_{kjt} + b_{jt} + v_{jt} = 0 \quad \text{for all } t \quad (INPT)$$

$$-i_{j0} + TINV_{jt} \leq \sum_{\tau=1}^t m_j^v v_{j\tau} + \sum_{\tau=1}^t -w_{j\tau} \leq SCAP_j - i_{j0} \quad \text{for all } t \quad (STOR)$$

$$-m_j^w w_{jt} - b_{jt} + \sum_k x_{jkt} = 0 \quad \text{for all } t. \quad (OUTP)$$

$$MNDI_{jt} DAYS_t \leq v_{jt} \leq MXDI_{jt} DAYS_t,$$

$$DAYS_t MNDW_{jt} \leq w_{jt} \leq MXDW_{jt} DAYS_t \quad \text{for all } t.$$

Potential storage node  $j$ :

$WINV, IJIV, INPT, STOR, OUTP,$  and

$$-i_{j0} + TINV_{jt} \leq \sum_{\tau=1}^t m_j^v v_{j\tau} + \sum_{\tau=1}^t -w_{j\tau} \quad \text{for all } t \quad (INVT)$$

$$w_{jt} - DAYS_t \bar{w}_{jT} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (VMDW)$$

$$MNCP_{jT} \leq \bar{i}_{jT} \leq MXCP_{jT} \quad \text{for all } T$$

$$MNMW_{jT} \leq \bar{w}_{jT} \leq MXMW_{jT} \quad \text{for all } T$$

$$0 \leq w_{jt} \quad \text{for all } t$$

$$MNDI_{jT} DAYS_t \leq v_{jt} \leq MXDI_{jT} DAYS_t \quad \text{for all } t \in T, \text{ for all } T.$$

Demand node  $j$ :

$$\sum_k -m_{kj} x_{kjt} + \sum_k x_{jkt} \leq -DMDM_{jt} + s_{jt}^D \quad \text{for all } t. \quad (DMDM)$$

Junction node  $j$ :

$$\sum_k -m_{kj}x_{kjt} + \sum_k x_{jkt} = 0 \quad \text{for all } t. \quad (JUNC)$$

Existing transportation agreement  $jk$ :

$$MNDT_{jkt} DAYS_t \leq x_{jkt} \leq MXDT_{jkt} DAYS_t \quad \text{for all } t.$$

Potential transportation agreement  $jk$ :

$$x'_{jkt} + x''_{jkt} \leq JCAP_{jkt} DAYS_t \quad \text{for all } t \in T, \text{ for all } T \quad (JCAP)$$

$$x'_{jkt} - DAYS_t y'_{jkt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (MXMF)$$

$$x''_{jkt} - DAYS_t INTA_{jkt} y''_{jkt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (MXMI)$$

$$0 \leq y'_{jkt} \leq MXMF_{jkt}, 0 \leq y''_{jkt} \leq MXMI_{jkt} \quad \text{for all } T$$

$$0 \leq x'_{jkt} \leq DAYS_t MXMF_{jkt}, 0 \leq x''_{jkt} \leq DAYS_t MXMI_{jkt} INTA_{jkt} \quad \text{for all } t \in T, \text{ for all } T.$$

Side constraints on fixed arcs:

$$-s_l^- + LHSV_l \leq \sum_{t \in T_l} \sum_{(j,k) \in A} a_{jkt} x_{jkt} \leq RHSV_l + s_l^+ \quad \text{for all } l \in L, \text{ for all } L. \quad (ALNK)$$

### Peak-Day Constraints

Demand node  $j$ :

$$\sum_k -m_{kj} \hat{x}_{kjt} + \sum_k \hat{x}_{jkt} \leq -DMDM_{jt} LOAD_{jt} / DAYS_t + \hat{s}_{jt}^D \quad \text{for all } t. \quad (PDMD)$$

Simple junction node  $j$ :

$$\sum_k -m_{kj} \hat{x}_{kjt} + \sum_k \hat{x}_{jkt} = 0 \quad \text{for all } t. \quad (PJNC)$$

Spot purchase node  $j$  or existing pipeline contract node  $j$ :

$$0 \leq \sum_k \hat{x}_{jkt} \leq MXDP_{jt} RELF_{jt} \quad \text{for all } t. \quad (PMDP)$$

Potential pipeline contract node  $j$ :

$$\sum_k \hat{x}_{jkt} - RELF_{jt} u_{jt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (PMMD)$$

$$\sum_{j' \in J_j} \sum_k \hat{x}_{j'kt} + \sum_k \hat{x}_{jkt} - RELF_{jt} \hat{u}_{jt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (PJMM)$$

$$\sum_{j' \in J_j} \sum_k \hat{x}_{j'kt} - FCMX_{jT} \hat{u}_{jT} \leq 0 \quad \text{for all } t \in T, \text{ for all } T. \quad (PJCC)$$

Existing storage node  $j$ :

$$\sum_k -m_{kj} \hat{x}_{kjt} - m_j^w \hat{w}_{jt} + \sum_k \hat{x}_{jkt} = 0 \quad \text{for all } t \quad (PSTR)$$

$$\frac{1}{f_{jt}} \hat{w}_{jt} + \sum_{\tau=1}^t -m_j^v v_{j\tau} + \sum_{\tau=1}^t w_{j\tau} \leq i_{j0} + \frac{h_{jt}}{f_{jt}} \quad \text{for all } t \quad (PINV)$$

$$\hat{w}_{jt} + \sum_{\tau=1}^t -m_j^v v_{j\tau} + \sum_{\tau=1}^t w_{j\tau} \leq i_{j0} \quad \text{for all } t \quad (PTIV)$$

$$0 \leq \hat{w}_{jt} \leq RELF_{jt} MXDW_{jt} \quad \text{for all } t.$$

Potential storage node  $j$ :

*PSTR*, *PINV*, *PTIV*, and

$$\hat{w}_{jt} - RELF_{jt} \bar{w}_{jt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (PVMW)$$

$$0 \leq \hat{w}_{jt} \quad \text{for all } t.$$

Existing transportation agreement:

$$MNDT_{jkt} \leq \hat{x}_{jkt} \leq MXDT_{jkt} RELF_{jkt} \quad \text{for all } t.$$

Potential transportation agreement  $jk$ :

$$\hat{x}'_{jkt} + \hat{x}''_{jkt} \leq JCAP_{jkt} \quad \text{for all } t \quad (PTCP)$$

$$\hat{x}'_{jkt} - RELF_{jkt} y'_{jkt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (PMXF)$$

$$\hat{x}''_{jkt} - RELI_{jkt} y''_{jkt} \leq 0 \quad \text{for all } t \in T, \text{ for all } T \quad (PMXI)$$

$$0 \leq \hat{x}'_{jkt}, 0 \leq \hat{x}''_{jkt} \quad \text{for all } t.$$

### Costs

$x'_{jkt}$   $r_t CTFM_{jkt}$ , or  $r_t CTFM_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;

$x''_{jkt}$   $r_t CTIN_{jkt}$ , or  $r_t CTIN_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;

- $x_{jkt}$   $r_t CTRN_{jkt}$ , or  $r_t CTRN_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;  
 $\hat{x}'_{jkt}$   $r_t CTFM_{jkt}$ , or  $r_t CTFM_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;  
 $\hat{x}''_{jkt}$   $r_t CTIN_{jkt}$ , or  $r_t CTIN_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;  
 $\hat{x}_{jkt}$   $r_t CTRN_{jkt}$ , or  $r_t CTRN_{jkt} + r_t CCOM_{jt}$  if  $j$  is a supply node;  
 $v_{jt}$   $r_t CIN_j + \sum_{\tau=t}^T m_j^v r_\tau DAYS_\tau CIN_j$ ;  
 $w_{jt}$   $r_t CWDR_j - \sum_{\tau=t}^T r_\tau DAYS_\tau CIN_j$ ;  
 $\bar{i}_{jT}$   $r_T CCAP_{jT}$ ;  
 $\bar{w}_{jT}$   $r_T CMDW_{jT}$ ;  
 $b_{jt}$  no cost;  
 $\hat{w}_{jt}$   $r_t CWDR_j$ ;  
 $y'_{jkt}$   $\sum_{t=T}^{\bar{T}} r_t CDFM_{jkt}$ ;  
 $y''_{jkt}$   $\sum_{t=T}^{\bar{T}} r_t CDIN_{jkt}$ ;  
 $u_{jT}$   $\sum_{t=T}^{\bar{T}} r_t CDMD_{jT}$ ;  
 $z_{jT}$   $\sum_{t=T}^{\bar{T}} r_t CDMS_{jT}$ ;  
 $\tilde{u}_{jT}$   $\sum_{t=T}^{\bar{T}} r_t CJDD_{jT}$ ;  
 $\tilde{z}_{jT}$   $\sum_{t=T}^{\bar{T}} r_t CJDS_{jT}$ ;  
 $s_{jt}^D$   $r_t CPEN_j$ ;  
 $s_{jS}^M$   $r_S CDMD_j MNPP_{jS}$ ;  
 $\hat{s}_{jt}^D$   $r_t CPEN_j$ ;  
 $s_t^-$   $CLPN_t$  times appropriate discount rate;  
 $s_t^+$   $CRPN_t$  times appropriate discount rate.

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