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(U) Identifying Targets from Filtering Effects

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Abstract

The response of a radar or sonar target to a signal may be described by an impulse response function, which means that the target may be considered as a filter acting on a signal. It is known that filters are not exactly invertible, and this lack of invertibility may be used to identify the particular target that reflected a signal. We apply techniques from nonlinear dynamics to determine the probability that a function exists between 2 signals. If 2 identical signals are reflected by the same target, then our statistic will indicate a high probability that a function exists between the 2 signals; if the 2 signals were reflected by different targets, then the statistic will show a low probability that the 2 signals are related by a function. We demonstrate target identification with both numerical simulations and acoustic experiments.

Keywords: radar, target identification

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1.0 Introduction

Considering a radar (or sonar) target as more than a simple point scatterer brings up the possibility of identifying the target based on the reflected signal [1]. The response of a target to an incoming signal can be described by an impulse response function [2]

$$h(t, p) = \sum_{l=1}^L a_l(p) \delta[t - T_l(p)] + \sum_{m=1}^M b_m(p) \exp\{s_m[t - T_m(p)]\} \Theta[t - T_m(p)] \quad (1.1)$$

where p indicates that the quantity is dependent on the aspect angle. The index l is used for the specular reflections, while m is used for resonances. The coefficients b_m and s_m are complex, and Θ is the step function. The T 's are the delay times for each of the scatterers and resonances.

Based on eq. (1) a target may be identified based on either the specular terms or on the resonant terms. To use the specular terms, a range-cross range profile is generated by matched filtering the signal reflected from the target and creating a pattern of points from the large peaks in the output of the matched filter [1]. Pattern recognition tools are then used to associate a particular pattern with a target. The patterns are highly dependent on aspect angle, so it may be necessary to compare many patterns for a particular target.

The resonant terms from eq. (1) may also be used as identifiers [2-4]. The target is illuminated with a pulse of length T_g . After a time $t > T_g + T_m$ and $t > T_g + T_i$, only the complex exponentials remain in eq. (1). These exponential terms are not aspect angle dependent, so measuring the decaying exponential response of the target allows one to identify the target for any aspect angle. The problem with identifying the target using this transient information is that the transient soon decays to a level below the noise background, so it's difficult to measure a long enough portion of the transient signal to identify the target. Other approaches to target identification use adaptive waveform design [5-8].

2.0 Filter ID Method

Because the target response may be described by an impulse response function (1), one may describe the target as a filter acting on a signal. Exact inversion of a filter is not possible, meaning that there is no function that can input a filtered signal and output the unfiltered signal. We are able to use this lack of invertibility as a means to identify a particular filter, and therefore a particular target. If a signal s is reflected from target A to give a reflected signal $s(A)$, and s is also reflected from target B to get $s(B)$, then the fact that there is no function from $s(A)$ to s , and no function from $s(B)$ to s implies that there is no function from $s(B)$ to $s(A)$ unless A and B are the same target (and the same signal s is used).

The impulse response function (eq. 1) has both angle dependent and angle independent terms. The identification method described in this paper may be trained using target responses collected over a window of aspect angles, so that the angle dependent terms are averaged out, and identification is dominated by angle independent terms. Target ID methods that depend on resonances also use these angle independent terms [2-4] but unlike those methods that are based on short pulses, the method described in this paper uses a long modulated pulse, allowing us to signal average over a long pulse to improve the signal to noise ratio.

If the signals of interest are generated by deterministic dynamical systems, which is always the case for radar or sonar signals, then the signals may be analyzed using methods from the field of dynamical systems. There are algorithms from this field for determining the probability that 2 signals are related by a function [9]. These algorithms depend on the concept of predictability; if there is a function that inputs A and outputs $f(A)$, then one should be able to predict $f(A)$ if A is known.

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A deterministic dynamical system has a finite number of variables and a set of rules that describes how these variables evolve over time. We may analyze the behavior of the dynamical system by plotting the trajectory of the system in a phase space (or state space), in which each axis corresponds to one of the variables from the system. If the dynamical system is dissipative, this trajectory forms a pattern in phase space called an attractor. Radar or sonar transmitters are examples of finite dimensional deterministic dynamical systems, so radar or sonar signals may be analyzed using dynamical systems methods.

Knowledge of the attractor for a system can be used to determine if one signal is predictable from another signal. A typical dynamical system based on ordinary differential equations, with a set of variables $\mathbf{x} = (x_1, x_2, \dots, x_d)$ may be described by

$$\frac{d\mathbf{x}}{dt} = h[\mathbf{x}] \quad (1.2)$$

We also have the set of variables $\mathbf{y} = (y_1, y_2, \dots, y_d)$, where

$$\mathbf{y} = f(\mathbf{x}) \quad (1.3)$$

Consider a set of variables near \mathbf{x} , $\mathbf{x}_a = (x_1 + \delta_1, x_2 + \delta_2, \dots, x_d + \delta_d)$. If f is continuous and differentiable, then as

$$\delta \rightarrow 0, f(\mathbf{x}_a) \rightarrow f(\mathbf{x}) = \mathbf{y} \quad (1.4)$$

If we have the signals \mathbf{x} and \mathbf{y} , and would like to determine if $\mathbf{y} = f(\mathbf{x})$, we apply the definition of eqs. 2-4) by searching in the phase space of \mathbf{y} for points that are within some distance ε of \mathbf{y} . We then locate the points with the corresponding time indices on \mathbf{x} and measure the average distance δ between this set of points and \mathbf{x} . The size of this phase space radius δ is a measure of the probability that a function exists from \mathbf{x} to \mathbf{y} . The smaller δ , the higher the probability there is a function. How to define "small" is a statistical question that we won't consider here, because we are only interested in relative measurements, i.e. given signals \mathbf{y} and \mathbf{z} , which of these signals is more likely to be a function of \mathbf{x} .

2.1 Embedding

There is usually only 1 available signal from a radar or sonar, the signal that is transmitted and reflected, but an object that is equivalent in many ways to the signal vector \mathbf{x} may be reconstructed from only 1 of the variables. It has been proven that it is possible to reconstruct an object that is topologically equivalent to the attractor for the dynamical system by using the method of delays [10, 11]. Using a digitized version $s(i)$ of a continuous time signal $s(t)$, a series of delay vectors is created from $s(i)$:

$$\mathbf{s}(i) = [s(i), s(i + \tau_1), \dots, s(i + \tau_{d-1})] \quad (1.5)$$

where the dimension d and delay times τ_j may be chosen by known methods [12, 13]. The set of these vectors over time form a reconstruction (also called an embedding) of the signal $s(i)$ in a phase space.

2.2 Function statistic

When looking to see if a function exists between 2 signals embedded in phase space, we estimate the probability that there is a continuous function between the 2 signals by considering small neighborhoods in the phase space. If we want to know if there is a function that transforms signal Y into signal X , we locate a group of points on signal X that all fall within some small radius in phase space, and ask if the corresponding points on signal Y also fall within some small region. It is necessary to define what is meant by "small", but the general idea is that the smaller the region on signal Y , the more likely signal X is to be a function of Y . If signals Y and X are both produced by applying the same filter to signal Z , then the small region in Y will be at a minimum which is determined by the nearest neighbor distances on Y .

Our target ID algorithm treats a reflection from a radar target as a filter acting on signal Z . The first time we see the target, we transmit signal Z and record signal X . Later, we again transmit signal Z and receive signal Y . We then want to know if signals X and Y are reflections from the same target.

In order to detect a difference between signals, we first embed a digitized reflected signal x in a d dimensional phase space (or state space) by creating a series of delay vectors $\mathbf{x}(i) = [x(i), x(i + \tau_1), \dots, x(i + \tau_{d-1})]$. We then randomly pick some index $i_1, 1 \leq i_1 \leq P - \tau_{d-1}$, where P is the number of points on the time series. The embedded delay vector corresponding to this index is $\mathbf{x}(i_1)$.

We then look for points that are nearby neighbors to $\mathbf{x}(i_1)$ in phase space. By neighbors, we mean the set of points for which the distance

$$d_{mi} = \sqrt{\sum_{j=0}^{d-1} [x(i_1 + \tau_j) - x(i_{mi} + \tau_j)]^2} \quad (1.6)$$

is smallest. We find a group of M nearby phase space neighbors $\{\mathbf{x}(i_{m1}), \mathbf{x}(i_{m2}), \dots, \mathbf{x}(i_{mM})\}$ and record the indices of all points: $R(i_1) = \{i_1, i_{m1}, i_{m2}, \dots, i_{mM}\}$. We then choose a new index point and find a new set of M neighbors, continuing until we have examined some number N of index points. The recorded sets of indices $\{R(i_1) \dots R(i_N)\}$ form a reference which may later be used to identify this particular target.

To apply this reference, we then take a signal y which has been reflected from an unknown target and embed it in a d dimensional phase space. To make sure that the signal y is properly aligned in time with the reference x , we find the cross correlation between x and y to find out τ_y , the amount by which y leads or lags x . We then translate y by this amount so that it is aligned in time with x .

Next, from our previously recorded reference $\{R(i_1) \dots R(i_N)\}$, we retrieve the indices of an index point and a set of neighbors. We examine the points $\{y(i_1), y(i_{m1}), y(i_{m2}), \dots, y(i_{mM})\}$ on the unknown signal corresponding to these indices. If the unknown target is the same as the target used to create the reference, these points will be neighbors, and will be close together in phase space. If the unknown target is different from the reference target, the points on the unknown signal will not be neighbors, and they will be far apart in phase space. We define the phase space radius as

$$\varepsilon = \frac{1}{M} \sum_{j=1}^M |y(i_{mj}) - y(i_1)| \quad (1.7)$$

where the differences are Euclidean distances. The average of the phase space radius over the entire signal is $\langle \varepsilon \rangle$.

2.3 Aspect Angle Dependence

As described in eq. (1), part of the signal reflected from a target may be independent of the aspect angle Θ . It is possible to create a composite reference for a target that combines data from a window of angles to produce a reference that works for any angle within that window.

To produce a composite reference for a target, we follow roughly the same set of steps as above, but we use reflected signals $x(\Theta_j, t)$ from N_{ang} different angles Θ_j . We choose an index point i_1 and find the embedded points $\mathbf{x}(\theta_j, i_1)$, $j = 1, 2, \dots, N_{ang}$ on each of the signals corresponding to this index. We then

search for a group of indices $\{i_{nn1}, i_{nn2}, \dots, i_{nnM}\}$ whose corresponding points $\{\mathbf{x}(\theta_j, i_{nn1}), \mathbf{x}(\theta_j, i_{nn2}), \dots, \mathbf{x}(\theta_j, i_{nnM})\}$, $j = 1, 2, \dots, N_{ang}$ fall within some minimum radius of the center point for all the signals, or for every angle,

$$\max_k \left(\left\| \mathbf{x}(\theta_j, i_{nnk}) - \mathbf{x}(\theta_j, i_1) \right\| \right) \leq \delta \quad j = 1, 2, \dots, N_{ang} \quad (1.8)$$

where k is the index of a nearest phase space neighbor. These points may have different locations on different signals. The distance δ is then the smallest distance for which we can find M points that satisfy eq. (8) for every angle (every value of j). For an individual signal, this set of indices probably does not give the closest M points to the center point.

As previously, these point indices are recorded and then a new index is chosen for a new set of center points. The set of recorded indices then forms a composite reference for the particular shape. When an unknown signal y is received, this composite reference is used to calculate $\langle \varepsilon \rangle$.

2.4 Phase space strands for noise mitigation

The effect of noise on the phase space distance $\langle \varepsilon \rangle$ was discussed in [14]. In [15], instead of searching for nearby points in phase space, a search was made for continuous portions of phase space trajectories called phase space strands. Phase space strands may also be used with the target identification algorithm to aid in distinguishing targets in noisy situations. The strand method is essentially the same as our phase space target identification algorithm, except that we compare distances between continuous sets of points in phase space, rather than just single points.

Phase space strands were first used for dimension estimation [16]. A strand is a group of consecutive points on a trajectory in phase space, rather than just 1 point. In order to create a reference from the reflected signal x , we first choose an index point on x . The strand of length n_s starting with $\mathbf{x}(i_0)$ is defined as

$$\mathbf{s}[\mathbf{x}(i_0)] = [\mathbf{x}(i_0), \mathbf{x}(i_0 + 1), \dots, \mathbf{x}(i_0 + n_s - 1)] \quad (1.9)$$

We then search for a number of strands $\mathbf{s}[\mathbf{x}(j)]$ on the signal x which have the smallest Euclidean distance from $\mathbf{s}[\mathbf{x}(i_0)]$. For a reference, we store the indices of the starting points on these strands.

When we want to compare to an unknown signal y , we construct the strands $\mathbf{y}(i_0)$ and $\mathbf{y}(j)$ and find the radius

$$\varepsilon = \frac{1}{N_s} \sum_{j=1}^{N_s} \sqrt{\sum_{k=1}^d [y(j + \tau_d) - y(i_0 + \tau_d)]^2} \quad (1.10)$$

which is then averaged over all the indices in the reference to get $\langle \varepsilon \rangle$. As with the single point method, a composite reference for multiple angles may be produced by finding the set of strands that minimize the distance for the signals from all the angles used to produce the reference (eq. 8).

3.0 Finite Difference Time Domain Simulations

We tested the target ID algorithm with finite difference time domain (FDTD) [17] simulations of several simple radar targets. In the FDTD code, we defined a target shape, including types of materials used (all conductors for these simulations), and an incident RF signal. The FDTD code then simulated the electromagnetic scattering problem for the target and output a reflected signal. We began with 2 similar shapes, a shape that approximated a 130 mm artillery shell, and a cylinder of the same size. These 2 shapes are shown in figure 1. Both objects were 13 cm in diameter and 67 cm long.

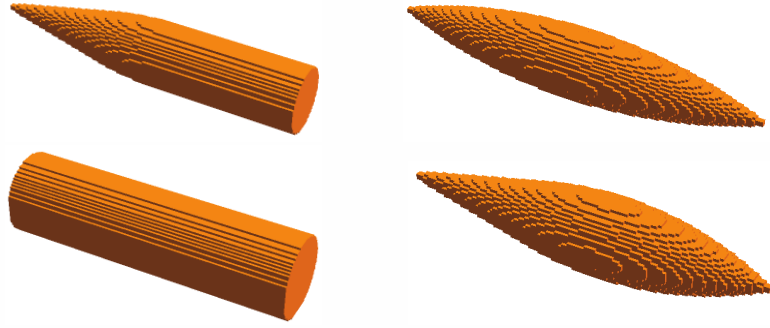


Figure 1. Shell (top), cylinder (bottom), double ogive (top) and cone ogive (bottom) models used in the FDTD simulations. All objects are 13 cm in diameter and 67 cm long

We created a frequency modulated signal based on the chaotic map

$$z(n+1) = 2.1z(n) \bmod 1 \quad (1.11)$$

The frequency modulated signal was created from a series of concatenated sinusoids, with the frequency of the n 'th sinusoid determined by $\zeta(n) = 1.0 + \beta(z(n) + 0.5)$, where β was used to vary the modulation bandwidth. The sinusoids were matched in phase when one sinusoid ended and the other began. Mathematically, the n 'th sinusoid of s was given by

$$s(i) = \sin(2\pi t / [20\zeta(n)]) [i = 0, 1, 2, \dots, i < 20\zeta(n)] \quad (1.12)$$

If $\zeta(n) = 1$, then the period of the sinusoid was 20 points.

The signal used with the FDTD simulations had a center frequency of 2 GHz and a bandwidth of approximately 400 MHz. The length of 1 pulse was 5.97×10^{-7} s, or 1200 sinusoids (just over 63,000 points). The range resolution of the pulse, estimated from its autocorrelation, was about 37.5 cm. We are not actually imaging the target, but matching it to a reference, which is why we can use a low resolution signal. The FDTD code transmitted each pulse as linearly polarized electromagnetic wave with the electric field polarization in the plane of the target. The wave was transmitted at 0^0 elevation and azimuthal angles from 0^0 to 20^0 . The mono-static back scattered pulse was collected in each instance and employed in this analysis. The grid size used to model the missiles was 5 mm.

3.1 Identification

We attempted to identify the different shapes when noise was present. For identification in the presence of noise, we used 4 different shapes, a cylinder, a shell, a shape consisting of 2 ogives back to back (the double ogive), and a shape containing a cone and an ogive back to back (the cone ogive).

For the noise studies, bandpass filtered noise was added to all signals. The noise was filtered to have the same bandwidth as the signals. We fitted composite references to signals covering angular windows of widths from 1 to 8 degrees. We then compared unknown signals recorded for individual angles within the same windows to these composite references. For a given reference, if an unknown signal other than the correct unknown signal (the signal reflected from the same shape used to create the reference) gave a smaller phase space distance $\langle \varepsilon \rangle$, we counted the result as an error. We then divide the number of errors by the number of comparisons; there are 4 targets, and each is compared to the other 3 possible targets, so there are 12 chances to make an error. We also use 100 different realizations of the noise for each angle comparison. The results are shown in figure 2.

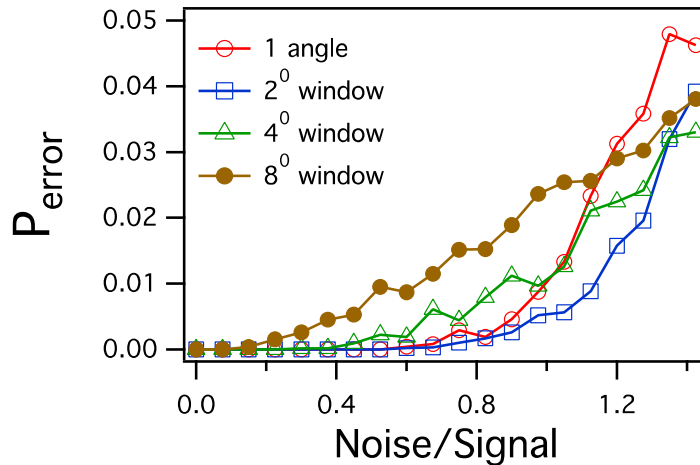


Figure 2. Fraction of errors in determining which of 4 shapes is present, as a function of added noise level. References for each shape were fitted for 1 angle or an angular window.

In figure 2, the largest error fraction sometimes occurs when a reference is generated from only a single angle. The errors are smaller for larger windows because when a reference is fit over a range of angles, the noise for each individual angle is averaged out, leading to a better reference. We confirmed this observation by repeating the noise simulations when no noise was added to the signals used to create the references, in which case the single angle reference gave the smallest error.

For all of these signals, the fraction of errors is only about 4 % for windows up to 8° wide, even when the noise level is larger than the signal. The 4 shapes used are very similar to each other, so distinguishing them is difficult, but the phase space radius method is able to tell the difference even in the presence of noise.

4.0 Experiments

We used acoustic experiments as an inexpensive substitute for radar experiments. The target ID method was tested in acoustic experiments using objects with similar shapes and sizes. The targets were 2 glass vases and 2 bottles, shown in Figure 2.

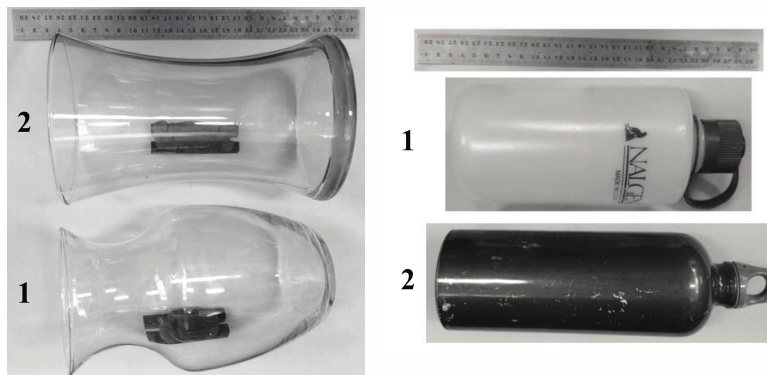


Figure 3. The 2 glass vases (1 and 2) and the 2 bottles (1 and 2) that were used as targets. The ruler at the top of the picture was 30 cm long. The dark objects under the vases are mounting posts used to mount the vases.

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A speaker was driven with a frequency modulated signal based on a chaotic map. The map was described by eq. (11).

The frequency modulated signal was created from a series of concatenated sinusoids, with the frequency of the n 'th sinusoid determined by $\zeta(n) = 1.0 + \beta(z(n) + 0.5)$, where β was used to vary the modulation bandwidth. The sinusoids were matched in phase when one sinusoid ended and the other began. Mathematically, the n 'th sinusoid of s was given by

$$s(i) = \sin(2\pi t / [50\zeta(n)]) [i = 0, 1, 2, \dots, i < 50\zeta(n)] \quad (1.13)$$

If $\zeta(n) = 1$, then the period of the sinusoid was 50 points.

The center frequency of the signal driving the speaker was 15 KHz, and the bandwidth was 2 KHz. The targets were mounted on a rotator that was approximately 2 m from the speaker. The reflected signal was recorded by a microphone placed just below the speaker. Because of the short distance from speaker to target, the transmitted signal was divided into 15 different pulses of 4 ms each. Each reflected pulse was digitized at a rate of 500 KHz. The pulses were combined after digitization by embedding all of them in the same phase space. Because of the large background noise, each set of 15 pulses was transmitted 10 times, and the reflections were coherently averaged in time to reduce noise. We used sound absorbing materials to lessen the acoustic clutter in the experiment, but we did not use an anechoic chamber, so clutter and noise were still present.

For each target, a set of references was created by reflecting signals from the target for each target rotation angle from 0 to 360 degrees. Each target was then dismounted from the rotator. Unknown signals were then created by once again mounting each target on the rotator and again reflecting signals from the target for angles from 0 to 360 degrees. Creating separate reference and unknown signals allowed us to test if the target ID method was robust to errors in target alignment and the different noise signals present at different times. The signal to noise plus clutter ratio for the reflected signals varied from +3 dB to -10 dB, depending on the target rotation angle.

As described in above, additive noise increases the phase space distances between points or strands for an embedded signal. The changing signal to noise ratio for different aspect angles of the targets will therefore affect the measurements of phase space radius, altering the results of target identification. In order to minimize this effect, filtered noise was added to the signals to equalize the signal to noise ratios. The distance between closest strands in an embedded signal is proportional to the noise level [15], so the distances to nearest neighbor strands for all signals were used as an indicator of the signal to noise ratio; the smaller the distance, the smaller the signal to noise ratio. A Gaussian random noise signal was filtered to match the bandwidth of the reflected signals, and was added to each signal with an amplitude proportional to the difference between the nearest strand distance for that signal and the largest value of the nearest strand distance for all signals.

We reflected signals from each of the 4 targets as before, creating separate reference and unknown signals. For a given unknown signal, we calculated the average phase space radius for each of the 4 references. For each comparison, we compared a target return signal from one angle to references from every angular window for every target. If a reference from the correct target gave the smallest phase space radius, we recorded a correct identification; otherwise, we recorded an error. We repeated the identification process for composite references that spanned windows from 1 to 90 degrees. The references did not overlap, so the total number of references ranged from 180 to 2.

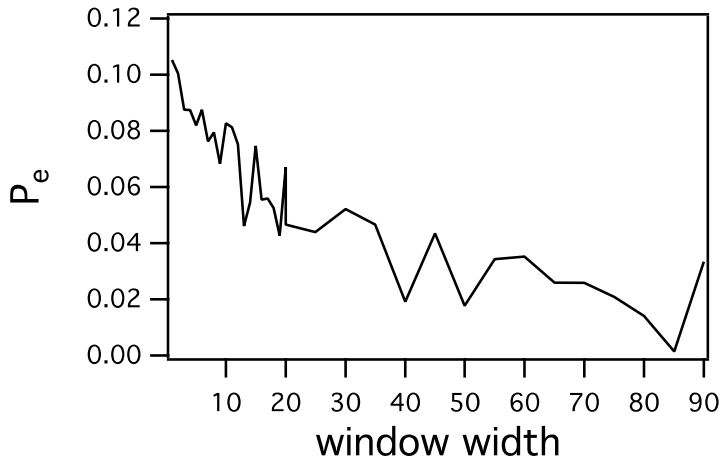


Figure 4. Probability of making an error (P_e) when determining which of 4 targets was present, as reference window widths were increased from 1 degree to 90 degrees. Target returns from each individual angle were compared to composite references fitted over larger windows. The probability of error decreases as window width increases because the larger reference windows result in averaging of the reference which reduces noise and alignment errors.

Figure 4 shows the probability of making an error when deciding which of the 4 targets is present from the acoustic experimental data. The probability of error drops as the window width increases because using composite references for a large range of angles averages out noise and target alignment errors. The fact that the error probability drops as the window width increases also indicated that this target ID method is most sensitive to resonances in the target, because these resonances are independent of the aspect angle.

5.0 Conclusions

We were able to identify different targets based on the properties of their impulse response functions. The results show that this target ID method is similar to ID methods that use target resonances, but the current method uses long modulated pulses, allowing for time averaging to improve the signal to noise.

Because of limits in our experimental system, the signals we used for training our method (the reference signals) were contaminated by noise. If we had low noise training data, this method should be able to yield a lower probability of error.

The theory behind this method depends on all signals coming from low dimensional dynamical systems, although the method will also work with randomly modulated signals, and random signals have an infinite number of dimensions. The random signal must be modulated onto a carrier in some way however, and while the global dimension of this carrier may be infinite, the local dimension is finite. so the target ID algorithm still works.

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