



Coastal Engineering Technical Note



DIRECTIONAL WAVE SPECTRA USING NORMAL SPREADING FUNCTION

PURPOSE: To present a parameterized model of a directional spectrum of the sea surface using an energy spectrum and a value for the mean wave direction. The model used in this Technical Note is the Normal Spreading Function.

BACKGROUND: With the growing interest in directional spectral estimation and modeling of so-called "Spreading Functions," there is a need for methods by which the directional spectral density can be easily computed using various types of data. If one has data from directional type measuring devices, it is then possible to estimate a directional spectrum directly. In many cases, the raw data are not available but have been reduced to estimates of the energy spectrum and a mean direction estimate. This is the situation that will be addressed in this note.

There are many functional models in the literature with which to represent the directional spectra of the sea surface. This CETN will present the normal spreading function which is one of the most simple and easily used models. This model has been shown to fit prototype data (Sand 1984). For a more detailed explanation of directional wave spectra, see Panicker (1974).

DIRECTIONAL SPECTRA: The directional spectrum of the sea surface is modeled as the energy spectrum times a function of angular spreading, usually called a spreading function. The directional spectrum is:

$$E(f, \theta) = E(f) D(f, \theta)$$

where

$E(f)$ = energy spectral density function

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14. ABSTRACT With the growing interest in directional spectral estimation and modeling of so-called "Spreading Functions," there is a need for methods by which the directional spectral density can be easily computed using various types of data. If one has data from directional then possible to estimate a directional spectrum raw data are not available but have been reduced spectrum and a mean direction estimate. This is addressed in this note.					
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$D(f, \theta)$ = spreading function

$E(f, \theta)$ = directional spectral density function

f = frequency in cycles per second

θ = direction in radians

The function $D(f, \theta)$ can be represented by a wrapped normal function of the following form:

$$D(f, \theta) = \sum_{k=-1}^1 \frac{\exp \left[-\frac{1}{2} \left(\frac{\theta - \theta_0 - 2\pi k}{\sigma} \right)^2 \right]}{\sqrt{2\pi} \sigma}$$

where

σ = the directional spreading parameter in radians

θ_0 = the mean wave direction in radians

for $\sigma < \pi/3$ radians.

If θ_0 is not close to zero (or 2π), then the spreading function can be approximated by a normal bump centered at θ_0 , and the terms for $k = -1$ and $k = 1$ may be dropped. Thus

$$D(f, \theta) = \frac{\exp \left[-\frac{1}{2} \left(\frac{\theta - \theta_0}{\sigma} \right)^2 \right]}{\sqrt{2\pi} \sigma}$$

This is the well known normal probability function where θ_0 and θ are functions of frequency. The most commonly used spreading function is the COSINE 2s model (Long 1980). The normal model is used here as an approximation due to its mathematical simplicity.

EXAMPLE: Suppose a given location is partially sheltered by an obstacle such that only the wave energy associated with the angular band θ_1, θ_2 is of interest (Mathiesen 1984) (see Figure 1). For this example, assume that no diffraction or refraction occurs at the inlet entrance. If we are given the energy spectrum of the sea surface and a mean direction θ_0 offshore of the

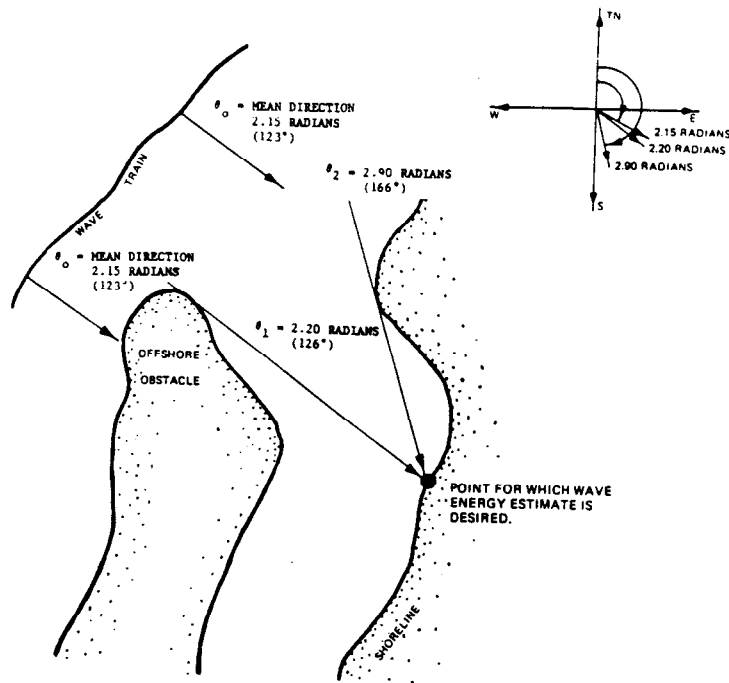


Figure 1. Partially sheltered location

obstacle, then the energy E associated with the angle interval θ_1 to θ_2 is approximately

$$E = m_0 \int_{\theta_1}^{\theta_2} D(\theta) d\theta \quad (1)$$

where m_0 is the variance of the sea surface

$$m_0 = \int_{-\infty}^{\infty} E(f) df$$

The dependency on frequency has been dropped from $D(f, \theta)$ leaving $D(\theta)$. This implies that all frequencies have the same mean direction and speed. The validity of this assumption depends on the narrow bandedness of the energy spectral density function. For fairly narrow spectra (e.g., a swell train), the assumption is good. For situations where the spectral density is not

narrow banded (e.g., a locally generated sea train) this method will usually yield incorrect results.

The energy reduction factor due to sheltering A can be shown as the integral in the following manner:

$$A = \int_{\theta_1}^{\theta_2} D(\theta) d\theta \quad (2)$$

This integral is evaluated in the following way:

$$A = \int_{\frac{\theta_1 - \theta_o}{\sigma}}^{\frac{\theta_2 - \theta_o}{\sigma}} z(x) dx, \quad (3)$$

where $Z(x)$ is the standard normal probability function. The integral in equation 3 is further reduced to

$$\begin{aligned} A &= \int_{-\infty}^{\frac{\theta_2 - \theta_o}{\sigma}} Z(x) dx, - \int_{-\infty}^{\frac{\theta_1 - \theta_o}{\sigma}} Z(x) dx, \\ &= P\left(\frac{\theta_2 - \theta_o}{\sigma}\right) - P\left(\frac{\theta_1 - \theta_o}{\sigma}\right) \end{aligned} \quad (4)$$

The values for $P(Z)$ are available in standard normal tables such as the CRC Standard Mathematical Tables (Selby 1981).

Suppose that $\theta_2 = 2.90$ radians and $\theta_1 = 2.20$ radians where $\theta_o = 2.15$ radians. A value of $\sigma = 0.524$ radians (based on 30° spread) is assumed. If an approximate value for σ is not known and one is unwilling to assume a value, then a general range of 15 to 60 deg may be assumed and the analysis computed at the end points of this range to obtain upper and lower bounds on

the resulting wave energy estimates. This range corresponds to the range for the COSINE 2s model that has been shown to hold for data obtained under various conditions (Sand 1981).

Then equation 4 becomes:

$$A = P(1.43) - P(0.10)$$

$$A = 0.92 - 0.54 = 0.38$$

Finally, $E = AM_0 = 0.38 M_0$ represents the approximate energy associated with the angles of interest. This represents a 62 percent reduction in wave energy due to sheltering.

CONCLUSION: The directional spectrum is useful in applications where the angular distribution of wave energy is of interest. One such example has been presented. The wrapped normal spreading function is a good model for obtaining an approximation to the directional spectrum of the sea surface. This is due to its relative simplicity, and it can be evaluated using existing tables.

ADDITIONAL INFORMATION: Contact Mr. J. Michael Hemsley, CERC Prototype Measurement and Analysis Branch, (601) 634-2075.

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