Signal Processing for Graphs

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Graph Exploitation Symposium

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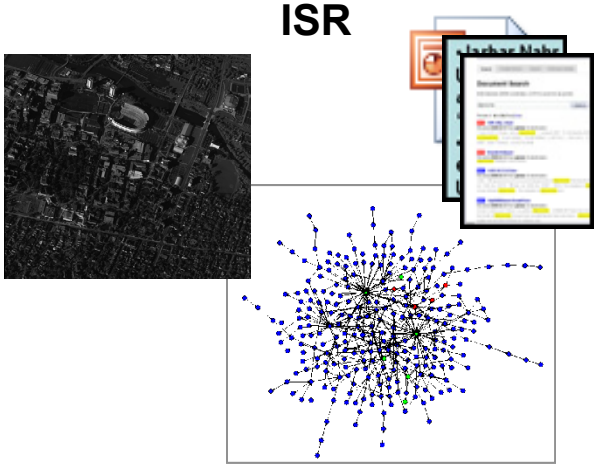
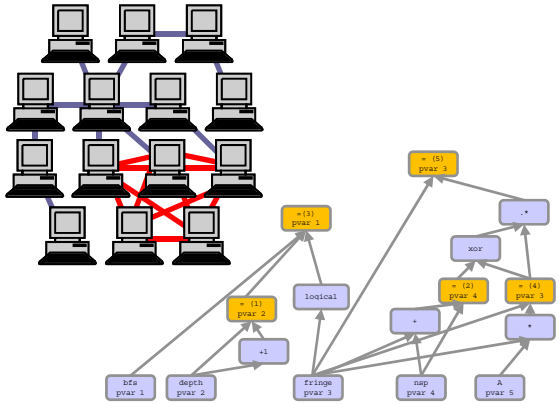
Applications of Graph Analysis
From Data to Relationships

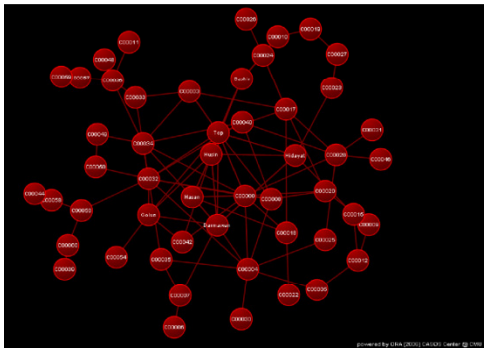
ISR



CYBER SECURITY



SOCIAL NETWORK ANALYSIS



- Extract entities detected from multi-INT sources
- Analyze relationships between entities

- Analyze communication patterns
- Analyze software execution patters
- Find behaviors consistent with cyber attack

- Consider interactions between individuals
- Locate anomalies and threat activity

Data encoding relationships between entities (graphs or networks) are highly relevant to a wide range of defense and intelligence applications

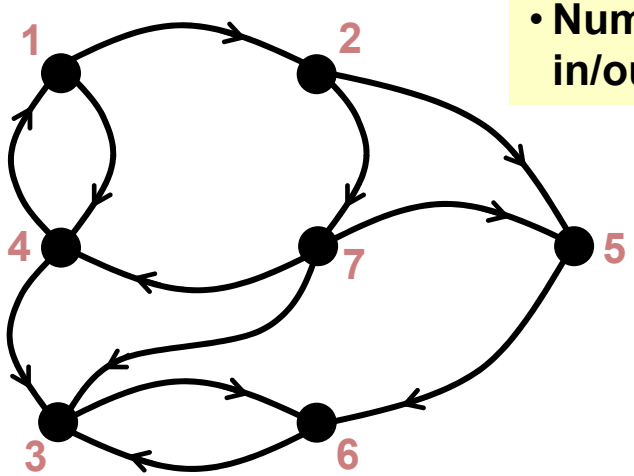
Graph Preliminaries (Formal Definition)

Graph G = (V,E) where
- **V = set of vertices**
- **E = set of edges**

Adjacency matrix representation:
- **Entry A(i,j) is non-zero if there exists an edge between vertices i and j**

Degree
- **Number of edges in/out of vertex v**

Graph G



Adjacency Matrix A

	1	2	3	4	5	6	7	Degree
1		●		●				2
2					●		●	2
3						●		1
4	●		●					2
5						●		1
6			●					1
7			●	●	●			3

GRAPH CHARACTERISTIC	ADJACENCY MATRIX REPRESENTION
G is undirected	A is symmetric
G is directed	A may or may not be symmetric
G is unweighted	A(i,j) is either 0 or 1
G is weighted	A(i,j) is a weight
G is a multi-graph	A is a multidimensional array

Time domain or multi-INT representation

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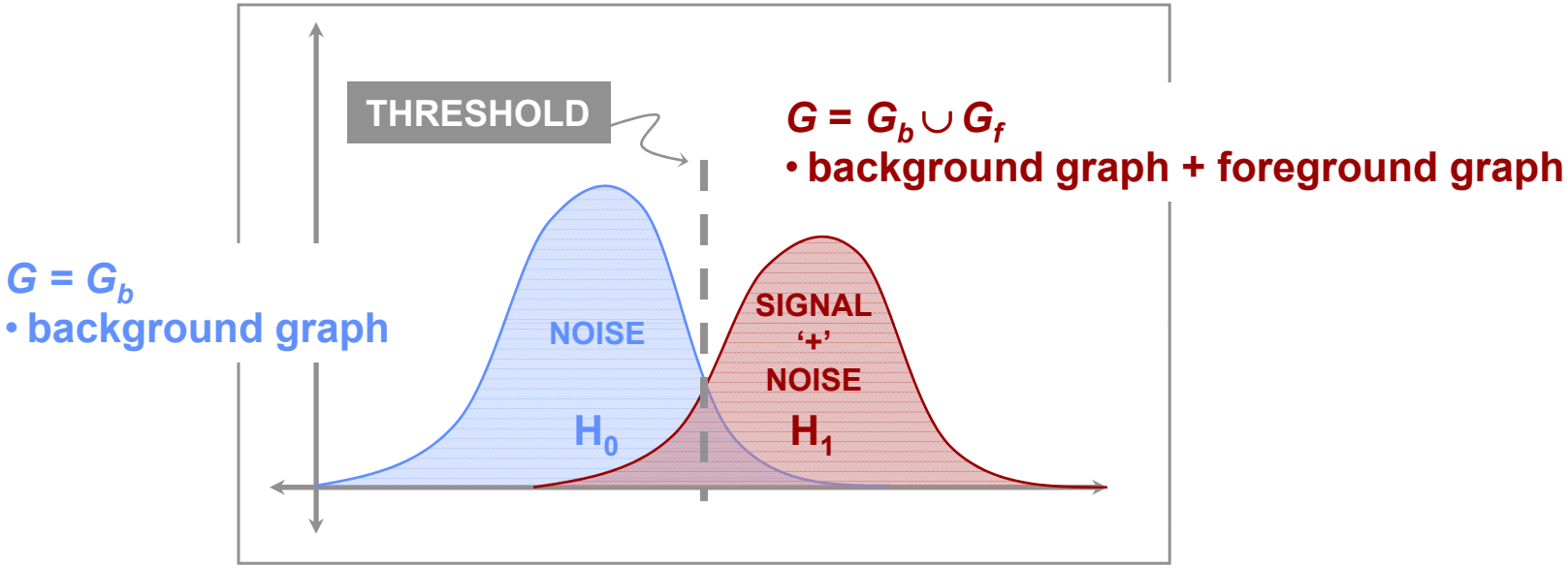
Subgraph Detection Problem



- H_0 : $G = G_b$
- H_1 : $G = G_b \cup G_f$
- Detection problem:
  - Given G, is H_0 or H_1 true?

Graph Detection Challenges:

- Background models
- Foreground models
- High-dimensional space

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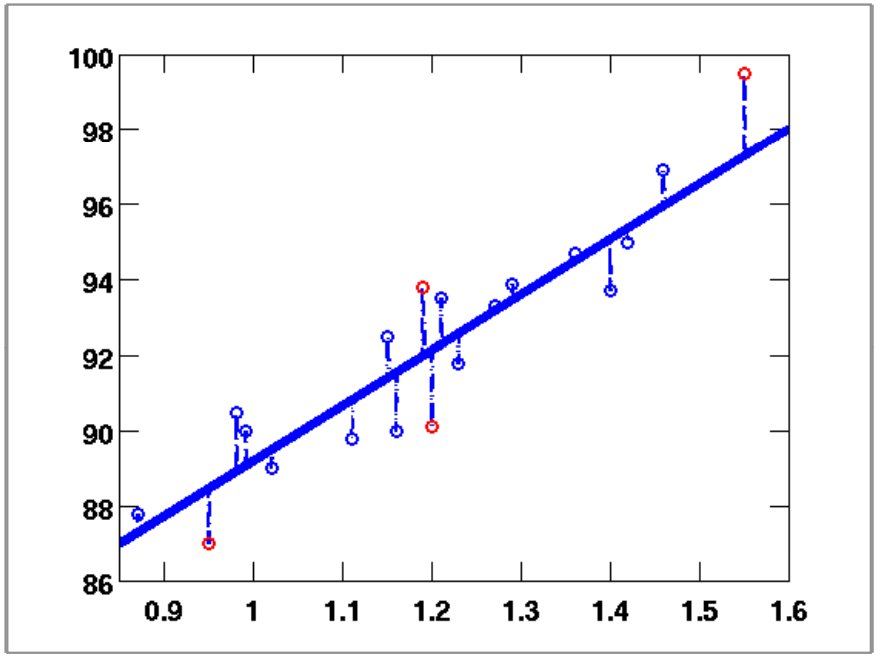
Outline

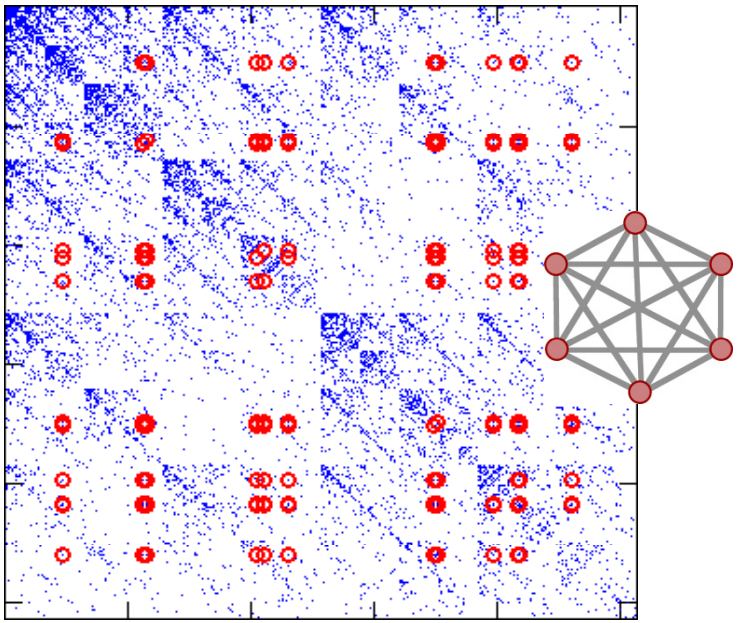
- Introduction
- **Approach**
- Results
- Summary

Graph Based Residuals Analysis

Linear Regression



Graph "Regression"



• Least-squares residuals from a best-fit line	• "Residuals" from a best-fit graph model
• Analysis of variance (ANOVA) describes fit	• Analysis of variance from expected topology
• "Explained" vs "unexplained" variance → signal/noise discrimination	• Unexplained variance in graph residuals → subgraph detection

ANALYSIS OF MODULARITY

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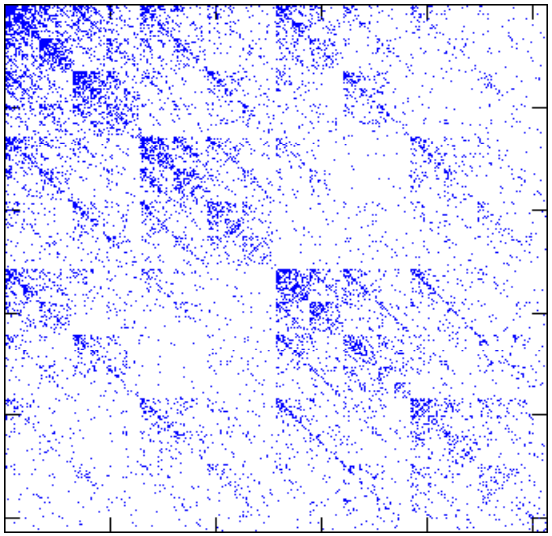
Subgraph Detection Algorithm Overview

Processing chain for subgraph detection analogous to a traditional signal processing chain

MODULARITY MATRIX CONSTRUCTION → EIGEN DECOMPOSITION → COMPONENT SELECTION → DETECTION → IDENTIFICATION

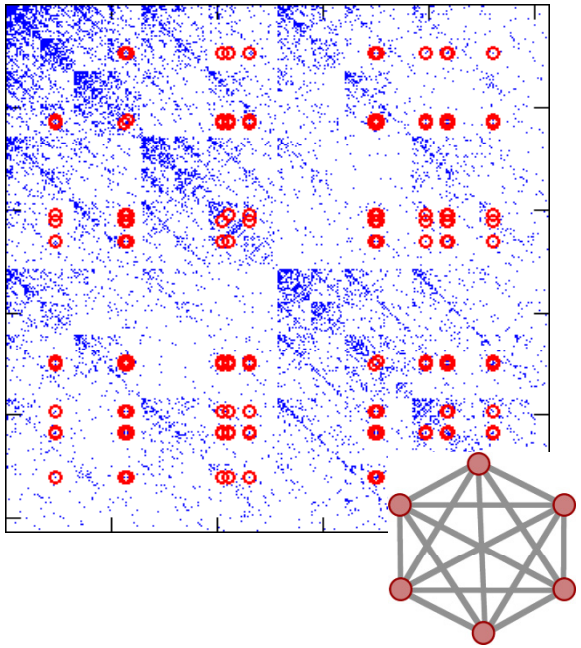


Input:

- A, adjacency matrix representation of G
- No cue

Output:

- v_s, set of vertices identified as belonging to subgraph G_f



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Modularity Matrix* Construction

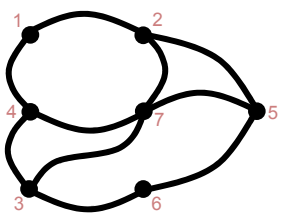
MODULARITY MATRIX CONSTRUCTION → EIGEN DECOMPOSITION → COMPONENT SELECTION → DETECTION → IDENTIFICATION

$$B = A - \frac{KK^T}{2M}$$

ADJACENCY MATRIX

DEGREE VECTOR

EXAMPLE: GRAPH G



$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & & \bullet & & \bullet & & & \\ 2 & \bullet & & & & \bullet & & \bullet \\ 3 & & & & \bullet & & \bullet & \bullet \\ 4 & \bullet & & \bullet & & & & \bullet \\ 5 & & \bullet & & & & \bullet & \bullet \\ 6 & & & \bullet & & \bullet & & \\ 7 & & \bullet & \bullet & \bullet & \bullet & & \end{array}$$

$$- \frac{1}{20} \left\{ \begin{bmatrix} 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 2 \\ 4 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 & 3 & 3 & 2 & 4 \end{bmatrix} \right\}$$

NUMBER OF EDGES

- Commonly used to evaluate *quality* of division of a graph into communities
- Observed minus "expected" edges, yielding residuals

*M. E. J. Newman. Finding community structure in networks using the eigenvectors of matrices. Phys. Rev. E, 74:036104, 2006.

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Eigen Decomposition

MODULARITY MATRIX CONSTRUCTION → **EIGEN DECOMPOSITION** → COMPONENT SELECTION → DETECTION → IDENTIFICATION

$$B = UDU^T$$

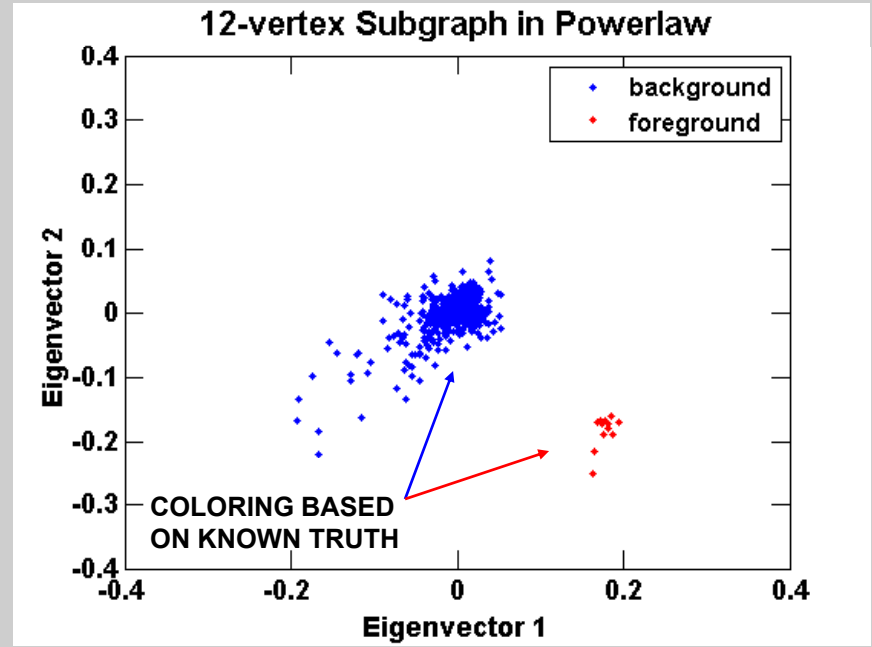
Eigenvalues, sorted by magnitude

$$\begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \cdots & & \\ & & & \lambda_{N-1} & \\ & & & & \lambda_N \end{bmatrix}$$

Corresponding eigenvectors

$$[\mathbf{u}_1 \mid \mathbf{u}_2 \mid \ldots \mid \mathbf{u}_{N-1} \mid \mathbf{u}_N]$$

Principal components



- Each point represents a vertex in *G*
- Vertices in *G*: 1024
- Vertices in G_f: 12
- Uncued background/foreground separation

Projection onto principal components of the modularity matrix yields good separation between background and foreground

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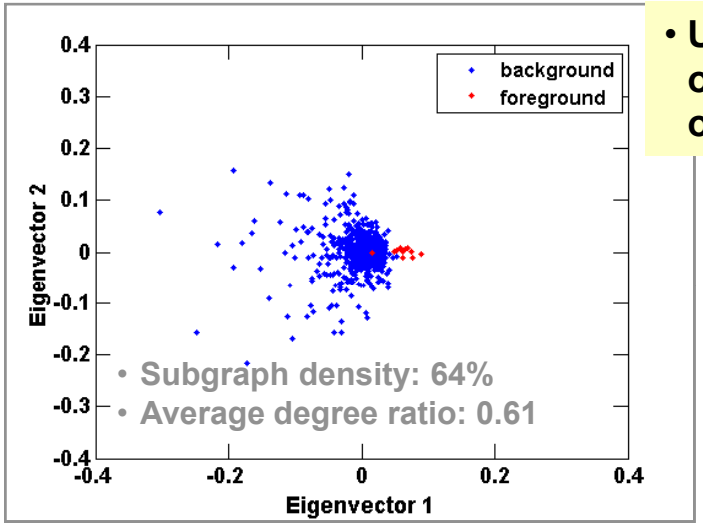
Component Selection
Detection of Weak Signals

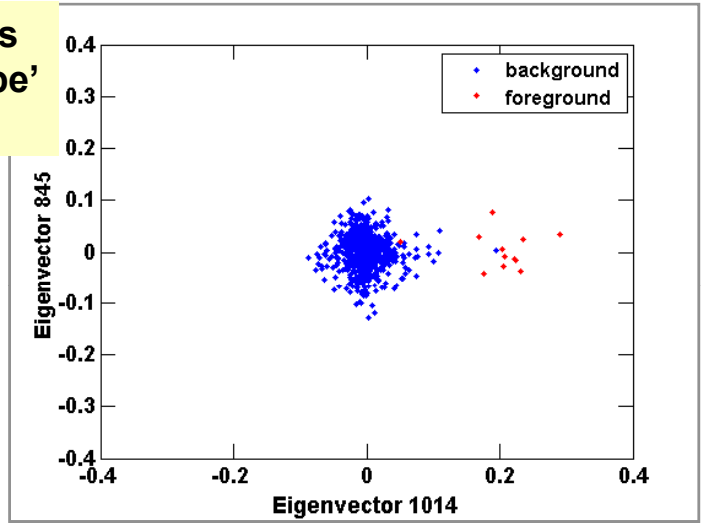
MODULARITY MATRIX CONSTRUCTION → EIGEN DECOMPOSITION → **COMPONENT SELECTION** → DETECTION → IDENTIFICATION



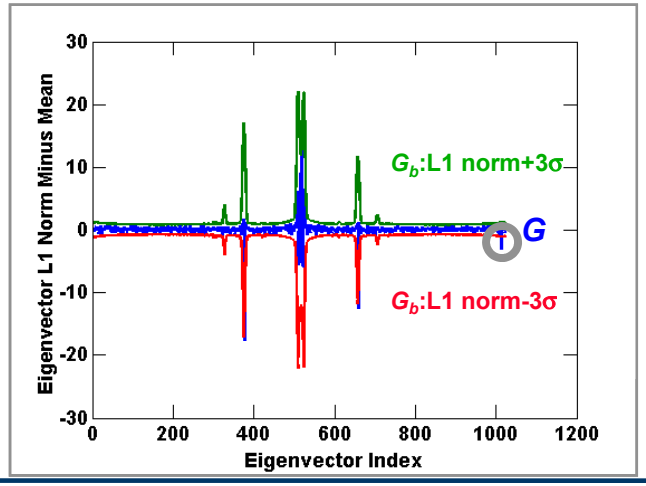
- Use L_1 norms of eigenvectors of G_b to generate an 'envelope' of normal graphs

L_1 Norm of vector u

$$\| \mathbf{u} \|_1 = \sum | u |$$



- Principal component analysis unable to separate weak foreground graph signature



- Select eigenvectors outside the envelope for projection
- Selected components yield good separation

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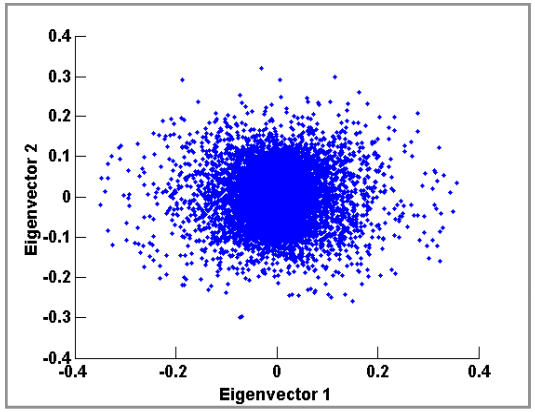
Toy Detection Example

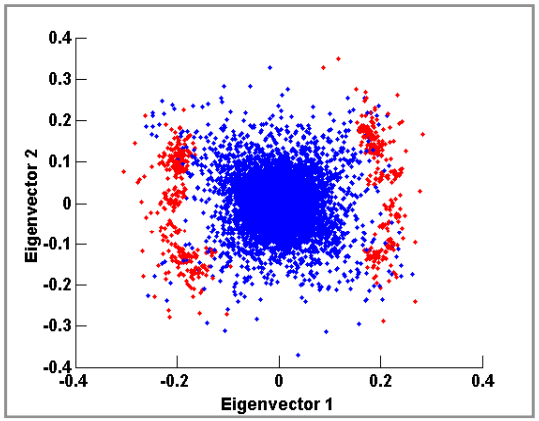
MODULARITY MATRIX CONSTRUCTION → EIGEN DECOMPOSITION → COMPONENT SELECTION → **DETECTION** → IDENTIFICATION

H_0



Background (normal behavior), G_b only

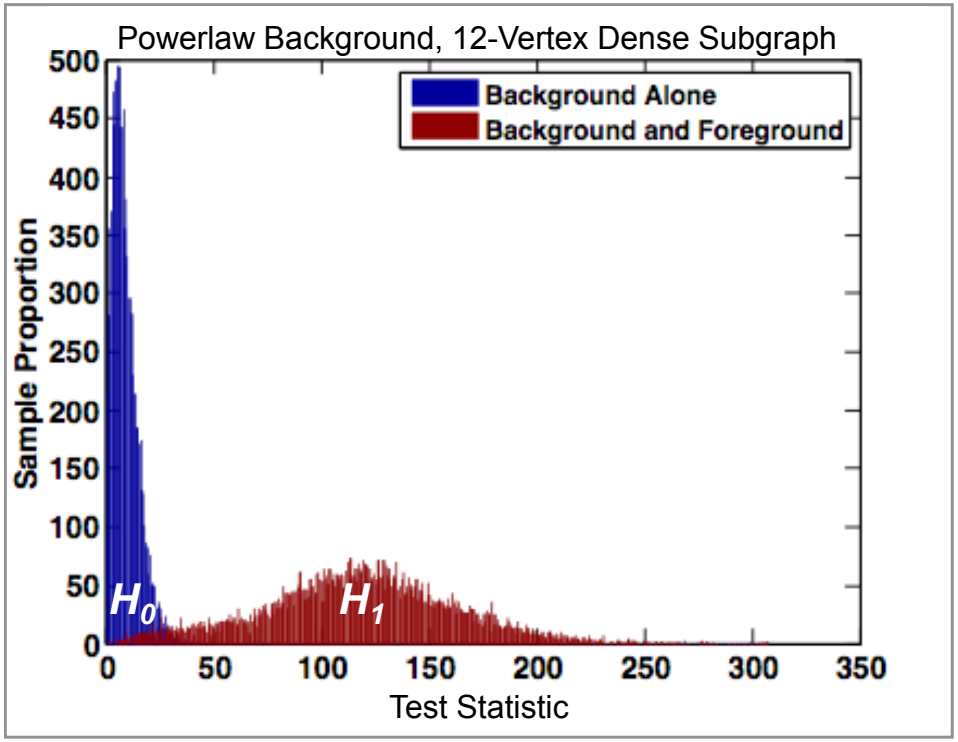
H_1



Background and foreground (includes anomalous behavior), $G_b + G_f$

TEST STATISTIC:

SYMMETRY OF THE PROJECTION ONTO SELECTED COMPONENTS



H_0 and H_1 distributions are well separated

B.A. Miller, N.T. Bliss and P.J. Wolfe, "Toward signal processing theory for graphs and non-Euclidean data," in *Proc. ICASSP*, 2010.

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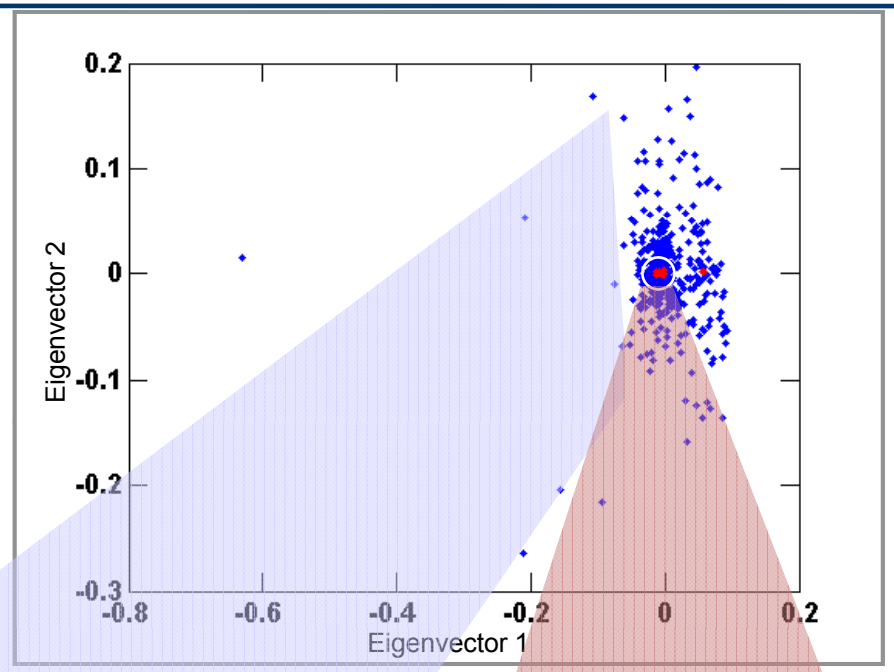
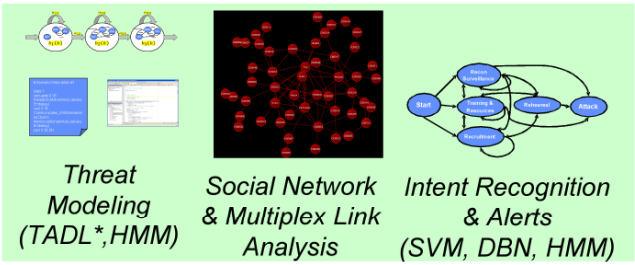
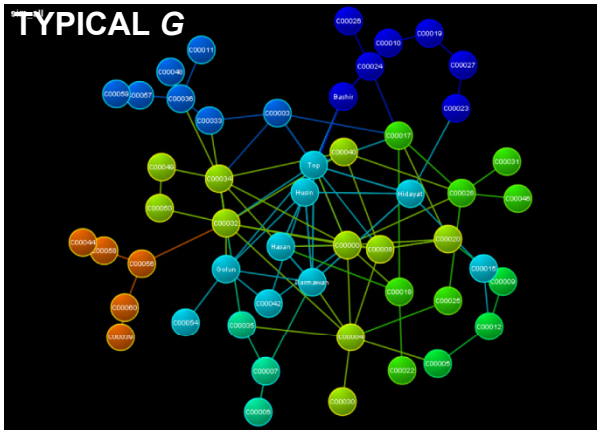
Social Network Analysis

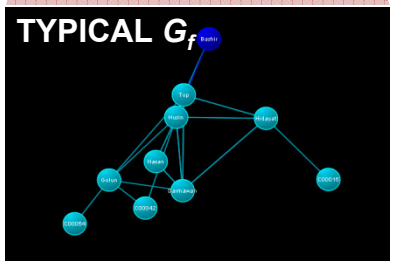
- Threat network dataset generated by the CT-SNAIR program
- Low density threat network embedded in a power law background
- Weak signature subgraph of interest
  - Average degree ratio: 0.44
  - Max eigenvalue ratio: 0.36
- Principal component analysis yields no foreground separation



Counter Terror Social Network Analysis and Intent Recognition







Networks generated from real scenarios

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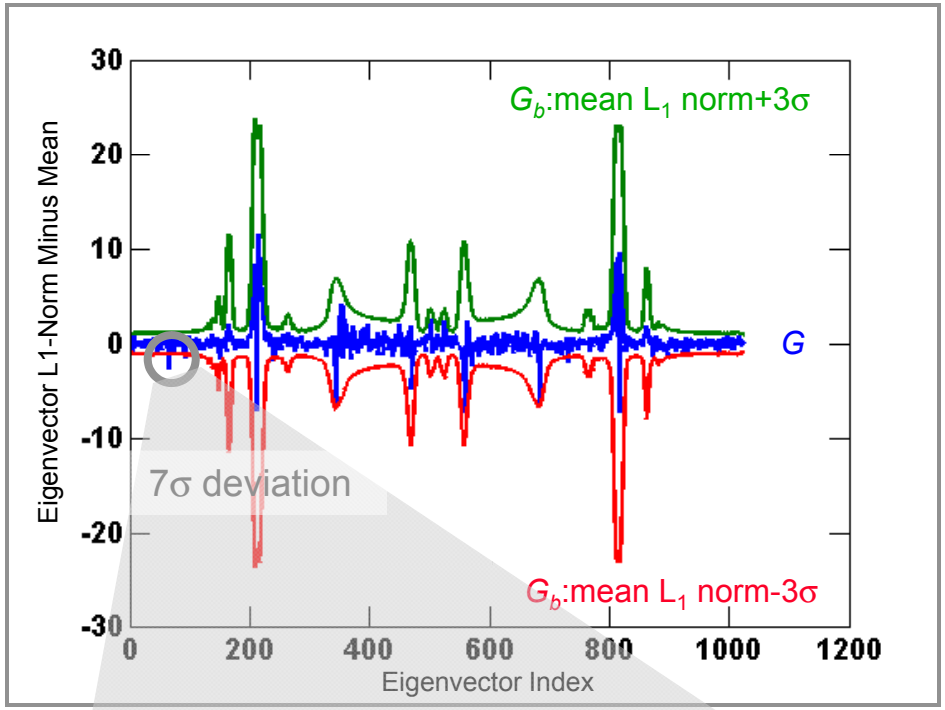
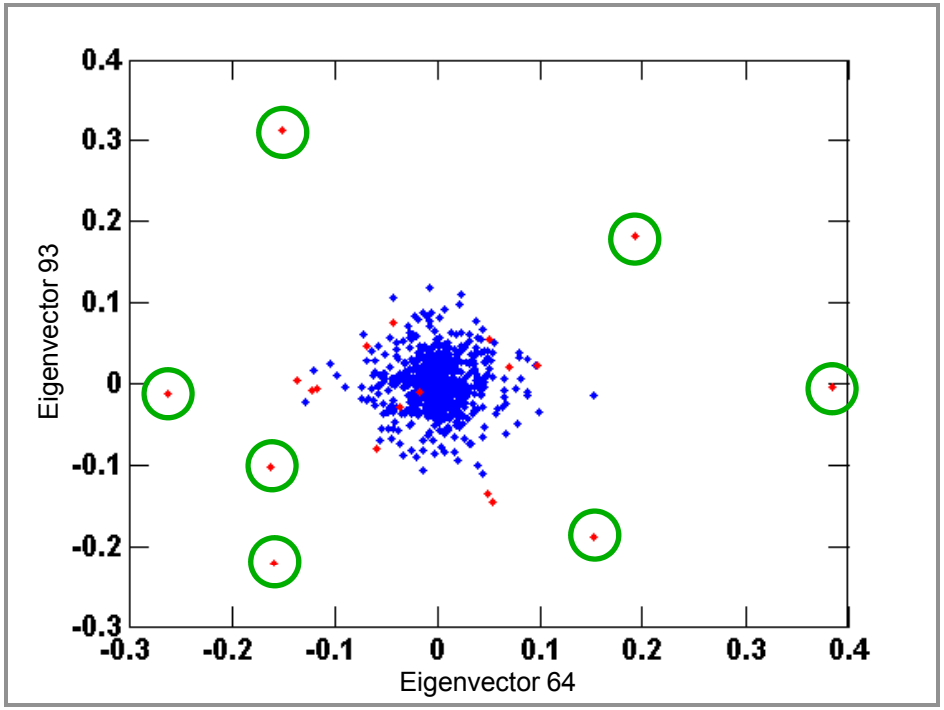
Threat Network Identification

L_1 Norm of Eigenvectors of Residuals Matrix



Scatter Plot Using Selected Components



Background statistics (power law) allow detection of statistically significant deviation in the L_1 norm

- **Significant number of threat vertices are separable and are identified by clustering**
- **Identified individuals can be used as cues**

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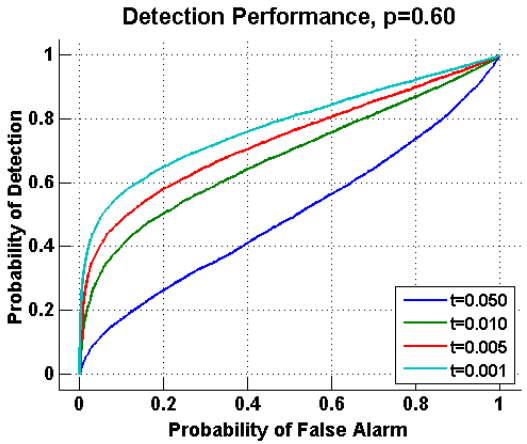
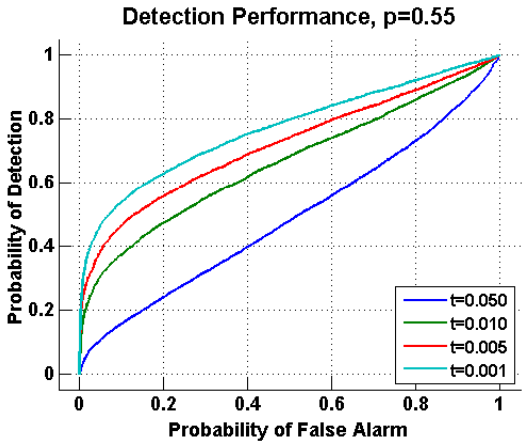
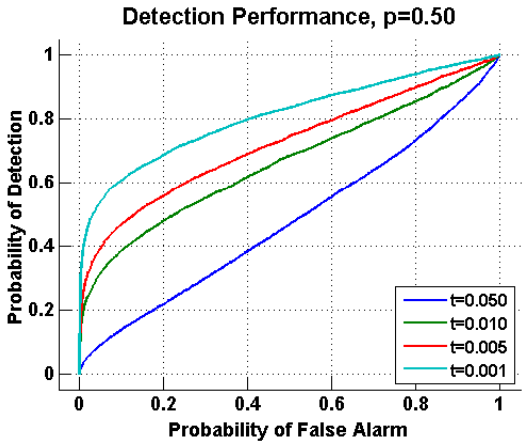
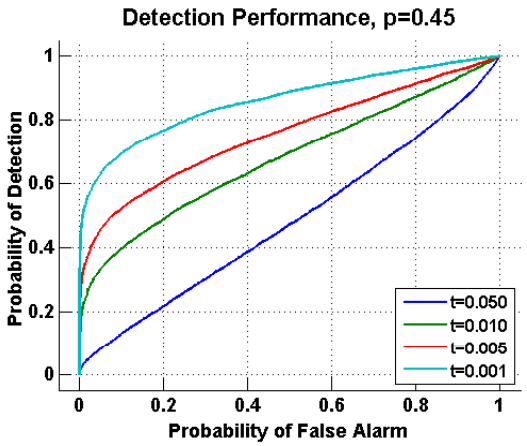
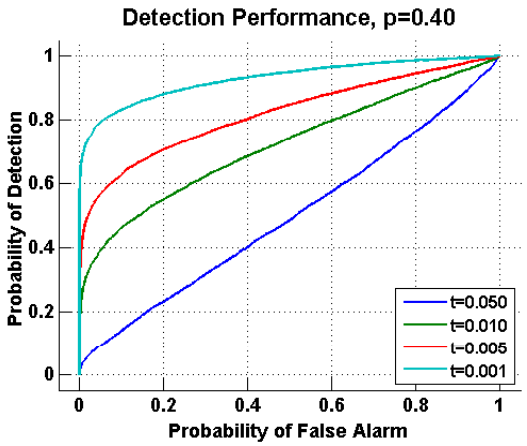
Detection Performance

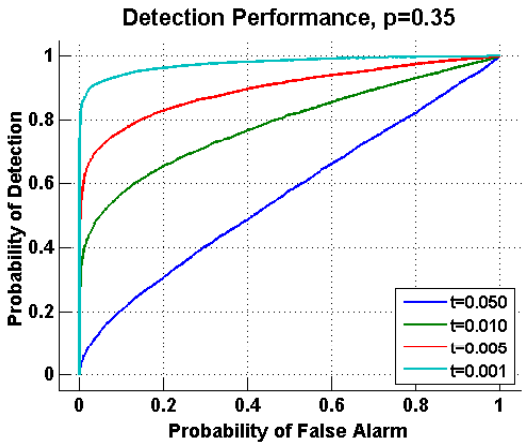












Detection performance improves as background strength decreases (across plots) and correlation with the background decreases (within plots)

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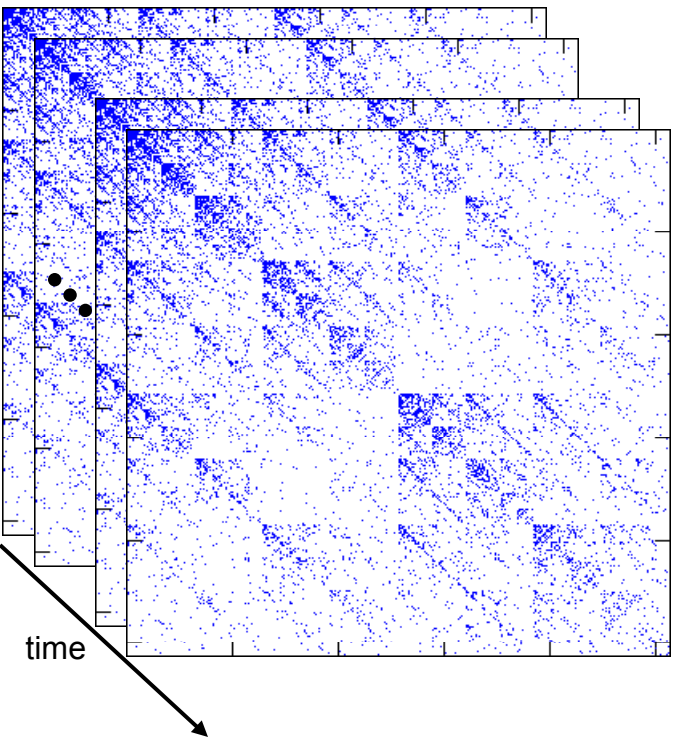
Dynamic Graph Processing Chain

Processing chain for *dynamic* subgraph detection incorporates *temporal integration*

MODULARITY MATRIX CONSTRUCTION → TEMPORAL INTEGRATION → EIGEN DECOMPOSITION → COMPONENT SELECTION → DETECTION → IDENTIFICATION

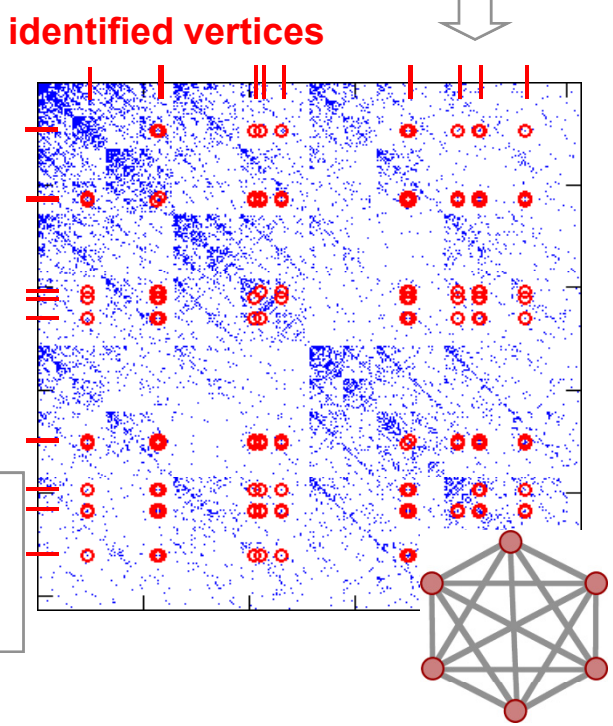


Input:

- ***A(t)*, *time-varying* adjacency matrix representation of *G***
- ***No cue***

Output:

- **v_s, set of vertices identified as belonging to subgraph G_f**



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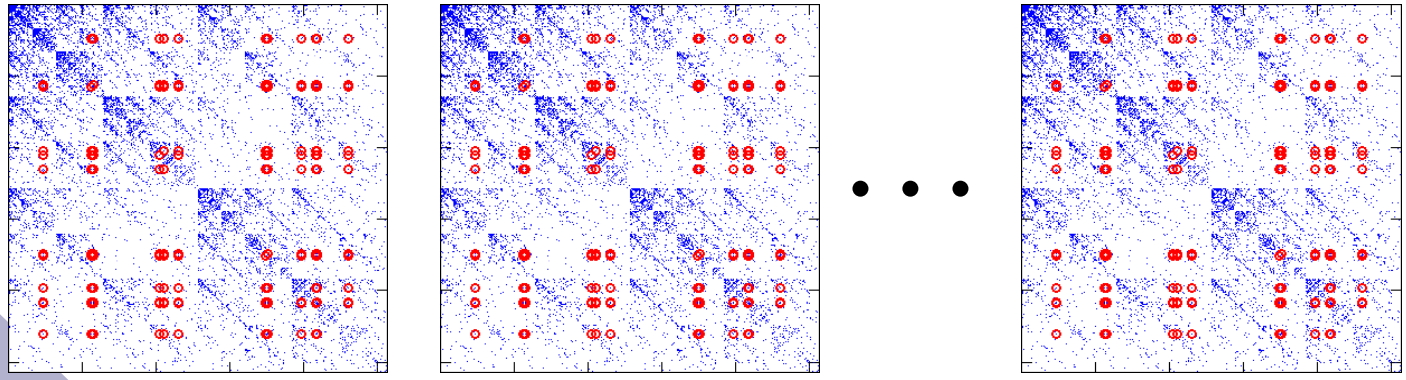
Temporal Integration

MODULARITY MATRIX CONSTRUCTION → **TEMPORAL INTEGRATION** → EIGEN DECOMPOSITION → COMPONENT SELECTION → DETECTION → IDENTIFICATION



Modularity (residuals) matrix at time *n*

$$\frac{B(n)}{h(0)} + \frac{B(n-1)}{h(1)} + \cdots + \frac{B(n-\ell+1)}{h(\ell-1)}$$

Filter coefficients (scalars)

$$\tilde{B}(n) = \sum_{k=0}^{\ell-1} h(k) B(n-k)$$: **Weighted sum of residuals**

Temporal integration achieved by creating a linear combination of individual time-step graphs

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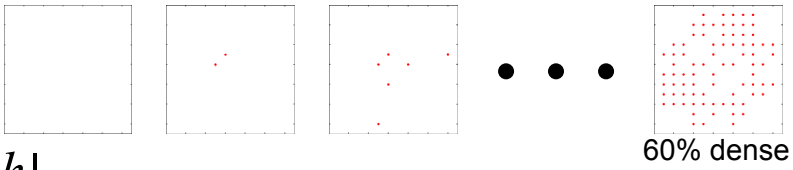
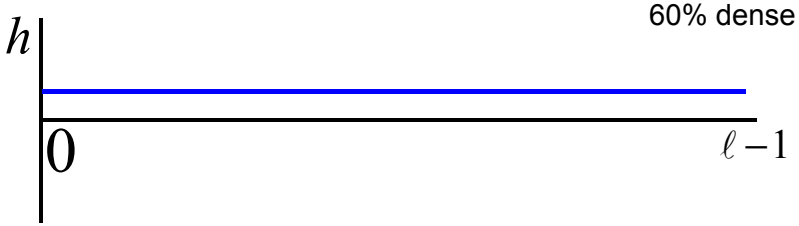
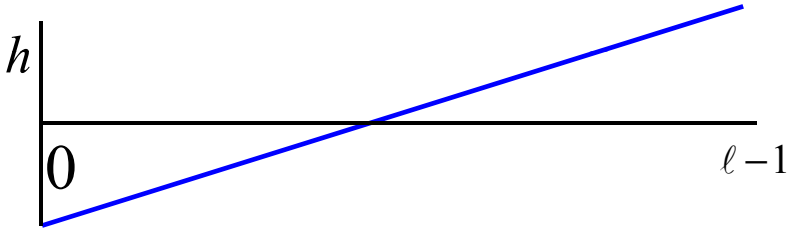
Filter Design

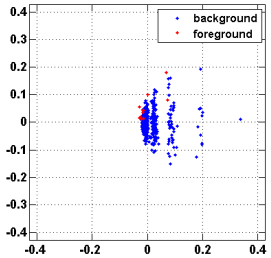
Foreground: subgraph with linearly increasing density

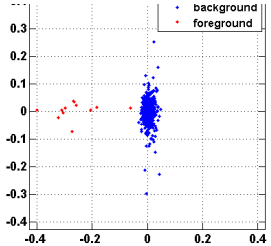


No cue **to foreground vertices; entire graph is filtered**









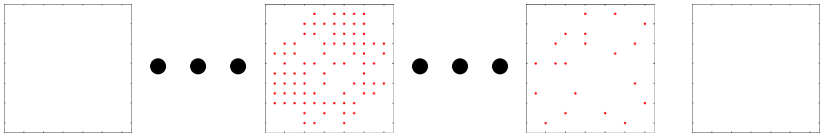
Test Statistic

0.2

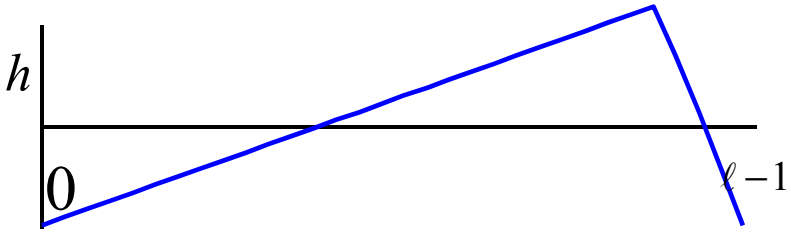
210.6

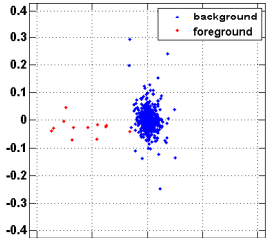
- FG vertex
- BG vertex
- ⬆ test stat. better

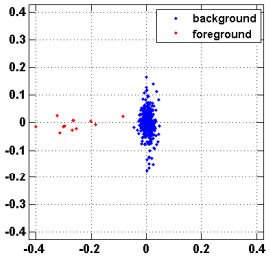
Foreground: subgraph that densifies and disperses



Foreground behavioral information yields better separation







51.2

183.0

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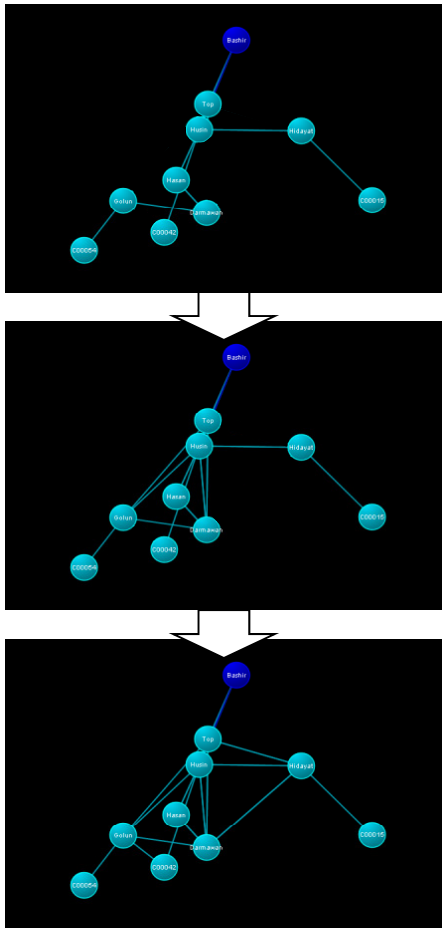
Application: Threat Network Evolution Detection

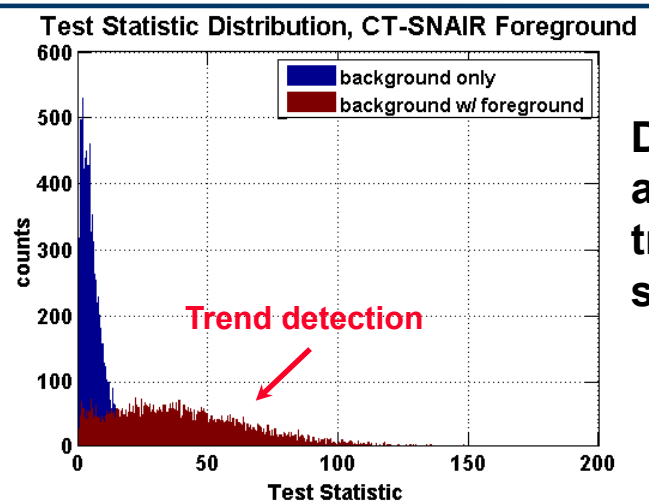
recruitment

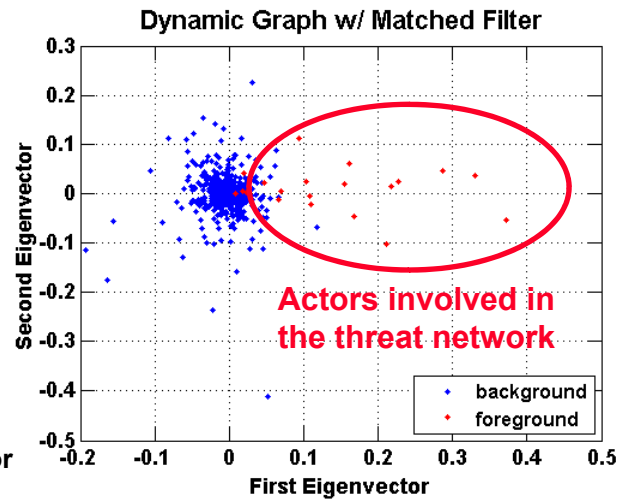
planning

execution



*Threat network foreground created under the Counter Terror Social Network Analysis and Intent Recognition effort





CONOPS:

Detect anomalous trends in a social network

Identify actors (individuals) involved

Time integration allows for identification of weak signature coherent activity

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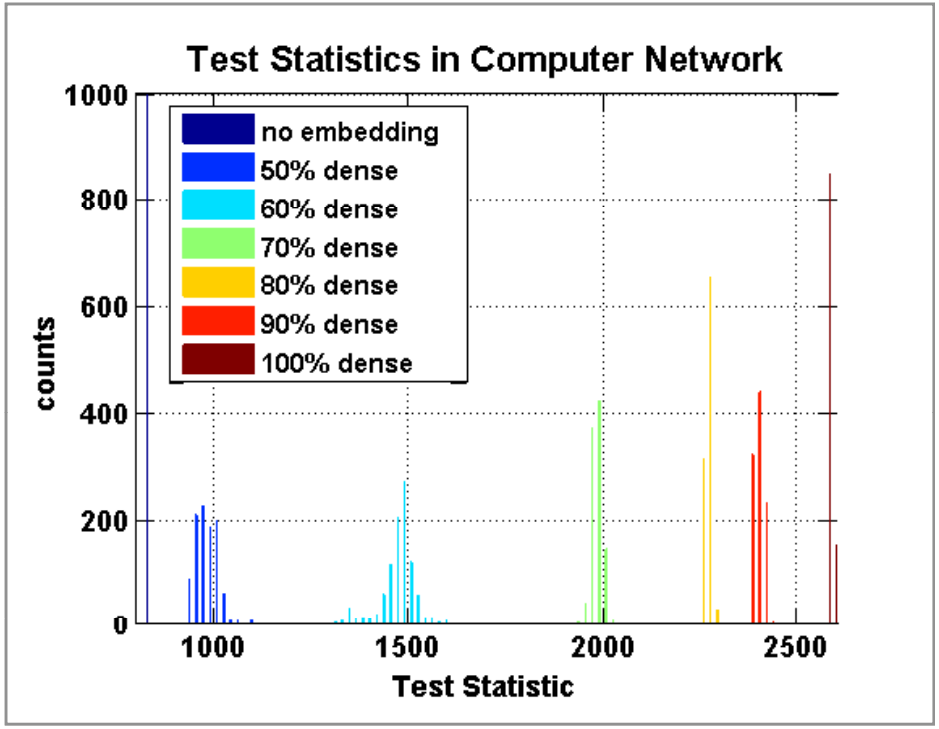
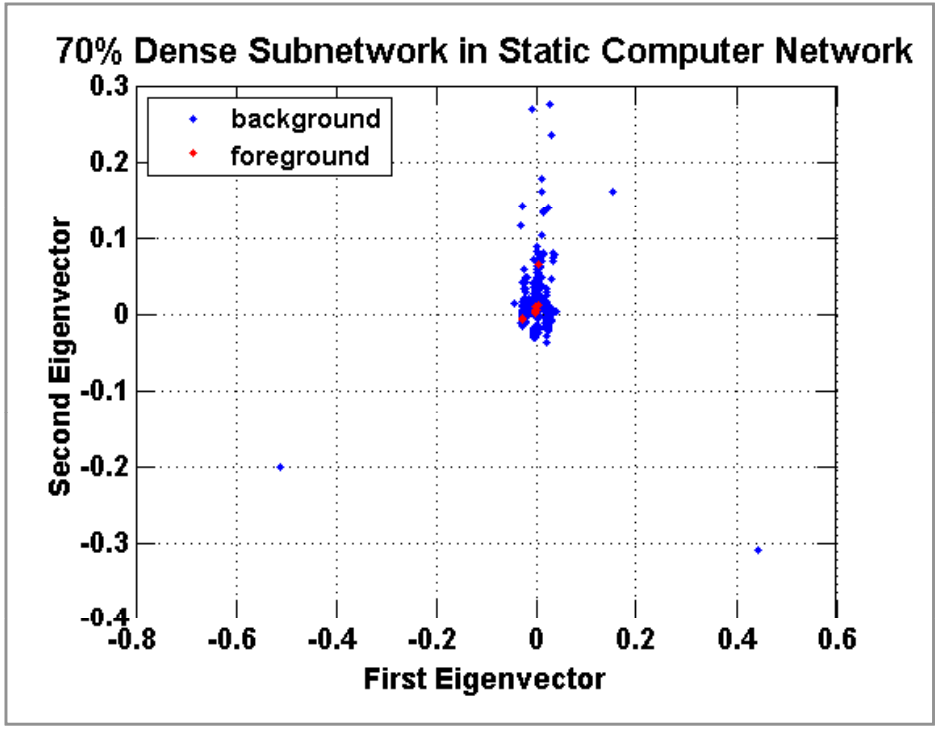
Application: Cyber Security





- Anomaly detection in computer networks can help counter cyber attacks
- Background: Real-life, dynamic, publicly-available* computer network (~3200 nodes, powerlaw)
- Foreground: Increasingly dense subgraph; simulating a subnetwork with exceptionally high inter-node communication

*Available on SNAP: http://snap.stanford.edu

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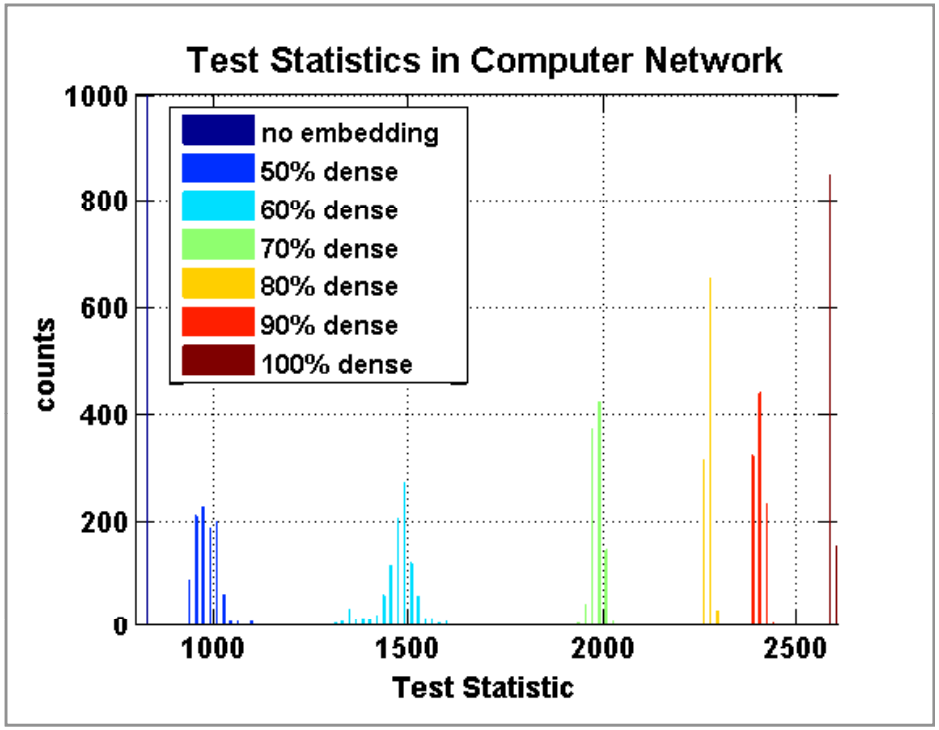
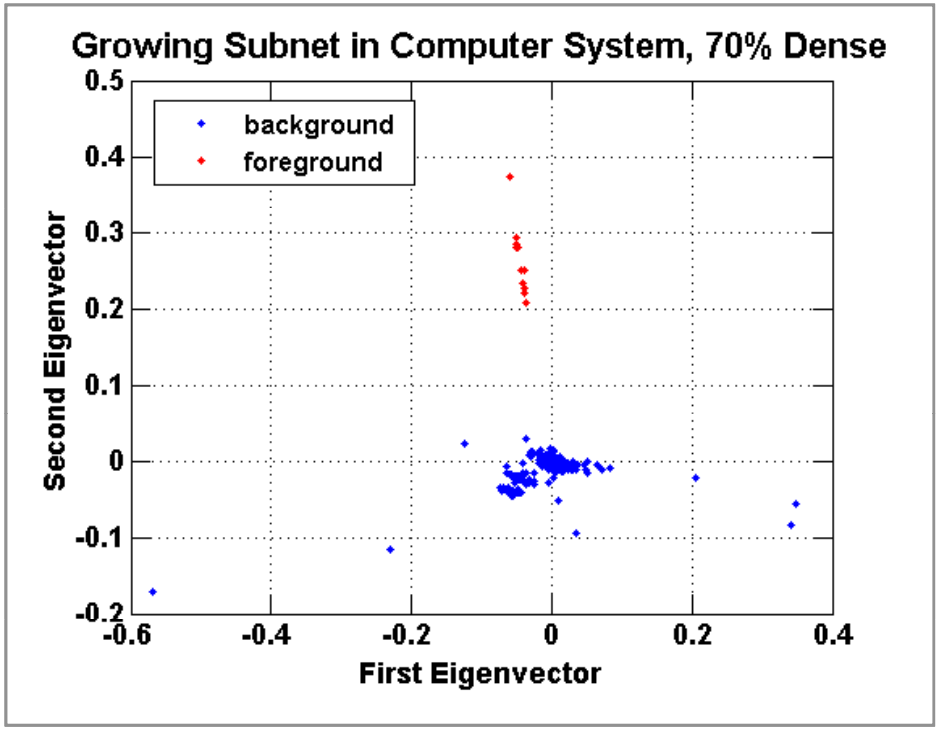
Application: Cyber Security





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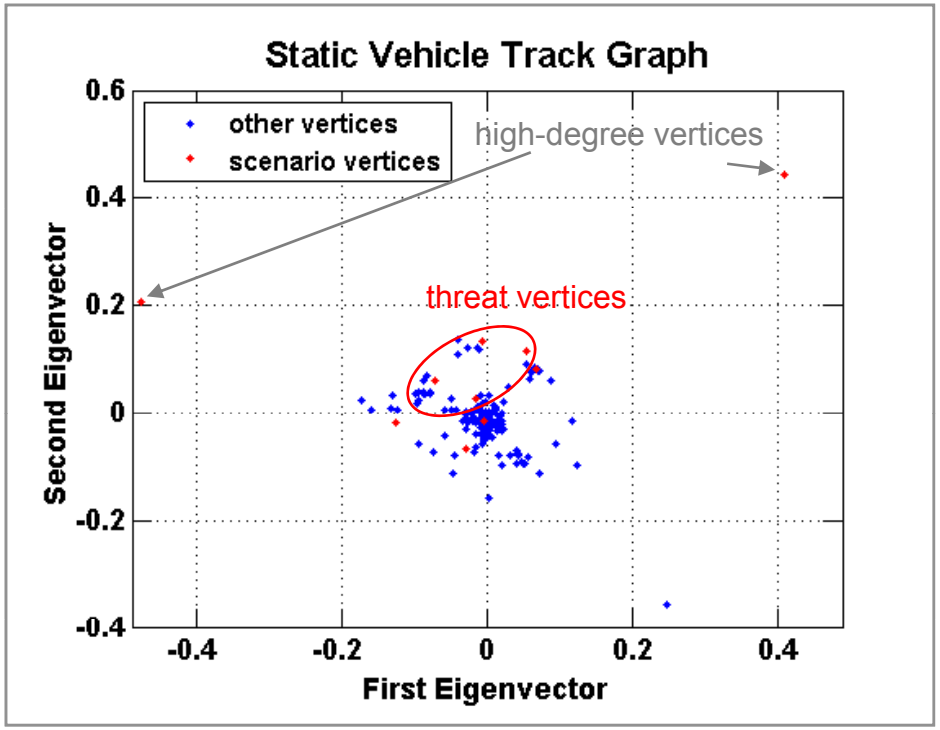
Application: Vehicle Track Analysis

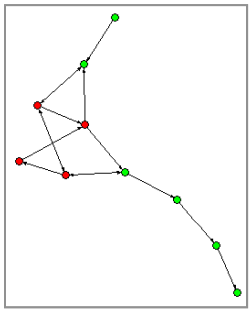


- Vehicle track graph generated using tracks from an EO dataset
- Vertices are sites, edges represent tracks
- Temporal data (i.e., when the tracks begin and end) is also recorded
- Scripted threat scenario among tracks

Scripted scenario:

threat vertices

other vertices



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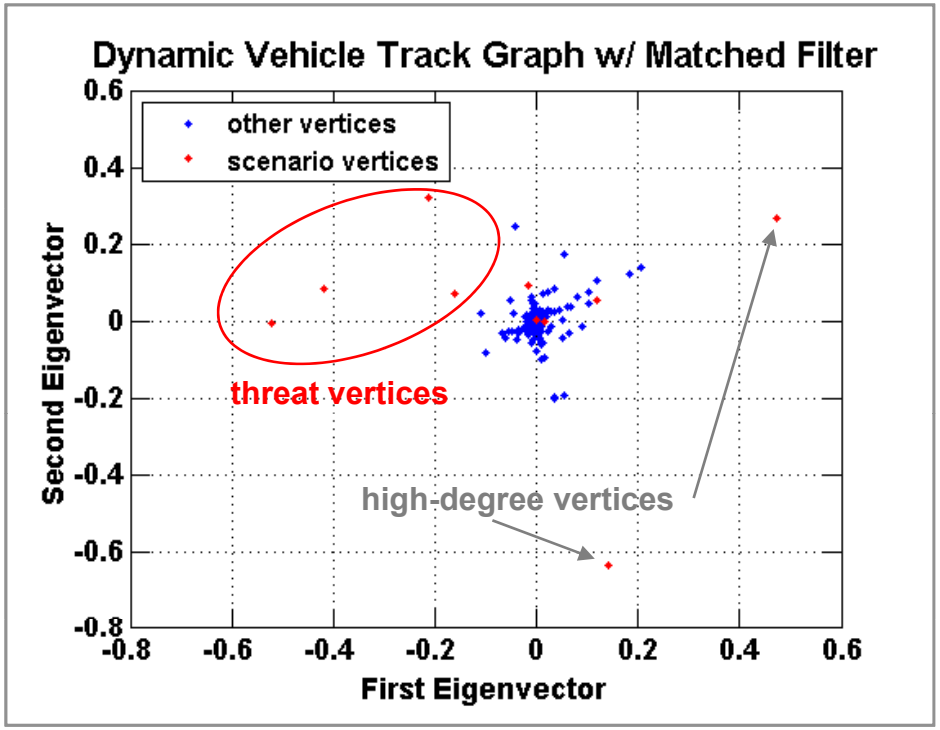

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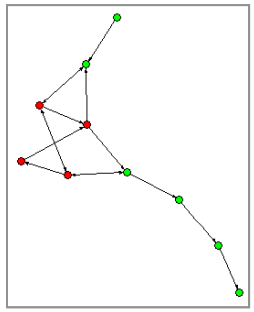

Summary

- Graphs and networks are key data structures for representation and analysis of relationships in complex multi-INT datasets
- Subgraph detection is an important problem for IC and DoD
- Detection framework for graphs enables algorithms and metrics development
- Results on simulated and real datasets demonstrate the power of the approach
- Background statistics (noise and clutter model) and time domain integration are of key importance in detecting weak signatures