

# Accuracy/Precision of USNO Predicted Clock Estimates for GPS Satellites

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**Abstract**—The United States Naval Observatory (USNO) Earth Orientation Department GPS Analysis Division (GPSAD) processes data from hundreds of GNSS receivers daily, computing multiple GNSS carrier-phase-based product sets 365 days/year. Quantities estimated typically include high-precision (1-3 cm) GNSS satellite orbits/clock corrections, receiver clock corrections, station coordinates, earth-orientation parameters (EOPs), and troposphere-delay estimates.

The goal of the research presented herein is to improve the accuracy of USNO “ultra-rapid” satellite clock predictions. Reason: Precise point positioning (PPP [1]) experiments conducted in Summer/Fall 2012 using USNO ultra-rapid predicted satellite clocks, satellite orbits and EOPs [2][3] indicated that improved position accuracy (output) would require improved satellite clock prediction accuracy (input).

The current USNO clock prediction algorithm uses linear modeling. A quadratic prediction algorithm tested degraded the accuracy of satellite clock predictions rather than improving it. Error analysis of results obtained using the extant linear prediction algorithm suggested that prediction errors for Block IIA satellites equipped with Cs clocks were larger than prediction errors associated with other satellites, consistent with results obtained in [2]. We were unable to identify an error source uniquely impacting those satellites. However, we demonstrated that PPP positioning might be improved by deleting measurements from Block IIA satellites. This suggests that PPP position errors caused by poorly estimated satellite clocks could be mitigated by de-weighting or deleting the measurements associated with those satellites.

## I. INTRODUCTION

The United States Naval Observatory (USNO) Earth Orientation Department hosts an International GNSS Service (IGS) analysis center in its GPS Analysis Division (GPSAD). GPSAD processes data from hundreds of IGS GNSS receivers daily, computing multiple GNSS carrier-phase-based product sets 365 days/year. Quantities estimated typically include high-precision (1-3 cm) GNSS satellite orbits/clock corrections, receiver clock corrections, station coordinates, earth-orientation parameters (EOPs), and troposphere-delay estimates.

We focus on USNO “ultra-rapid” satellite clock predictions in this paper. USNO ultra-rapid orbit/clock/EOP estimates span 48 hours, with 24 hours of post-processed values derived from the GNSS measurements and 24 hours of predictions. These estimates/predictions are (a) completed less than two hours after the last GNSS measurements are recorded, and (b) updated every six hours, meaning high-accuracy values are continuously available for use in real-time applications. Tests conducted comparing the precision of USNO ultra-rapid predictions to values available in the GPS broadcast message over 1 June 2010 – 31 May 2011 showed 44 mm RMS for USNO orbit predictions vs 909 mm for GPS broadcast orbits [4].

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In Summer 2012, USNO tested positioning/timing uncertainty obtained when performing PPP processing [2] using USNO ultra-rapid predicted orbits/satellite clocks/EOPs as input. This exercise yielded coordinate precision of 7-9 cm, compared to  $\sim 1$  cm precision obtained using post-processed IGS Final [5] orbits/satellite clocks/EOPs. Subsequent error analysis [2], [3] demonstrated that improvement in coordinate precision/accuracy required reduction in satellite clock (prediction) error.

In this study, we investigate the quality of USNO satellite clock predictions in more detail, in order to ascertain which issues limit our ability to predict satellite clock corrections and how these issues might be addressed.

## II. METHOD

We analyzed clock predictions for 14-17 Oct 2012 (GPS week 1710; MJDs 56214-17), assessing prediction error for the entire 24 hours of the prediction, rather than the first six hours as was done in [2], [4], [6], because we wished to know if users could meet PNT requirements if forced to rely on USNO ultra-rapid products through the end of the products' prediction span. Such a situation could occur, e.g., in a data-transfer outage. We computed the error in USNO predicted satellite clock values by comparing the USNO predicted estimate of (PRN XX – USN3) to the IGS Final estimate of the same quantity.<sup>1</sup> “USN3” refers to UTC(USNO) as disseminated through the IGS GPS receiver labeled “USN3.”

USNO estimates/predicts GPS carrier-phase-based satellite-clock values using the *Bernese GPS Software* (version 5.0 at this writing) [9]. Post-processed satellite- and receiver-clock values for the first 24 h of the ultra-rapid time span are estimated in a network processing mode using a zero-average-correction of all system clocks as the reference. Measurements from approximately 34 ground-based receivers are used. Then satellite- and receiver-clock values are predicted for the subsequent 24 h. At present, a linear fit/prediction algorithm is used; twice-per-satellite-orbit periodic terms are also estimated. Both the degree of the polynomial fit/prediction and the number of per-orbit periodic terms can be varied.

In this study, we first attempted to improve satellite clock predictions by using a quadratic rather than a linear model to fit the data and provide the predictions. This seemed logical because satellite clocks sometimes exhibit quadratic drift in addition to a linear frequency difference with respect to system time, and removing this drift reduces residuals by a few ns – the size of the satellite clock prediction error we wish to mitigate. (See Fig. 1 for an example.)

But as the results will show, and for reasons postulated in “Discussion,” our implementation of a quadratic fit/prediction algorithm degraded rather than improved prediction error. We therefore performed further error analysis on results obtained using the existing linear modeling.

## III. RESULTS

### A. Test of quadratic prediction algorithm

Using a quadratic function to predict satellite clock values caused greater error in satellite clock predictions than did using the current linear function. Fig. 2 compares results obtained using linear and quadratic prediction for PRN 1 and 18. Whereas the PRN1-USN3 prediction error obtained using the USNO linear prediction algorithm is on the order of 200 ps, the prediction errors obtained using the test quadratic algorithm are biased and grow as large as 9 ns. Similar results (0-2 ns linear; up to 10 ns quadratic) are shown for PRN 18. It is interesting that on MJD 56214, the linear and quadratic algorithms produce similar errors.

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<sup>1</sup> The IGS released new values of its Final Products (clock values, satellite orbits, EOPs) for GPS weeks 1702-1715 after the conclusion of this paper's research, citing a processing error [7]. Correspondence with the IGS Analysis Center Coordinator [8] indicated that the new final clock values lay within 1-2 ps RMS of the originally-released values, thus not altering the conclusions of this paper.

Fig. 3 shows the median of the daily RMS errors for each satellite for both the linear and quadratic prediction algorithms. The RMS errors are again visibly larger when the quadratic algorithm is used, with an across-day-across-satellite median RMS error of 5 ns for the quadratic algorithm compared to 1 ns for the linear algorithm. Testing of the quadratic prediction algorithm was stopped at this point.

#### *B. Error sources in the present linear prediction algorithm*

To discover what problem needed solving, we searched the linear-prediction-algorithm results for satellites for whom the USNO clock-prediction error exceeded 10 ns. PRN 3, 8, 9, 10, and 30 met this criterion, i.e., all of the Block IIA satellites equipped with Cs clocks plus PRN 30 (Block IIA with Rb clock).

This is consistent with [2] in which the median RMS agreement of USNO predicted satellite-clock estimates with IGS Final Clock Estimates was approximately 1 ns when computed across all satellites, but approximately 4 ns when computed across Block IIA satellites only.

Figs. 4a-c show the satellite clock prediction errors; those of PRN 9 and 30 are the largest. Fig. 5 shows the RMS satellite prediction error for each satellite and each day studied. It is clear, particularly from Fig. 5, that prediction errors are largest for the Block IIA satellites.

We then began to examine why USNO algorithms are less accurate when handling Block IIA measurements.

We first asked if USNO post-processed estimates for Block IIA clocks were markedly less accurate than those for other blocks, hypothesizing that this might drive prediction inaccuracy. But Fig. 6, which shows the error in both the post-processed and predicted portion of each satellite's clock estimate, indicates no clear correlation between prediction error and post-processed accuracy. In fact, some satellites with large prediction errors have comparatively small post-processed errors.

We next hypothesized that Block IIA predictions were inaccurate relative to the (IGS Final Estimate) truth standard because the associated post-processed values were biased relative to that truth standard. Fig. 7 shows however that post-processed clock estimates of satellites from several different blocks were biased – not just those from the Block IIAs – and so this is unlikely to be the sole cause of the problem. (A table linking PRN number to block is available at [10].)

The mechanism causing USNO Block IIA clock predictions to be estimated less accurately than those of other blocks remains undiscovered. However, we have as of late devised a processing approach potentially mitigating the impact of Block IIA clock-prediction errors.

The goal of the present experiment was to improve satellite clock prediction so as to improve PPP positioning accuracy obtained using predicted satellite clocks. Fig. 8 shows results of a study [3] in which measurements simulated using IGS Final satellite clocks/orbits/EOPs were processed using IGS predicted satellite clocks (in tandem with the IGS Final orbits/clocks used in measurement simulation), with Fig. 9 showing the satellite clock error introduced by doing so (and again that Block IIAs are the worst offenders). Fig. 8a shows that introducing these clock errors induces RMS north, east, up positioning errors of 6, 14 and 8 cm, respectively. Fig. 8b shows however that simply omitting Block IIA observations from the data processing halves these errors to 3, 7 and 4 cm, respectively without negatively impacting coordinate precision.

A PPP processing strategy in which the data from satellites whose clocks are poorly estimated are de-weighted or deleted merits further investigation.

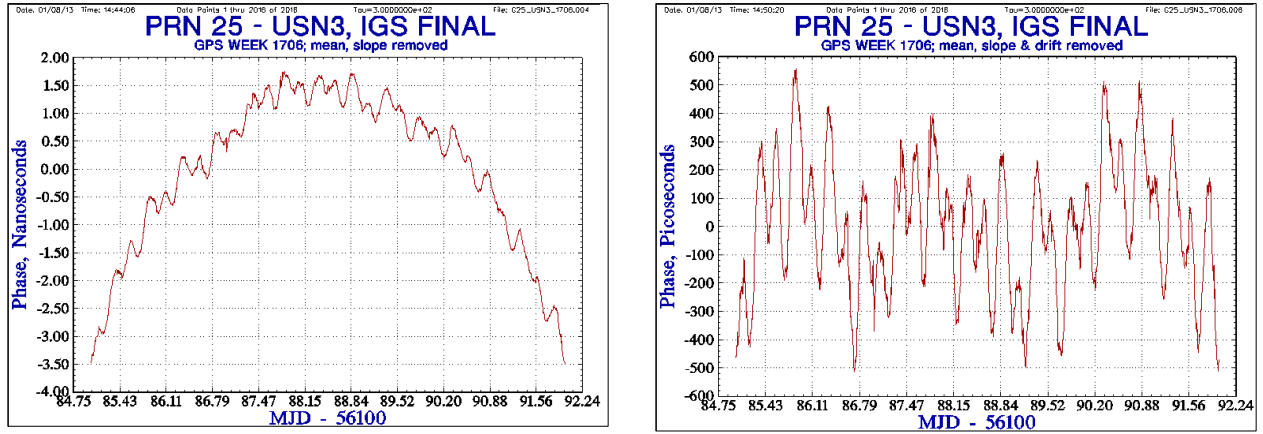


Figure 1. Motivation for testing quadratic rather than (current) linear fit/prediction regime: removing a linear fit from PRN 25 – USN3 leaves a 5-ns quadratic residual (left); further removing quadratic drift reduces the residuals to a few hundred ps (right). GPS week 1706 (16–22 Sep 2012; MJDs 56185–91) shown above. Calculations performed using *Stable 32*; <http://www.stable32.com>.

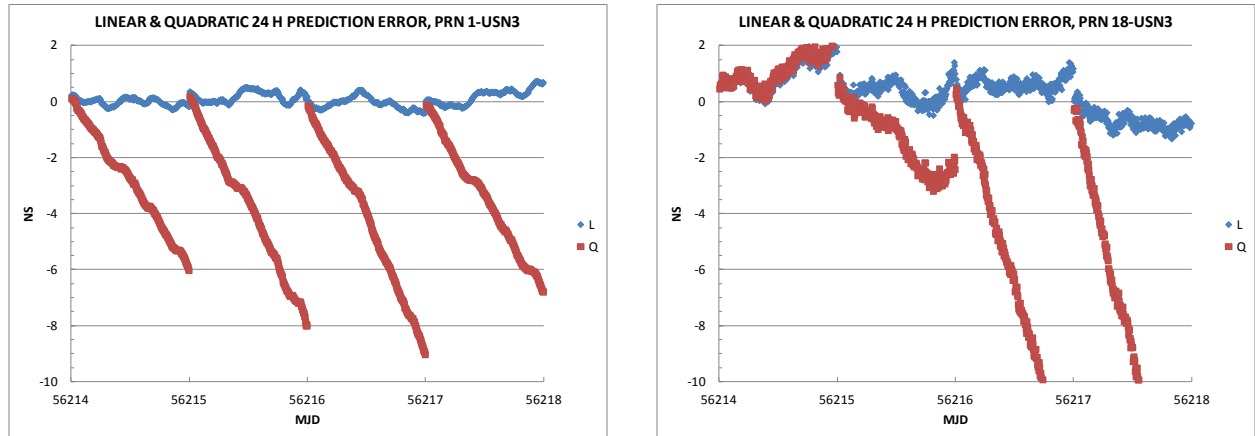


Figure 2. Prediction errors, PRN 1 and 18, obtained using linear (“L”) and quadratic (“Q”) fit/extrapolation algorithms, 14–17 Oct 2012. In each plot, the IGS final estimate of (PRN XX – USN3) is subtracted from the USNO predicted value thereof.

#### IV. DISCUSSION

The quadratic drift of PRN 25 shown in Fig. 1 notwithstanding, [11] indicates that the GPS Control Segment [12] removes the drift of rubidium clocks aboard GPS satellites prior to said clocks being used to broadcast a signal. The systematic effect caused by the residual drift would then lie below the stochastic noise of the clock; measurements broadcast using this clock and then recorded by a GPS receiver would thus show virtually no satellite-clock drift relative to stochastic satellite-clock noise. The poor results of the quadratic experiment could be explained if we were indeed attempting to remove systematic signals subsumed by stochastic noise from a significant fraction of the GPS constellation.

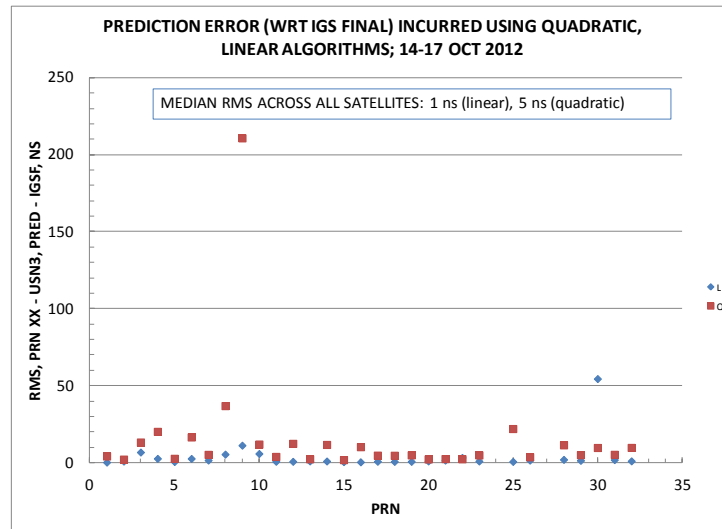
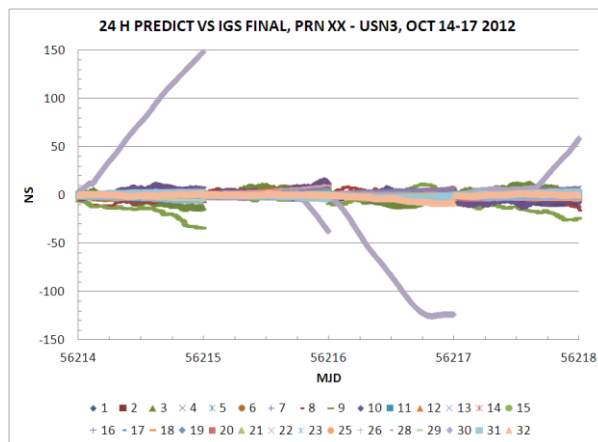
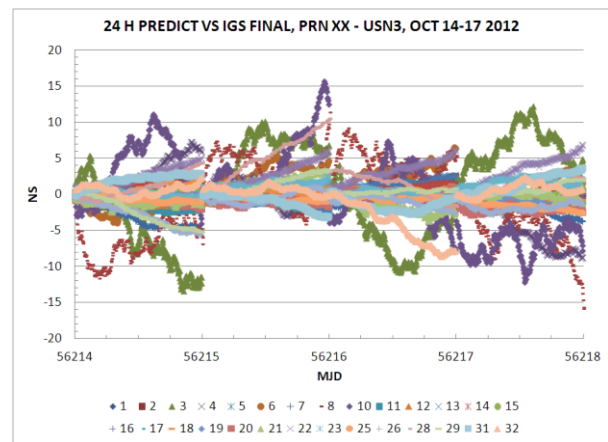


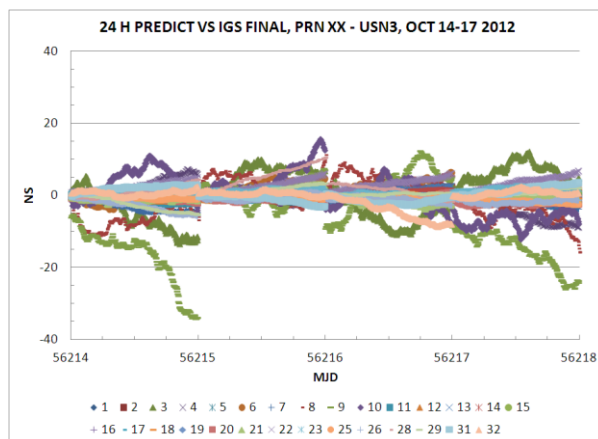
Figure 3. Median of daily RMS prediction errors obtained using linear (“L”) and quadratic (“Q”) fit/extrapolation algorithms, 14-17 Oct 2012. IGS Final estimates used as truth standard.



a) All satellites



c) PRN 30, 9 removed



b) PRN 30 removed

Figure 4. Satellite clock prediction errors obtained using USNO’s current linear fit/extrapolation algorithm, 14-17 Oct 2012. Legend numbers refer to satellite PRN code.

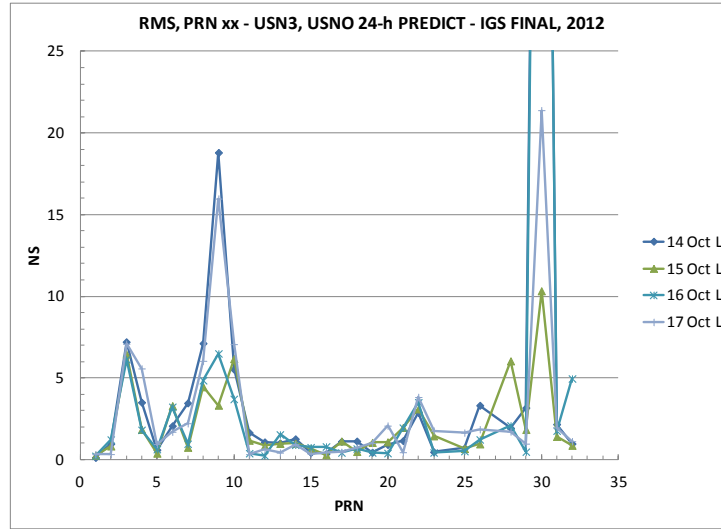


Figure 5. RMS of satellite clock prediction errors shown in Fig. 4, sorted by satellite and day.



Figure 6. RMS error of USNO clock estimates, post-processed (“post”) and predicted (“pred”), each day of 15-17 Oct 2012 and overall. Vertical lines mark PRNs 3, 8, 9, 10 and 30. Prediction error does not seem correlated with error of post-processed values.

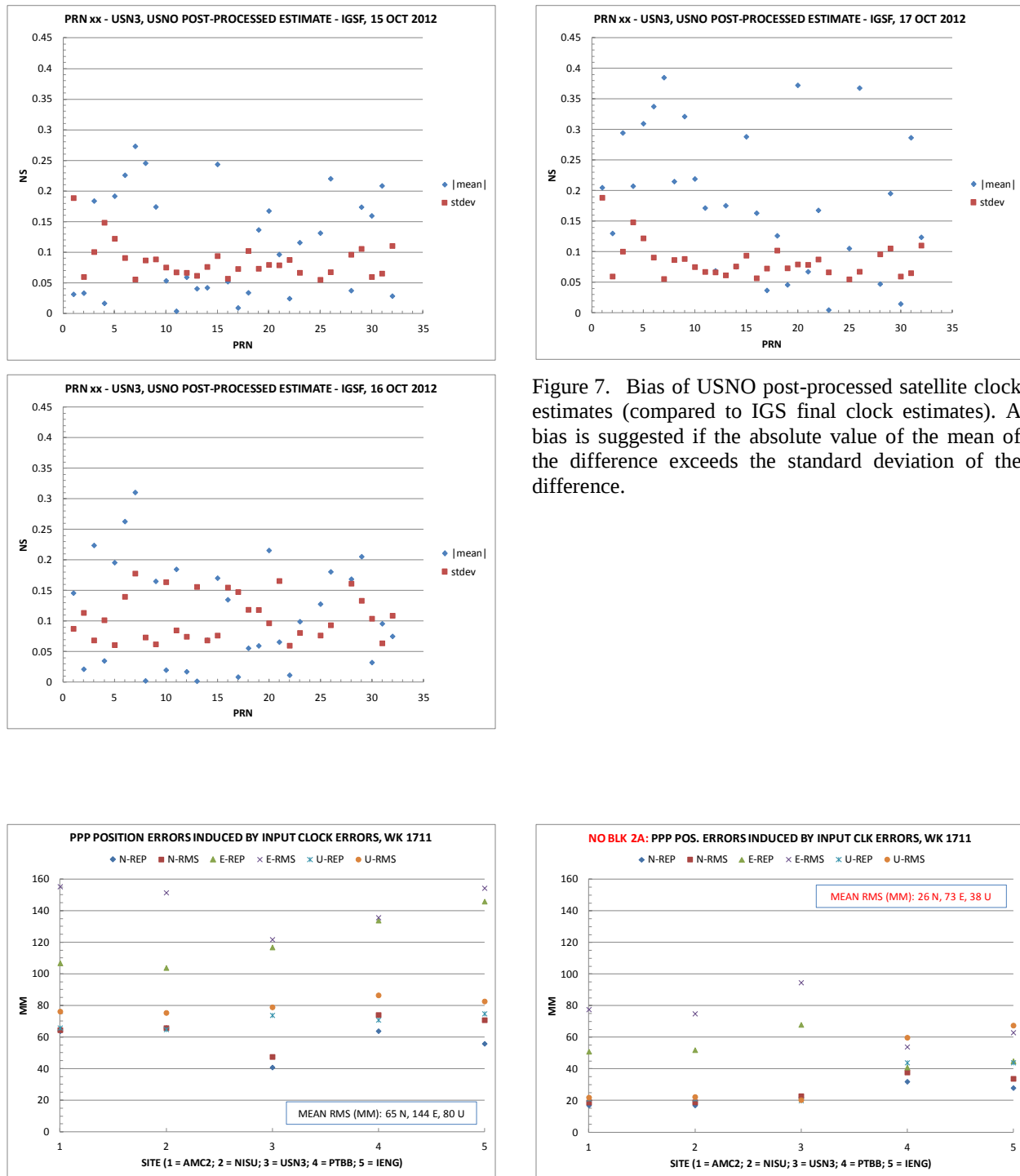


Figure 8. Mitigation of PPP positioning errors through deletion of Block IIA measurements, 21-27 October 2012 [3]. “REP” denotes coordinate repeatability (precision) in the north (N), east (E) or up (U) direction; “RMS” indicates RMS position accuracy. In this simulation study, mean RMS position error was halved without degrading precision by removing Block IIA measurements. Receiver AMC2 is located in Colorado Springs, CO; NISU in Boulder, CO; USN3 in Washington, DC; PTBB in Braunschweig, Germany and IENG in Torino, Italy.

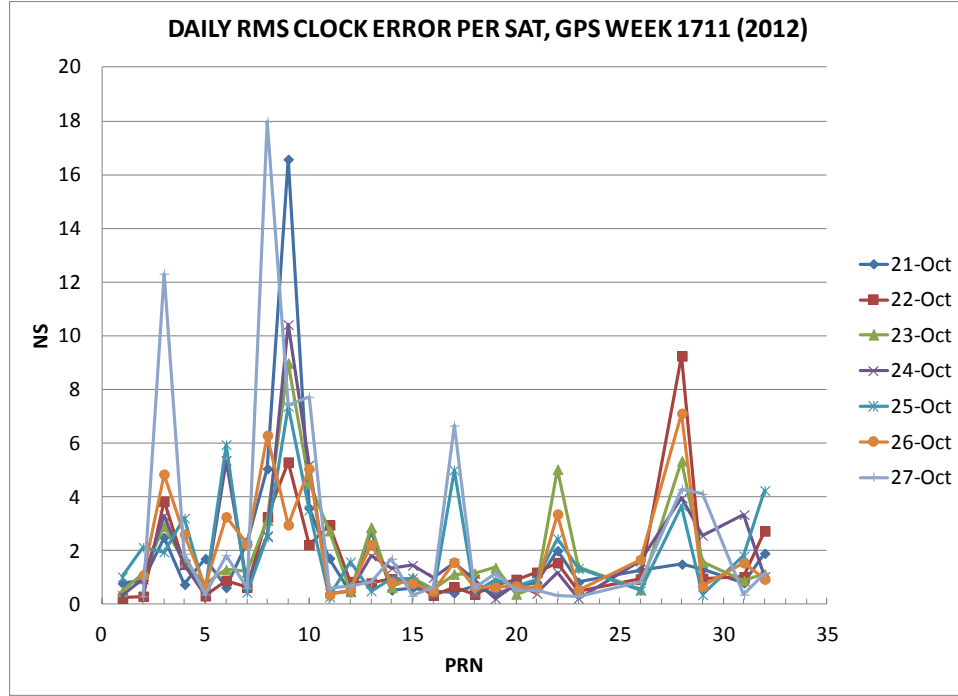


Figure 9. Daily RMS error of IGS ultra-rapid satellite clock predictions, 21-27 October 2012 (GPS week 1711; MJDs 56221-7). Error was computed by subtracting IGS ultra-rapid predicted and IGS final estimates of (PRN XX – USN3).

## V. CONCLUSIONS

Attempts were made to mitigate the errors in USNO satellite clock predictions, with the goal of improving PPP positioning accuracy achieved using USNO predicted satellite orbits, satellite clocks and earth-orientation parameters.

The current USNO clock prediction algorithm uses linear modeling. A quadratic prediction algorithm tested degraded the accuracy of satellite clock predictions rather than improving it. Error analysis of results obtained from the present linear prediction algorithm suggested that prediction errors for Block IIA satellites equipped with Cs clocks are larger than prediction errors associated with other satellites. We were not able to identify an error source affecting only these satellites. However, we showed that PPP positioning might be improved by deleting measurements from Block IIA satellites. This suggests that PPP position errors caused by poorly estimated satellite clocks could be mitigated by de-weighting or deleting the measurements associated with those satellites.

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## DISCLAIMER

Although some software and models are identified for the purpose of scientific clarity, their use is not an endorsement by the USNO or the Department of the Navy.

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