

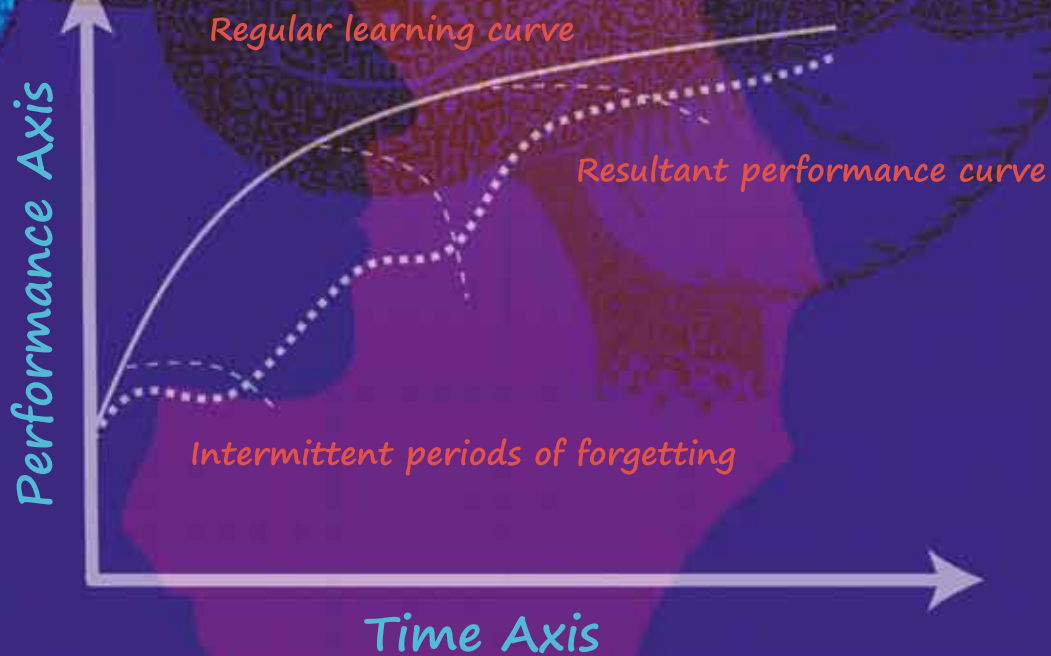
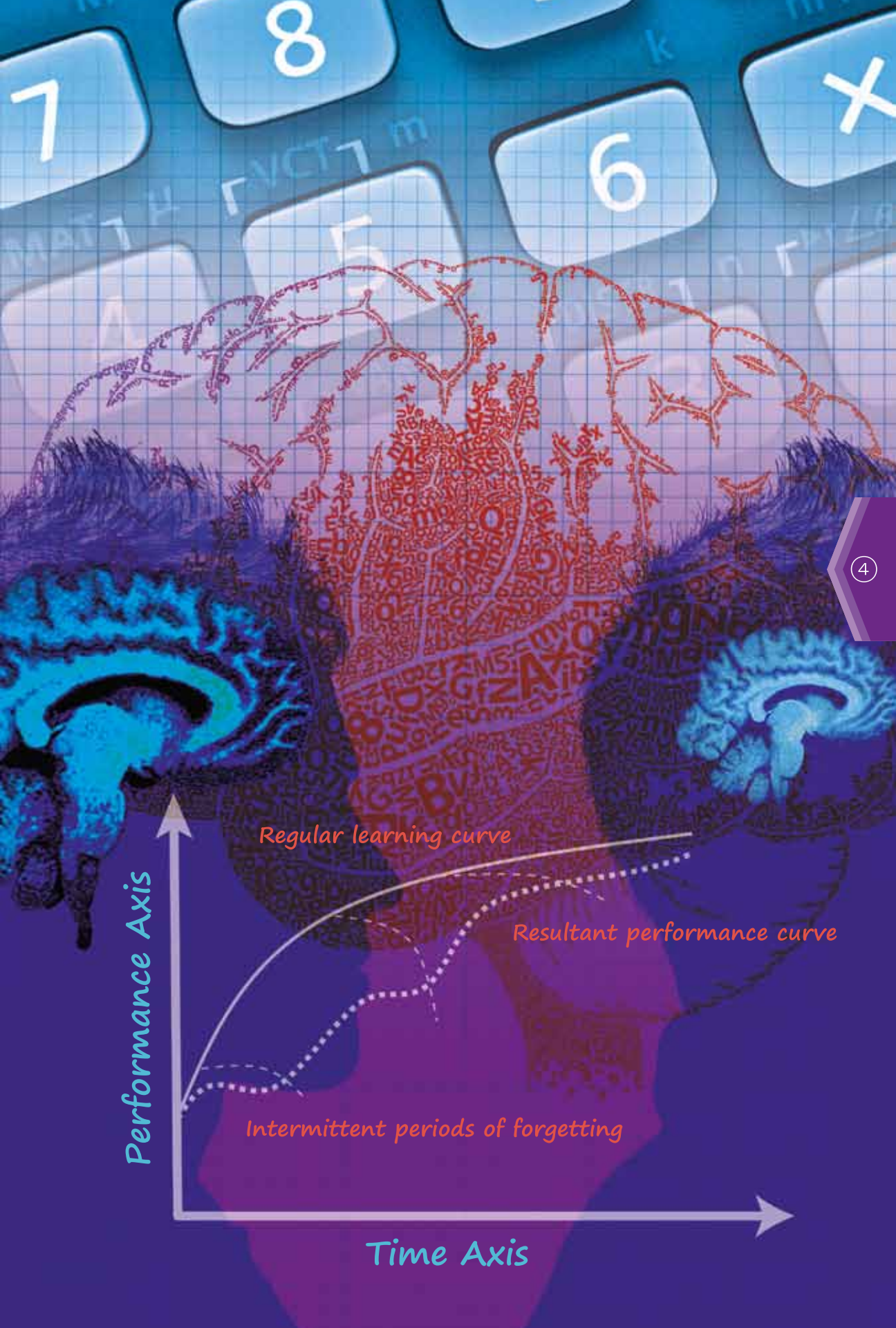
Keywords: *Learning Curve, Half-Life, Learn-Forget Models, Performance, Training, Acquisition*

Half-Life Learning Curves in the Defense Acquisition Life Cycle

 **Adedeji B. Badiru**

Learning curves are useful for assessing performance improvement due to the positive impact of learning. In recent years, the deleterious effects of *forgetting* have also been recognized. Workers experience forgetting or decline in performance over time. Consequently, contemporary learning curves have attempted to incorporate forgetting components into learning curves. An area of increasing interest is the study of how fast and how far the forgetting impact can influence overall performance. This article introduces the concept of half-life analysis of learning curves using the concept of growth and decay, with particular emphasis on applications in the defense acquisition process. The computational analysis of the proposed technique lends itself to applications for designing training and retraining programs for the Defense Acquisition Workforce.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE JUL 2012		2. REPORT TYPE		3. DATES COVERED 00-00-2012 to 00-00-2012	
4. TITLE AND SUBTITLE Half-Life Learning Curves in the Defense Acquisition Life Cycle				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Defense Acquisition University,9820 Belvoir Road, Suite 3,Fort Belvoir,VA,22060-5565				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 26	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			



THE ILLITERATE OF THE 21ST CENTURY WILL NOT BE
THOSE WHO CANNOT READ AND WRITE, BUT THOSE
WHO CANNOT LEARN, UNLEARN, AND RELEARN.
—ALVIN TOFFLER

Formal analysis of learning curves first emerged in the mid-1930s in connection with the analysis of the production of airplanes (Wright, 1936). Learning refers to the improved operational efficiency and cost reduction obtained from repetition of a task. Learning curves have been used for decades to assess improvement achieved over time due to the positive impacts of learning. Early analytical modeling of learning curves focused on reduction in cumulative average cost per unit as production level doubles. Several alternate models of learning curves have been presented in the literature of the decades. The classical models have been successfully applied to a variety of problems. In recent years, the deleterious effects of forgetting have also been recognized. It has been shown that workers experience forgetting or decline in performance even while they are making progress along a learning curve. Consequently, contemporary learning curves have attempted to incorporate forgetting components into learning curves. An area of increasing interest is the study of how fast and how far the forgetting impact can influence overall performance.

This article presents the concept of half-life analysis of learning curves (Badiru, 2010), using the concept of *growth* and *decay* of learning in the acquisition environment. Half-life is the amount of time it takes for a quantity to diminish to half of its original size through natural processes. Although the common application of half-life is in natural sciences, the computational analysis lends itself to applications to learning curves. Several research and application studies have confirmed that human performance improves with reinforcement or frequent and consistent repetitions. Badiru (1992, 1994) provides a computational survey of learning curves as well as industrial application to productivity and performance analysis. Reductions in operation processing times achieved through learning curves can directly translate to cost savings. In today's technology-based operations, retention of learning may be threatened by fast-paced shifts in operating requirements. Thus, those involved in computational analysis of learning curves may find it of benefit to study the half-life properties of learning curves. Informa-

tion about the half-life can tell us something about the sustainability of learning-induced performance. This is particularly useful for designing training programs and assessing workers' performance.

Concept of Growth and Decay

Growth and decay occur naturally in many processes. We often speak of “twice as much” and “half as much” as benchmarks for process analysis. In economic and financial principles, the “rule of 72” refers to the length of time required for an investment to double in value. These common “double” or “half” concepts provide the motivation for examining half-life properties of learning curves. The usual application of half-life is in natural sciences. For example, in Physics, the half-life is a measure of the stability of a radioactive substance. In practical terms, the half-life of a substance is the time it takes for the substance to decay to half of its initial size. The longer the half-life of a substance, the more stable it is. This provides a good analogy for modeling learning curves with the recognition of increasing performance or decreasing cost with respect to the passage of time. For purposes of this article, the following key definitions are provided:

- *Half-life of a learning curve* is the incremental production level required to reduce cumulative average cost per unit to half of its initial level.
- *Half-life of a forgetting curve* is the amount of time it takes for performance to decline to half of its initial level.

Literature on Learning Curves

Although an extensive collection exists of classical studies of performance *improvement* due to learning curves, only very limited attention has been paid to performance *degradation* due to the impact of forgetting. Some of the classical works on process improvement due to learning include Smith (1989); Belkaoui (1976, 1986); Nanda (1979); Pegels (1976); Richardson (1978); Towill, and Kaloo (1978); Womer (1979, 1981, 1984); Womer and Gullledge (1983); Camm, Evans, and Womer (1987); Liao (1979); McIntyre (1977); Smunt (1986); Sule (1978); and Yelle (1976, 1979, 1983). Only in recent years has the recognition of “forgetting” curves begun to emerge, as can be seen in more recent literature (Badiru, 1995; Jaber & Sikstrom, 2004; Jaber, Hemant, & Darwin, 2003; Jaber &

Bonney, 2003, 2007; Jaber & Guiffrida, 2008). The new and emerging research on the forgetting components of learning curves provides the motivation for studying half-life properties of learning curves. Performance decay can occur due to several factors, including lack of training, reduced retention of skills, lapse in performance, extended breaks in practice, and natural forgetting.

The Acquisition Learning Framework

It is a natural process for people to learn, unlearn, and relearn. Capturing this process in a quantitative framework is essential for making effective decisions in any operation, particularly in the defense acquisition environment, where human-machine interfaces are common.

Defense acquisition endeavors often get behind schedule, exceed cost baselines, and/or exhibit poor performance. Many of these problems have their sources in the human elements within the acquisition life cycle. Ward (2010, 2012), using his FIST (Fast, Inexpensive, Simple, and Tiny) model, calls for rapid acquisition using the concept of “80% now is better than 100% later.” This perfectly fits the learning curve approach proposed in this article.

Because the degradation of learning does not follow a linear path, it is essential to monitor the various stages of the learning, unlearning, and relearning processes. This article presents an analytical modeling of the stage where a learning profile has degraded to half of its initial value. This is useful for predicting the magnitude and behavior of learning over time. The article points out that the half-life point is of most interest in tracking the degradation path of learning. That half-life point can be used for acquisition training and retraining purposes. With the techniques in this article, something similar to a break-even analysis of learning can be done because the upswing of learning and the downswing of learning conceptually intercept at some point. Of particular note in the decision process is whether that interception point occurs before or after the half-life point. For the purpose of training in acquisition operations, an organization can use the half-life computational technique to estimate what fraction of training retention remains after some point in time and what level of retraining might be needed during the acquisition life cycle.

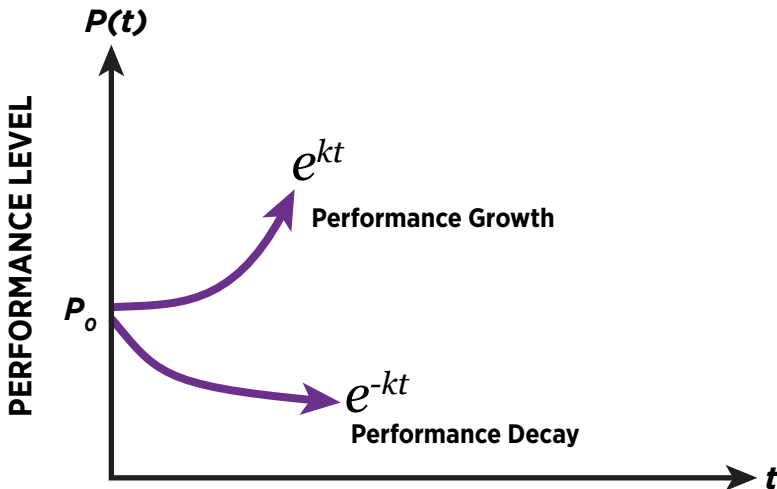
General Half-Life Profile

Figure 1 shows a graphical representation of performance as a function of time under the influence of forgetting (i.e., performance decay). Performance decreases as time progresses. The objective is to determine when performance has decayed to half of its original level. Based on the law of radioactive disintegration, the Law of Learning Decay is proposed here.

The rate of decay of learning due to the effect of forgetting is proportional, at any instant, to the incipient learning level.

The Law of Learning Decay is formulated mathematically in subsequent sections of this article. A mathematical abstraction of the physical process of learning and forgetting is formulated by considering the rate of change in performance (P), which is a function of the learning rate (L). While learning itself is difficult to quantify and measure, its output and performance can be measured as a physical quantity of production. The discrete process is approximated by a continuous curve.

FIGURE 1. REPRESENTATION OF SYSTEM RESPONSE TIME WITH RESPECT TO PASSAGE OF TIME



Thus, the following mathematical formulation emerges: At time t , a certain level of learning L , yields a certain level of performance denoted as P . Denote this transformation as:

$$L \rightarrow P$$

The rate of decay of P can be written as:

$$\frac{d[P]}{dt} = -k[P],$$

where k = decay coefficient. This has the general form of an initial value problem in first-order linear equations, and it has the following general solution:

$$P(t) = P_0 e^{-kt},$$

where P_0 = initial level of performance. The half-life of P is computed as the value of t at which P decays to half of its original level. That is:

$$P(t_{1/2}) = P_0 e^{-kt_{1/2}} = \left(\frac{1}{2}\right) P_0 e^{-kt_0},$$

which is solved to obtain the half-life as:

$$t_{1/2} = \frac{1}{k} \ln 2.$$

To illustrate the application of half-life computations, consider an engineering reactor that converts the relatively stable uranium 238 into the isotope plutonium 239. After 15 years, it is determined that 0.043 percent of the initial amount A_0 of the plutonium has disintegrated. Determining the half-life of the isotope is the point of interest. From Physics, the initial value problem is stated as:

$$\frac{dP}{dt} = kP$$

with $P(0) = P_0$. This has a general solution of the form:

$$P(t) = P_0 e^{-kt}$$

If 0.043 percent of the atoms in A_0 have disintegrated, then 99.957 percent of the substance remains. To find k , we will solve:

$$\alpha P_0 = P_0 e^{-15k}$$

where α = remaining fraction of the substance. With α we obtain $k = 0.00002867$. Thus, for any time t , the amount of the plutonium isotope remaining is represented as:

$$P(t) = P_0 e^{-0.00002867t}$$

This has a general decay profile similar to the plot of $P(t)$ in Figure 1. Computation can now be done of the half-life as corresponding value at time t for which $P(t) = P_0/2$. That is:

$$\frac{P_0}{2} = P_0 e^{-0.00002867t}$$

which yields t (half-life) value of 24,180 years. With this general knowledge of the half-life, several computational analyses can be done to predict the behavior and magnitude of the substance over time. As another example, consider a radioactive nuclide, which has a half-life of 30 years. Suppose the interest lies in computing the fraction of an initially pure sample of this nuclide that will remain undecayed at the end of a time period, say 90 years. From the equation of half-life, the solution for k can be deduced:

$$\frac{P_0}{2} = P_0 e^{-kt_{\text{half-life}}}$$

$$k = \frac{\ln 2}{t_{\text{half-life}}}$$

Which gives $k = 0.0231049$. Now, we can use this value of k to compute:

$$\frac{P_0}{P} = e^{-(0.0231049)(90)} = 0.125$$

Similarly, let us consider a radioactive isotope with a half-life of 140 days. The number of days it would take for the sample to decay to one-seventh of its initial magnitude can be computed:

$$\frac{P_0}{2} = P_0 e^{-kt_{\text{half-life}}}$$

$$k = \frac{\ln 2}{t_{\text{half-life}}}$$

Which yields $k = 0.004951$ and results in:

$$P = 1/7 P_0$$

$$\frac{1}{7} P_0 = P_0 e^{-kt}$$

$$t = \frac{\ln 7}{k} = 393 \text{ days}$$

For learning curves, similar computational analysis can be performed to assess the forgetting-induced properties of the curves. Thus, a comparative analysis of the different models can be conducted.



Half-Life Application to Learning Curves

Wright (1936) documented the “80 percent learning” effect, which indicates that a given operation is subject to a 20 percent productivity improvement each time the activity level or production volume *doubles*. The proposed half-life approach is the antithesis of the double-level milestone. Some of the classical learning curve models are:

- *Log-linear model*
- *S-curve model*
- *Stanford-B model*
- *DeJong’s learning formula*
- *Levy’s adaptation function (Levy, 1965)*
- *Glover’s learning formula (Glover, 1966)*
- *Pegels’ exponential function (Pegels, 1976)*
- *Knecht’s upturn model (Knecht, 1974)*
- *Yelle’s product model*

The basic log-linear model is the most popular learning curve model. It expresses a dependent variable (e.g., production cost) in terms of some independent variable (e.g., cumulative production). The model states that the improvement in productivity is constant (i.e., it has a constant slope) as output increases. That is:

$$C(x) = C_1 x^{-b}$$

Where:

$C(x)$ = cumulative average cost of producing x units

C_1 = cost of the first unit

x = cumulative production unit

b = learning curve exponent

Notice that the expression for $C(x)$ is practical only for $x > 0$. This makes sense because learning effect cannot realistically kick in until at least one unit ($x \geq 1$) has been produced. For the standard log-linear model, the expression for the learning rate, p , is derived by considering two production levels where one level is double the other. For example, given the two levels x_1 and x_2 (where $x_2 = 2x_1$), the following expressions emerge:

$$C(x_1) = C_1(x_1)^{-b}$$

$$C(x_2) = C_1(2x_1)^{-b}$$

The percent productivity gain, p , is then computed as:

$$p = \frac{C(x_2)}{C(x_1)} = \frac{C_1(2x_1)^{-b}}{C_1(x_1)^{-b}} = 2^{-b}$$

The performance curve, $P(t)$, shown earlier in Figure 1 can now be defined as the reciprocal of the average cost curve, $C(x)$, and as a function of production level, x . Thus, we have

$$P(x) = \frac{1}{C(x)}.$$

The application of half-life analysis to learning curves can help address questions such as:

- How fast and how far can system performance be improved?
- What are the limitations to system performance improvement?
- How resilient is a system to shocks and interruptions to its operation?
- Are the performance goals that are set for the system achievable?

Derivation of Half-Life of the Log-Linear Learning Curve

Figure 2 shows a pictorial representation of the basic log-linear model, with the half-life point indicated as $x_{1/2}$. The half-life of the log-linear model is computed as follows:

C_0 = Initial performance level

$C_{1/2}$ = Performance level at half-life

$$C_0 = C_1 x_0^{-b} \text{ and } C_{1/2} = C_1 x_{1/2}^{-b}$$

$$\text{But } C_{1/2} = \frac{1}{2} C_0$$

$$\text{Therefore, } C_1 x_{1/2}^{-b} = \frac{1}{2} C_1 x_0^{-b}, \text{ which leads to } x_{1/2}^{-b} = \frac{1}{2} x_0^{-b},$$

Which, by taking the $(-1/b)^{\text{th}}$ exponent of both sides, simplifies to yield the following expression as the general expression for the standard log-linear learning curve model,

$$x_{1/2} = \left(\frac{1}{2} \right)^{-\frac{1}{b}} x_0, \quad x_0 \geq 1$$

where $x_{1/2}$ is the half-life and x_0 is the initial point of operation; $x_{1/2}$ is then referred to as the *First-Order Half-Life*.

The *Second-Order Half-Life* is computed as the time corresponding to half of the preceding half. That is:

$$C^1 x_{1/2(2)}^{-b} = \frac{1}{4} C_1 x_0^{-b},$$

which simplifies to yield:

$$x_{1/2(2)} = \left(\frac{1}{2} \right)^{-\frac{2}{b}} x_0.$$

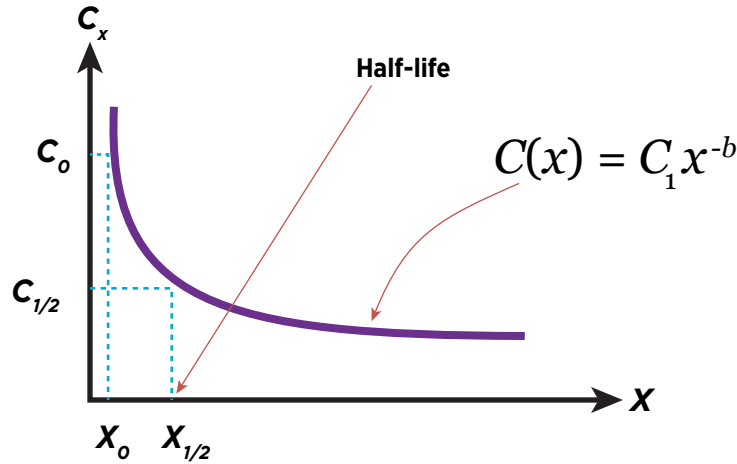
Similarly, the Third-Order Half-Life is derived to obtain:

$$x_{1/2(3)} = \left(\frac{1}{2} \right)^{-\frac{3}{b}} x_0,$$

In general, the k^{th} -Order Half-Life for the log-linear model is represented as:

$$x_{1/2(k)} = \left(\frac{1}{2}\right)^{-\frac{k}{b}} x_0.$$

FIGURE 2. GENERAL PROFILE OF THE BASIC LEARNING CURVE MODEL



Computational Examples

Figure 3 shows a comparison of learning curve profiles of the log-linear model with $b = 0.75$ and $b = 0.3032$ respectively. The graphical profiles reveal the characteristics of learning, which can dictate the half-life behavior of the overall learning process. Knowing the point where the half-life of each curve occurs can be very useful in assessing learning retention for the purpose of designing training programs or designing work.

For $C(x) = 250x^{-0.75}$, the First-Order Half-Life is computed as:

$$x_{1/2} = \left(\frac{1}{2}\right)^{-\frac{1}{0.75}} x_0, \quad x_0 \geq 1$$

If the above expression is evaluated for $x_0 = 2$, the first-order half-life yields $x_{1/2} = 5.0397$, which indicates a fast drop in the value of $C(x)$. The specific case of $x_0 = 2$ shows $C(2) = 148.6509$ corresponding to a half-life

of 5.0397. Note that $C(5.0397) = 74.7674$, which is about half of 148.6509. The conclusion from this analysis is that if we are operating at the point $x = 2$, we can expect this particular curve to reach its half-life decline point at $x = 5$.

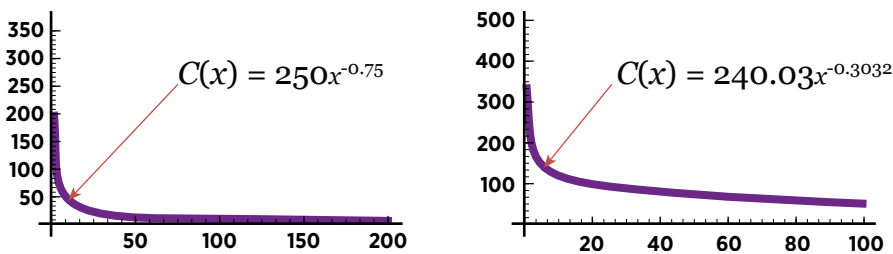
For $C(x) = 240.03x^{-0.3032}$, the First-Order Half-Life is computed as:

$$x_{1/2} = \left(\frac{1}{2}\right)^{-\frac{1}{0.3032}} x_0, \quad x_0 \geq 1$$

If we evaluate the above function for $x_0 = 2$, the First Order Half-Life yields $x_{1/2} = 19.6731$. This does not represent as precipitous a drop as the other curve. The half-life analysis can be applied to learning curves to determine when each cost element of interest will decrease to half of its starting value. This information can be useful for product pricing purposes, particularly for technology products that are subject to rapid price reductions due to declining product cost. Several models and variations of learning curves have been reported in the literature (Badiru, 1992; Jaber & Guiffreda, 2008). Models are developed through one of the following approaches:

1. Conceptual models
2. Theoretical models
3. Observational models
4. Experimental models
5. Empirical models

FIGURE 3. COMPARISON OF LOG-LINEAR CURVES FOR $b = -0.75$ AND $b = -0.3032$



Half-Life Derivations for Classical Learning Models

The S-Curve model. The S-Curve (Towill & Cherrington, 1994) is based on an assumption of a gradual start-up. The function has the shape of the cumulative normal distribution function for the start-up curve and the shape of an operating characteristics function for the learning curve. The gradual start-up is based on the fact that the early stages of production are typically in a transient state with changes in tooling, methods, materials, design, and even changes in the workforce. The basic form of the S-Curve function is:

$$C(x) = C_1 + M(x + B)^{-b}$$

$$MC(x) = C_1 \left[M + (1 - M)(x + B)^{-b} \right]$$

Where:

$C(x)$ = learning curve expression

b = learning curve exponent

$M(x)$ = marginal cost expression

C_1 = cost of first unit

M = incompressibility factor (a constant)

B = equivalent experience units (a constant).

Assumptions about at least three out of the four parameters (M , B , C_1 , and b) are needed to solve for the fourth one. Using the $C(x)$ expression and derivation procedure outlined earlier for the log-linear model, the half-life equation for the S-Curve learning model is derived to be:

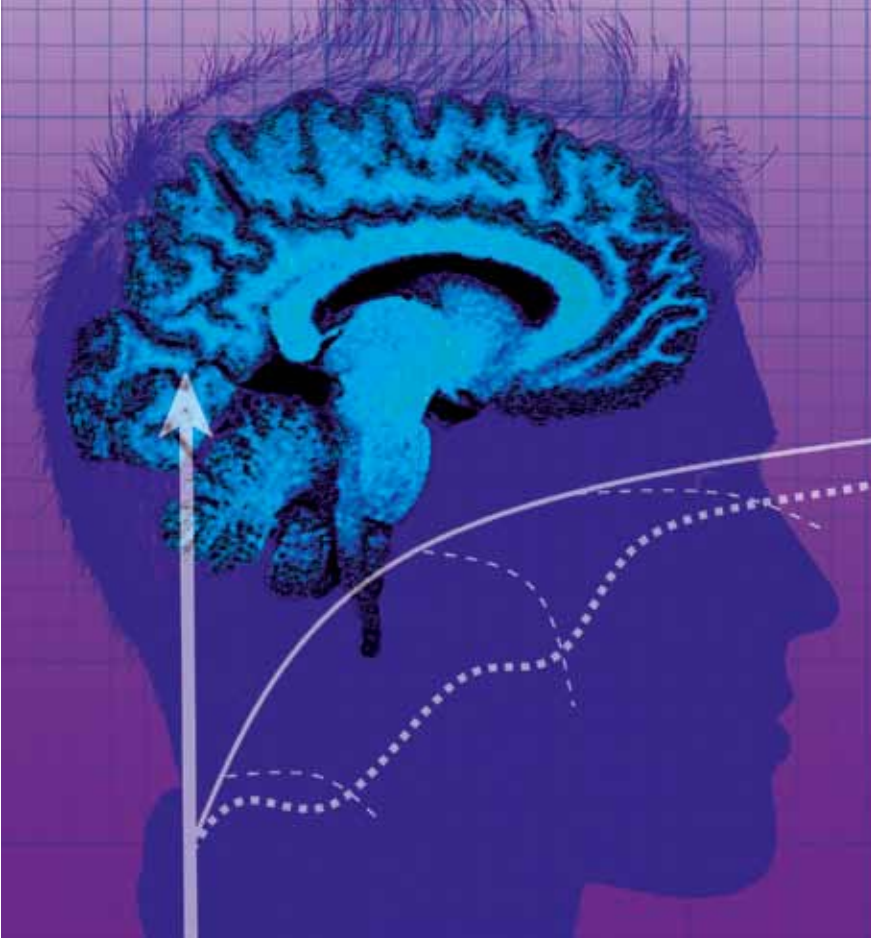
$$x_{1/2} = (1/2)^{-1/b} \left[\frac{M(x_0 + B)^{-b} - C_1}{M} \right]^{-1/b} - B$$

Where:

$x_{1/2}$ = half-life expression for the S-Curve Learning Model

x_0 = initial point of evaluation of performance on the learning curve

In terms of practical application of the S-Curve, consider when a worker begins learning a new task. The individual is slow initially at the tail end of the S-Curve. But the rate of learning increases as time goes on, with additional repetitions. This helps the worker to climb the steep-slope segment of the S-Curve very rapidly. At the top of the slope, the worker is classified as being proficient with the learned task. From then on, even if the worker puts much effort into improving upon the task, the resultant learning will not be proportional to the effort expended. The top end of the S-Curve is often called the slope of *diminishing returns*. At the top of the S-Curve, workers succumb to the effects of *forgetting* and other performance-impeding factors. As the work environment continues to change, a worker's level of skill and expertise can become obsolete. This is an excellent reason for the application of half-life computations.



The Stanford-B model. An early form of learning curve is the Stanford-B model, which is represented as:

$$UC(x) = C_1(x + B)^{-b}$$

Where:

$UC(x)$ = direct cost of producing the x^{th} unit

b = learning curve exponent

C_1 = cost of the first unit when $B = 0$;

B = slope of the asymptote for the curve;

B = constant ($1 < B < 10$). This is equivalent units of previous experience at the start of the process, which represents the number of units produced prior to first unit acceptance. It is noted that when $B = 0$, the Stanford-B model reduces to the conventional log-linear model. Figure 4 shows the profile of the Stanford-B model with $B = 4.2$ and $b = -0.75$. The general expression for the half-life of the Stanford-B model is derived to be:

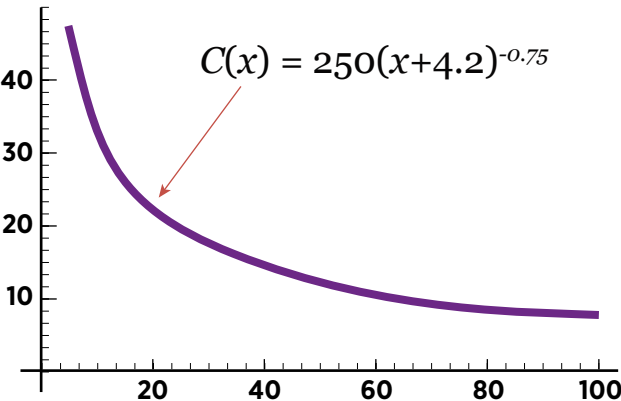
$$x_{1/2} = (1/2)^{-1/b}(x_0 + B) - B$$

Where:

$x_{1/2}$ = half-life expression for the Stanford-B Learning Model

x_0 = initial point of evaluation of performance on the learning curve

FIGURE 4. STANFORD-B MODEL WITH PARAMETERS $B = 4.2$ AND $b = -0.75$



Multifactor Half-Life Derivation

Badiru (1994) presents applications of learning and forgetting curves to productivity and performance analysis. One example presented used production data to develop a predictive model of production throughput. Two data replicates are used for each of 10 selected combinations of cost and time values. Observations were recorded for the number of units representing double production levels. The resulting model has the functional form below and the graphical profile shown in Figure 5.

$$C(x) = 298.88x_1^{-0.31}x_2^{-0.13}$$

Where:

$C(x)$ = cumulative production volume

x_1 = cumulative units of Factor 1

x_2 = cumulative units of Factor 2

b_1 = First learning curve exponent = -0.31

b_2 = Second learning curve exponent = -0.13

A general form of the modeled multifactor learning curve model is:

$$C(x) = C_1x_1^{-b_1}x_2^{-b_2}$$

and the half-life expression for the multifactor learning curve was derived to be:

$$x_{1(1/2)} = (1/2)^{-1/b_1} \left[\frac{x_{1(0)}x_{2(0)}^{b_2/b_1}}{x_{2(1/2)}^{b_2/b_1}} \right]^{-1/b_1}$$

$$x_{2(1/2)} = (1/2)^{-1/b_2} \left[\frac{x_{2(0)}x_{1(0)}^{b_1/b_2}}{x_{1(1/2)}^{b_1/b_2}} \right]^{-1/b_2}$$

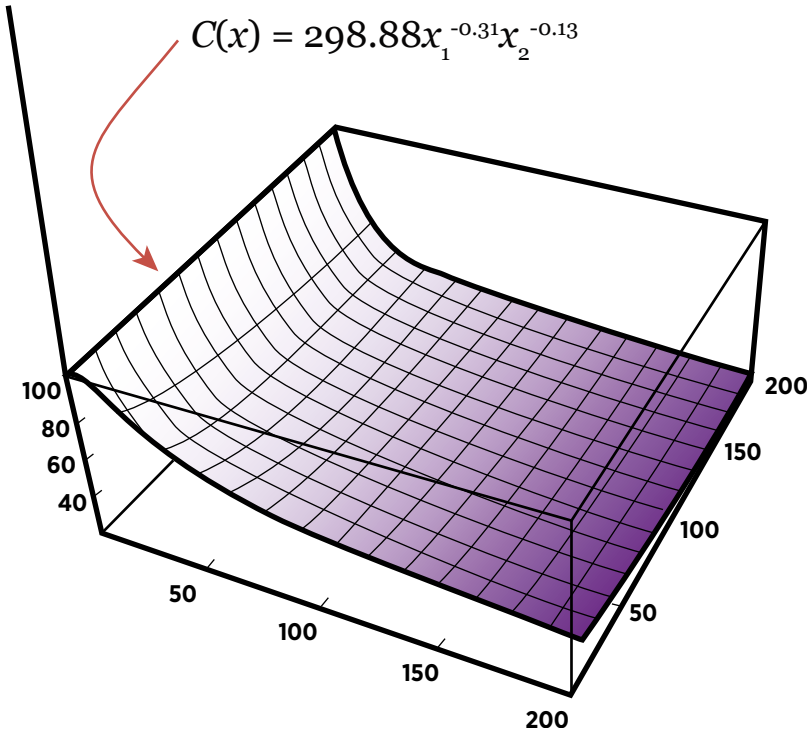
Where:

$x_{i(1/2)}$ = half-life component due to Factor i ($i = 1, 2$)

$x_{i(0)}$ = initial point of Factor i ($i = 1, 2$) along the multifactor learning curve

Knowledge of the value of one factor is needed to evaluate the other factor. Just as in the case of single-factor models, the half-life analysis of the multifactor model can be used to predict when the performance metric will reach half of a starting value.

FIGURE 5. BIVARIATE MODEL OF LEARNING CURVE



Incorporation of Forgetting Functions into Learning Curves

Several factors can influence learning rate in practice. A better understanding of the profiles of learning curves can help in the development of forgetting intervention programs and for the assessment of the sustainability of learning. For example, shifting from learning one operational process to another can influence the half-life profile of the original learning curve. Important questions that half-life analysis can address include the following:

1. What factors influence learning retention and for how long?
2. What factors foster forgetting and at what rate?
3. What joint effects exist to determine the overall learning profile for worker performance and productivity?
4. What is the profile of and rate of decline of the forgetting curve?

The issues related to the impact of forgetting in performance and productivity analysis are brought to the forefront by Badiru (1994, 1995) and all the references therein. Retention rate and retention capacity of different workers will determine the nature of the forgetting function to be modeled for the workers. Whenever interruption occurs in the learning process, as in scheduled breaks (Anderlohr, 1969), it results in some forgetting. The resulting drop in performance rate depends on the initial level of performance and the length of the interruption. The following three potential cases illustrate how forgetting may occur:

Case 1: Forgetting may occur continuously throughout the learning process.

Case 2: Forgetting may occur discretely over distinct bounded time intervals.

Case 3: Forgetting may occur over intermittent and/or random time intervals where the time of occurrence and duration of forgetting are described by some probability distribution.

Any operation that is subject to interruption in the learning process is susceptible to the impact of forgetting. Sule (1978) postulated that the forgetting model can be represented as:

$$y_f = x_f r_f^{-b_f},$$

where:

y_f = number of units that could be produced on r^{th} day in the presence of forgetting.

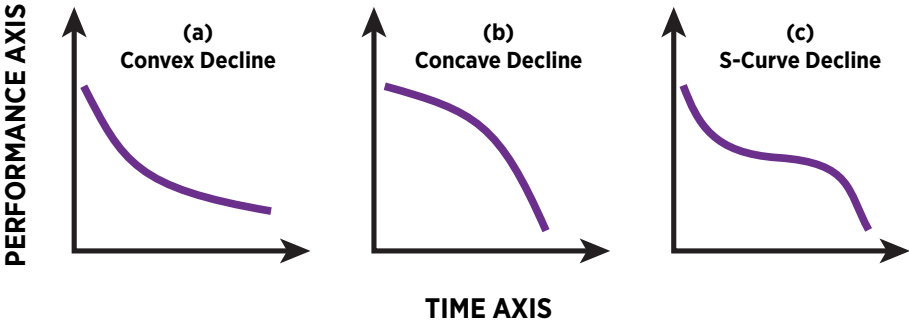
x_f = equivalent production on first day of the forgetting curve.

r_f = cumulative number of days in the forgetting cycle.

b_f = forgetting rate.

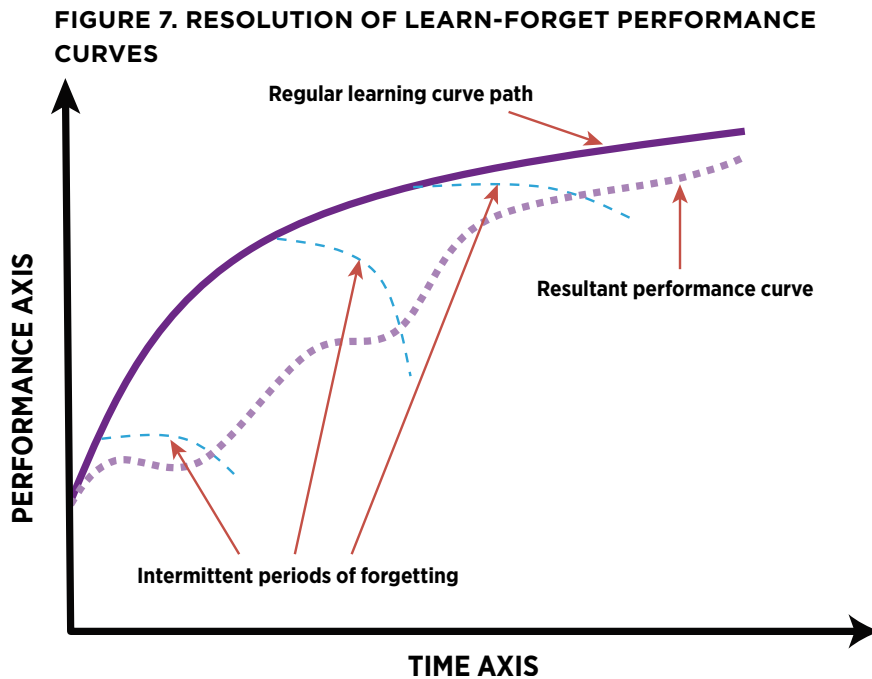
The forgetting function has the same basic form as the standard learning curve model, except that the forgetting rate will be negative, indicating a decay process. Figure 6 shows some of the possible profiles of the forgetting curve. Profile (a) shows a case where forgetting occurs rapidly along a convex curve. Profile (b) shows a case where forgetting occurs more slowly along a concave curve. Profile (c) shows a case where the rate of forgetting shifts from convex to concave along an S-Curve.

FIGURE 6. ALTERNATE PROFILES DECLINING IMPACT OF FORGETTING



The profile of the forgetting curve and its mode of occurrence can influence the half-life measure. This is further evidence that the computation of half-life can help distinguish between learning curves, particularly if a forgetting component is involved. The combination of

the learning and forgetting functions presents a more realistic picture of what actually occurs in a learning process. The combination is not necessarily as simple as resolving two curves to obtain a resultant curve. The resolution may particularly be complex in the case of intermittent periods of forgetting. Figure 7 shows representations of periods where forgetting occurs and the resultant learn-forget profile.



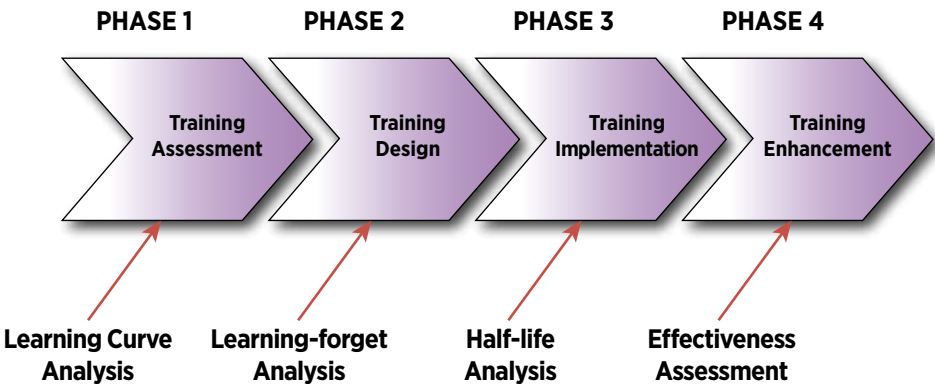
Applications to Training and Worker Effectiveness Analysis

Learning curves are traditionally used for diagnostic and planning purposes in installed operations. The premise of this article is that learning curve analysis, learn-forget modeling, and half-life analysis can be used proactively to design or enhance training programs, thereby improving worker effectiveness. Training is a capital-intensive overhead cost that is often difficult to justify in terms of revenue production. There are two aspects of justifying training programs: effectiveness and efficiency of the training program. Effectiveness refers to the benefits an organization derives from training the workforce to meet organizational objectives. Efficiency refers to the process of determining the resources required for the training versus the expected output. In this process, it is essential to provide the resources required at the right time, in the

right form, and in the right quantity. An understanding of the half-life characteristics of the learning process can make the resources allocation process more effective.

In practice, there is a lack of a structured approach to ensuring training effectiveness and efficiency. Sawhney, Badiru, and Niranjan (2004) present a structured model training. The model is adapted here to show where learning curve analysis may be important and how half-life analysis can be incorporated. Figure 8 shows the streamlined training process incorporating learning curve analysis, forgetting analysis, and half-life analysis. The first phase is to assess the alignment of the training program to the organizational strategic goals in light of the learning curve impact. Phase 2 involves specific design of the training program with recognition of the learn-forget phenomenon. Phase 3 addresses training implementation with respect to the limit of the learning effect, half-life properties of learning, and the limit of retention. Phase 4 finalizes the process with training enhancement activities. This can involve resource realignment, output evaluation, and risk mitigation for the subsequent rounds.

FIGURE 8. INCORPORATION OF LEARNING, FORGETTING, AND HALF-LIFE ANALYSIS INTO TRAINING PROCESS



Conclusions

Degradation of performance occurs naturally either due to internal processes or externally imposed events, such as extended production breaks. For productivity assessment purposes, it may be of benefit to determine the length of time it takes a production metric to decay to half

of its original magnitude. For example, for career planning strategy, one may be interested in how long it takes for skills sets to degrade by half in relation to current technological needs of the workplace. The half-life phenomenon may be due to intrinsic factors, such as forgetting, or due to external factors, such as a shift in labor requirements. Half-life analysis can have application in intervention programs designed to achieve reinforcement of learning. It can also have application for assessing the sustainability of skills acquired through training programs. Further research on the theory of half-life of learning curves should be directed to topics such as the following:

- Half-Life Interpretations
- Training and Learning Reinforcement Program
- Forgetting Intervention and Sustainability Programs

In addition to the predictive benefits of half-life expressions, they also reveal the ad hoc nature of some of the classical learning curve models that have been presented in the literature. The author recommends that future efforts to develop learning curve models should also attempt to develop the corresponding half-life expressions to provide full operating characteristics of the models. Readers are encouraged to explore half-life analysis of other learning curve models not covered in this article.

Author Biography



Professor Adedeji Badiru is head of Systems and Engineering Management at the Air Force Institute of Technology. He is a registered professional engineer, a certified Project Management Professional (PMP), and a Fellow of the Institute of Industrial Engineers. He has BS and MS degrees in Industrial Engineering (IE), an MS in Mathematics from Tennessee Technological University, and a PhD in IE from the University of Central Florida. He has authored several books and journal articles.

(E-mail address: adedeji.badiru@afit.edu)

References

- Anderlohr, G. (1969). What production breaks cost. *Industrial Engineering*, 1(9), 34–36.
- Badiru, A. B. (1992). Computational survey of univariate and multivariate learning curve models. *IEEE Transactions on Engineering Management*, 39(2), 176–188.
- Badiru, A. B. (1994). Multifactor learning and forgetting models for productivity and performance analysis. *International Journal of Human Factors in Manufacturing*, 4(1), 37–54.
- Badiru, A. B. (1995). Multivariate analysis of the effect of learning and forgetting on product quality. *International Journal of Production Research*, 33(3), 777–794.
- Badiru, A. B. (2010). Half-life of learning curves for information technology project management. *International Journal of IT Project Management*, 1(3), 28–45.
- Belkaoui, A. (1976). Costing through learning. *Cost and Management*, 50(3), 36–40.
- Belkaoui, A. (1986). *The learning curve*. Westport, CT: Quorum Books.
- Camm, J. D., Evans, J. R., & Womer, N. K. (1987). The unit learning curve approximation of total cost. *Computers and Industrial Engineering*, 12(3), 205–213.
- Glover, J. H., (1966). Manufacturing progress functions: An alternative model and its comparison with existing functions. *International Journal of Production Research*, 4(4), 279–300.
- Jaber, M. Y., & Bonney, M. (1996). Production breaks and the learning curve: The forgetting phenomena. *Applied Mathematical Modelling*, 20(2), 162–169.
- Jaber, M. Y., & Bonney, M. (2003). Lot sizing with learning and forgetting in setups and in product quality. *International Journal of Production Economics*, 83(1), 95–111.
- Jaber, M. Y., & Bonney, M. (2007). Economic Manufacture Quantity (EMQ) model with lot size dependent learning and forgetting rates. *International Journal of Production Economics*, 108(1–2), 359–367.
- Jaber, M.Y., & Guiffrida, A. L. (2008). Learning curves for imperfect production processes with reworks and process restoration interruptions. *European Journal of Operational Research*, 189(1), 93–104.
- Jaber, M. Y., Hemant, K. V., & Darwin D. J. (2003). Countering forgetting through training and deployment. *International Journal of Production Economics*, 85(1), 33–46.
- Jaber, M. Y., & Sikstrom, S. (2004). A numerical comparison of three potential learning and forgetting models. *International Journal of Production Economics*, 92(3), 281–294.
- Knecht, G. R. (1974). Costing, technological growth, and generalized learning curves. *Operations Research Quarterly*, 25(3), 487–491.
- Levy, F. K. (1965). Adaptation in the production process. *Management Science*, 11(6), B136–B154.
- Liao, W. M. (1979). Effects of learning on resource allocation decisions. *Decision Sciences*, 10(1), 116–125.
- McIntyre, E. V. (1977). Cost-volume-profit analysis adjusted for learning. *Management Science*, 24(2), 149–160.
- Nanda, R. (1979). Using learning curves in integration of production resources. *Proceedings of the 1979 IIE Fall Conference*, 376–380.
- Pegels, C. C. (1976). Start up or learning curves—Some new approaches. *Decision Sciences*, 7(4), 705–713.
- Richardson, W. J. (1978). Use of learning curves to set goals and monitor progress in cost reduction programs. *Proceedings of 1978 IIE Spring Conference*, 235–239.

- Sawhney, R., Badiru A. B., & Niranjana, A. (2004). A model for integrating and managing resources for technical training programs. In Y. A. Hosni and T. M. Khalil (Eds.), *Internet Economy: Opportunities and Challenges for Developed and Developing Regions of the World*, pp. 337–351. Boston, MA: Elsevier Ltd.
- Smith, J. (1989). *Learning curve for cost control*. Norcross, GA: Industrial Engineering and Management Press.
- Smunt, T. L. (1986). A comparison of learning curve analysis and moving average ratio analysis for detailed operational planning. *Decision Sciences*, 17(4), 475–495.
- Sule, D. R. (1978). The effect of alternate periods of learning and forgetting on economic manufacturing quantity. *AIIE Transactions*, 10(3), 338–343.
- Towill, D. R., & Cherrington, J. E. (1994). Learning curve models for predicting the performance of advanced manufacturing technology. *International Journal of Advanced Manufacturing Technology*, 9(3), 195–203.
- Towill, D. R., & Kaloo, U. (1978). Productivity drift in extended learning curves. *Omega*, 6(4), 295–304.
- Ward, D. (2010). Faster, better, cheaper revisited: Program management lessons from NASA. *Defense AT&L magazine*, 213(2), 48–52.
- Ward, D. (2012). Faster, better, cheaper: Why not pick all three? *National Defense*. Retrieved from <http://www.nationaldefensemagazine.org/archive/2012/April/Pages/Faster,Better,CheaperWhyNotPickAllThree.aspx?PF=1>
- Womer, N. K. (1979). Learning Curves, production rate, and program costs. *Management Science*, 25(4), 312–219.
- Womer, N. K. (1981). Some propositions on cost functions. *Southern Economic Journal*, 47(4), 1111–1119.
- Womer, N. K. (1984). Estimating learning curves from aggregate monthly data. *Management Science*, 30(8), 982–992.
- Womer, N. K., & Gullledge, T. R. (1983). A dynamic cost function for an airframe production program. *Engineering Costs and Production Economics*, 7(3), 213–227.
- Wright, T. P. (1936). Factors affecting the cost of airplanes. *Journal of Aeronautical Science*, 3(2), 122–128.
- Yelle, L. E. (1976). Estimating learning curves for potential products. *Industrial Marketing Management*, 5(2/3), 147–154.
- Yelle, L. E. (1979). The learning curve: Historical review and comprehensive survey. *Decision Sciences*, 10(2), 302–328.
- Yelle, L. E. (1983). Adding life cycles to learning curves. *Long Range Planning*, 16(6), 82–87.